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COSMIC-RAY MUONS AS A CALIBRATION SOURCE FOR HIGH-ENERGY
GAMMA-RAY DETECTORS

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ABSTRACT

In this paper a measurement of the directional distribution of cosmic-ray muons, at the latitude of Stockholm, is reported. In fitting the measured flux to a simple analytical expression, the distribution was found to be symmetric around a line approximately to the northwest at 4.2 ± 0.7 degrees from zenith. The east-west asymmetry amounted to a difference in the total intensity of 20 ± 4 % at the zenith angle of 45 degrees.

The spectra of energies deposited by the muons in a BGO-detector orientated at different angles, are obtained through a Monte Carlo-simulation, where the muon distribution is used as a weight function for sampling muons in different directions.

1. INTRODUCTION.

The nuclear physics group at Stockholm University has over the last few years been studying gamma radiation from heavy-ion fusion reactions. These reactions give rise to two processes, generally considered independent of one another, the formation and the decay of compound nuclei.

The deexcitation of a compound nucleus takes place through the emission of photons and nuclear particles. Close studies have been made of the resulting spectra, but not until 1981 did one take a more pronounced interest in the high-energy part of such gamma spectra. [ref. 1]

In the beginning of the eighties, the nuclear physics group investigated the decay of compound nuclei, which were created with a ^{12}C beam and targets of natural indium and terbium. The gamma-ray detectors used in these experiments, performed at the Manne Siegbahn Institute in Stockholm, were cylindrical, 8 cm diam. x 25 cm long, NaI-detectors. Due to poor energy resolution, they are unsuitable for detection of gamma rays in the high-energy region above 15 MeV, and this made some of the results uncertain. To meet the requirements of energy and time resolution in this type of experiments, a set of six scintillation detectors was developed.

They have been employed in experiments at The Svedberg Laboratory in Uppsala, investigating bremsstrahlung emission in collisions between heavy ions. [ref. 2]

Each of the detectors consists of two different scintillators optically isolated from one another, one large bismuth germanate (BGO) crystal and one small built-in barium fluoride (BaF_2) crystal. The BGO crystal has a high absorption probability and the BaF_2 is characterized by good timing properties. [ref. 2]

In the low-energy region the calibration of the BGO-detector is done in a traditional way with radioactive sources. A calibration point at high energy can be obtained from cosmic-ray muons passing through the detector. The assumption made for the Monte Carlo-simulations of the spectra, was that the energy deposition is approximately proportional to the distance covered by the muons in the BGO-material, which can be justified by the fact that cosmic-ray muons are strongly penetrating particles.

To make full use of the possibilities of the six detectors in e.g. directional correlation experiments, it is necessary to be able to put them in any chosen direction.

However, owing to the directional distribution of the muons and the irregular geometry of the detector, the energy deposited in the detector is dependent on the orientation of the detector. This was one of the problems encountered in calibrating these new detectors.

The present report describes an experimental determination of the directional distribution of muons at the latitude of Stockholm. Furthermore, a program, BGOSIM, has been developed, which simulates muon spectra in the BGO-detector for any chosen orientation.

1.1 CONTENTS.

Section 2 contains a description of the experimental set-up and measurements, followed by a short discussion concerning these. In section 3 the results of a fit to the muon distribution is presented.

A brief description of the Monte Carlo method used in BGOSIM is given in section 4, in addition to an overview of BGOSIM. Here is also described the different subroutines and the somewhat complicated geometry of the detector. Finally, section 5 contains two simulated spectra, and a concluding discussion.

2. THE MUON EXPERIMENT.

2.1 BACKGROUND.

It is well known that the flux of cosmic rays is latitude dependent, which is explained by the variation of the geomagnetic field. The angular variation is twofold, on one hand there are marked disparities in intensities at different zenith angles, on the other hand there are also small variations of intensities with the azimuth angle.

The measurements performed in this experiment, of the directional distribution of muons, were made in Stockholm, situated between the 59th and 60th parallels.

Since the energy detected by the BGO was assumed to be approximately proportional to the distance covered by the muon inside the BGO, regardless of the momentum of the particle, no measurements were made of the momentum spread. The average energy of cosmic-ray muons is about 2 GeV. Some have as much energy as a few TeV [ref. 3]. The decrease of the total intensity with increasing zenith angle is connected with a variation of the momentum spectrum with zenith angle.

The flux of cosmic-ray muons is an interesting subject in its own right. However, in the present case the main purpose was to obtain a formula, that could be used as a weight function for the sampling of incident muons. Little emphasis has therefore been put on the discussion of the origins of the asymmetries found in the east-west as well as in the north-south directions.

2.2 THE EXPERIMENT.

Two cylindrical, 5 cm diam. x 5 cm long, NaI-detectors were mounted aligned with each other, connected to separate photo-multiplier tubes (PM-tubes). They were 6 cm apart. The energy threshold for detection was set to 10 MeV. Requiring a coincidence condition between the two detectors selected one direction at a time.

The measurements were made in the vertical direction and at four zenith angles towards the east, west, north and south. At the zenith angles 15 and 45 degrees the flux was measured at four additional azimuthal angles, namely to the northwest, the southwest, the southeast and the northeast.

In the horizontal plane the detectors were placed in the east-west and the north-south directions. However, in the latter cases the statistics are rather poor, due to the rare occurrence of muons incident in the horizontal plane.

A reservation must be made concerning the azimuthal angles, the steel reinforcement bars and the abundance of electric equipment in the laboratory introduce a systematic uncertainty that is estimated to be less than 5 degrees.

TABLE 1

DAY [COUNTS]	NIGHT [COUNTS]
307±18	345±19
331±18	277±17
	311±18

In addition to the directional distribution, investigations were made of a possible variation of the flux of muons with the time of day or night. No statistically significant difference was seen for measurements made in the vertical direction in eight-hour intervals, starting at 9.00 and 18.00

respectively [c.f. table 1]. To separate eventual accidental coincidences from the true ones, measurements were made with the NaI-detectors a few meters apart. The chance coincidences were 20 during $5 \cdot 10^5$ s, which in the vertical direction gives the ratio random/true coincidences ≈ 0.0038 .

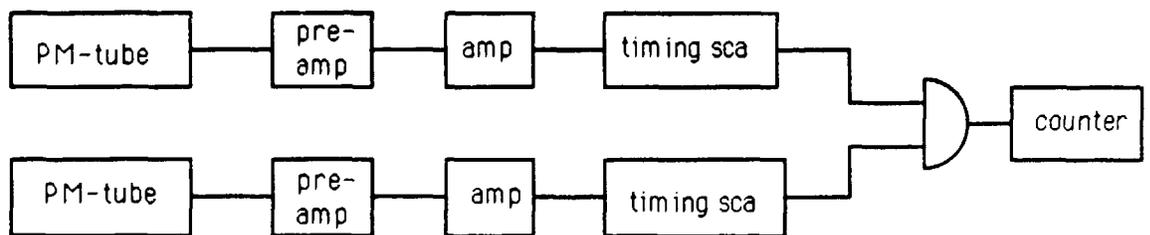


Fig. 1 Block scheme of the experimental set-up.

2.3 RESULTS. DIRECTIONAL DISTRIBUTION.

The east-west asymmetry that was found was expected, since there is a predominance of positively charged particles in the primary cosmic radiation. This fact results in a slightly larger number of positive muons at ground level, giving a charge ratio of approximately 1.25-1.30. [ref. 3]

In the geomagnetic field positively charged particles are deviated towards the east, and the opposite effect is the case for the negative particles.

The deflection of the muons is negligible in the north-south directions, but the primary cosmic radiation is effected by the magnetic field of the earth. At a given latitude the magnetic cut-off of primary energies varies with zenithal and azimuthal angles.

At the zenith angle of 45 degrees the discrepancy in favor of muons coming from the west, was measured to be $20 \pm 4\%$. This result is in agreement with the charge ratio mentioned above. [c.f. table 2]

In the north-south plane, at the same zenith angle, the intensity also varied approximately $20 \pm 3\%$.

What causes this deviation is more uncertain, but one reason could be the greater flux of cosmic radiation at the poles. This is due to the relative ease with which less energetic particles reach the ground, coming as they do along the magnetic field. The total intensity would in this way be greater close to the poles.

TABLE 2

ASYMMETRIES IN THE EAST/WEST AND THE NORTH/SOUTH PLANES

<u>θ (degrees)</u>	<u>I(E)/I(W) [%]</u>	<u>I(S)/I(N) [%]</u>
15	90.9 \pm 3.2	89.6 \pm 3.1
30	86.1 \pm 3.5	86.0 \pm 2.4
45	80.3 \pm 4.0	79.9 \pm 2.8
60	84.8 \pm 5.3	78.9 \pm 5.1

3. FITTED MUON DISTRIBUTION.

The program FIT.PAS is adapted to the already existing FORTRAN routine FUNFIT, which adjusts the parameters in a given expression to experimental data.

The relation that describes the variation of the intensity I of the muon flux, has been found to be of the form [ref. 4]

$$(3.1) \quad I = A \cdot \cos^b \vartheta$$

where A is the vertical intensity, ϑ is the zenith angle and b is a constant for a given location and momentum.

Since the experimental data in this case showed that the total intensity varies with φ_μ , the azimuthal angle, the assumption was made that there is a coordinate system for which this relation holds. This system was found to be slightly rotated with respect to the geomagnetic system and we call it the symmetry system. [c.f. fig 2]

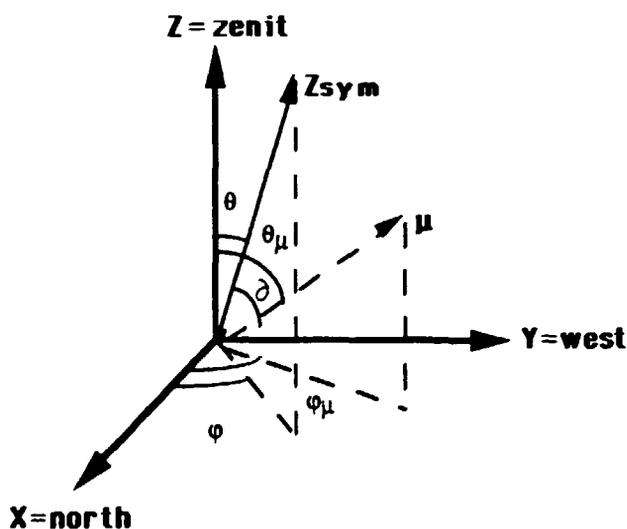


Fig. 2 The geomagnetic system, θ_μ and φ_μ denote the direction of the incident muon.

In the geomagnetic system then

$$(3.2) \quad I = A \cdot \cos^b \vartheta + C$$

where C is the intensity at $\vartheta=90$ degrees and

$$\begin{aligned} \cos \vartheta = Z_{sym} \cdot \mu = & \sin \theta \cos \varphi \sin \theta_\mu \cos \varphi_\mu + \\ & \sin \theta \sin \varphi \sin \theta_\mu \sin \varphi_\mu + \\ & \cos \theta \cos \theta_\mu \end{aligned}$$

Z is the z-axis in the symmetry system and μ is the unit vector in the opposite direction of the incidence of the muon, both expressed in spherical coordinates of the geomagnetic system. [c.f. fig. 2]

The symmetry axis turned out to lie at zenith angle 4.2 ± 0.7 degrees and at azimuthal angle 44.7 ± 9.2 degrees, which is almost exactly to the northwest. Looking at the experimental data this seems reasonable, the deviations of the intensity to the west and to the north were comparable.

The reduced chi-square of the fitted distribution is 1.96, which indicates that the relation (3.2) is not altogether appropriate as a model for the directional distribution. One possible reason for this is, that it is the total intensity that has been measured without considering the momentum spectra.

In table 3 the experimental data are given together with the estimated values according to the equation (3.2), using the parameters listed below.

THE PARAMETERS USED IN BGOSIM

$$A = 0.0100 \pm 0.0002 \text{ s}^{-1} = 6.64 \cdot 10^{-3} \text{ cm}^{-2} \text{ s}^{-1} \text{ sterad}^{-1}$$

$$b = 1.7404 \pm 0.0693$$

$$C = 0.0007 \pm 0.0001 \text{ s}^{-1} = 0.465 \cdot 10^{-3} \text{ cm}^{-2} \text{ s}^{-1} \text{ sterad}^{-1}$$

$$\theta = 0.0741 \pm 0.0126 \text{ radians} = 4.2 \pm 0.7 \text{ degrees}$$

$$\varphi = 0.7841 \pm 0.1646 \text{ radians} = 44.9 \pm 9.4 \text{ degrees}$$

The values in the unit [$\text{cm}^{-2} \text{ s}^{-1} \text{ sterad}^{-1}$] are obtained through the simplification that the aperture angle is ≈ 18 degrees. The trajectory of the muon is assumed to pass the full length of both the detectors. That is, the solid angle is seen from the center of the flat end of one of the cylindrical detectors, and the flat surface furthest away of the other detector is the approximate area with which the solid angle is calculated.

4. THE PROGRAM BGOSIM.

The first subroutines which are called are ALFATABLE and PATHTABLE. They set up tables to be used later by other routines.

The next subroutine to be called is RANDMUON, which samples the direction and the coordinates of one incident muon, and makes the necessary coordinate transformations.

Then SURFACES calls secondary subroutines to calculate points of intersection of the muon trajectory with the surfaces of the detector, whether or not the incident muon passed the detector, and finally if it did, the distance it covered inside the BGO.

If the muon in question did pass through the detector, the subroutine SPECTRUM updates the histogram of distances. The number of sampled particles is checked, and if not all done, RANDMUON is called again. To finish, subroutines handling the output of the histogram are executed.

4.1 SAMPLING OF THE MUONS AND MONTE CARLO IN BGOSIM.

The routine RANDMUON starts off with sampling the two angles ϑ and β to determine the direction of the muon in the symmetry system [c.f. fig 2]. The angle ϑ is the angle between the symmetry axes and the direction of the particle, the angle β is the angle in a plane perpendicular to the plane of the symmetry axes. (Corresponding to the ordinary spherical coordinates θ and ϕ).

The direct Monte Carlo method is to solve an integral equation of the form:

$$(4.1) \quad r = \int_{x_{\min}}^x f(x) dx = H(x)$$

and solve for x: $x = H^{-1}(r)$

where r is a random number and $f(x)$ is the probability distribution. In the simulation process a random number r is generated and put into the formula to turn out the x in a direct manner. [ref. 5]

However, this is sometimes inconvenient or even impossible, as integral equations are not always algebraically solvable. In some cases one can make use of tables, and this is done in the current program.

Our equation (3.2) gives the probability distribution, (the intensity), as a function of the variable $\cos \vartheta$, ϑ varying from 0 to $\pi/2$. The total probability is normalized to 1, which gives:

$$\begin{aligned}
 (4.2) \quad 1 &= k_1 \cdot \int_0^{\pi/2} (A (\cos \vartheta)^b + C) d\Omega \\
 &= k \cdot \int_0^{\pi/2} (A (\cos \vartheta)^b + C) \sin \vartheta d\vartheta \\
 &= k \cdot (A/(b+1) + C) ;
 \end{aligned}$$

that is the normalization constant $k = 1/(A/(b+1) + C)$.

To set up a table, the range of definition is, (in this case, $0 < \vartheta < \pi/2$), subdivided into small intervals, $d\vartheta$. We define and calculate

$$\begin{aligned}
 (4.3) \quad Q_i &= k \cdot \int_0^{\vartheta_i} (A \cos^b \vartheta + C) \sin \vartheta d\vartheta \\
 &= -k \cos \vartheta_i \cdot (A/(b+1) \cdot \cos^b \vartheta_i + C) + k \cdot (A/(b+1) + C)
 \end{aligned}$$

That is, Q_i is the probability to obtain an outcome between 0 and a certain ϑ_i . The sampled ϑ is then found by searching the right interval, for which the condition is

$$(4.4) \quad Q_{i-1} < r < Q_i$$

In the program the random generator is called for a number, a binary search routine finds the right interval in the table of Q_i , and returns an index number i with which the angle ϑ is calculated.

The sought outcome of ϑ is then

$$(4.5) \quad \vartheta = (\vartheta_i + \vartheta_{i-1})/2$$

Due to the cylindrical symmetry of this system, the angle β is easily obtained by the formula

$$(4.6) \quad \beta = r \cdot 2\pi \quad \text{where } r \text{ is a new random number.}$$

The direction vector of the muon is then transformed from spherical to rectangular coordinates.

For a muon of a given incident direction, the probability to hit a certain point is equal for all points on a surface perpendicular to the given direction. Another two random numbers r_1 and r_2 are obtained. Calculated in a plane perpendicular to the direction of incidence of the muon, the x_μ - and y_μ -coordinates in this plane are simply:

$$(4.7) \quad x_\mu = -\text{maxcoord} + r_1 \cdot 2 \cdot \text{maxcoord}$$

$$y_\mu = -\text{maxcoord} + r_2 \cdot 2 \cdot \text{maxcoord}$$

where maxcoord is the maximum coordinate possible to cover the greatest cross-section of the BGO-cone. The coordinates are immediately transformed to the symmetry system. [c.f. fig. 2]

In order to get the equation of the trajectory of the muon expressed in the detector system, the routine makes two transformations with the aid of directional cosines. The first one to the geomagnetic system, and the second transformation takes us to the system of the detector. [c.f. appendix]

4.3 THE DETECTOR AND THE PATH OF THE MUON.

The detector itself consists, strictly speaking, of two separate detectors, one made of BGO and the other of BaF₂. The BaF₂-crystal and parts of the PM-tube coupled to it are inserted into the BGO-crystal.

The BGO itself is a truncated cone, two circles of radii 40 mm and 60 mm forming the flat ends. The BaF₂ is half-cylindric, half-cubic, the radius of the cylinder is 20 mm and the side of the cube is 40 mm. The cavity containing the PM-tube is similar in shape but a little bigger, the radius of the cylinder being 28 mm.

The origo of the detector system is placed inside the BaF₂, at the point where the diagonals of a circumscribed cube would intersect. This is the coordinate system that yields the least complicated equations for the surfaces enclosing the detector. [c.f. fig. 3]

The defining surfaces of the detector are:

$$(4.8) \quad z=5 \cdot \sqrt{(x^2 + y^2)} - 220 ; \quad z=-20 ; \quad z=80 ; \quad (\text{THE BGO})$$

$$z=\sqrt{(28^2 - y^2)} ; \quad x=-20 ; \quad y=-28 ; \quad y=28 ; \quad (\text{THE PM-TUBE})$$

$$z=\sqrt{(20^2 - y^2)} ; \quad x=-20 ; \quad y=-20 ; \quad x=20 ; \quad y=20 ; \quad (\text{THE BaF}_2)$$

The trajectory of the muon is described by global variables in parameter form:

$$(4.9) \quad (x_0, y_0, z_0) + t \cdot (a_0, b_0, c_0)$$

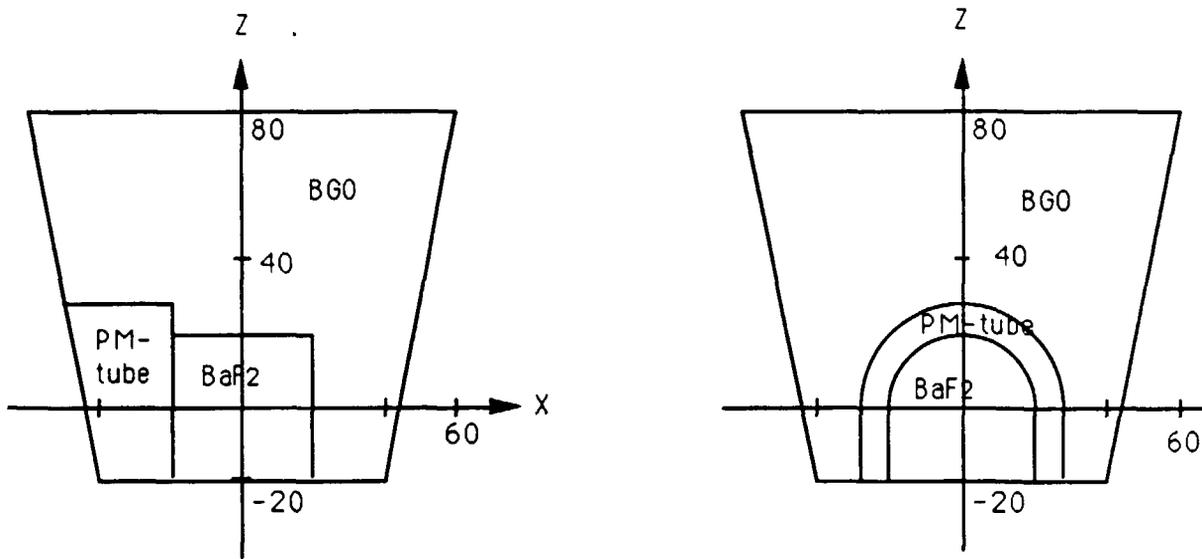


Fig. 3 Longitudinal cross-sections of the detector.

In the procedure SURFACES all surfaces that enclose the detector are investigated for possible points of intersection. At this stage the extension of the detector itself is not taken into account. The coordinates of all points of intersection, no matter where they are situated in space, are sorted into a linked list according to growing z -coordinate. If the trajectory of the muon has no component along the z -axis, the sorting is done along either the y - or the x -axis.

Three types of surfaces are defined to limit the detector from the inside as well as from the outside: the plane, the cylinder, the cone. It would be easy to modify the program to fit another geometry.

The procedure PATH calculates a point that lies in between two consecutive points of intersection. Then this in-between-point is tested in yet another routine, INBGO, to tell if the point lay inside or outside the BGO-crystal.

It turned out to be practical to formulate the conditions from within: The function INBGO starts from the inside, checking if the point-in-between lies in the BaF_2 or in its PM-tube, and if in neither, whether the point is situated in the detector at all.

If INBGO, which is a boolean function, is true, the distance between the two actual points of intersection is calculated and added to a sum variable. Then the procedure goes on to the next two points, if there are any left in the linked list.

To finish, SPECTRUM simply adds the number one to the appropriate interval in the histogram table, and EMPTYLIST makes everything ready for a new muon to be sampled.

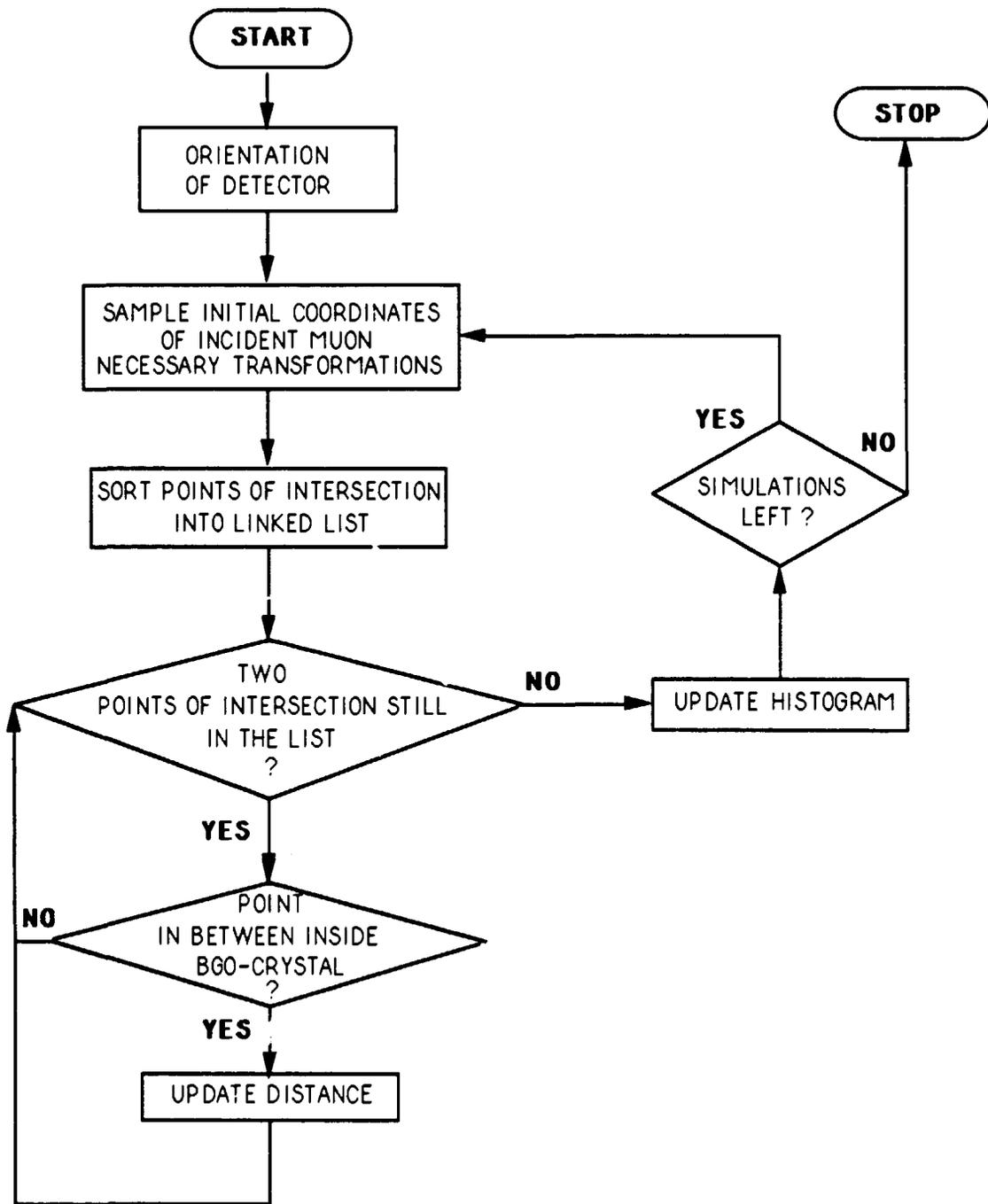


Fig. 4 Flowchart of the main program.

5. RESULTS.

5.1 SIMULATED SPECTRA.

The results of two simulations are shown in fig. 5 and 6. The first spectrum is obtained when the detector is standing in an upright position, the large cut-off surface of the cone being the upper end and the PM-tube pointing to the south. In fig. 6 the detector is lying down, the PM-tube of the BaF₂-detector still towards the south but the large flat end of the cone is directed to the west.

What can be seen in these two simulations is that the shapes of the spectra are similar. However, there is a characteristic spike at 100 mm with the detector in the upright position, which probably originates from the rather great area exposed to vertical muons passing 100 mm of BGO-crystal.

The spike is smeared out when the detector is lying down. The spectra also differ in the slope of the intensity towards the maximum distance covered by the muons. As could be expected the spectra are rather smooth, since almost all distances less than maximum, are represented in all directions with just a small fluctuation of probability.

5.2 DISCUSSION.

The final calibration of the combined BGO-BaF₂ detector, requires a comparison of a real spectrum to a simulated one. This remains to be done.

As for the flux of cosmic-ray muons, it would perhaps be useful to make more diversified measurements, and take the differential momentum spectra at various zenith angles into consideration. The weight function could possibly be improved if another equation was used as a model, or more measurements made and eventual systematic errors removed.

The assumption made that the energy deposited by the muons in the BGO-material is proportional to the distance covered by the muons in the material, is of course a simplification, which might result in some deviations from the real spectra.

To perfect the calibration these issues should be taken into account.

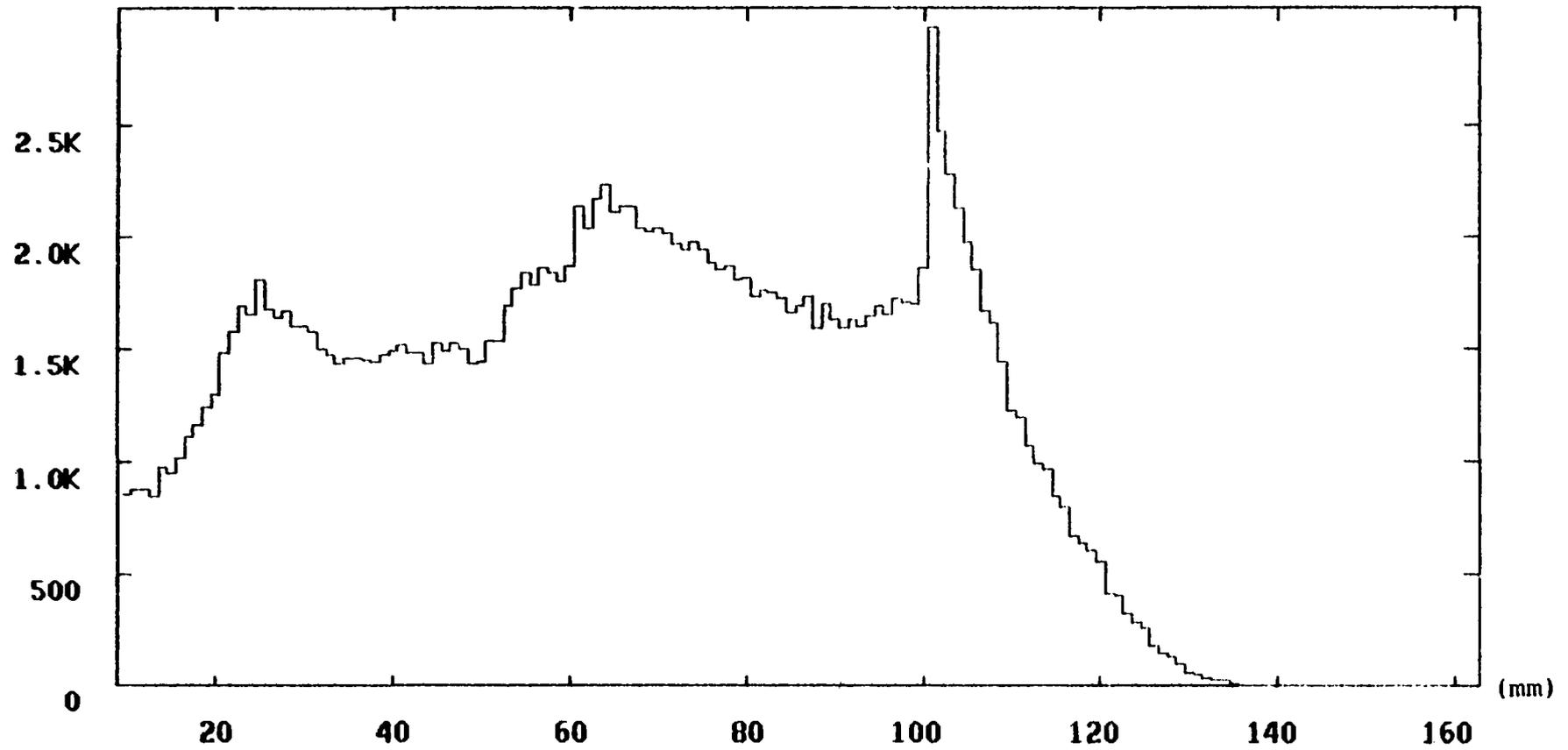


Fig.5 A simulated spectra of the trajectories of the muons through the detector when it is placed in an upright position.

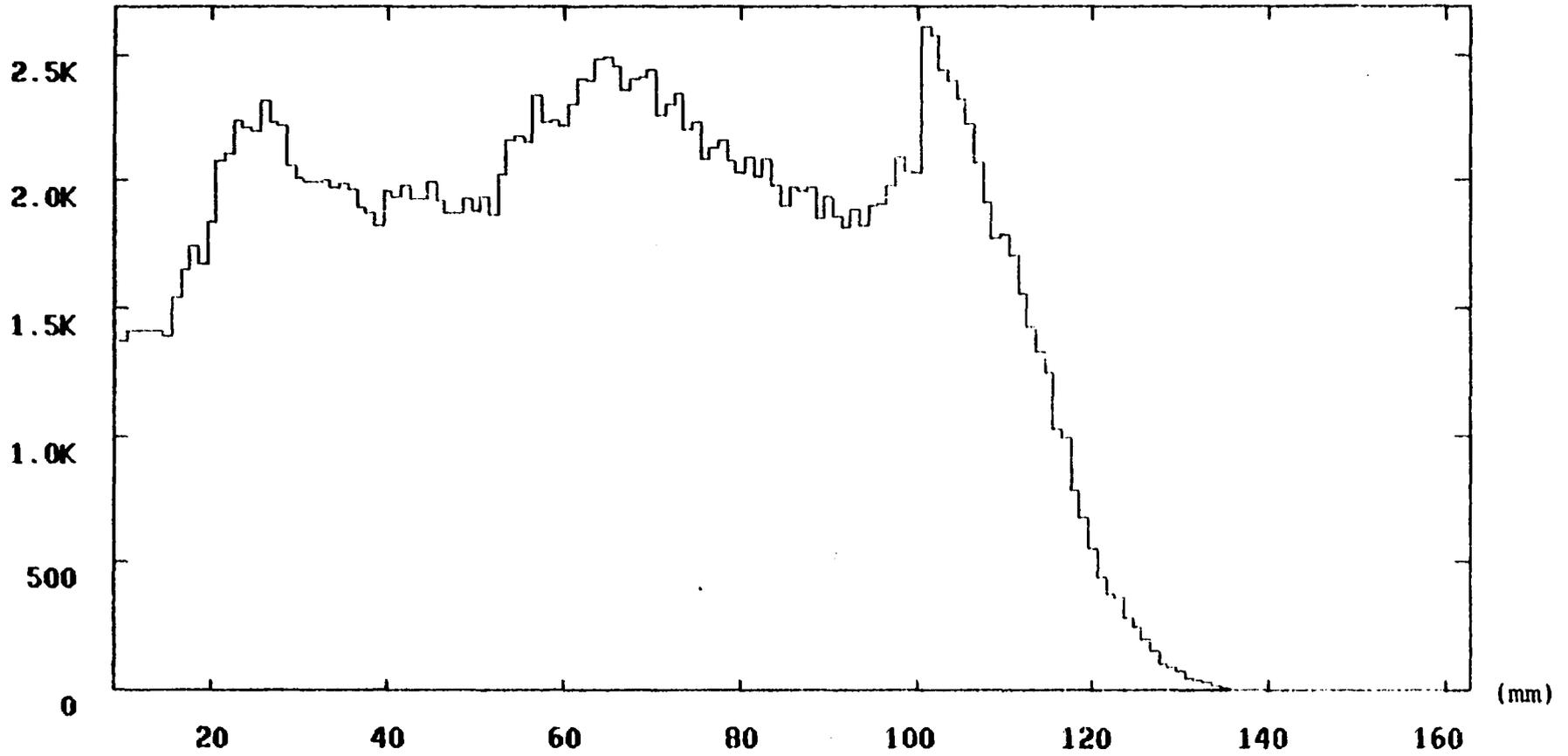


Fig.6 The simulated spectra is obtained through a transformation of coordinates, as if the detector was lying down.

APPENDIX: TRANSFORMATION OF COORDINATES IN BGOSIM.

The two transformations of coordinates in BGOSIM are done by means of direction cosines.

For a line joining points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ direction cosines are defined as:

$$(A.1) \quad \begin{aligned} l &= \cos \alpha = (x_2 - x_1) / d \\ m &= \cos \beta = (y_2 - y_1) / d \\ n &= \cos \gamma = (z_2 - z_1) / d \end{aligned}$$

where α, β, γ are the angles that the line P_1P_2 make with the positive x, y, z axes respectively, and d is the distance between P_1 and P_2 .

1) TRANSFORMATION TO THE GEO-SYSTEM FROM THE SYMMETRY SYSTEM.

Coordinates of the x'', y'', z'' unit vectors of the symmetry system, expressed in the geomagnetic system are (c.f. fig. 7).

$$(A.2) \quad \begin{array}{lll} x'' : x' = \cos \theta \cdot \cos \varphi & y' = \cos \theta \cdot \sin \varphi & z' = -\sin \theta \\ y'' : x' = -\sin \varphi & y' = \cos \varphi & z' = 0 \\ z'' : x' = \sin \theta \cdot \cos \varphi & y' = \sin \theta \cdot \sin \varphi & z' = \cos \theta \end{array}$$

Consequently we obtain the direction cosines for the x'', y'' and z'' axis as defined above:

$$(A.3) \quad \begin{array}{lll} l_1 = \cos \theta \cdot \cos \varphi & l_2 = -\sin \varphi & l_3 = \sin \theta \cdot \cos \varphi \\ m_1 = \cos \theta \cdot \sin \varphi & m_2 = \cos \varphi & m_3 = \sin \theta \cdot \sin \varphi \\ n_1 = -\sin \theta & n_2 = 0 & n_3 = \cos \theta \end{array}$$

Transformation of coordinates involving pure rotation yields the new coordinates of the geomagnetic system according to the following:

$$(A.4) \quad \begin{aligned} x' &= l_1 \cdot x'' + l_2 \cdot y'' + l_3 \cdot z'' \\ y' &= m_1 \cdot x'' + m_2 \cdot y'' + m_3 \cdot z'' \\ z' &= n_1 \cdot x'' + n_2 \cdot y'' + n_3 \cdot z'' \end{aligned}$$

2) TRANSFORMATION TO THE DETECTOR SYSTEM.

Finally the coordinates of the detector system are obtained in an analogous manner (c.f. fig. 8):

$$(A.5) \quad \begin{aligned} x &= \cos \alpha_1 \cdot x' + \cos \alpha_2 \cdot y' + \cos \alpha_3 \cdot z' \\ y &= \cos \beta_1 \cdot x' + \cos \beta_2 \cdot y' + \cos \beta_3 \cdot z' \\ z &= \cos \gamma_1 \cdot x' + \cos \gamma_2 \cdot y' + \cos \gamma_3 \cdot z' \end{aligned}$$

where α_1-3 are the angles between the x', y', z' axes relative the x -axis, β_1-3 are the angles between the x', y', z' axes relative the y -axis, and γ_1-3 are the angles between the x', y', z' axes relative the z -axis.

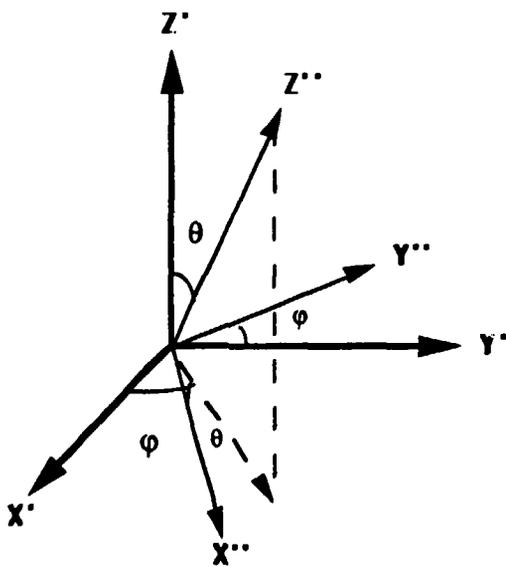


Fig. 7 The y'' -axis is fixed in the $x'y'$ -plane.

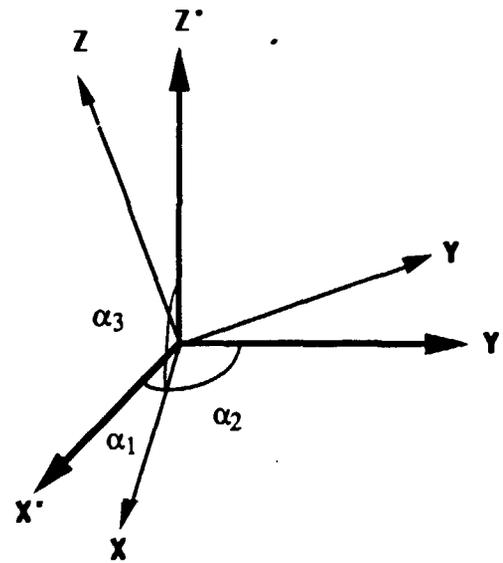


Fig. 8 No axis is fixed in the last transformation.

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ACKNOWLEDGEMENTS.

I want to thank the members of the nuclear physics group for their welcoming and supporting attitude, especially Kjell Fransson, P.-E. Tegnér, Lars Sandberg and Chr. Bargholtz have been most helpful at different stages of the progress of my examination assignment.