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TITLE: INDENTIFICATION AND DETERMINATION OF SOLITARY WAVE STRUCTURES IN NONLINEAR WAVE PROPAGATION

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IDENTIFICATION AND DETERMINATION OF SOLITARY WAVE STRUCTURES IN NONLINEAR WAVE PROPAGATION

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ABSTRACT

Nonlinear wave phenomena are characterized by the appearance of "solitary wave coherent structures" traveling at speeds determined by their amplitudes and morphologies. Assuming that these structures are briefly noninteracting, we propose a method for the identification of the number of independent features and their respective speeds. Using data generated from an exact two-soliton solution to the Korteweg-de-Vries equation, we test the method and discuss its strengths and limitations.

I. INTRODUCTION

A striking qualitative feature of typical spatially extended nonlinear systems, revealed in both laboratory and computer experiments, is the appearance of "pattern." Often, however, a quantitative description of the precise nature of the apparent patterns remains elusive. One central reason for this elusiveness is that "pattern," as discerned by the human eye, in fact reflects a multitude of distinct phenomena which differ in many fundamental ways (Thompson, 1942). Some pattern phenomena are intrinsically regular and deterministic while others exhibit stochastic behavior; some patterns are spatially or temporally stationary, while others show clustering in space or time; and some have scale invariant scaling properties such as self-similarity (again either regular or stochastic) while others possess isolated "coherent structures," such as solitary waves. To distinguish in a reliable, quantitative manner among these possibilities, it is vital to identify the potential symmetries, statistical behavior, invariants, and other features characteristic of the specific pattern to be described. Once this is done, robust empirical descriptions or models of computer or laboratory experiments. can be developed. Ideally, these descriptions should be applicable even in the highly nonlinear regime and in the presence of external noise.

In the recent literature, there have been several successful examples of these "pattern specific" approaches. Using Karhunen-Loeve techniques, a variety of chaotic flows evolving in *a priori* high-dimensional phase spaces have been described by the dynamics of "coherent structures" in low-dimensional spaces (Aubry *et al.*, 1988, 1989; Sirovich, 1989). Properties of the boundaries between crystalline regions exhibiting cellular spatial regularities have been quantified using a "vector pattern recognition" technique (Ourmazd *et al.*, 1989). Patterns which exhibit "fractal" or "self-similar" character have recently been extensively studied using the "wavelet transform" (Goupillaud, Grossmann, *et al.*, 1985; Mallat, 1989; Meyer, 1990; Strang, 1989) and their dynamics have been analyzed using hierarchical modeling (Newman and Turcotte, 1990; Newman and Wasserman, 1990).

In the present article we focus on another particularly significant form of pattern, observed in situations ranging from solid state physics to plasmas, from ocean waves to astrophysics: nonlinear "solitary" traveling wave motion, in which (perhaps several) localized disturbances propagate at (approximately) uniform speeds (typically) related to their amplitudes. Such multi-dimensional wave-fronts are neither homogeneous nor

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isotropic. Nonetheless, the patterns they present are "regular" and hence not appropriately described by statistical or stochastic techniques. Further, despite a formal mathematical equivalence between individual traveling waves and self-similar solution (Barenblatt, 1979; Barenblatt and Zel'Dovich, 1972), the complex translational character of a multiple "solitary wave" solutions suggests that an approach different from the wavelet transform applied to self-similar phenomena should be developed. In certain cases – namely, those physical systems *known* to be well described by an integrable nonlinear wave equation – one can hope to use a "soliton" basis (Whitham, 1974; Ablowitz and Segur, 1981; Drazin, 1984) to describe the solitary waves. A celebrated case in point is the use of periodic solutions to the Korteweg-de Vries equation to describe shallow-water ocean surface waves (Osborne and Segre, 1990; Osborne *et al.*, 1990).

Surprisingly, in the general case, the identification and quantitative description of the solitary waves proves quite non-trivial. Consider as an illustration that can link both the specific and general cases the familiar two-soliton solution of the Korteweg-de-Vries (KdV) equation. In Figure 1, we show the amplitude function Φ as a function of spatial coordinate x at uniform intervals of time t. The eye readily discriminates the two pulse-like features in $\Phi(x,t)$, which we observe are traveling at different speeds. As noted above, if one knows *a priori* that one is observing the evolution of solutions to the Korteweg-de Vries equation, then one can employ a variant of the inverse scattering transform to deduce immediately the speeds of the two pulses. Suppose, on the other hand, that the function evolution displayed in Figure 1 were obtained during a laboratory or computational experiment and that the underlying equation were unknown. How then would one proceed? Assuming that Φ in the "vicinity" of one of the pulses could be represented as an ideal traveling wave leads to the functional dependence $\Phi(x - vt)$, where v is the speed of the pulse's propagation; Φ would then satisfy the equation

$$\frac{\partial \Phi}{\partial t} + v \cdot \frac{\partial \Phi}{\partial x} = 0 \qquad . (1)$$

Thus, if one could reliably determine derivatives by numerical means in the vicinity of the pulse, one would obtain

$$v = - \frac{\partial \Phi}{\partial t} / \frac{\partial \Phi}{\partial x} \qquad (2)$$

as an estimate for the velocity. Unfortunately, it is evident, particularly for experimental data where small differences between noisy measurements must be evaluated, that this procedure would be particularly ill-conditioned and that a more sophisticated approach, preferably motivated by intuition, is needed. In the next section we outline such an approach.

II. MODEL

When the eye detects wave-motion, it recognizes a collection of features that undergo translation at relatively uniform rates over short intervals of time. Thus, it is reasonable to approximate the situation described by Figure 1 over a short interval of time by an expression of the forrn

$$\Phi(x,t) = \sum_{m=0}^{N} \phi_m (x - v_m t) + \zeta(x,t) \qquad . (3)$$

where ζ is an error term which describes the departure from N traveling waves. The quantities v_m describe the velocities of each of the N respective traveling waves, while the functions ϕ_m describe the (possibly independent) shapes of the superposed waves. When the eye identifies a set of features that travel comparatively unchanged and with relatively fixed velocities, it implicitly associates what is seen with a *linear superposition* of waves; this provides the intuitive motivation for the representation in equation (3). The problem at hand, then, is to find the most efficient way of isolating the number N of independent solitary waves, their velocities v_m, $m = 1, \ldots, N$, and their shapes ϕ_m, $m = 1, \ldots, N$, given a limited knowledge of $\Phi(x,t)$. Such situations could include having a sequence of "snapshots" at a set of discrete, presumably uniformly-spaced intervals of time, or having measurements of a limited number of spatial Fourier components of $\Phi(x,t)$ at discrete intervals of time.

We wish to develop a criterion for best estimating the functions ϕ_m in equation (3). For a given set of estimates ϕ_m, $m = 1, \ldots, N$ of the traveling waves, the error term $\zeta(x,t)$ is defined as the departure of the measured signal $\Phi(x,t)$ from the superposed combination of the estimated traveling waves in (3). Thus, it will become necessary to relate the optimality criterion, which we have yet to develop, to the minimization of some form of norm, such as the L_2-norm which provides an estimate of the "power" resident in the residual error or noise. Before we can contemplate establishing such a criterion, we must somehow decouple the spatial and temporal variation contained within each of the estimates $\phi_m(x - v_m t)$.

The Fourier transform is the simplest manipulation for separating the spatial variation from the temporal behavior. We define the usual spatial Fourier transform pair,

$$\begin{aligned} \hat{\Phi}(k,t) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} \Phi(x,t)\ exp(-ikx)\ dx \\ \Phi(x,t) &= \int_{-\infty}^{\infty} \hat{\Phi}(k,t)\ exp(ikx)\ dk \end{aligned} \tag{4}$$

where the caret indicates the Fourier transformed function. With this definition, it follows in a natural way that equation (3) can be written

$$\begin{aligned} \hat{\Phi}(k,t) &= \frac{1}{2\pi}\sum_{m=1}^{N}\int_{-\infty}^{\infty} \phi_m(x - v_m t)\ exp(-ikx)\ dk\ +\ \hat{\zeta}(k,t) \\ &= \sum_{m=1}^{N} \hat{\phi}_m(k)\ exp(ikv_m t)\ +\ \hat{\zeta}(k,t) \end{aligned} \tag{5}$$

where we have adopted the use of carets to indicate Fourier transforms. In particular, we note for any choice of the wavenumber k that the time variation has the form of a complex exponential. Thus, the task of identifying the number of superposed traveling waves is reduced to that of identifying the number of sinusoidal oscillators that are present at any wavenumber. This, we shall presently show, can be reduced to the problem of identifying the number of roots and their value of a particular polynomial. Consider now uniform samples in time at intervals Δt of $\hat{\Phi}(k,t)$, say $\hat{\Phi}^{(n)}(k)$, defined by

$$\begin{aligned} \hat{\Phi}^{(n)}(k) &\equiv \hat{\Phi}(k,t_n) \\ t_n &= t_0 + n\cdot\Delta t \end{aligned} \tag{6}$$

Then, we observe that

$$\hat{\Phi}^{(n)}(k) = \sum_{m=1}^{N}\left[\hat{\phi}_m(k)\ exp(ikv_m t_0)\right]\ exp(ikv_m\Delta tn)\ +\ \hat{\zeta}^{(n)}(k) \tag{7}$$

For convenience, let us write

$$\begin{aligned} \hat{\phi}_m(k)\ exp(ikv_m t_0) &\equiv \varphi_m(k) \\ z_m(k) &\equiv exp(ikv_m\Delta t) \end{aligned} \tag{8}$$

Then, we obtain that

$$\hat{\Phi}^{(n)}(k) = \sum_{m=1}^{N}\varphi_m(k)\ z_m^n\ +\ \hat{\zeta}^{(n)}(k) \tag{9}$$

where we have dropped the wavenumber dependence of z_m. We illustrate one approach to analyzing this sum in the ensuing section.

III. WIENER PREDICTION FILTER THEORY

For a given value of k, this transformed model equation becomes an ideal candidate for the application of Wiener prediction filter theory (Newman, 1977; Wiener, 1950). Wiener's theory was designed to minimize the effects of noise in the case in which a signal composed of a superposition of pure sinusoids, the "message," was additively contaminated by noise. Equation (5) has a similar form, albeit with no specific information about the nature of ζ. Wiener formulated the problem by assuming that the signal, which contained a message together with noise, could be filtered to predict the message alone or, conversely, the noise alone. In particular, Wiener conceived of the "signal" $\hat{\Phi}^{(j)}$ to be composed of a "message" $s^{(j)}$ and "noise" $\xi^{(j)}$, that is

$$\hat{\Phi}^{(j)} = s^{(j)} + \xi^{(j)} \quad . \quad (10)$$

(Here, the superscript (j) indicates that the relevant message and noise quantities were sampled at time $t_0 + j\Delta t$.) Filtering, then, is the process of modifying a signal in order to remove the noise and recapture the original message.

Wiener assumed that the two components of the signal would have very different properties. The noise component was supposed to be a random quantity which varies from one observer to another and, therefore, describes a *stochastic* process. The message, on the other hand, is the component of the signal that would be common to any observer and, we say, describes a *deterministic* process. Since the message is a well-defined, repeatable quantity, it should normally possess a Taylor series or a Fourier series representation. Hence, we might expect that we can extrapolate from the message's past history in order to predict with reasonable accuracy its future behavior*.

This may be achieved by taking a linear combination of previous measurements of the signal to predict the j^{th} message, say (where $\dot{s}^{(j)}$ denotes the predicted message)

$$\dot{s}^{(j)} = \sum_{k=1}^{M} g_k \, \hat{\Phi}^{(j-k)} \quad ; \quad (11)$$

this expression is said to describe an "M-point predictive signal filter." We must demand that the filter should not only estimate the message from previous signals, it must do so in a manner which minimizes the cumulative effect of the noise resident in the previous signals. It is convenient to define coefficients $\Gamma_0, \ldots, \Gamma_M$ according to

$$\Gamma_k = \begin{cases} 1 & k = 0 \\ -g_k^* & k = 1, \ldots, M \end{cases} \quad . \quad (12)$$

Then, for a given value of the wavenumber k equations (11) and (12) become, where $\dot{\xi}^{(j)} = \hat{\Phi}^{(j)} - \dot{s}^{(j)}$ denotes the predicted noise,

$$\xi^{(n)}(k) = \sum_{m=0}^{M} \Gamma_m \, \hat{\Phi}^{(n-m)}(k) \quad . \quad (13)$$

This expression is said to describe an "M+1 point noise prediction filter" for a given wavenumber k. In particular, let the Γ's be selected such that

$$\sum_{m=0}^{\mathrm{M}} \Gamma_m z_p^{-m} = 0 \quad (14)$$

for $p = 1, \ldots, M$, a set of equalities that we can satisfy as a consequence of the Fundamental Theorem of Algebra (Rudin, 1976). Then, we observe that

$$\dot{\xi}^{(n)}(k) = \sum_{m=0}^{\mathrm{M}} \Gamma_m \, \hat{\zeta}^{(n-m)}(k) \quad , \quad (15)$$

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* For simplicity, we use here the conventional *linear* extrapolation methods; exciting recent developments (see, e.g., Farmer and Sidorowich, 1987, 1988) on *nonlinear* forecasting methods may permit one to distinguish (high-dimensional) "true" stochastic noise from (low-dimensional) "deterministic chaos"; in principle, we could combine these temporal methods with our spatial scheme.

which completes the association between the Γ coefficients and the Fourier transformed "errors" $\zeta^{(n)}$. In the ideal case of traveling waves, $\zeta \equiv 0$, and $\xi^{(n)} = 0$ for all n, *if* $M = N$: that is, if we have exactly the number of free coefficients as there are solitary waves. Of course, *a priori* we do not know this number, and hence we must vary M to determine the best fit (which obviously should be for $M = N$). Recalling that $\Gamma_0 = 1$ and assuming that the error is small, we can estimate as well as *extrapolate* the signal, by writing

$$\hat{\Phi}^{(n)}(k)^{est} = -\sum_{m=1}^{M} \Gamma_m \hat{\Phi}^{(n-m)}(k) \qquad , \quad (16)$$

for *any* n. It is useful to rewrite equation (15) as

$$P_M = \left\langle \left|\xi^{(n)}(k)\right|^2 \right\rangle = \left\langle \left|\Gamma_m \, \hat{\Phi}^{(n-m)}(k)\right|^2 \right\rangle \qquad , \quad (17)$$

where P_M is the estimated noise power for the M+1 point noise prediction filter. Expectation values here are population averages, i.e. the mean of an arbitrarily large number of independent samplings of the same stationary process. We observe that the left-hand side describes the estimated amount of power resident in the noise, while the right-hand side has contributions from both the noise *and* the message. Then, it follows by varying the Γ coefficients (subject to $\Gamma_0 \equiv 1$) so that the left-hand side is minimized, that we are in an approximate sense eliminating any contribution of the message to the estimate of the noise power. Assuming that the "ergodic hypothesis" of information theory holds (Brillouin, 1962), we can replace these with time averages*. It is assumed throughout that the process is "stationary." Physically, this is equivalent to requiring that the data be collected only over an interval where wave-like behavior with unvarying speeds is maintained.

IV. VARIATIONAL PRINCIPLE AND IMPLEMENTATION

In principle, we recognize equation (17) as being a variational problem where we vary the Γ's, apart from $\Gamma_0 \equiv 1$, in order to make the right hand side a minimum (Wiener, 1950). This variational approach is the basis of a class of prediction schemes known as "autoregressive" methods (Marple Jr., 1987). The minimization process for the Γ coefficients is a classical least squares problem where the underlying matrix enjoys certain symmetries (i.e. the matrix is "Toeplitz") which makes it possible to construct solution schemes that are particularly computationally efficient (Golub and Loan, 1989; Levinson, 1946), as well as giving polynomials associated with the Γ coefficients, notably equation (14), some remarkable analytic properties (Edward and Fitelson, 1973). Other analytic properties of the autoregressive method cause it to be methodologically identical to the *maximum entropy method* of spectral analysis in information theory (Burg, 1967; Burg, 1972; van den Bos, 1971) and to be related conceptually to fundamental statistical mechanical principles (Jaynes, 1957a; Jaynes, 1957b).

When time averages replace expectation values, the variational principle is modified to minimize "end of data set" effects and has come to be known as the Burg-Levinson-Wiener algorithm (Burg, 1967). This algorithm also assures that the roots of the Γ polynomial are always on or inside the unit circle. The reasons for this are quite technical, but physically they correspond to the requirement that predictions be "stable" or non-increasing. The roots of the polynomial correspond to the $exp\,(ikv_m\Delta t)$ terms and, hence, give estimates of v_m. Importantly, since this model is based on the expectation of traveling waves, it has the capability of deducing v_m with resolution much better than $\frac{1}{Nk\Delta t}$, which is the usual limit obtained for a windowed periodogram via the "uncertainty principle" of information theory (Bracewell, 1986; Brillouin, 1962). It is important to point out that one is not getting "something for nothing" here, since one is assuming the form of the answer by using the theoretical model (3) and looking explicitly for it. The appearance of N velocities implies that there are N free parameters Γ_m. All of the requisite information is contained within these Γ coefficients.

* In nonlinear dynamical contexts, this assumption may be suspect, but it can both be checked and, if necessary, relaxed using some of the recently developed "deterministic chaos" approaches; see Farmer and Sidorowich, 1987, 1988.

Two practical issues now emerge in this problem. First, how can a computational procedure be developed that assures that genuine wave-like features have roots that emerge precisely on the unit circle? And second, how many wave-like features are present in the data? The first of these questions is formally identical to the signal processing problem of identifying sinusoids immersed in noise. This issue was addressed rather elegantly by Pisarenko (Pisarenko, 1973), but the associated computational problem in Pisarenko's method is cumbersome and requires relatively accurate initial estimates. Owing to approximations made in the Burg-Levinson-Wiener algorithm to assure numerical stability (Fougere, Zawalick, et al., 1976), the polynomial roots which lie on the unit circle will shift inside the unit circle and the presence of noise can cause individual roots to be misidentified as closely spaced *multiple* roots. The question of how large M must be can be addressed via statistical methods, e.g. Akaike's method (Akaike, 1969; Akaike, 1974; Tong, 1975). Essentially, Akaike's "information criterion" (AIC) is based upon the observation that the estimated noise power in equation (17) systematically diminishes as the length M of the filter increases. This remains true even when the filter contains more terms than there are independent signal components as a consequence of the excess coefficients effectively fitting the noise. The AIC allows the user to discriminate between filter coefficients that fit the message and those which fit the noise, and estimate the optimal number of filter coefficients. Fougere *et al.* (1976) developed a scheme for combining, where necessary, Pisarenko's method and the Akaike information criterion with initial estimates of the filter coefficients obtained from the Burg-Levinson-Wiener method. Methodologically, this provides a practical approach to obtaining the number M and phases of the polynomial roots $z_m(k), m = 1, \ldots, M$ associated with the velocities v_m of the individual traveling waves.

Thus, with a small number of well-resolved spatial Fourier transforms, the velocities (and the number of traveling waves) can be accurately estimated. As a consequence, this method is computationally optimal to the task of estimating the number of traveling waves and their respective wave speeds. Singular value decomposition (SVD) methods (Golub and Loan, 1989; Lawson and Hanson, 1974), which have found application to other pattern recognition problems (Aubry *et al.* 1988, 1989), could also be employed in the Fourier-transform domain but not in the original configuration space. However, SVD methods would be much more computationally intensive since they are designed to solve much more general problems, in contrast with the specific problem posed here. (The standard SVD formulation will result in an eigenvalue problem, the nature of whose roots parallels in part that of the polynomial in Γ.)

Once the number and numerical values of the Γ coefficients are known, it is a straightforward (but tedious and possibly ill-conditioned) problem to determine the polynomial (14) roots z_m, $m = 1, \ldots, M$. The corresponding values for v_m will have a real component, i.e. the estimate of the velocity of the m^{th} wave, and an imaginary positive-valued component which can be regarded as a measure of the uncertainty of the wave-speed. In particular, if the imaginary component is numerically "large" i.e. $\approx (k\Delta t)^{-1}$, then one should regard that particular root as not being well-defined or correspond not to a solitary wave feature but to noise features. It also follows that the (real parts of the) velocities v_m are uncertain modulo $\frac{2\pi}{k\Delta t}$, and estimates of velocity given data from a given wavenumber k can be "aliased" by some multiple of this amount. This potential uncertainty can be alleviated by employing data derived from several wavenumbers and/or by sampling more frequently, i.e. reducing the value of Δt, in order to satisfy the "sampling theorem" (Bracewell, 1986).

Sometimes, it is more useful to employ a graphical measure of the velocities and their uncertainties. It is conceptually simple to construct such a graphical device by recalling that this methodology was adapted from Wiener's scheme for estimating power spectra (Newman, 1977; Wiener, 1950). In particular, if we assume that the noise estimator (13) is stationary and decorrelated, then the quantity analogous to the spectrum, namely

$$S(v) \equiv \frac{P_N \Delta t}{\left| \sum_{\ell=0}^{N} \Gamma_\ell exp(-2\pi i \ell \Delta t k v) \right|^2} \quad , \quad (18)$$

will have well-resolved peaks or line-like features at the appropriate velocities. The width of the features can be regarded as a measure of their uncertainty. Aliasing can result in their being displaced by an amount equivalant to the width of the spectral window, i.e. $\frac{2\pi}{k\Delta t}$. If the Burg-Levinson-Wiener algorithm is used, without the refinements due to Pisarenko and to Akaike, then it is possible that certain of the line features will become degenerate, splitting into a number of closely-spaced components. Fougere's procedure (Fougere, Zawalick, et al., 1976) can be employed to eliminate this degeneracy.

V. COMPUTATIONAL RESULTS AND CONCLUSIONS

To illustrate this method in one of the simplest possible contexts, we apply it to the identification and velocity estimation of the realization of an exact two-soliton solution to the Korteweg-de Vries equation

$$\Phi_t + \frac{1}{6}\Phi\Phi_x + \Phi_{xxx} = 0 \tag{19}$$

(Hirota, 1971; Whitham, 1974), namely

$$\Phi = \frac{72}{F^2}\{FF_{xx} - F_x^2\} \tag{20}$$

where the subscript x denotes partial differentiation with respect to x and

$$F = 1 + f_1 + f_2 + \frac{(\alpha_2 - \alpha_1)^2}{(\alpha_2 + \alpha_1)^2} f_1 f_2 \tag{21}$$

and where

$$f_j \equiv exp\{-\alpha_j(x - s_j) + \alpha_j{}^3 t\} \tag{22}$$

for *any* $\alpha_j, s_j, j = 1, 2$. In Figure 1, we plot $\Phi(x,t)$ for $\alpha_1 = 1, \alpha_2 = \frac{3}{2}$ and with $s_1 = 0$ and $s_2 = -1$ at nine intervals of time $\Delta t = \frac{1}{4}$. (Each successive plot is displaced by $\frac{1}{2}$ in amplitude to facilitate reading the figure.) The corresponding wave speeds, namely $-\alpha_j{}^2$, $j = 1, 2$, are -1 and $-\frac{9}{4}$. The eye can readily identify two features moving at different speeds, but the issue here is whether the methodology developed above finds two coherent structures traveling at different speeds. In Figure 2, we display the "spectrum" emerging from the Γ filter coefficients for a wavenumber k corresponding to $\frac{1}{8}$ of the length displayed of the amplitude function; note that the Akaike information criterion in this case *does* give $M = 2$, which is equal to N, as it should be. The precision with which our method determined the two velocities was repeated at other wavelengths, but diminished if the associated wavelength was very different from the spatial extent of the two coherent structures. Comparable accuracy was obtained using different time sequences so long as the two structures were distinct. When the included times when the two structures were completely mixed, the method yielded velocity estimates that were seemingly drawn toward each other just as the two structures appeared to lose their independence.

We have explored in a preliminary way the performance of this method in applications to "noisy" sequences of data generated in computer experiments, as well as investigated its performance for the interaction of substantial numbers of coherent structures with widely different amplitudes and wave speeds. One important aspect of this latter problem is what happens when the Akaike predicted M is *not* equal to N. Fortunately, we find that in general, the approach is satisfactorily robust; we shall discuss this in detail in a forthcoming paper.

The above discussion presents a preliminary overview of one natural methodology for identifying and evaluating the speed of solitary wave structures traveling in a medium given limited spatial spectral information. This methodology differs substantially from others currently in vogue, in particular, a variety of schemes based on the (orthogonal) modal decomposition of measured data (Aubry *et al.* 1988, 1989; Sidorovich, 1989). Our preliminary results show substantial promise so long as there are only a limited number of "coherent structures" present and the data available satisfies reasonable constraints derived from information theory. However, further work is needed to develop a better understanding of the robustness of this scheme.

We are also exploring other schemes for investigating this problem: for example, methods based on various moments of the distribution. Qualitatively similar results have emerged there for conceptually related reasons. In another direction, we are considering ways to exploit the formal equivalence of partial differential equations describing self-similar and of traveling-wave behavior (Barenblatt, 1979; Barenblatt and Zel'Dovich, 1972). In so doing, we hope to develop methods for identifying coherent traveling-wave like structures employing schemes that can be conceptually related to the wavelet transform.

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FIGURE CAPTIONS

Figure 1. Amplitude function $\Phi(x, t)$.

Figure 2. "Power Spectrum" as function of v.

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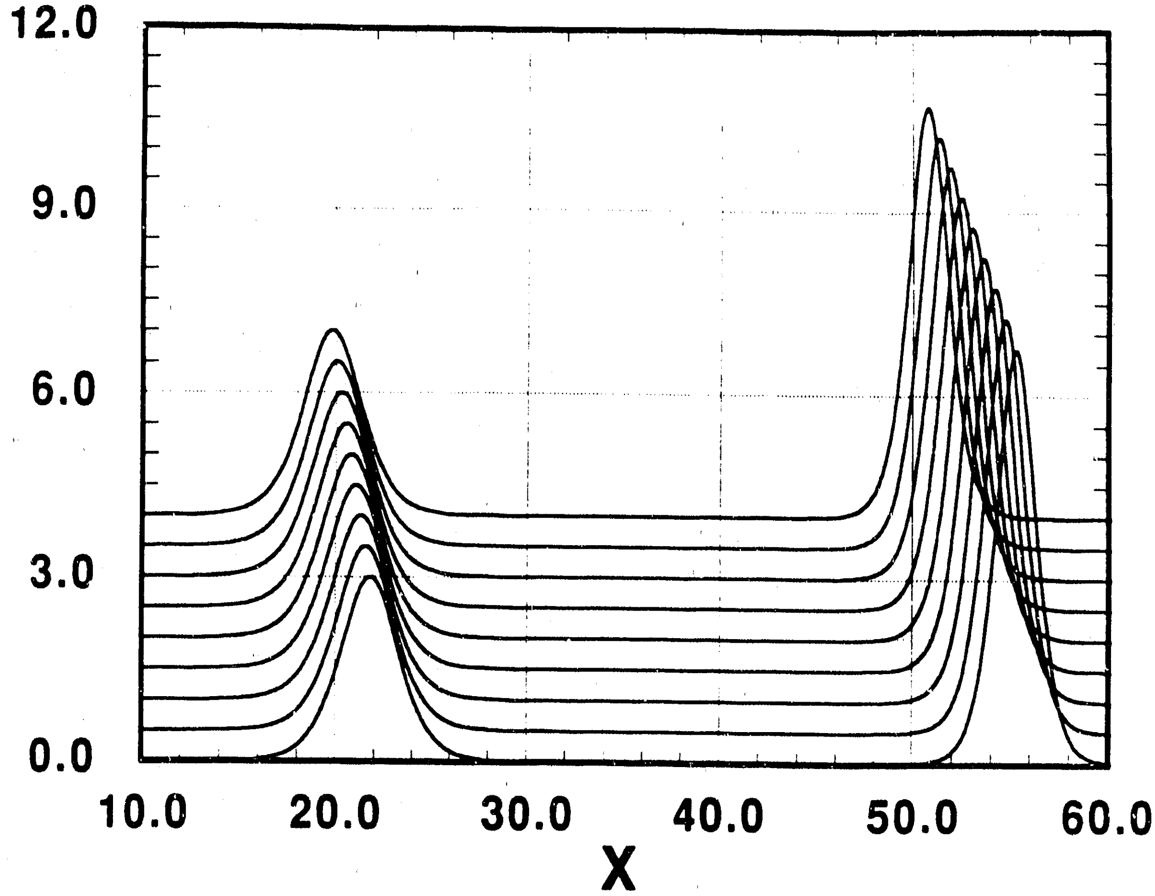
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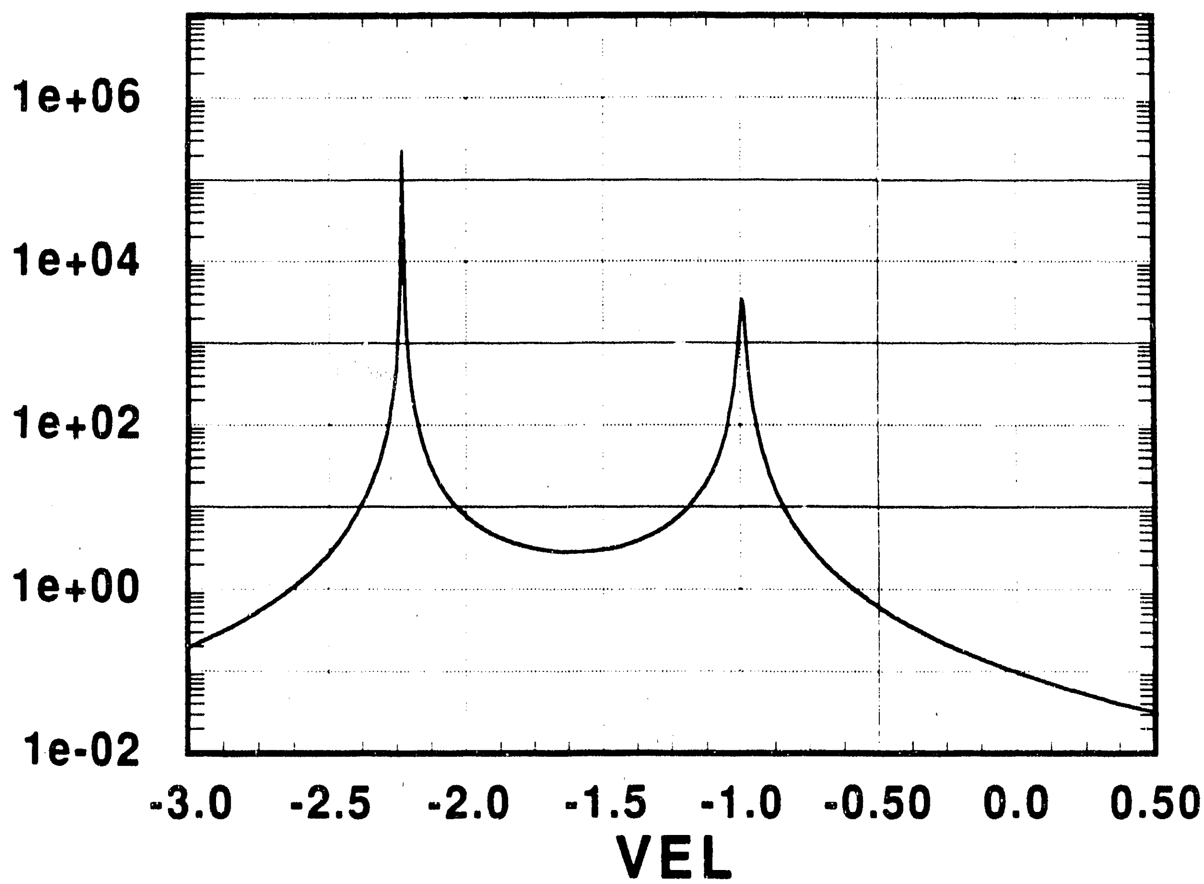
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