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Damped Time Advance Methods for Particles and EM Fields*

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Recent developments in the application of damped time advance methods to plasma simulations include the synthesis of implicit and explicit "adjustably damped" second order accurate methods for particle motion and electromagnetic field propagation.[1] The prototypical scheme applies to an ODE for the position x of a particle as a function of time:

$$\frac{x_{n+1} - 2x_n + x_{n-1}}{\Delta t^2} = \frac{a_{n+1} + \bar{A}_{n-1}}{2} ;$$

$$\bar{A}_{n-1} \equiv (\theta/2)a_n + (1 - \theta/2)\bar{a}_{n-2} ;$$

$$\bar{a}_{n-1} \equiv (1 - \theta/2)a_n + (\theta/2)\bar{a}_{n-2} .$$

Here, subscripts denote time levels and the a's denote accelerations; $a_n \equiv (q/m)E_n(x_n)$. The $\bar{a}$'s are lag-averaged accelerations, the $\bar{A}$'s temporary quantities defined for convenience, and θ is a user-specified damping parameter. For $\theta = 1$ the familiar "d1" scheme is recovered, while for $\theta = 0$ an undamped "c0" scheme results.

A form of implicit advance which uses offset x and v (as in a leapfrog) is:

$$v_{n+1/2} = v_{n-1/2} + (\Delta t/2)\left[a_{n+1} + \bar{A}_{n-1}\right] ;$$

$$x_{n+1} = x_n + \Delta t\, v_{n+1/2} .$$

The dispersion properties of this scheme, some tests, and some observations concerning the dispersion properties of any implicit scheme which interpolates the "future" field at an inexact (e.g., "free streaming") location, have been presented in Reference 1.

An explicit variant of the particle mover is readily derived. One starts with the implicit advance, then replaces the "true" future acceleration a_{n+1} with an extrapolated acceleration based only on explicitly known quantities:

$$a_{n+1} \Longrightarrow 2a_n - a_{n-1} .$$

The dispersion relation for the resulting explicit scheme is cubic. Defining $z \equiv e^{-i\omega\Delta t}$,

$$(z-1)^2(z - \frac{\theta}{2}) = -(\omega_0\Delta t)^2\left[(1 + \frac{\theta}{4})z(z - \frac{\theta}{2}) - \frac{1}{2}(z - \frac{\theta}{2}) + \frac{z}{2}(1 - \frac{\theta}{2})^2\right] .$$

An alternative form of this is:

$$\left(\frac{2}{\omega_0\Delta t}\sin\frac{\omega\Delta t}{2}\right)^2 = 1 + \frac{\theta(\cos\omega\Delta t - 1)}{2e^{-i\omega\Delta t} - \theta} .$$

The transition to instability occurs at a critical timestep size:

$$(\omega_0 \Delta t_c)^2 = 4 \; \frac{2+\theta}{2+3\theta} \; .$$

Special cases are: leapfrog ($\theta = 0$), $\omega_0 \Delta t_c = 2$; explicit d1 ($\theta = 1$), $\omega_0 \Delta t_c = \sqrt{2.4} \approx 1.5492$.

The damping at small timestep has the same frequency dependence as the corresponding implicit scheme:

$$\frac{\gamma}{\omega_0} = -\frac{1}{2} \, \frac{\theta}{(2-\theta)^2} \, (\omega_0 \Delta t)^3 + \cdots \; .$$

The error in the real frequency differs from that of the implicit scheme. One finds:

$$\frac{\omega_r}{\omega_0} = 1 + (\omega_0 \Delta t)^2 \left[\frac{1}{24} - \frac{\theta}{4(2-\theta)} \right] + \cdots \; .$$

By choosing $\theta = 2/7$, one can cause the error in the real frequency to cancel to lowest order. In this case the damping per timestep is: $\gamma \Delta t = -\,(7/144)\,(\omega_0 \Delta t)^4$. For $\omega_0 \Delta t = 0.1$ (period of 20π steps), if one allows a 1% decay in oscillation amplitude one can take 2067 steps, and follow the motion for 32.9 periods.

Implicit and explicit EM field advance schemes have also been worked out. A damped explicit algorithm which requires only a single extra set of arrays (for $\bar{\mathbf{E}}$) is:

(1) Update the lag-averaged field $\bar{\mathbf{E}}$ using:

$$\bar{\mathbf{E}}_{n-1} = (1 - \theta/2)\; \mathbf{E}_n + (\theta/2)\; \bar{\mathbf{E}}_{n-2} \; .$$

(2) Advance the electric field using:

$$\mathbf{E}_{n+1} = \mathbf{E}_n + c\Delta t \nabla \times \mathbf{B}_{n+1/2} - 4\pi \Delta t \mathbf{J}_{n+1/2} \; .$$

(3) Then, in the same loop so that $\mathbf{E}_n$ is still available,

$$\mathbf{B}_{n+3/2} = \mathbf{B}_{n+1/2} - \frac{c\Delta t}{2} \; \nabla \times \left[\left(2 + \frac{\theta}{2}\right) \mathbf{E}_{n+1} - \mathbf{E}_n + \left(1 - \frac{\theta}{2}\right) \bar{\mathbf{E}}_{n-1} \right] .$$

Many codes also need $\mathbf{B}_{n+1}$ (for the particle advance). This is obtained using a step like (3) with $c\Delta t/4$ instead of $c\Delta t/2$.

A dispersion relation for the explicit EM advance can readily be derived if one assumes centered spatial differencing in 1d. One method is to proceed by analogy to the particle advance, making the correspondence:

$$\omega_0^2 \;\Longrightarrow\; c^2 k^2 \frac{\sin^2(k\Delta x/2)}{(k\Delta x/2)^2} \;=\; \left(\frac{2\Omega}{\Delta t}\right)^2 ,$$

where $\Omega \equiv (c\Delta t/\Delta x) \sin(k\Delta x/2)$.

The critical timestep size (for a mode with $\sin(k\Delta x/2) = 1$) has been verified by numerical experiment[2]; it is given by:

$$\frac{c\Delta t_c}{\Delta x} = \sqrt{\frac{2+\theta}{2+3\theta}} ,$$

For small timestep size, one finds:

$$\omega_r \Delta t = 2\Omega \left[1 \ + \ \Omega^2 \left(\frac{1}{6} - \frac{\theta}{2-\theta} \right) \ + \ \cdots \right] ;$$

$$\gamma \Delta t = - \ \frac{8\theta}{(2-\theta)^2} \, \Omega^4 \ + \ \cdots .$$

This electromagnetic advance has been tested[2] as a remedy for dispersion-induced noise in the propagation of an EM signal. It works fairly well, but is not as effective as Flux Corrected Transport in preserving a square pulse. However, for many problems only smoothness really matters, and the techniques should be comparably effective. The damped mover is much easier to implement, especially for bounded systems where the extended "stencil" of high-order spatial differencing is problematic.

The damped explicit EM advance has been implemented in the CONDOR code at LLNL, and applied to the modeling of injector behavior in the ATA induction linear accelerator. The results are very encouraging. In the run, EM pulses are sent down ten gaps. They add, creating the applied voltage across the anode-cathode gap. The question to be answered was, "How do timing errors in the pulses affect the gap voltage, beam current, and beam energy?" In Fig. 1, the beam current at a particular axial location is plotted as a function of time. Noise in the voltage leads to fluctuations in the electron injection; hence the beam current history is easily corrupted by noise. The leapfrog run (Fig. 1a) is quite noisy; the damped run (Fig. 1b), with $\theta = 0.8$, is much quieter. This damping parameter was chosen because it was close to the maximum stable value for this problem. Damping parameters as small as 0.1 were almost as effective.

It should be straightforward to derive higher-order damped advances, in pursuit of more accurate dispersion. They might prove useful for the EM field advance and for other hyperbolic systems on a mesh, where retention of several time levels might obviate a large spatial stencil.

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1 A. Friedman, "A Second Order Implicit Particle Mover With Adjustable Damping," *J. Comput. Phys.* 90, 292 (1990).

2 P. W. Rambo, J. J. Ambrosiano, A. Friedman, and D. E. Nielsen, Jr., "Temporal and Spatial Filtering Remedies for Dispersion in Electromagnetic Particle Codes," *Proc. 13th Conf. on the Numerical Simulation of Plasmas*, R. J. Mason (Ed.), Santa Fe, Sept. 17–20, 1989 (Los Alamos National Laboratory and Sandia National Laboratory, unpublished).

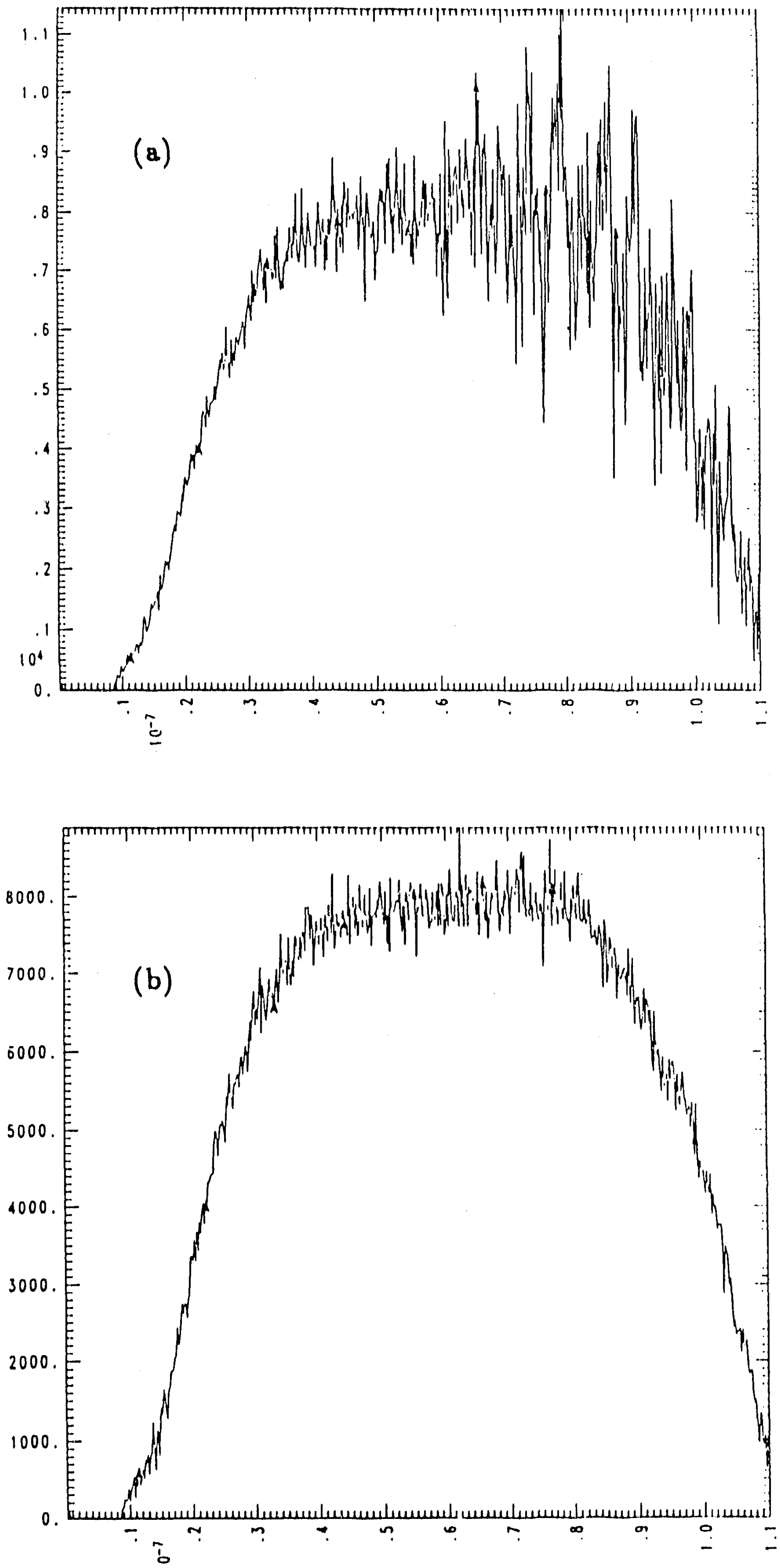

Figure 1. (a) Beam current versus time for the ATA injector problem, using leapfrog time advance for the EM field; (b) using damped time advance with $\theta = 0.8$.

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