PPPL--2734

DE91 005995

NONLINEAR THEORY OF TRAPPED ELECTRON TEMPERATURE GRADIENT DRIVEN TURBULENCE IN FLAT DENSITY H–MODE PLASMAS

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Abstract

Ion temperature gradient turbulence based transport models have difficulties reconciling the recent DIII-D H-mode results[1] where the density profile is flat, but $\chi_e > \chi_i$ in the core region. In this work, a nonlinear theory is developed for recently discovered ion temperature gradient trapped electron modes propagating in the electron diamagnetic direction. This instability is predicted to be linearly unstable for $L_{Ti}/R \lesssim k_\theta \rho_s \lesssim (L_{Ti}/R)^{1/4}$. They are also found to be strongly dispersive even at these long wavelengths, thereby suggesting the importance of the wave–particle–wave interactions in the nonlinear saturation phase. The fluctuation spectrum and anomalous fluxes are calculated. In accordance with the trends observed in DIII-D, the predicted electron thermal diffusivity can be larger than the ion thermal diffusivity.

I. Introduction

One of the outstanding problems in current tokamak microturbulence theory research is understanding the confinement properties of the flat density profile plasmas often found in H-mode discharges.[1,2] Recently, the ion temperature gradient (ITG) driven mode[3] in flat density regime has received considerable attention because it is a natural candidate for explaining the anomalous ion thermal transport in the absence of density gradient. The ITG–turbulence models typically predict χ_i(ion thermal diffusivity) > χ_e(electron thermal diffusivity), D(particle diffusivity) within its validity regime where the destabilizing influence is mainly from ion dynamics.

However, these ITG–turbulence models have difficulties reconciling the recent DIII-D H-mode results[1] where $\chi_e > \chi_i$ in the core region where the collisionality is relatively low. Meanwhile, the conventional electron drift instability (including trapped electron instability) becomes subdominant to ITG mode as the density gradient becomes weaker.[4]

In this paper, the challenging problem just described is addressed by considering the realistic trapped electron and ion dynamics with temperature gradients in toroidal geometry. The linear theory is based upon the recently analyzed ITG-trapped electron instability in toroidal geometry.[5] For this instability, the ion temperature gradient coupled to sound wave propagation provides the real frequency, meanwhile the electron temperature gradient coupled to the trapped electron dissipation provides the growth rate. This is in contrast to both conventional electron drift wave case where the real frequency is determined mainly by the density gradient (except for a small finite Larmor radius correction involving ∇T_i) and the ITG instability case where the growth rate is determined by the ion dynamics alone.

In this paper, a nonlinear theory is developed for both collisionless trapped electron (CTE) and dissipative trapped electron (DTE) regimes. Since the ITG–trapped-electron-mode[5] (ITGTEM) obeys a dispersion relation $\omega_r \simeq \omega_{*Ti}^{1/3}(Cs/2qR)^{2/3}$, and $\gamma < \omega_r$, the weak turbulence theory[6] applies. Due to its strong dispersion ($\omega \propto k_\theta^{1/3}$), the wave–particle–wave interaction (specifically, the ion Compton scattering) acts as the dominant nonlinear mechanism.

The principal results of this investigation include:

1. The density fluctuation spectral intensity, $I(k_\theta) \approx < \delta n^2(k_\theta)/n_0^2 >$, is obtained from the steady–state solution of the wave-kinetic equation.

   For DTE regime,

$$I(k_\theta) \sim \epsilon^{1/2}\left(\frac{C_s}{L_{Te}\nu_{ei}}\right)\left(\frac{T_i}{T_e}\right)^{2/3}\frac{q^{1/3}}{\hat{s}^{5/3}}\frac{\rho_s^3}{L_{Ti}^{5/6}R_0^{13/6}}\cdot(k_\theta\rho_s)^{-8/3}$$

   for $\left(\frac{2}{3}\right)^{3/2}\frac{L_{Te}^{3/2}}{L_{T_i}^{1/2}2qR}\left(\frac{T_i}{T_e}\right)^{1/2}\rho_s < k_\theta\rho_s < \left(\frac{L_{Ti}}{R}\right)^{1/4}$.

   For CTE regime, $I(k_\theta)$ is slowly decreasing with k_θ for $(T_i/T_e)^{1/2}(L_{Ti}/2qR)\rho_s < k_\theta\rho_s < k_M\rho_s$ and $I(k_\theta) \propto k_\theta^{-14/3}$ for $k_M\rho_s < k_\theta\rho_s < (L_{Ti}/R)^{1/4}$, where $k_M\rho_s \equiv (2/17G)^{3/2}(T_iL_{Te}/T_eL_{Ti})^{1/2}(1/2q)(R/L_{Te})^{1/2}$.

   For $k_\theta\rho_s < (T_i/T_e)^{1/2}\, L_{Ti}/2qR$, and for $k_\theta\rho_s > (L_{Ti}/R)^{1/4}$, the fluctuation spectral intensity is feeble.

2. The following anomalous fluxes have been calculated.

$$\frac{\Gamma_e}{n_0} \cong \begin{cases} \frac{8}{5}\frac{cT_e}{eB}\epsilon^3\left(\frac{C_s}{L_{Te}\nu_{ei}}\right)^2\left(\frac{T_i}{T_e}\right)^{2/3}\frac{1}{\hat{s}^{4/3}}\frac{\rho_s}{L_{T_i}^{3/4}R^{5/4}}, & \text{for DTEM regime} \\ \frac{15\pi^{3/2}}{2^{11/2}}\frac{cT_e}{eB}\epsilon\left(\frac{T_i}{T_e}\right)^{7/6}\frac{q^2}{\hat{s}^{4/3}}\left(\frac{R}{GL_{Te}}\right)^2G^{3/2}\frac{\rho_s}{L_{T_i}^{1/2}R^{3/2}}, & \text{for CTEM regime.} \end{cases}$$

$$Q_e \cong \begin{cases} 5T_e\Gamma_e & \text{for DTEM regime, and} \\ \frac{7}{4}T_e\Gamma_e & \text{for CTEM regime.} \end{cases}$$

$$Q_i \cong \frac{23}{4}T_i\Gamma_e$$

In particular, we note that $\chi_e^{\rm eff} > \chi_i^{\rm eff}$ is obtained for $L_{Ti} \ll L_{Te}$, in rough agreement with DIII-D H–mode results.[1]

The remainder of this paper is organized as follows. In Sec. II, our theoretical model is described with an emphasis on the basic linear properties and the governing nonlinear equations. A weak turbulence theory leading to the wave-kinetic equation is developed in Sec. III and the fluctuation spectrum is calculated from the wave–kinetic equation. The anomalous fluxes and heat exchange rate are calculated in Sec. IV. Finally, the implications of these results for H-mode plasma core confinement are discussed in Sec. V.

II. Theoretical Model

In this section, we describe our theoretical model and briefly review the linear theory of ITGTEM in a simple context focusing on the properties which would be useful in developing nonlinear theory. To simplify the analysis and to emphasize the existence of instability propagating in the electron (temperature) diamagnetic direction in the absence of the density gradient, we consider a plasma with flat density profile commonly found in H–mode discharges. The trapped electron dynamics is considered in a toroidal plasma with ion and electron temperature gradients. The untrapped electrons are described by the Boltzmann response, assuming $\omega \ll k_{||} v_{Te}$. We also consider a case where the effective collision frequency for trapped electrons, $\nu_{eff} = \nu_{ei}/\epsilon$ is smaller than the trapped electron bounce frequency $\omega_{be} = \epsilon^{1/2} v_{Te}/qR$, i.e., $\nu_{*e} \equiv \nu_{eff}/\omega_{be} < 1$.

Then, the trapped electron dynamics are described by the bounce averaged drift kinetic equation,

$$\begin{aligned} -i(\omega - \omega_{de} + i\nu_{\rm eff})h - i(\omega - \omega_{*Te}(E/T_e - 3/2))\Phi F_M \\ = \nabla\phi \times \hat{b} \cdot \nabla h, \qquad (1) \end{aligned}$$

where $\omega_{*Te} = cTek_\theta/eBL_{Te}$ is the electron diamagnetic frequency due to temperature gradient. Also, ϕ is the electrostatic potential fluctuation, $<\phi>$ is the bounce averaged value, h is the nonadiabatic part of the trapped electron distribution function, $k_\theta = nq/r$ is the poloidal component of the wave vector, $\omega_{de} = cT_e k_\theta G(\hat{s},\kappa)/eBR$ is the trapped electron precession drift frequency, where $G(\hat{s},\kappa) = (2\mathrm{E}/\mathrm{K}-1) + 4S(\mathrm{E}/\mathrm{K}+\mathrm{K}^2-1)$,[7] E and K are the elliptic functions. Note that the right hand side (RHS) is the $\mathrm{E}\times\mathrm{B}$ nonlinearity.

Meanwhile, the ion dynamics is described by the collisionless nonlinear gyrokinetic equation[8] neglecting trapped ion dynamics

$$-i(\omega - k_{\|}v_{\|} - \omega_{di})g + i\{\omega - \omega_{*Ti}(E/T_i - 3/2)\}J_0\phi F_M = \nabla\phi J_0 \times \hat{b}\cdot\nabla g \quad (2)$$

where $\tau = T_e/T_i$, $J_0 = J_0(k_\perp\rho_i)$ is the Bessel function of order zero, $\rho_i = c\sqrt{T_i M_i}/eB$ is the ion gyroradius. $k_\perp$ is the perpendicular component of wave vector. ω_{di} is the ion ∇B and curvature drift frequency, and g is the nonadiabatic part of the perturbed ion distribution function. The RHS is the $\mathrm{E}\times\mathrm{B}$ nonlinearity.

Now, we briefly review the linear theory. Details can be found in Ref. 5. The linear eigenmode equation for ITGTEM can be obtained by linearizing Eqs. (1)-(2) and taking perturbed density moments based upon the appropriate ordering.

The electron density response can written as

$$\frac{\delta n_e}{n_o} = [1 - i\delta]e\phi/T_e. \quad (3)$$

For dissipative trapped electron regime where $\omega_{be} > \nu_{eff} > \omega, \omega_{*Te} > \omega_{de}$,

$$\delta = \delta^{DTE} \equiv 4\sqrt{\frac{2}{\pi}}\epsilon^{3/2}\frac{\omega_{*Te}}{\nu_{ei}}[\frac{3}{2} - \frac{\omega}{\omega_{*Te}}] \quad (4)$$

where ν_{ei} is evaluated at the electron thermal energy. It is also straightforward to derive the response in CTE regime, where $\omega_{be} > \omega_{*Te}, \omega \gtrsim \omega_{de} > \nu_{eff}$,

$$\delta^{CTE} = 2(2\pi\epsilon)^{1/2}(\frac{\omega}{\bar{\omega}_{de}})^{1/2}e^{-\omega/\bar{\omega}_{de}}[\frac{\omega_{*Te}}{\bar{\omega}_{de}}(\frac{\omega}{\bar{\omega}_{de}} - \frac{3}{2}) - \frac{\omega}{\bar{\omega}_{de}}] \tag{5}$$

from the trapped electron precession resonance.[9]

The ion density response is obtained from Eq. (2) for the long wavelength fluid regime, $k_\perp \rho_i \ll 1$, and $\omega > k_\| v_{Ti}, \omega_{di}$,

$$\frac{\delta n_i}{n_0} = \frac{\omega_{*Ti}}{\omega}(\frac{k_\|^2 v_{Ti}^2}{\omega^2} - k_\perp^2 \rho_i^2 - \frac{\omega_{di}}{\omega})\frac{e\phi}{T_i}. \tag{6}$$

Finally, the quasineutrality condition $\delta n_e = \delta n_i$ yields the eigenmode equation. The eigenmode equation has been solved in Ref. 5, via two scale analysis in ballooning coordinate. The essence of their result is that the role of ω_{di} is to introduce a quasi periodicity on the connection length scale ($\sim qR$) along the magnetic field lines and consequently set the parallel wavelength of the field line at $2qR$. Since ω_{di} oscillates along the field line its effect on the frequency is found to be negligible.

The following dispersion relation has been obtained from the dominant balance between the electron response and the ion acoustic contribution, the first term on RHS of Eq. (6),

$$1 - i\delta = \omega_{*Ti}\frac{(Cs/2qR)^2}{\omega^3}. \tag{7}$$

The role of finite Larmor radius (FLR) term, $k_\perp^2 \rho_i^2$ is to determine the width of the fluctuation envelope along the field line. In the absence of trapped electron dynamics ($\delta \rightarrow 0$), an overstable root propagation in the ion diamagnetic direction is the well-known ITG mode. On the other hand, a marginally stable root propagating in the electron diamagnetic direction is expected to be heavily damped in a sheared magnetic field by ion Landau damping,[10] and has not gotten a lot of attention. In Ref. 5, it has been shown that this root can be destabilized by trapped electron dynamics ($\delta > 0$) in a toroidal geometry. It should be emphasized that the modification of mode structure due to toroidal geometry is crucial in allowing the instability. A similar example is well-known for electron drift wave with density gradient.[11]

For typical case of $\delta \ll 1$, the linear dispersion rclation becomes,

$$\omega_r = \omega_{*Ti}^{1/3}(\frac{Cs}{2qR})^{2/3}, \tag{8a}$$

$$\gamma = (\frac{\delta}{3})\omega_r. \tag{8b}$$

We note that the linear growth region in k_θ is determined by the following constraints. First, it should be remembered that the previously mentioned toroidal mode structure and the corresponding dispersion relation is applicable only for $k_\theta \rho_s < (L_{Ti}/R)^{1/4}$ in both collisionality regimes considered here. Second, if k_θ becomes very small, the mode becomes stabilized. In DTE regime, $\delta < 0$ for $\omega/\omega_{*Te} > 3/2$. This happens for small enough k_θ since $\omega \propto k_\theta^{1/3}$ and $\omega_{*Te} \propto k_\theta$, and imposes the following condition on k_θ for the instability,

$$\left(\frac{2}{3}\right)^{3/2} \frac{L_{Te}^{3/2}}{L_{Ti}^{1/2} 2qR} \left(\frac{T_i}{T_e}\right)^{1/2} \rho_s < k_\theta \rho_s. \tag{9}$$

For CTE regime, the fluid–ion approximation breaks down at very low k_θ. The inequality $\omega > v_{Ti}/2qR$ imposes the following condition on k_θ for the instability,

$$k_\theta \rho_s > \left(\frac{T_i}{T_e}\right)^{1/2} \frac{L_{Ti}}{2qR}. \tag{10}$$

The linear eigenmode structure in the ballooning coordinate η has been found to be

$$\phi(\eta) = \phi(0) \cos\left(\frac{\eta}{2}\right) \exp[-\left(\frac{q^4 (k_\theta \rho_s)^4}{4\tau \epsilon_{Ti}}\right)^{1/3} \hat{s}\eta^2] \tag{11}$$

where $\cos(\eta/2)$ is the factor which varies on the connection length scale introduced by ω_{di}, and the last exponential factor is the slowly varying envelope. To facilitate the nonlinear analysis without complication due to nonlinear ballooning formalism, we inverse–Fourier–transform the eigenmode structure to configuration space to get

$$\phi_{n,m}(X) = \phi_{n,m} \frac{\left\{ e^{-(X-\Delta n/2)^2/2\Delta X^2} + e^{-(X+\Delta n/2)^2/2\Delta X^2} \right\}}{2}, \tag{12}$$

where

$$X = r - r_{n,m},$$

$r_{n,m}$ is the radial location of the rational surface defined by $q(r_{n,m}) = m/n$,

$$\Delta_n = \frac{1}{k_\theta \hat{s}},$$

$$\Delta X = \frac{2\pi}{\hat{s}} \rho_s (4\epsilon_{Ti})^{-1/6} \left(\frac{q^2 \hat{s}}{k_\theta \rho_s} \right)^{1/3}.$$

III. Nonlinear Analysis

In this section, the wave kinetic equation which describes the nonlinear evolution of fluctuation spectrum is derived. The ITGTEM we are considering has some properties which are distinct from those of the usual (trapped particle) drift type instabilities. First, the real frequency $\omega(k_\theta)$ is proportional to $k_\theta^{1/3}$ in contrast to most drift–type modes where $\omega(k_\theta) \propto k_\theta$ at long wavelength. This fact has a significant impact on the nonlinear evolution of the instability. Since this mode is strongly dispersive ($\partial\omega/\partial k_\theta \neq$ const) even at long wavelength, the three-wave-decay nonlinearity is ineffective (it is impossible to satisfy the matching condition of real frequencies and the wave vectors of three waves (i.e., $\omega'' = \omega - \omega'$, and $\vec{k}'' = \vec{k}' - \vec{k}$). Therefore, in the $\delta \ll 1$ limit, weak turbulence theory based on the wave-particle-wave interactions is applicable. Second, since the instability exists only in a relatively long ($k_\theta \rho_s \lesssim (L_{Ti}/R)^{1/4}$) wavelength regime, the real frequency is monotonically increasing in the linearly unstable region in k_θ. Therefore, there is no possibility of distant interaction and the trapped particle scattering (resonant interaction between the low frequency beat wave and the trapped particle) is weak even in the collisionless trapped electron regime. This is in contrast to the usual CTEM case with density gradient,[12,13] where the distant interaction between two modes with similar frequencies but disparate wavelength is strong. In the DTEM regime, the trapped electron dynamics can be treated linearly.[14] Because of these facts, the ion Compton scattering is the dominant nonlinear saturation mechanism of ITGTEM. Starting from the nonlinear gyrokinetic equation, Eq. (2), it is straightforward to carry out the weak turbulence expansion up to the 3rd order, to get the following nonlinear response.[6,13-15] In the expansion we have kept the only dominant "bare" contribution,

$$g_{\vec{k}}^{(3)} = (C_s\rho_s)^2 \sum_{\vec{k}'=\vec{k}-\vec{k}''} (\vec{k}\times\vec{k}'\cdot\hat{b})^2 \int d^3v \frac{1}{\omega} \frac{\tau}{\omega''-k_{\|}''v_{\|}} \left(\frac{\omega_{*Ti}}{\omega} - \frac{\omega_{*Ti'}}{\omega'}\right)\left(\frac{E}{T_i}-\frac{3}{2}\right) F_M |\phi_{\vec{k}'}|^2 \phi_{\vec{k}}. \tag{13}$$

In Eq. (13), the fluid ion approximation has been made for the test k and background k′ eigenmodes, meanwhile the driven low frequency beat modes k″ have been treated kinetically including the nonlinear ion Landau damping (ion Compton scattering). The nonlinear coupling coefficients $(\vec{k}\times\vec{k}'\cdot\hat{b})^2$ is just a local expression and must be generalized to an appropriate operator in a realistic geometry. From Eq. (12),

$$\phi_{n,m}(X) = \frac{\phi(n,m)}{2}\left\{e^{-(X-\Delta n/2)^2/2\Delta X^2} + e^{-(X+\Delta n/2)^2/2\Delta X^2}\right\}$$

i.e., each $\phi_{n,m}(X)$ has two maxima which is located between neighboring rational surfaces for fixed n as illustrated in Fig. 1. For approximate treatement of nonlinear term, we evaluate $g_{\mathrm{k}}^{(3)}$ at the maxima of the fluctuation amplitude, i.e., $X = \pm\Delta n/2$. Noting that the nonlinear coupling $(\vec{k}\times\vec{k}'\cdot\hat{b})^2$ has been originated from the familiar E × B convective nonlinearity, and that $\partial/\partial x\, \phi_{n,m}(X)$ vanishes at $X = \pm\Delta n/2$, we can find that the appropriate nonlocal expression for $(\vec{k}\times\vec{k}'\cdot\hat{b})^2|\phi_{\vec{k}'}|^2\phi_{\vec{k}}$ is $k_\theta^2|\partial/\partial X'\phi_{\vec{k}'}|^2\phi_{\vec{k}}$. Therefore, from Eq. (21), we get

$$\left|\frac{\partial}{\partial X'}\phi_{\vec{k}'}\right|^2 = \frac{1}{4}\Delta X_{n'}^{-4}|\phi_{n',mm'}|^2\left\{\left(X'-\frac{\Delta n'}{2}\right)e^{-(X'-\Delta n'/2)^2/2\Delta X'^2} + \left(X'+\frac{\Delta n'}{2}\right)e^{-(X'+\Delta n'/2)^2/2\Delta X'^2}\right\}^2. \tag{14}$$

Noting that the imaginary part of the nonlinear density response contributes to the wave–kinetic equation which describes the time evolution of fluctuation spectrum, we have the following expression from Eq. (13) after velocity space resonant integral involving $\mathrm{Im}\{1/(\omega'' - k''_{\parallel} v_{\parallel})\}$.

$$\mathrm{Im}\frac{\delta n_i^{(3)}}{n_0} = \int d^3 v J_0 \mathrm{Im} g_{\vec{k}}^{(3)}$$

$$= (C_s \rho_s)^2 \sum_{\vec{k}'} \frac{\tau}{\omega} \left(\frac{\omega_{*Ti}}{\omega} - \frac{\omega_{*Ti}}{\omega'}\right) \frac{\pi}{|k''_{\parallel}|} \frac{1}{(2\pi)^{1/2} v_{Ti}}$$

$$\cdot \left(\frac{(\omega''/k''_{\parallel} v_{ti})^2 - 1}{2}\right) e^{-(1/2)(\omega''/k''_{\parallel} v_{Ti})^2} k_\theta^2 \Delta X_{n'}^{-4} \frac{I(k'_\theta)}{4}$$

$$\cdot \left\{ (X' - \frac{\Delta_{n'}}{2}) e^{-(X' - \Delta n'/2)^2/2\Delta X'^2} + (X' + \frac{\Delta_{n'}}{2}) e^{-(X' + \Delta'_n/2)^2/2\Delta X'^2} \right\}^2, \quad (15)$$

where J_0^2 has been approximated by 1, and $I(k'_\theta) \equiv |\phi(n', m')|^2$. The summation over $\vec{k}'$ is performed by using the continuum approximation[16]

$$\sum_{k'} \cong \int dn' dm' \cong \frac{r\hat{s}}{q} \int dk'_\theta |k'_\theta| \int dX',$$

yielding

$$\mathrm{Im}\frac{\delta n_i^{(3)}}{n_0} = -(C_s \rho_s)^2 \frac{r_o \hat{s}}{q} \int_0^\infty dk'_\theta k'_\theta k_\theta^2$$

$$\frac{\tau}{\omega} \left(\frac{\omega_{*Ti}}{\omega} - \frac{\omega'_{*Ti}}{\omega'}\right) \frac{1}{8} \sqrt{\frac{\pi}{2}} \frac{1}{v_{Ti}} \Delta X_{n'}^{-4} I(k'_\theta)$$

$$\int_{-\infty}^{\infty} dX' \frac{1}{|k''_{\parallel}|} \left(\left(\frac{\omega''}{k''_{\parallel} v_{Ti}} \right)^2 - 1 \right) e^{-(1/2)(\omega''/k''_{\parallel} v_{Ti})^2}$$

$$\left\{ \left(X' - \frac{\Delta n'}{2} \right) \right\} e^{-(X' - \Delta n'/2)^2/2\Delta X'^2} + \left(X' + \frac{\Delta n'}{2} \right) e^{-(X' + \Delta n'/2)^2/2\Delta X'^2} \Big\}^2. \quad (16)$$

Since $k''_{\parallel} = \frac{1}{L_s}(k_\theta X - k'_\theta X')$, at $X = \Delta n/2$ where $\phi_{\mathbf{k}}$ is maximum, $k''_{\parallel} = \frac{1}{2qR} - \frac{k'_\theta X'}{L_s}$. Then $\int dX'$ integral in Eq. (16) can be approximately evaluated noting the following. The last factor inside $\{\}$ of Eq. (16) varies on the $\Delta X'$ scale while other factors involving $k''_{\parallel}$ varies on the $\Delta_{n'}$ (i.e., longer) scale. It is easily found that the integrand is maximum at $X' \cong -(\Delta n'/2 \pm \Delta X)$. Then evaluating $|k''_{\parallel}|$ at $X' = -\Delta n'/2$, $\int dX'$ can be asymptotically evaluated over the mode structure factor.

Using the expressions for $\Delta X_{n'}$ and ω_{*Ti}/ω from Eqs. (8) and (12), Eq. (16) simplifes to

$$
\begin{aligned}
-\operatorname{Im}\frac{\delta n_i^{(3)}}{n_0} &= -(C_s\rho_s)^2\frac{r_0\hat{s}}{q}k_\theta^2\frac{\tau}{\omega}\frac{1}{v_{Ti}}\frac{\pi qR}{16\sqrt{2}} \\
&\cdot\left(\frac{2q}{\epsilon_{Ti}\tau}\right)^{2/3}\left(\frac{\hat{s}}{2\pi}\right)\rho_s^{-1}(4\epsilon_{Ti})^{1/6}(q^2\hat{s})^{-1/3} \\
&\cdot\int_0^\infty dk'_\theta k'_\theta[(k_\theta\rho_s)^{2/3}-(k'_\theta\rho_s)^{2/3}](k'_\theta\rho_s)^{1/3} \\
&[2^{-4/3}\left(\frac{qR}{v_{Ti}}\right)^{2/3}\left(\frac{C_s}{L_{Ti}}\right)^{2/3}\{(k_\theta\rho_s)^{1/3}-(k'_\theta\rho_s)^{1/3}\}^2-1] \\
&\exp[-2^{-7/3}\left(\frac{qR}{v_{Ti}}\right)^{2/3}\left(\frac{C_s}{L_{Ti}}\right)^{2/3}\{(k_\theta\rho_s)^{1/3}-(k'_\theta\rho_s)^{1/3}\}^2] \\
&\cdot I(k'_\theta)\phi_{\vec{k}}. \qquad (17)
\end{aligned}
$$

This expression, if combined with the appropriate electron linear driving terms, would result in the wavekinetic equation in the form of an integral equation for $I(k_\theta)$. Here, we make a differential approximation on $I(k'_\theta)$, i.e., $I(k'_\theta) \simeq I(k_\theta) + (k'_\theta - k_\theta)\partial I/\partial k_\theta$. The validity of this approximation requires that the kernel of k'_θ integration should vary smoother than $I(k'_\theta)$ itself. Expanding the kernel of k'_θ–integration around $k'_\theta \simeq k_\theta$, the final expression for $\operatorname{Im}\delta n_i^{(3)}/n_0$ becomes,

$$
\begin{aligned}
\operatorname{Im}\frac{\delta n_i^{(3)}}{n_0} &= -\sqrt{\pi}2^{2/3}3^2\tau^{2/3}\hat{s}^{5/3}q^{-1/3}L_{Ti}^{5/6}R^{7/6}r_0\rho_s^{-3} \\
&(k_\theta\rho_s)^{14/3}\frac{\partial I}{\partial(k_\theta\rho_s)}\phi_{\mathbf{k}}. \qquad (18)
\end{aligned}
$$

Finally, the wave kinetic equation is obtained by imposing the quasineutrality condition. We note that in the fluctuation energy populated zone in k_θ (i.e., $(L_{Ti})/R \lesssim k_\theta\rho_s \lesssim (L_{Ti})/R)^{1/4}$), the linear ion Landau damping is negligible. Therefore, the dominant balance exists between the trapped electron density response and the ion nonlinear contribution from the Compton scattering i.e., Eq. (18). For DTE–regime, from Eqs. (4)

and (18),

$$4\sqrt{\frac{2}{\pi}}\epsilon^{3/2}\frac{\omega_{*Te}}{\nu_{ei}}\left(\frac{3}{2}-\frac{\omega}{\omega_{*Te}}\right)$$
$$= -\pi^{1/2}2^{2/3}3^2\tau^{2/3}\hat{s}^{5/3}q^{-1/3}L_{Ti}^{5/6}R^{7/6}r_0\rho_s^{-3}(k_\theta\rho_s)^{14/3}\frac{\partial I}{\partial(k_\theta\rho_s)}. \qquad (19)$$

This is an inhomogeneous first order differential equation in $k_\theta\rho_s$, which is straightforward to solve. The appropriate boundary condition is $I(k_\theta) \rightarrow 0$ as $k_\theta \rightarrow \infty$ which is justified by the direction of fluctuation energy transfer (from short wavelength to long wavelength) for the ion Compton scattering. Meanwhile, $I(k_\theta)$ must be also negligible for $k_\theta\rho_s \ll k_c\rho_s$, where $k_c\rho_s = (T_i/T_e)^{1/2}L_{Ti}/2qR$, since the wave is heavily damped by the linear ion Landau damping. The solution is given by,

$$I(k_\theta) = \left(\frac{2^{11/6}}{9\pi}\right)\epsilon^{1/2}\left(\frac{C_s}{L_{Te}\nu_{ei}}\right)^{2/3}\left(\frac{T_i}{T_e}\right)^{2/3}\frac{q^{1/3}}{\hat{s}^{5/3}}\frac{\rho_s^3}{L_{Ti}^{5/6}R^{13/6}}$$
$$\cdot\int_{k_\theta\rho_s}^{\infty} d\kappa\kappa^{-11/3}\left(\frac{3}{2}-(\frac{T_i}{T_e})^{1/3}\frac{L_{Te}}{L_{Ti}^{1/3}(2qR)^{2/3}}\kappa^{2/3}\right). \qquad (20)$$

The shape of the spectral intensity, $I(k_\theta)$, is shown in Fig. 2. We note that for the most of populated region $(k_c < k_\theta < (L_{Ti}/R)^{1/4})$, $I(k_\theta)$ decays monotonically in k_θ according to the power law $k_\theta^{-8/3}$ which is rather steeper than the simple heuristic mixing length estimates $I \sim (\delta n/n_0)^2 \propto k_\theta^{-2}$. For long wavelength region $k_\theta < k_c$, $I(k_\theta)$ is suppressed due to linear damping with some overspill from the ion Compton scattering.

In the collisionless trapped electron regime, the trapped electron response is given in Eq. (5). A similar consideration as the one for the DTE regime gives us an approximate solution,

$$I(k_\theta) = \frac{1}{9\cdot 2^{1/6}}\left(\frac{R}{GL_{Te}}\right)\left(\frac{T_i}{T_e}\right)^{7/6}\frac{G^{-3/2}\rho_s^3}{q^{2/3}\hat{s}^{5/3}L_{Ti}^{4/3}R^{7/6}r_0^{1/2}}$$
$$\cdot\int_{k_\theta\rho_s}^{\infty} d\kappa\kappa^{-17/3}\exp[-G^{-1}(\frac{T_iL_{Te}}{T_eL_{Ti}})^{1/3}(2q)^{-2/3}(\frac{R}{L_{Te}})^{1/3}]\kappa^{-2/3}]. \qquad (21)$$

For the long wavelength, $\delta^{CTE} \propto e^{-\omega/\bar{\omega}_{de}}$ becomes extremely small, and more importantly the fluid ion approximation ($\omega > k_{\|}v_{\|}$) breaksdown for $k_\theta < k_c$. Therefore, we should impose a low-k_θ cut off near $k_\theta = k_c$. Although a detailed shape of $I(k_\theta)$ for $k_\theta < k_c$ cannot be calculated, it is expected that $I(k_\theta) \ll I(k_c)$ for $k_\theta \ll k_c$, and $I(k_\theta)$ monotonically increases as $k_\theta \to k_c$. Since the integrand of Eq. (21) has a rather sharp maximum near

$$k_M \rho_s \equiv (\frac{2}{17G})^{3/2}(\frac{T_i L_{Te}}{T_e L_{Ti}})^{1/2}\frac{1}{2q}(\frac{R}{L_{Te}})^{1/2}.$$

$I(k_\theta)$ has a shape shown in Fig. 3. For $k_c < k_\theta < k_M$, $I(k_\theta)$ is slowly decreasing as k_θ is increased, while for $k_\theta > k_M$, the exponential factor in the integrand is essentially 1, and $I(k_\theta)$ decays according to a power law, $I(k_\theta) \propto k_\theta^{-14/3}$.

IV. Fluctuation–induced Transport

In this section, the anomalous particle and thermal fluxes of both electrons and ions induced by the nonlinearly saturated ion temperature gradient trapped electron driven flucutation are calculated. The untrapped electron transport is very small because the ITGTEM fluctuation resonates only with the extremely low velocity part of the electron distribution function, due to its small parallel component of the phase velocity. Therefore, the trapped electrons are transported preferentially. Since the flux is a product of the radial component of $E \times B$ velocity involving the fluctuating E_θ and the density fluctuation, the trapped electron particle flux is given by

$$\Gamma_e = Re\sum_k ik_\theta c\Phi_{-k}\int d^3v h_k^{(1)}/B_0, \tag{22}$$

where $\sum_k$ is essentially the spatial average in toroidal and poloidal angle variables, transformed to k–space summation via Parseval's relation. Equation (20) can be written in an integral form adopting the continuum approximation[16] and straight-forward velocity space integration,

$$\Gamma_e = -\frac{c}{B_0}\sum_{\mathbf{k}} k_\theta Im\epsilon^{(1)}(\omega, k)|\phi_{\mathbf{k}}|^2$$
$$= -(\frac{c}{4B_0})(r\hat{s}/q)\int_0^\infty dk_\theta |k_\theta|^2 I(k_\theta)\int_{-\infty}^\infty dx\left\{e^{-\frac{(X-\Delta n/2)^2}{2\Delta X^2}} + e^{-\frac{(X+\Delta n/2)^2}{2\Delta X^2}}\right\}^2 \delta, \quad (23)$$

where δ is given in Eq. (4) or Eq. (5) depending on the trapped electron collisionality. A radial integration over the mode width yields

$$\Gamma_e = \frac{c}{B_0}\frac{r\hat{s}}{q}\int_0^\infty dk_\theta I(k_\theta)\delta\frac{\pi^{3/2}}{\hat{s}}\rho_s(4\epsilon_{Ti})^{-1/6}(q^2\hat{s})^{1/3}(k_\theta\rho_s)^{-1/3}. \quad (24)$$

Using the calculated spectral intensity given in Eqs. (20) and (21), we obtain the following results,

$$\frac{\Gamma_e}{n_o} = \begin{cases} \frac{8}{5}\frac{cT_e}{eB_0}\epsilon^3(\frac{C_s}{L_{Te}\nu_{ei}})^2(\frac{T_i}{T_e})^{2/3}\frac{1}{\hat{s}^{4/3}}\frac{\rho_s}{L_{T_i}^{3/4}R^{5/4}}, & \text{for DTEM regime} \\ \frac{15\pi^{3/2}}{2^{11/2}}\frac{cT_e}{eB}\epsilon(\frac{T_i}{T_e})^{7/6}\frac{q^2}{\hat{s}^{4/3}}(\frac{R}{L_{Te}})^2 G^{-1/2}\frac{\rho_s}{L_{T_i}^{1/2}R^{3/2}}, & \text{for CTEM regime.} \end{cases} \quad (25)$$

Due to the ambipolarity constraint, the ion particle flux Γ_i is identical to Γ_e. The electron thermal flux Q_e is calculated by the same method. By taking the energy moment of the perturbed distribution function rather than the number density moment, we can easily find that there exists the following approximate relation between Q_e and Γ_e.

$$Q_e \cong \begin{cases} 5T_e\Gamma_e & \text{for DTEM regime, and} \\ \frac{7}{4}T_e\Gamma_e & \text{for CTEM regime.} \end{cases} \quad (26)$$

For most of the k_θ–range of interest where the fluctuation energy is populated, the magnetic shear induced ion Landau damping is feeble. Therefore, the linear ion Landau damping has no significant direct effect on the ion thermal transport and the ion Compton scattering plays a dominant role in determining the amount of ion heat flux driven by the fluctuation. Keeping the dominant nonlinear (ion Compton scattering) contribution only,

$$Q_i = -\frac{cT_e}{eB_0}\sum_{\mathbf{k}} k_\theta \mathrm{Im}\int d^3v\frac{1}{2}mv^2 g_{\mathbf{k}}^{(3)} J_0\phi_{-\mathbf{k}} \quad (27)$$

where the expression for $g^{(3)}$ is available from Eq. (13). For the relevant long wavelength ($k_\theta \rho_s < (L_{Ti}/R)^{1/4}$) fluctuation, the Bessel function J_0 can be approximated by 1, and the expression for the velocity space resonant integral involving the nonlinear Landau damping for the Maxwellian background distribution function can be straightforwardly evaluated.

The remaining dX and dk_θ integrals for mode summations are similar to the previous calculation for Γ_e, and are not repeated here. The final expression for Q_i is given by

$$Q_i \cong \frac{23}{4} T_i \Gamma_e. \tag{28}$$

Finally, the microturbulence can also produce anomalous heat exchange between electrons and ions.[13,17] The anomalous electron cooling rate is given by

$$\begin{aligned} Q_{ei} &= Re < \phi \frac{\partial}{\partial t} \tilde{n}_e >= -Re < i\phi\omega_r \tilde{n}_e > \\ &= \begin{cases} -(\frac{T_i}{T_e})^{2/3} \frac{3T_e}{(2q)^{2/3} R^{1/2} L_{T_i}^{1/2}} \Gamma_e & \text{for DTE regime} \\ -\frac{7}{4}(\frac{T_e}{T_i})^{1/6} \frac{(2q)^{1/3} G^{3/2} L_{T_i}^{1/6}}{R^{7/6}} T_e \Gamma_e & \text{for CTE regime,} \end{cases} \end{aligned} \tag{29}$$

where $\gamma_{NL} = 0$, $\omega_r \cong v_* k_\theta$ have been used and Γ_e is given in Eq. (25). This implies that the electrons lose energy from the fluctuation while the ions gain energy from the fluctuation at the same rate so as to keep the fluctuation level at saturated value.

V. Summary

A nonlinear theory is developed for the ion temperature gradient trapped electron instability in a flat–density–profile toroidal plasmas. This wave is driven unstable by the trapped electron dissipation in the presence of the electron temperature gradient. The real frequency is given by a geometric mean of the sound wave frequency and the ion temperature gradient diamagnetic frequency, weighted toward the former ($\omega \approx (C_s/2qR)^{2/3}\omega_{*T_i}^{1/3}$), and consequently has a strong dispersion ($\omega \propto k_\theta^{1/3}$). Therefore, the three wave resonant interaction is very weak and the ion Compton scattering (wave–ion–wave interation) is a dominant nonlinear process. The density fluctuation as well as the temperature fluctuations are produced by the electron and ion temperature gradients. The spectral intensity of the density fluctuation for the nonlinearly saturated state is calculated and its shape is sketched in Figs. 2 and 3.

Various fluctuation–induced anomalous fluxes are calculated and presented in Eqs. (25), (26), and (28) for both dissipative trapped electron (DTE) and collisionless trapped electron (CTE) regimes. It is worth noting that $\chi_i^{eff}/\chi_e^{eff} \simeq C_R L_{Ti}/L_{Te}$ with $C_R = 23/20$ for DTE regime and $C_R = 23/7$ for CTE regime. This is in rough agreement with DIII-D H–mode results where $\chi_e > \chi_i$ in the core region where $L_{Ti} \ll L_{Te}$, and $\chi_e < \chi_i$ in the outer region where $L_{Ti} \sim L_{Te}$. We also note that the anomalous fluxes are only weakly dependent on L_{Ti}, allowing an accessiblity of high $T_i(0)$ value without drastic confinement degradation. This result may suggest a relevance of the fluctuation propagating in the electron diamagnetic direction (but not a **usual** electron drift wave) in determining the confinement properties of the flat–density H–mode plasmas.

ACKNOWLEDGEMENTS

The author would like to thank Drs. L. Chen and W.M. Tang for useful discussions and for their encouragement. This work was supported by the United States Department of Energy Contract No.DE–AC02–76–CHO3073.

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Figures

FIG. 1. Plot of mode structure, $\Phi_{n,m}(X)$, in configuration space. $\Phi_{n,m}(X)$, has two maxima shifted away from the reference mode rational surface where $q(r) = m/n$.

FIG. 2. Plot of spectral intensity, $I(k_\theta)$, in logarithmic scale for dissipative trapped electron (DTE) collisionality regime. Three different asymptotic regions in k_θ are identified. The region "a" corresponds to an extremely long wavelength regime where fluctuations are suppressed due to linear ion Landau damping. The region "b" corresponds to a long wavelength regime where most of the fluctuation energy is populated and $I(k_\theta)$ decays according to a power law, $k_\theta^{-8/3}$. Finally, the region "c" corresponds to a relatively short wavelength regime, $k_\theta \rho_s > (L_{Ti}/R)^{1/4}$, where $I(k_\theta)$ decays more steeply.

FIG. 3. Plot of spectral intensity, $I(k_\theta)$, in logarithmic scale for collisionless trapped electron (CTE) collisionality regime. Four different asymptotic regions in k_θ are identified. The region "a" corresponds to an extremely long wavelength regime where fluctuations are suppressed due to linear ion Landau damping. The region "b" corresponds to a long wavelength regime where most of the fluctuation energy is populated and $I(k_\theta)$ is relatively flat.

In the region "c", $I(k_\theta)$ decays according to a power law $I(k_\theta) \propto k_\theta^{-14/3}$. Finally, the region "$d$" corresponds to a relatively short wavelength regime, $k_\theta \rho_s > (L_{Ti}/R)^{1/4}$, where $I(k_\theta)$ decays more steeply.

PPPL #90T0067

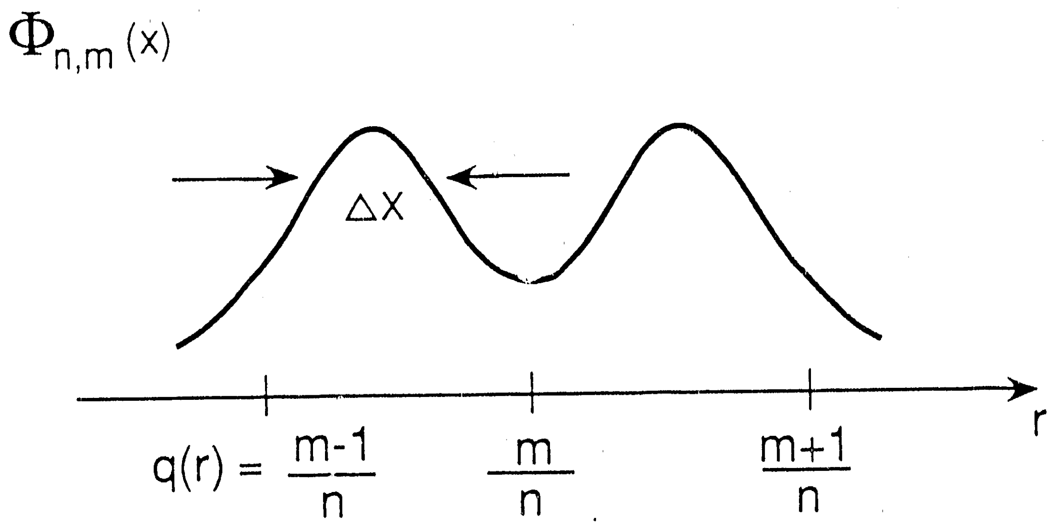

FIG. 1

PPPL #90T0069

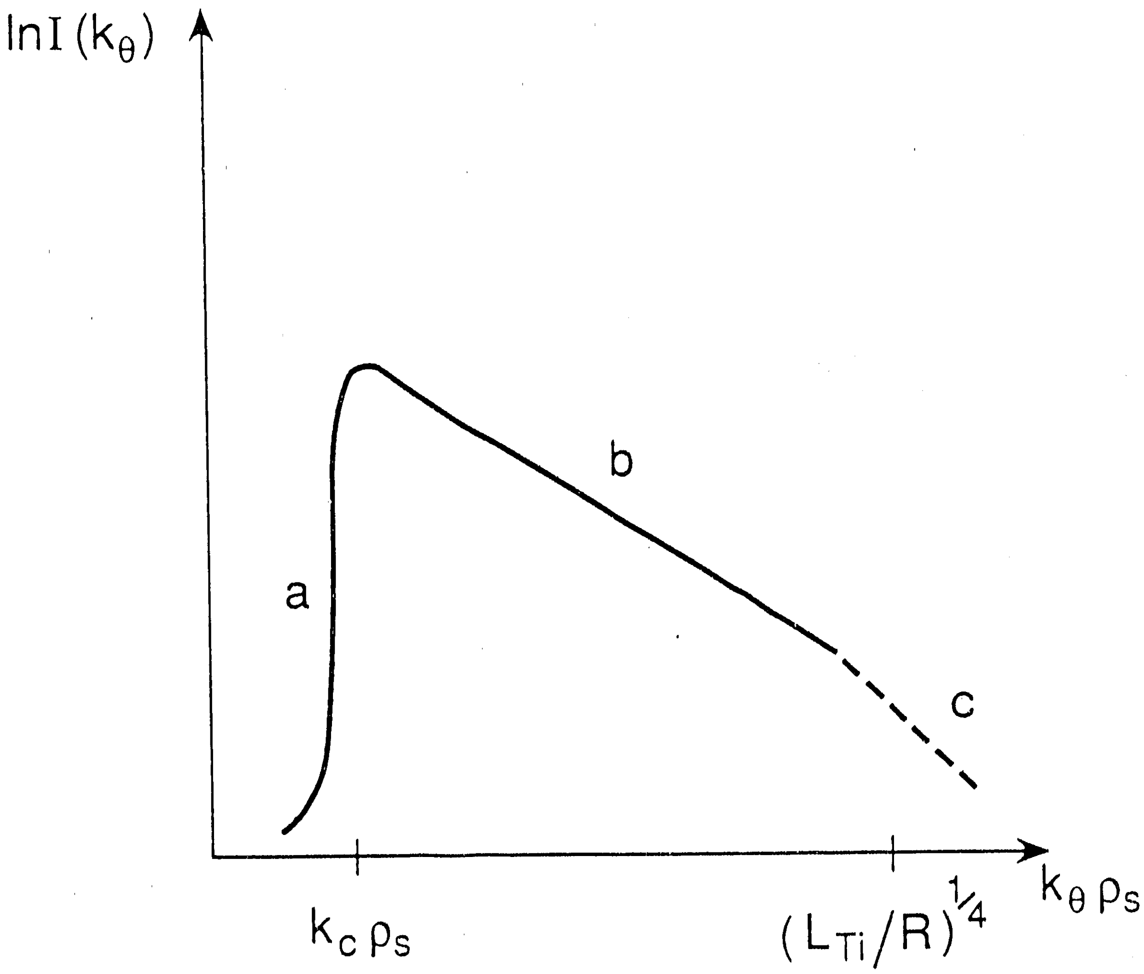

FIG. 2

PPPL #90T0068

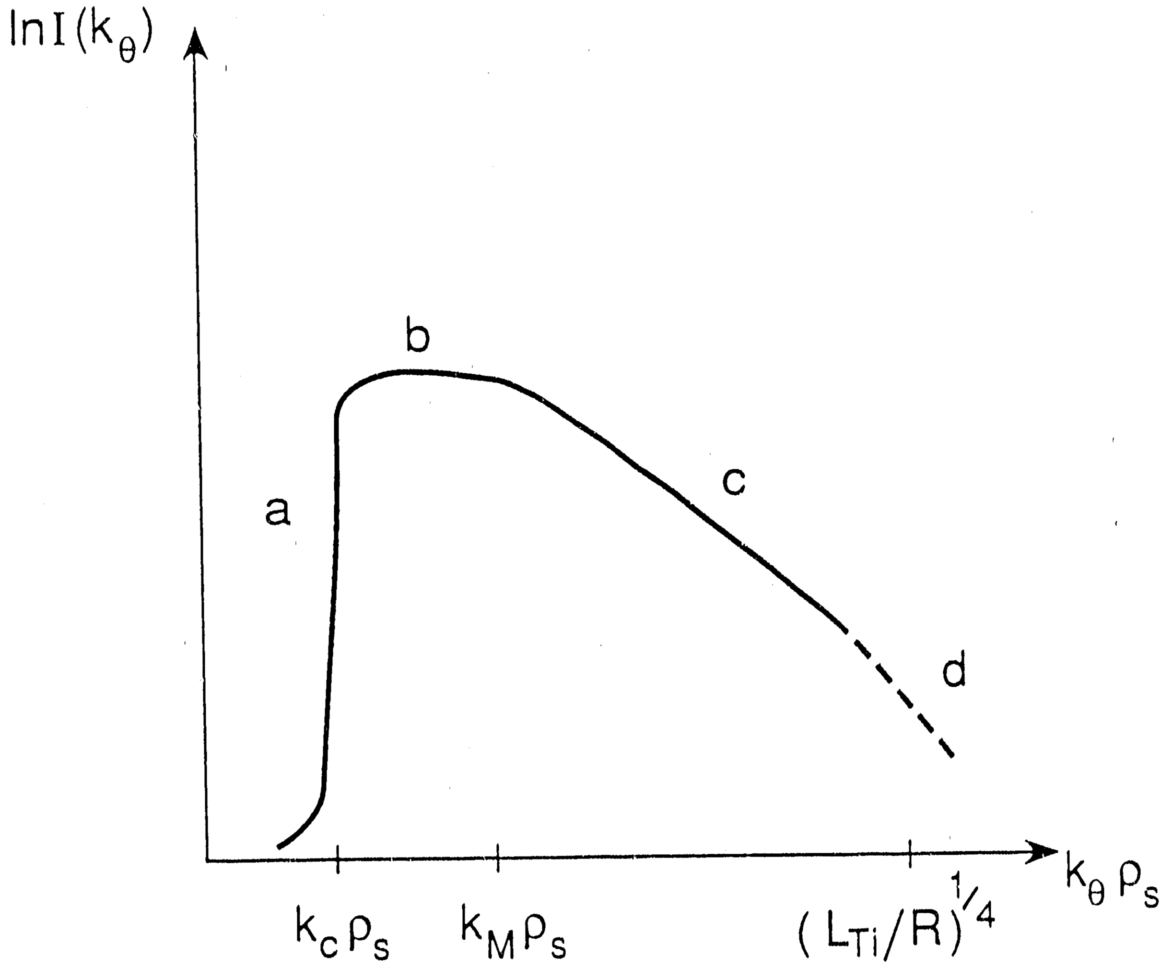

FIG. 3

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