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GREEN'S FUNCTIONS OF SOLITONS IN HEAT BATH

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Soliton Green's functions at nonzero temperature are studied. Considering various model examples we show that the Green's function pole position does not coincide generally speaking with free energy of a soliton. We discuss, in particular, the Frölich polaron and the t'Hooft-Polyakov monopole the Green's function for which is in general a poorly defined concept as it involves an infinite imaginary part connected to the infinite total cross section of monopole scattering by electric charge.

The pole position of the Green's function of the collective sphaleron excitation in the Glashow-Weinberg-Salam model does not as well coincide with the sphaleron free energy $F_{sph}(T) = M_{sph}\sqrt{1-\frac{T^2}{T_c^2}}$ and contains the large imaginary contribution, $\sim i\,T^3/m_w^2$, connected with the sphaleron scattering by excitation in the medium.

Fig. – 9, ref. – 24

I. Motivation

The main reason which impelled me to perform the present investigation were very interesting recent works where it was found that at high temperatures (of order of the temperature T_c of phase transition with restoration of SU(2)xU(1) gauge invariance or higher) there are fast processes with the baryon charge nonconservation[1-2]. This leads to difficulties in explaining the observable present baryon asymmetry of the Universe.

The physical picture of this phenomenon is the following. At high temperatures sphalerons - the solutions of the classical equations of motion in GWS theory which correspond to the local extremum of energy but contain an unstable mode with $\omega_-^2 \sim -m_W^2$ [3] - can be spontaneously created in the medium. Sphalerons have zero-energy fermion modes and in the process of sphaleron creation and decay the level crossing effect may occur which results in effective quark [illegible] lepton transitions. The probability of sphaleron creation at nonzero temperature contains the Gibbs exponential

$$W_{\Delta n \neq 0} \sim \exp\left(-F^{sph}(T)/T\right). \tag{1}$$

The main reason inducing fast baryon nonconservation transitions is the fact that at $T \leq T_c$ the exponent in the r.h.s. of eq.(1) is not small (the exponential factor calculated in ref.[2] does not change the qualitative conclusion that baryon-nonconservation processes are fast). And this in turn is due to the fact that the sphaleron free energy falls down with increase of T an

$$F^{sphal}(T) = M^{sph}(0)\sqrt{1-T^2/T_c^2}\,. \qquad (2)$$

(We assume $T \gg m_W$) and vanishes at $T = T_c$. The result (2) can be easily obtained looking for the extremum of the effective hamiltonian of GWS theory at T ≠ 0 found in refs.[4-5]:

$$H^{eff} = \int d\bar{x} \left\{ \frac{1}{4}\left(G^a_{ij}G^a_{ij} + H_{ij}H_{ij}\right) + \overline{\mathcal{D}_i\varphi}\,\mathcal{D}_i\varphi + \right. \qquad (3)$$

$$\left. + \lambda(\bar{\varphi}\varphi - v^2)^2 + \frac{8\lambda + 3g^2 + g'^2}{16}\bar{\varphi}\varphi T^2 \right.$$

One can easily see that the only influence of finite temperature is renormalizing the scalar vacuum average: $v_T = v\sqrt{1-T^2/T_c^2}$ where

$$T_c = 2v\sqrt{\frac{1}{1+(3g^2+g'^2)/8\lambda}} \qquad (4)$$

and the sphaleron free energy proportional to v_T is given by eq.(2).

In all further estimates we assume $g'^2 = 0$, $g^2 \sim \lambda \ll 1$.

One may, however, describe sphaleron transitions in alternative, kinetical language. In fact, we deal with a resonance process of the type

$$\mathcal{N}(W,H) \to \text{Sphaleron} \to \mathcal{N}'(W,H) + qqq\bar{\ell} \qquad (5)$$

where $\mathcal{N}' \sim \mathcal{N}' \sim 1/g^2$ (see ref.[2]). In this case the sphaleron is interpreted as a resonance with the mass $\sim m_w/\alpha_w$ and the width $\sim m_w$. At finite temperature the initial, finite and intermediate states of the process (5) should be understood as collective excitations in the temperature medium. In this paper our task is to find the pole position and to compare it with the free energy (2). We say in advance that in this problem like in some other similar problems these two definitions of temperature-dependent mass do not coincide.

In the next sections we disucss a number of model examples. In Sec.2 we consider the Frölich polaron at $T \neq 0$ and note that in this case the polaron free energy does not in the least coincide with the pole position of the polaron Green's function. In Sec.3 we consider the heavy charged particle in the photon heat bath. On the one-loop level the contribution to the free energy of the system connected with interaction of the particle with photon bath coincides with the temperature shift of the Green's function pole in the main nonrelativistic approximation. When accounting for the relativistic corrections $\sim T/M$ this values are not the same anymore. On the two-loop level the Green's function acqure an imaginary part whereas free energy is a real quantity by definition. We discuss also a more realistic problem of a heavy charged particle in relativistic plasma and show that in this case the two-loop imaginary part exceeds significantly the one-loop contribution determined earlier.

In Sec.4 we pass on to consideration of the t'Hooft-Polyakov monopole and the sphaleron.

By close analogy with the previously discussed problem, the real part of the temperature shift of the soliton Green's function calculated in the lowest nontrivial order of perturbation theory coincides with the temperature shift of the soliton free energy. However, the Green's function involves also a large imaginary part connected with the soliton scattering by excitations of the medium. For sphaleron the imaginary part of the Green's function pole $\mathrm{Im}\,\omega$ is of order of $\sim T^3/m_w^2$ which is much larger than the real part shift $\Delta \mathrm{Re}\,\omega \sim T^2/m_w$ in the region $T \gg m_w$.

The monopole (in contrast to sphaleron) contains the power quasi-Coulomb tail in gauge field dependence which leads to the infinite total cross section of monopole scattering by a charged particle and as a consequence, to the infinite imaginary contribution in the monopole Green's function.

In Sec.5 we discuss the results obtained and make some final notes.

2. Polaron at Finite Temperature

The Fröhlich polaron, i.e. the electron interacting with long-wave optical phonons in a polar crystal is known as one of the most simple and beautiful problems of solid state physics where the methods of quantum field theory are much efficient and the quantitative comparison of theoretical predictions with experiment is possible[6-7]. The hamiltonian of electron in polar crystal has the form

$$H = \frac{\bar{P}^2}{2m^*} + \omega_{ph} \int \frac{d\bar{k}}{(2\pi)^3} a^+_{\bar{k}} a_{\bar{k}} + i(2\pi\alpha\sqrt{2})^{1/2} \cdot \left(\frac{\omega^3_{ph}}{m^*}\right)^{1/4} \int \frac{d\bar{k}}{(2\pi)^3} \frac{1}{|\bar{k}|} \left(a^+_{\bar{k}} e^{-i\bar{k}\bar{x}} - a_{\bar{k}} e^{i\bar{k}\bar{x}}\right), \quad (6)$$

where $\bar{x}$ and $\bar{p}$ are the electron position and momentum, m^* is the effective electron mass withour account of its interaction with long-wave modes, ω_{ph} is the optical phonon frequency (it is assumed not to depend on the phonon momentum) and $a^+_{\bar{k}}$, $a_{\bar{k}}$ are the operators of phonon creation and annihilation, $\hbar = c = 1$ convention is adopted. At T = 0 the polaron energy is the sum of the energy of the electron stripped of the long-wave phonons and the polarization energy depending on the spacial distribution of the polarization vector:

$$P(\bar{x}) = \int d\bar{k} \frac{\bar{k}}{|\bar{k}|} \left(a^+_{\bar{k}} e^{-i\bar{k}\bar{x}} - a_{\bar{k}} e^{i\bar{k}\bar{x}}\right). \quad (7)$$

When the coupling constant α is large, this distribution has a complicated soliton-like form. It is explained in the reviews [6-7] how to find the total polaron energy and its effective mass as a function of α using various field-theory methods (Perturbation theory at small α and the path integral approach at large α).

If the temperature of the crystal is nonzero the situation is much less clear. Different generalizations of the concept of the polaron energy for nonzero temperature have proven to lead to different answers. Thus, at $T \ll \omega_{ph}$ and $\alpha \ll 1$ the polaron free energy (defined as the free energy of the hamiltonian (6) substracted by the free energy of the free phonon gas and the free energy of an isolated electron) has the form [8-9]

$$F^{int}(T) = -\alpha\omega_{ph}\left[1+\frac{T}{4\omega_{ph}}+O\left(\frac{T^2}{\omega_{ph}^2}\right)\right]+O(\alpha^2) \quad (8)$$

The term $-\alpha\omega_{ph}$ in the r.h.s. of eq.(8) is just the zero-temperature renormalization of the polaron energy due to electron-phonon interactions. The second term $\sim -\alpha T/4$ arises at nonzero temperature due to renormalization of the polaron mass, i.e. the coefficient at $\bar{p}^2$ in the polaron T = 0 dispersion law:

$$E(\bar{p}) = E_0 - \alpha\omega_{ph} + \frac{\bar{p}^2}{2m^*}(1-\alpha/6) + O(\bar{p}^4, \alpha^2). \quad (9)$$

Really, the partition function of dressed polaron is

$$Z_{pol} \sim \left(m^*(1+\tfrac{\alpha}{6})T\right)^{3/2}\exp\left\{-\frac{E_0-\alpha\omega_{ph}}{T}\right\} \quad (10)$$

and we see immediately that the free energy $F_{pol} = -T\ln Z_{pol}$ differs from the free energy of the bare electron by the value (8). The terms $\sim\bar{p}^4$, $\sim\bar{p}^6$ etc. in the dispersion law (9) contribute in the higher orders of expansion of the free energy on the parameter T/ω_{ph} .

As for the temperature shift of the polaron Green's function pole it is entirely different. In the lowest order in α the temperature-dependent contribution in the polaron Green's function is given by the graph of Fig.1 where the crossed phonon line describes the contribution of real phonon excitations and contains the factor

$$n_\beta^{ph}(k) = \frac{1}{e^{\beta\omega_{ph}} - 1} \sim \exp(-\omega_{ph}/T) \qquad (11)$$

(see, e.g. ref.[5]). Thus the polaron Green's function pole at $\bar{p} = 0$ is shifted by the value

$$\Delta E(\alpha, T) = -\alpha\omega_{ph}[1 + C\exp(-\omega_{ph}/T)] \qquad (12)$$

(this result was originally observed in a slightly different language in the work[10]). At T =0 the values (8) and (12) coincide but their temperature dependence differs drastically.

For the polaron itself the question is a bit academical as neither of these values is viable to direct experimental measurement. The best way to study the temperature dependence of polaron properties is connected with the cyclotron resonance experiment. It turns out however, that at $T \neq 0$ the cyclotron line width is always of the same order or larger than the temperature shift of the resonance maximum, the latter being not connected directly with the Green's function pole or with the coefficient at $\bar{p}^2$ in the momentum dependence of the polaron free energy. In other words, there is no universal concept of the polaron mass at $T \neq 0$[11].

However, the mere fact that the soliton free energy and the pole of the soliton Green's function are different concept and their quantitative values can differ drastically has a significant methodical importance. In each particular case a special investigation is required to find out whether these concepts are interchangable.

3. Heavy Charged Particle in Photon Bath

let us study one more model example after which we shall be able to pass on to discussion of monopoles and sphalerons we are interested in. Consider the charged spinless particle with the mass M interacting with the photon gas at temperature T. We assume $T \ll M$ so that nonrelativistic expansion on the parameter T/M is possible.

In the lowest order in e^2 the temperature correction to the Green's function of the heavy particle is given by the graph in Fig.2. The graphs with temperature insertion in the scalar particle line contains the exponential factor $\sim \exp(-M/T)$ and can be ignored safely. The most convenient gauge is $A_0 = 0$ where the temperature correction to the photon propagator has the form

$$D_{ij}(k) = \frac{2\pi i}{e^{\beta|k_0|}-1}\left(\delta_{ij} - \frac{k_i k_j}{\bar{k}^2}\right)\delta(k_0^2-\bar{k}^2). \quad (13)$$

In this gauge the graph in Fig.2a does not contribute at all to the polarization operator. Really, at nonzero external $\bar{p}$ this follows immediately from the transversality of the propagator (13) while at nonzero $\bar{p}$ the contribution of the Fig.2a graph to the polarization operator is proportional to the integral

$$\Pi_{Fig.2a}(E,\bar{p}) \sim e^2 \int \frac{d\bar{k}}{|\bar{k}|} \left(\bar{p}^2 - \frac{(\bar{p}\bar{k})^2}{\bar{k}^2}\right) \cdot$$

$$\cdot \frac{1}{E\omega - \bar{p}\bar{k}} \Big|_{\omega = \pm|\bar{k}|} \sim e^2 \int \frac{d\bar{k}}{|\bar{k}|} \left(\bar{p}^2 - \frac{(\bar{p}\bar{k})^2}{\bar{k}^2}\right) \bar{p}\bar{k} \cdot$$

$$\cdot \frac{1}{E^2 \bar{k}^2 - (\bar{p}\bar{k})^2}$$

which vanishes after angular integration.

The contribution of the graph in Fig.2b to $\Pi(E,p)$ can be easily calculated and is equal to

$$\Pi_{Fig.2b}(E,\bar{p}) = e^2 T^2/6. \qquad (14)$$

Note that the temperature contribution to the one-loop polarization operator can be presented in the form

$$\Pi^{1\,loop}(E,\bar{p}) = -2 \int A(\bar{k}) \frac{d\bar{k}}{(2\pi)^3 \, 2|\bar{k}|} \frac{1}{e^{\beta|\bar{k}|} - 1} \qquad (15)$$

where $A(\bar{k}) = -2e^2$ is the free amplitude of the forward compton scattering of the photon with momentum $\bar{k}$ by heavy particle. Later this observation is to be intensively exploited.

Thus, on the one-loop level the dispersive equation for the corresponding collective excitation is very simple,

$$E^2 - \bar{p}^2 = M^2 + e^2 T^2/6. \qquad (16)$$

And the temperature mass shift is

$$\Delta M(T) = e^2 T^2 / 12M. \tag{17}$$

Needless to say, the r.h.s. of eq.(16) contains the renormalized not bare mass. One can be convinced easily that the dispersion law for the heavy nonrelativistic charged fermion is given by eq.(16) too.

Let us find now the free energy of the heavy charged particle interacting with the photon heat bath. In the nonrelativistic approximation the hamiltonian of the system is

$$H = \frac{1}{2M}\Big(\bar{p} - e\int \frac{d\bar{k}}{(2\pi)^3\sqrt{2|\bar{k}|}} \sum_{r=1,2} \big[c^{+}_{\bar{k}r} e^{-i\bar{k}\bar{x}} + c_{\bar{k}r} e^{i\bar{k}\bar{x}}\big]\, \bar{e}_{\bar{k}r}\Big)^2 + \int \frac{d\bar{k}}{(2\pi)^3} \sum_{r=1,2} |\bar{k}|\, c^{+}_{\bar{k}r} c_{\bar{k}r} \tag{18}$$

(we substituted $A(x)$ by its plane wave expansion). Let us write down the partition function of the hamiltonian (18) as

$$Z = \sum_{\{n_i r_i \bar{p}\}} \exp\Big\{-\big[E^{(0)}_{\{n_i r_i\},\bar{p}} + \Delta E_{\{n_i r_i\},\bar{p}}\big]/T\Big\} \tag{19}$$

where $|\{n_i r_i\},\bar{p}\rangle$ is the eigenstate of the hamiltonian with n_i photons in the state with momentum $\bar{k}_i$ and polarization r_i ; $\bar{p}$ is the momentum of the heavy particle. The zero-order energy of this eigenstate is

$$E^{(0)}_{\{n_i r_i\},\bar{p}} = \sum_{\bar{k}r} n_{\bar{k}r} |\bar{k}| + \bar{p}^2/2M$$

(we go over from the integration over $d\bar{k}$ to the summation

over discrete modes by the rule $\int d\bar{k}/(2\pi)^3 \to V^{-1}\sum_{\bar{k}}$)
and $\Delta E_{\{n_i\},\gamma,\bar{p}}$ is the energy shift of the eigenstate due to perturbation

$$H^{int} = \int d\bar{x} \left[-\frac{e}{2M} \left(\bar{p}\bar{A}(\bar{x}) + \bar{A}(\bar{x})\bar{p} \right) + \frac{e^2}{2M} \bar{A}^2(\bar{x}) \right] \quad (20)$$

The terms $\sim e^2$ in $\Delta E_{\{n_i\},\gamma,\bar{p}}$ are determined by the first term in H^{int} in the second order of perturbation theory and the second term in the first order. In the main nonrelativistic approximation the contribution due to the term $\sim e\bar{p}\bar{A}$ is suppressed, $\sim T/M$, and we will not consider it for a while. Averaging the term $\sim e^2\bar{A}^2$ over the state $|\{n_i\},\gamma,\bar{p}\rangle$ we get

$$\Delta E_{\{n_i\},\gamma,\bar{p}} = \langle \{n_i\},\gamma \Big| \frac{e^2}{4MV} \sum_{\bar{k}_i r_i} \frac{c_{\bar{k}_i r_i} c^+_{\bar{k}_i r_i} + c^+_{\bar{k}_i r_i} c_{\bar{k}_i r_i}}{|\bar{k}_i|} \Big| \quad (21)$$

$$|\{n_i\},\gamma\rangle = \frac{e^2}{2MV} \sum_{\bar{k}r} \frac{n_{\bar{k}r} + 1/2}{|\bar{k}|} .$$

We see that the energy shift of the eigenstate $|\{n_i\},\gamma,\bar{p}\rangle$ is combined of two parts.

The first part is independent of the occupation numbers $\{n_i\}$ andis interpreted naturally as the renormalization of the heavy particle energy at T = 0. This value contains a square divergent sum over $\bar{k}$ which corresponds to the square divergence in the graph of Fig.2b type where the photon propagator at zero temperature is inserted. If we cut the sum arbitrarily at $|\bar{k}| \sim M$ the energy of all the states is shifted by the value $\sim e^2 M$. Naturally, this shift appears in the free energy of interaction

F^{int} which is defined in the same way as in the polaron case as the free energy of the hamiltonian (18) subtracted by the free energy of the hamiltonian (18) where interaction is switched off.

The second part in $\Delta E_{\{n\},\psi,\bar{p}}$ depends on the occupation numbers n_i and is interpreted as the sum of photon frequency shifts produced by their interaction with the heavy particle

$$\Delta\omega_{\bar{k}} = \frac{e^2}{4MV|\bar{k}|} \tag{22}$$

The corresponding contribution in F^{int} is given by the expression

$$F^{int}_{\Delta\omega} = 2\int \frac{d\bar{k}}{(2\pi)^3} \frac{e^2}{2M|\bar{k}|} \frac{1}{e^{\beta|\bar{k}|}-1} = \frac{e^2T^2}{12M} \tag{23}$$

which coincides with the contribution of the graph of Fig.2b in the temperature shift of the pole of the Green's function (cf. eq.(17)) The total calculated contribution in F^{int} is

$$F^{int} = Ce^2M + \frac{e^2T^2}{12M} \tag{24}$$

where the first term is the mass renormalization at $T = 0$ (which is, generally speaking, divergent).

Consider now relativistic corrections. For that sake one should account for the second perturbative correction due to the term $\sim e\bar{p}\bar{A}$ in H^{int} and also the first order correction due to the terms $\sim e^2(\bar{p}\bar{A})^2$ and $\sim e^2\bar{p}^2\bar{A}^2$ in the full relativistic

$$H^{int} = \sqrt{M^2+(\bar{p}-e\bar{A})^2} - \sqrt{M^2+\bar{p}^2}\,. \tag{25}$$

The result of the accurate calculation is that the expression (24) is multiplied by the factor $1-3\,T/M$. Note that the first term in the r.h.s. of eq.(14) multiplied by this factor yields the contribution $\sim e^2 T$ in F^{int} which is much larger than $e^2T^2/12M$ even without account of the divergence at hand. The origin of this contribution is the same as the origin of the contribution $\sim \alpha T$ in expression (18) for the polaron free energy and is due to the zero-temperature mass renormalization.

Thus, in this problem just as in the polaron problem the temperature dependence $F^{int}(T)$ differs susbtantially from the temperature dependence of the Green's function pole position. Note, however, that at $T \ll M$ the agreement can be achieved if redefining arbitrary F^{int} as the difference of the free energy of the full hamiltonian and free energy of the hamiltonian

$$\tilde{H} = \sqrt{\tilde{M}^2 + \bar{p}^2} + \sum_r \int \frac{d\bar{k}}{(2\pi)^3} |\bar{k}| c^+_{\bar{k}r} c_{\bar{k}r}$$

where $\tilde{M}$ is the renormalized mass and the interaction is thus partially accounted for. However, relativistic corrections $\sim e^2 T^3/M^2$ to F^{int} which are absent in the Green's function pole shift break down this agreement too.

Let us discuss now two-loop contributions in the polarization operator of a heavy particle. At $\bar{p} = 0$ there is a single essential graph depicted in Fig.3. We shall see that on the two-loop level a new qualitative effect appears - the polarization operator acquires an imaginary part and the Green's function pole is shifted away from the real axis and becomes complex. Really, the imaginary part of the graph of Fig.3 comes from the integration region where all three intermediate lines are on mass shell. At T ≠ 0 this is permitted kinematically as the prescription (3) allows the values k_0 and q_0 to be negative (note the difference with T = 0 Cutkovsky rules where k_0 and q_0 are necessarily positive). Physically, the imaginary part $\mathrm{Im}\,\Pi(E,0)$ we are going to estimate is connected not with the decay but with the scattering of a heavy particle by real quanta of the medium. Thus, we have

$$\mathrm{Im}\,\Pi(E,0) \sim \frac{e^4}{M}\int\frac{d\bar{k}}{|\bar{k}|}\,\frac{1}{e^{\beta|\bar{k}|}-1}\int\frac{d\bar{q}}{|\bar{q}|}\,\frac{1}{e^{\beta|\bar{q}|}-1}\,\cdot$$

$$\cdot\,\delta\left(|\bar{k}|-|\bar{q}|-\frac{(\bar{k}+\bar{q})^2}{2M}\right) \sim \frac{e^4}{M}\int k^2 dk\left(\frac{1}{e^{\beta k}-1}\right)^2 \sim \frac{e^4T^3}{M} \tag{26}$$

(at T << M the last term in the argument of δ -function in the integrand may be neglected). Hence the Green's function pole acquires an imaginary contribution

$$\mathrm{Im}\,E(0) \sim e^4T^3/M^2 \tag{27}$$

It is naturally interpreted as the inverse free path time of the particle in the photon medium: $\mathrm{Im}\,E \sim \tau^{-1} \sim n\sigma \sim T^3(e^4/M^2)$ where n is the typical photon density and σ is the typical cross section. The real part of the graph of Fig.3 is of the same order as the imaginary part.

Note that the shift (27) exceeds (by a large factor ~M/T) the term $\sim e^4$ of the expansion of the square root $\sqrt{M^2+e^2T^2/6}$ in the r.h.s. of eq.(16). In other words, the two-loop contribution is more important than the iteration of the one-loop contribution.

It is clear that on the two-loop level it is rather pointless to compare the complex shift of the Green's function pole and the real by definition value F^{int}.

If we consider a more realistic case when the heat bath includes not only photons but also light charged particles (this is in fact just the problem of heavy quark in quark-gluon plasma considered in ref.[12]) the disagreement is even more drastic. In this case the imaginary part of the Green's function pole $\mathrm{Im}\,\omega \sim \sigma n$ is determined not by the Compton cross section $\sigma \sim e^4/M^2$ but by the cross section of Coulomb scattering by light charged particles (the corresponding contribution in the Green's function is depicted in Fig.4). Formally, this cross section is infinite due to divergence at small transferred momenta $\bar{q}$. But really the divergence is cut off at $|\bar{q}| \sim eT$ due to Debye screening of Coulomb field of the heavy particle. As a result the typical cross section is

$$\sigma_{char} \sim e^4 \int_{|\bar{q}| \sim eT} d\bar{q}^2/\bar{q}^4 \sim e^2/T^2 \tag{28}$$

and $\mathrm{Im}\,\omega \sim T^3 \sigma_{char} \sim e^2 T$ (cf. ref.[12]) which exceeds the one-loop shift (17) by a factor $\sim M/T$ and has no correspondence in the free energy approach.

Note that energy losses of a heavy particle moving through relativistic plasma are determined not by the total cross section connected with the imaginary part of the Green's function but by the transport cross section $\sigma_{tr} \sim e^4 \ln e^{-1}/T^2$.

4. Monopole and Sphaleron Green's Functions at T ≠ 0.

Now we are in a position to discuss the problem of monopoles and sphalerons in heat bath which we are interested in. Let us consider sphalerons first. At T = 0 the sphaleron solution has the form[3]

$$\hat{A}_i(\bar{x}) = \frac{f(\xi)}{gr^2}\varepsilon_{aij}x_j\tau^a \qquad (29a)$$

$$\varphi(\bar{x}) = v\frac{h(\xi)}{r}i(\bar{x}\bar{\tau})\begin{pmatrix}0\\1\end{pmatrix} \qquad (29b)$$

where $\xi = gvr \sim m_{w,H}r$ and $f(0) = h(0) = 0$, $f(\infty) = h(\infty) = 1$ up to exponentially small $\sim e^{-\xi}$ terms. At spacial infinity this solution is just a gauge transformed vacuum. Thus, the sphaleron presents a compact formation of the size $\sim m_w^{-1}$ without the power quasi-Coulomb tail . characteristic of monopole solution.

The main difficulty is that we cannot give a strict definition of the Green's function of sphaleron as a quantum object since we are not aware of the simple operator which creates sphaleron out of the vacuum with a considerable residue. However, the sphaleron Green's function at T ≠ 0 can be defined phenomenologically if studying the processes of sphaleron scattering by real quanta of the medium. Define the quasi-one-loop sphaleron polarization operator making use of the analog of the representation (15) which corresponds to the phenomenological graphs of Fig.5. The task is now to find the elastic amplitude of forward scattering of W-bosons and Higgs-particles with energy $\sim T \gg m_W$ by sphaleron. Or, which is the same, the elastic amplitude of zero-angle scattering of the particle on the external sphaleron field.

In concrete calculation the use of the unitary gauge involving a single Higgs and 3.3 = 9 W-boson quantum degrees of freedom would be most explicit physically. The physical quanta scatter by the classical field which falls off exponentially at spacial infinity (though singular at $x = 0$) and is obtained from the solution (29) by the relations

$$\hat{A}_i^{unit}(\bar{x}) = U(\bar{x})\left[\hat{A}_i(\bar{x}) + \frac{i}{g}\partial_i\right]U^{-1}(\bar{x})$$

$$\chi^{unit}(\bar{x}) = U(\bar{x})\varphi(\bar{x}) - v\begin{pmatrix}0\\1\end{pmatrix} \tag{30}$$

where $U(\bar{x}) = \exp\{-\frac{i\pi}{2}(\bar{x}\bar{\tau})/r\} = i\,\bar{x}\bar{\tau}/r$. However, technically more convenient is the use of the "semiphysical" gauge $\hat{A}_0 = 0$ involving 3.2=6 vector and 4 scalar degrees of freedom. Naturally, the physical values such as the position of Green's function pole is independent of the gauge choice.

In the first order in external field the scattering amplitudes are given by the graphs of Fig.6. One can easily see, however, that for the forward scattering ($\bar{q} = 0$) the contribution of these graphs is zero. Really, the $\bar{q} = 0$ Fourier component of the field $\hat{A}_i^{sphal}$ is just $\int \hat{A}_i^{sphal}(\bar{x})\,d\bar{x}$ which turns to zero due to spherical symmetry of the solution

(29a).

In the second order in the external field the amplitudes are given by the graphs of Figs.7,8 where the graphs in Fig.7 describe the scattering by the classical scalar field (29b) while the graphs in Fig.8 describe the scattering by the classical gauge field (29a).

Consider first the graphs of Fig.7. We immediately see that the graphs of Fig.7a,b involve the contribution

$$\int d\bar{q}\ \bar{\varphi}^{sphal}(\bar{q})\,\varphi^{sphal}(-\bar{q}) \sim \int \bar{\varphi}\varphi^{(sph)}(\bar{x})\,d\bar{x} \qquad (31)$$

as a factor. One can be convinced that in the gauge $\hat{A}_0 = 0$ the graphs of Fig.7c does not contribute at all at $|\bar{p}| \sim T \gg |\bar{q}| \sim M_W$ and the graphs of Fig.7d involves the structure $\sim$

$$\sim (p-q)_i (p-q)_j \frac{\delta_{ij} - \frac{(p+q)_i (p+q)_j}{\bar{p}^2}}{q^2 + 2\bar{p}\bar{q}} \sim 1 + O\left(\frac{|\bar{q}|}{|\bar{p}|}\right)$$

which is independent of $\bar{q}$ and also results in the factor (31) in the corresponding amplitude.

The integral in eq.(31) diverges at spacial infinity. The origin of this divergence is quite clear being connected with inaccurate definition of the interaction vertex we have used implicitly. E.g. for the W-scattering in the sphaleron field we have in fact chosen the vertex in the form

$$\Gamma^{sphal}_{A_r} = g^2 \bar{\varphi}^{sphal} \hat{A}_r \hat{A}_r \varphi^{sphal} \qquad (32)$$

whereas the real vertex is obtained from eq.(32) by subtracting the term $g^2 v^2 A^a_\mu A^a_\mu /2$ which describes the W -mass generation due to Higgs condensate. After accounting for this fact the elastic amplitudes of zero-angle scattering of vector and scalar quanta in the external scalar sphaleron field (the scattering by the classical gauge field is to be considered a bit later) prove to be finite

$$\mathcal{A}^{sphal}_{A_\mu} = \frac{2g^2 C_F}{3} \int d\bar{x} \left[v^2 - \bar{\varphi}^{sphal} \varphi^{sphal} \right] \tag{33a}$$

$$\mathcal{A}^{sphal}_{\varphi} = \left(3\lambda + \frac{g^2 C_F}{2}\right) \int d\bar{x} \left[v^2 - \bar{\varphi}^{sphal} \varphi^{sphal} \right] \tag{33b}$$

where $C_F = t^a t^a = 3/4$. Passing on to the dimensionless amplitudes of particle scattering by sphaleron $\tilde{\mathcal{A}}^{sphal}$ which coincide with the amplitudes (33) multiplied by the factor $2M^{sphal}$, and substituting them into expression

$$\Pi^{sphal} = -6 \int \tilde{\mathcal{A}}^{sphal}_{A_\mu} \frac{d\bar{k}}{(2\pi)^3 2|\bar{k}|} \frac{1}{e^{\beta|\bar{k}|} - 1} - 4 \int \tilde{\mathcal{A}}^{sphal}_{\varphi} \frac{d\bar{k}}{(2\pi)^3 2|\bar{k}|} \frac{1}{e^{\beta|\bar{k}|} - 1} \tag{34}$$

which is quite similar to eq.(15) with the only difference that now we have two kinds of scattered particles (the factors at the integrals are just the numbers of vector and scalar degrees of freedom in the gauge $\hat{A}_0 = 0$) we finally get

$$\Pi^{sphal} = -T^2 M^{sphal} \left(\lambda + \frac{3g^2}{8}\right) \int d\bar{x} \left[v^2 - \bar{\varphi}^{sphal} \varphi^{sphal} \right] \tag{35}$$

At $M_W \ll T \ll T_c$ the shift of the pole position

$$\Delta E = -\frac{T^2}{2}\left(\lambda + \frac{3g^2}{8}\right)\int d\bar{x}\left[v^2 - \bar{\varphi}^{sphal}\varphi^{sphal}\right] \quad (36)$$

following from eq.(35) coincides with the sphaleron energy shift in the first order of perturbation thoory over the temperature-dependent term in the effective hamiltonian (3), the infinite energy shift of vacuum $\varphi = v\binom{0}{1}$ being subtracted (remind that we have set $g' = 0$ for simplicity). This shift coincides with the first term of the expansion on T^2/T_c^2 of the r.h.s. of eq.(2):

$$\Delta E = -\frac{T^2}{2T_c^2}M^{sph}(0). \quad (37)$$

Comparing eq.(36) and (eq.37) and taking into account eq.(4) we are led to an interesting relation

$$\int d\bar{x}\left[v^2 - \bar{\varphi}^{sph}\varphi^{sph}\right] = \frac{M^{sph}(0)}{4\lambda v^2}. \quad (38a)$$

As far as we know, this relation was mentioned first in ref. [13]. In refs. [14-16] where temperature correction to the free energy of monopoles, Abrikosov strings and walls, was calculated the fulfillment of relations of this kind was overlooked. For the monopole of Georgi-Glashow model the analogous relation holds:

$$\int d\bar{x}\left[\varphi^a_{mon}(x)\varphi^a_{mon}(x) - v^2\right] = -\frac{M_{mon}}{4\lambda v^2} \quad (38b)$$

(the scalar field potential energy is normalized at $\lambda(\varphi^a\varphi^a - v^2)^2$)
The elementary derivation of relations (38) is given in the Appendix.

Substituting eq.(38a) into eq.(35) and solving the dispersive equation we finally get

$$\omega^{sph}(0) = M^{sph}(T=0)\sqrt{1-T^2/T_c^2}\,. \tag{39}$$

Essentially, we could obtain nothing else. Calculating the graphs of Fig.7 and substituing them into integral (34) we reproduced in fact the calculation of the scalar fields effective potential in the real time technique[5] in the gauge $\hat{A}_0 = 0$.

Up to now we did not consider the graphs of Fig.8 describing the scattering by sphaleron gauge field. At first signt this may appear quite justified as these graphs seemingly correspond to the effective potential of vector fields which is known to be of order of $\sim g^2|\bar{q}|TA_i^2$ where $|\bar{q}|\sim m_w$ is the characteristic external field momentum (see, e.g., refs. [17-19]) and its contribution is suppressed $\sim m_w/T$. We shall see however, that this line of reasoning is not justified here.

Consider the graphs of Fig.8a,b describing the scattering of an energetic scalar particle by the sphaleron external gauge field. The scattering amplitude has the structure

$$\mathcal{A}_\varphi^{vect} \sim g^2\int d\bar{q}\, A_i^a(\bar{q})\, A_j^a(-\bar{q})\left[\delta_{ij} - \frac{(2p+q)_i(2p+q)_j}{\bar{q}^2+2\bar{p}\bar{q}-i0}\right] \tag{40}$$

The sphaleron field of eq.(29a) is transverse, i.e.

$$A_i^a(\bar{q})\, A_j^a(-\bar{q}) \sim F(\bar{q}^2)\left(\delta_{ij} - q_i q_j/\bar{q}^2\right) \qquad (41)$$

Substituting eq.(41) into eq.(40) and performing angular integration we are convinced that real part of the amplitude $\mathcal{A}_\varphi^{vect}$ is suppressed by the factor $|\bar{q}|_{char}/|\bar{p}|_{char} \sim m_w/T$ compared to $\mathcal{A}_\varphi^{scal}$ and $\mathcal{A}_{A_\mu}^{scal}$, indeed. Large contributions in $Re\,\mathcal{A}_\varphi^{vect}$ cancel out in spite of the presence of the singularity in the integrand at $z_{pq} = 0$. The cancellation is due to the remarkable relation

$$\lim_{\alpha \to 0} \int_{-1}^{1} \frac{dz\,(1+z^2)}{z^2 - \alpha^2} = 0. \qquad (42)$$

However, the amplitude (40) involves also the large imaginary part. It can be estimated as

$$Im\, \mathcal{A}_\varphi^{vect} \sim g^2 p^2 \int q^2 F(q^2)\, dq \int dz\, \delta(pqz) \sim \frac{T}{m_w^2} \qquad (43)$$

which is is much larger than eqs.(33) by a factor $\sim T/m_w$. Multiplying eq.(43) by $2M^{sph}$ and substituting it into eq.(34) we arrive at the estimate $Im\, \Pi^{sph} \sim T^3/m_w^2 \cdot M^{sph}$ and hence

$$Im\, \omega^{sph}(0) \sim T^3/m_w^2 \qquad (44)$$

which is much larger than the result (36) obtained earlier. The origin of this imaginary part can be traced back with an ease.

It is due to the inelastic scattering of the sphaleron by excitations of the medium and coincides with the estimate $\mathrm{Im}\,\hat{\omega} \sim n\sigma^{tot}$ used in the previous section. In this case the excitation density is $n \sim T^3$ while $\sigma^{tot} \sim 1/m_w^2$ which is just the square of the characteristic sphaleron size (the cross section does not involve an additional smallness as the small constant g of interaction of a real particle with external field is compensated by the large values of the field $A^{sph} \sim 1/g$). The analogous results can be obtained calculating the graphs of Fig.8c,d which describe the scattering of vector particles by the sphaleron gauge field.

Seemingly, we have come at the paradox - the large imaginary amplitude (43) should imply the correspondingly large imaginary contribution in the effective potential of static vector fields

$$\mathrm{Im}\, V_T^{eff}(A) \sim \frac{g^2 T^3}{|q|_{char}} (A_i^a)^2 \tag{45}$$

which is absent as we know.

This contradiction is solved in the following way. In fact, the real time technique involves two kinds of fields - physical fields and ghost fields — this is the essence of the Keldysh diagram technique [20-21] or thermo-field dynamics [22-23]. In this technique all Green's functions are 2x2 matrices. One may be convinced that the component Π_T^{11} of the transverse vector fields polarization operator does involve the large imaginary contribution $\sim i g^2 T^3 / |\bar{q}|_{char}$ which comes from the graph of Fig.9. However, the similar large imaginary contribution appears in the off-diagonal components Π_T^{12} . These large contributions cancel out in the most of physical quantities such as the dispersive law for collective vector excitations. However, in the case under consideration as well as in some other cases this large imaginary contribution proves to be essential (it is Π'' which contribute to the graph of Fig.4 describing two-loop contribution in the Green's function of the heavy charged particle in relativistic plasma). As for the effective potential which determines the sphaleron free energy, it is connected with Euclidean single-component Green's functions and vector fields. The Euclidean vector Green's function does not coincide with D^{11} of real time technique and, in particular, does not involve a large imaginary part.

We have restricted ourselves by considering the scattering graphs in the second order in the external field. Note, however, that this problem does not involve a small parameter (see discussion after eq.(44)) and the scattering graphs with larger number of external field insertions may be equally important. It is quite probable that higher order graphs yield large contribution of order of eq.(44) also in the real part. At any rate one thing is clear: the pole position of the Sphaleron Green's function does <u>not</u> coincide with the sphaleron free energy of eq.(2) *) .

For monopole the disagreement is even more drastic. The monopole solution (in contrast to sphaleron) involves a power quasi-Coulomb tail and the total cross section of the magnetic monopole scattering by an electrically charged particle is infinite. In contrast to the problem of heavy charged particle in relativistic plasma discussed at the end of the previous section where such a divergence was cut due to Debye screening no screening is present in this case as abelian magnetic field is not screened in plasma. Thus, the monopole Green's function involves an infinite imaginary part.

5. Discussion

Thus, our wish to try to describe sphaleron transitions in the kinetic language has not been met. Even the starting point of kinetic description - the Green's function of collective sphaleron excitation - proved to have no direct relation to the sphaleron free energy which is the value determining the sphaleron creation probability.

Indeed, the amplitude of a particular process like that of eq.(5) involves the sphaleron Green's function but the total

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*) An attentive reader may recollect that in the previous section we found large corrections to the free energy of the heavy charged particle in the photon heat bath connected with the zero temperature mass renormalization and may wonder if such a correction is important in the sphaleron case. The answer is no. Really, the quantum correction to the sphaleron mass is of order of m_W. Multiplied by the relativistic factor T/M^{sph} (cf. discussion after eq.(15)) it yields the correction to the sphaleron free energy $\sim g^2 T$ which is much less than the contribution (37) calculated earlier. In the case of heavy charged particle this correction was in contrast much greater than the corresponding one-loop contribution of eq.(17).

probability of baryon nonconserving transitions depends on all the possible processes. There are enormous number of channels and the task of resummation of corresponding amplitudes does not seem to be realistic.

The fact that the soliton Green's function at $T \neq 0$ is not an especially physical concept is most clearly illustrated by the monopole example. Its Green's function has an infinite imaginary part whereas the physically measurable quantities such as ionization losses (in this case not ionization but Landau damping losses) of the monopole moving through the medium are quite finite

In conclusion I would like to point to one more unsolved question in the subject of sphaleron transitions which now seems to me much more important than the question of possibility of kinetical description of these transitions studied in this paper. This is the question of the influence of fermions on the transition probability.

In the reasoning based upon the level crossing picture which leads to large possibility of processes with $\Delta B \neq 0$ (see, e.g. refs.[24] and references therein) the most important premise is masslessness of fermion (only Higgs induced mass which is of order of or much less than the W mass is allowed). However in the temperature medium fermions are not massless in fact as the spectrum of quark excitations acquire a gap of order of $g_s T$ where g_s is the strong interaction constant. At $T \lesssim T_c$ this dynamically acquired mass is much larger than the characteristic sphaleron inverse size $\sim m_W$.

We cannot exclude that it may lead to the exponential suppression $\sim \exp(-g_s T/m_W)$ of the probability of transitions with $\Delta B \neq 0$ so that $\Delta B \neq 0$ processes are much slower than it was suggested in refs.[1-2]. Certainly, no definite statement can be made at the present level of understanding. An accurate and if possible exhaustive study of this question is highly desirable.

I am deeply indebted to V.V.Lebedev, K.G.Selivanov and M.E. Shaposhnikov for very useful discussions.

Appendix

I present here an elementary derivation of relations (38) which was kindly communicated to me by K.G.Selivanov. At T = 0 the sphaleron mass is

$$M^{sph} = \int d\bar{x} \left[\frac{1}{4} G^a_{ij} G^a_{ij} + \overline{D_i \varphi} D_i \varphi + \lambda (\bar{\varphi}\varphi - v^2)^2 \right] = \tag{A.1}$$
$$= I_1 + I_2 + I_3 .$$

Consider the field configuration obtained from the solution (29) by the scale transformation

$$\begin{cases} x_i \to \alpha x_i \\ A^a_i \to \alpha^{-1} A^a_i . \end{cases} \tag{A.2}$$

The energy of this configuration is

$$E(\alpha) = \frac{1}{\alpha} I_1 + \alpha I_2 + \alpha^3 I_3 . \tag{A.3}$$

The sphaleron configuration realizes the minimum of energy in a given topological class so that $\partial E(\alpha)/\partial\alpha = 0$ which implies

$$I_1 = I_2 + 3I_3 . \tag{A.4}$$

On the other hand

$$I_2 = -\int \bar{\varphi}\, \mathcal{D}_i^2 \varphi \, d\bar{x} = 2\lambda \int d\bar{x}\, \bar{\varphi}\varphi(v^2 - \bar{\varphi}\varphi) \tag{A.5}$$

due to equations of motion. Substituting eqs.(A.4) and (A.5) into eq.(A.1) we get

$$M^{sph} = 4\lambda v^2 \int d\bar{x}\,(v^2 - \bar{\varphi}\varphi) . \tag{A.6}$$

Q.E.D. The relation (38b) is obtained just in the same way.

Figures

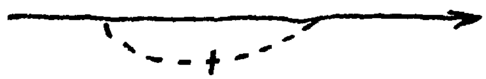

Fig.1. Temperature correction to polaron Green's function. Crossed dotted line corresponds to real phonon.

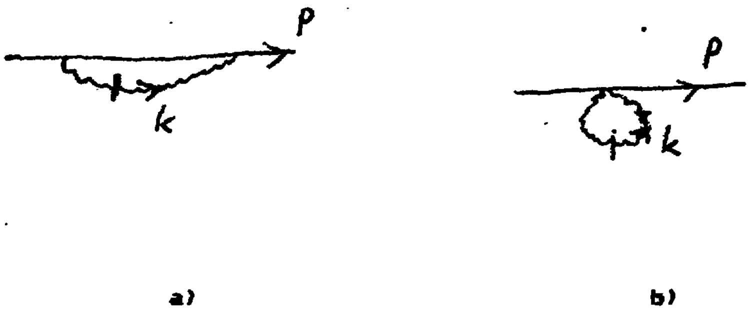

Fig.2. One-loop temperature contribution in the Green's function of heavy charge in photon bath.

Fig.3. Two-loop temperature contribution in the Green's function of heavy charge in photon bath.

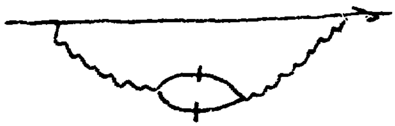

Fig.4. Two-loop temperature contribution in the Green's function of heavy charge in relativistic plasma.

Fig.5. Temperature contribution in the phenomenological Green's function of sphaleron.

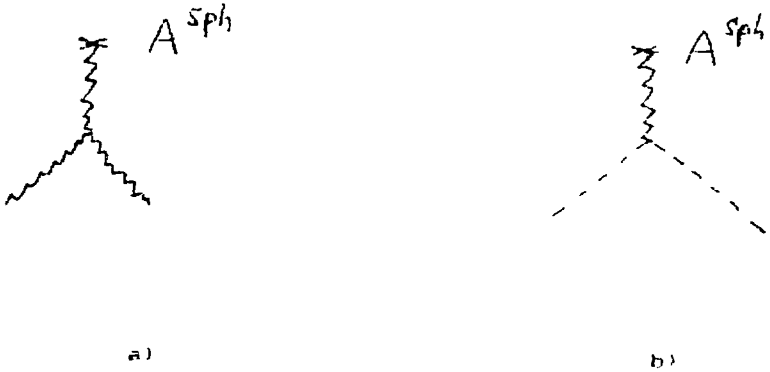

Fig.6. Forward scattering by the sphaleron field in the first order.

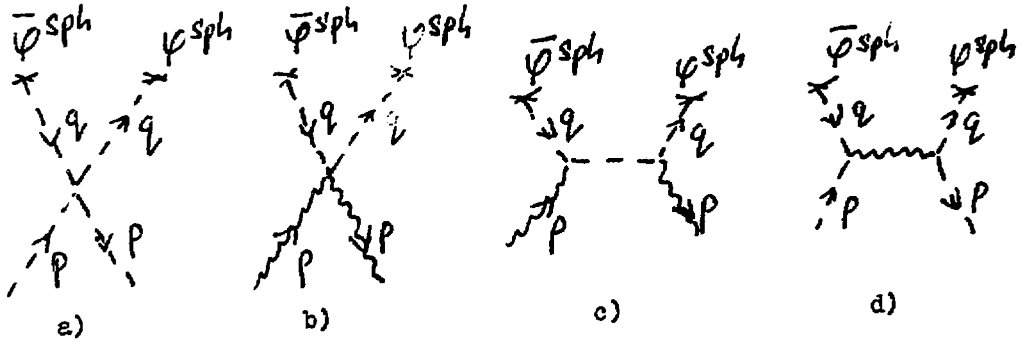

Fig.7. Forward scatterimg of scalar particles by the sphaleron field in the second order.

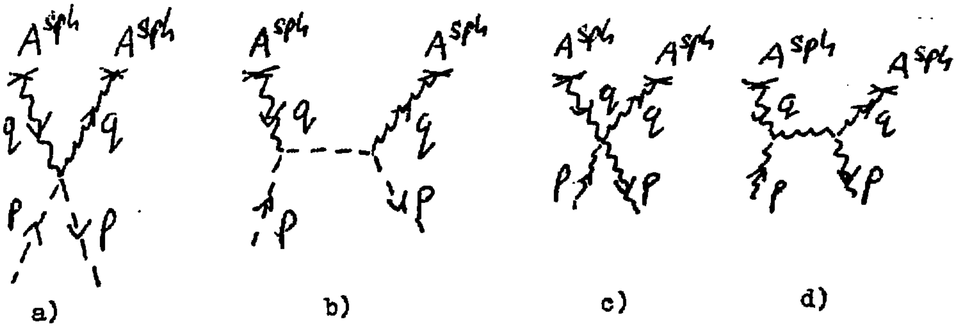

Fig.8. Forward scattering of vector particles by the sphaleron field in the second order.

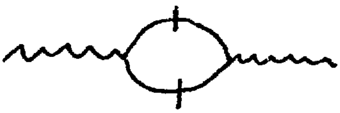

Fig.9. The graph which gives large imaginary contribution into Π^{11}_{T} .

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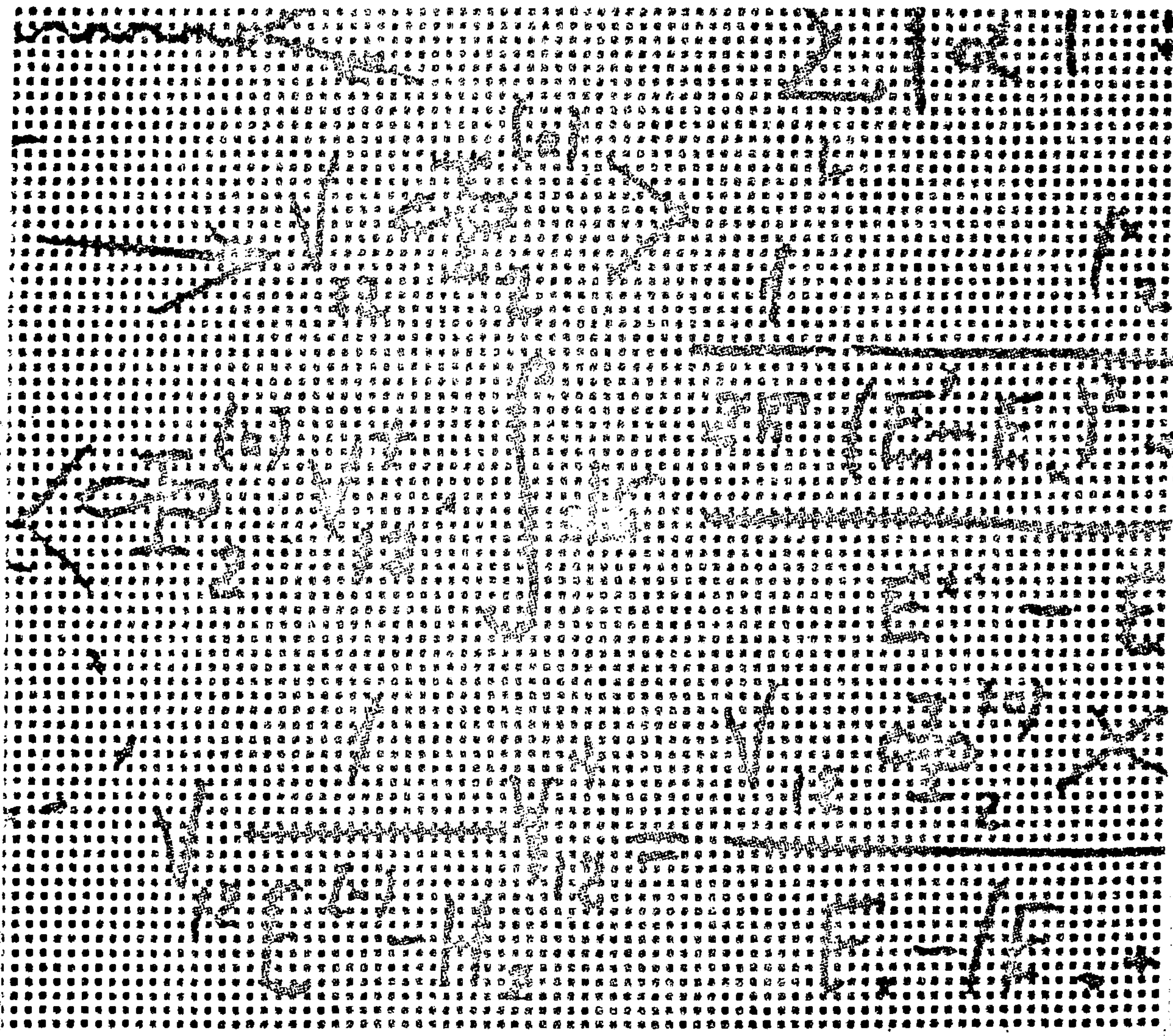

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