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IS THERE A HARD GLUONIC CONTRIBUTION TO THE FIRST MOMENT OF g_1?*

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ABSTRACT

We show that the size of the hard gluonic contribution to the first moment of the proton's spin-dependent structure function g_1 is entirely a matter of the convention used in defining the quark distributions. If the UV regulator for the spin-dependent quark distributions respects the gauge invariance of Green's functions (allows shifts of loop momenta) and respects the analyticity structure of the unregulated distributions, then the hard gluonic contribution to the first moment of g_1 vanishes. This is the case, for example, in dimensional regularization. By relaxing the requirement that the regulator allow shifts of loop momenta, we are able to obtain a nonvanishing hard gluonic contribution to the first moment of g_1. However, the first moments of the resulting quark distributions correspond to matrix elements that are either gauge variant or involve nonlocal operators and, hence, have no analogue in the standard operator-product expansion.

THE "SPIN CRISIS" AND A PROPOSED EXPLANATION

The hadronic part of the deep-inelastic scattering probability $W_{\mu\nu}$ is given by the forward matrix element in the proton state of two factors of the electromagnetic current:

$$W_{\mu\nu} = \sum_{\mathcal{X}} \langle \mathcal{X}|j_\nu|P,S\rangle^* \langle \mathcal{X}|j_\mu|P,S\rangle\, 2\pi\delta(M_{\mathcal{X}}^2 - (P+q)^2), \tag{1}$$

where $j_\mu = \overline{\psi}\gamma_\mu\psi$ is the electromagnetic current, ψ is the quark field, q is the four-momentum of the virtual photon, and P and S are the momentum and spin four-vectors of the proton, respectively. The spin-dependent part $\Delta W_{\mu\nu}$ can be decomposed into two form factors g_1 and g_2 as follows:

$$\Delta W_{\mu\nu} = \frac{4\pi i}{P\cdot q}\epsilon_{\mu\nu\lambda\sigma}q^\lambda M_P\left[S^\sigma g_1(x,Q^2) + (S^\sigma - \frac{S\cdot q}{P\cdot q}P^\sigma)g_2(x,Q^2)\right], \tag{2}$$

where $Q^2 = -q^2$ and $x = Q^2/(2P\cdot q)$. For longitudinally polarized protons, only $g_1(x)$ contributes to $\Delta W_{\mu\nu}$ in the scaling limit. The EMC collaboration[1] has measured $g_1(x)$ at $\langle Q^2\rangle = 10.7\ \mathrm{GeV}^2$.

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DISCLAIMER

In the naive parton model, the first moment of $g_1(x)$ is simply related to the first moments of the spin-dependent quark distributions:

$$\int_0^1 dx\, g_1(x) = {}^1\!/\!_2({}^4\!/\!_9\Delta u + {}^1\!/\!_9\Delta d + {}^1\!/\!_9\Delta s), \tag{3}$$

where $\Delta p = \int_0^1 dx\, \Delta p(x)$. The spin-dependent parton distribution $\Delta p(x)$ is defined as the difference between the probabilities to find the parton with spin parallel to the proton or antiparallel to the proton at longitudinal momentum fraction x. Gluonic radiative corrections of $O(\alpha_S)$ introduce a factor $(1 - \alpha_S/\pi)$ on the RHS of (3).

Usually, the spin-dependent quark distributions Δq_i are assumed to be related to the expectation value of the axial-vector current:

$$2M_P S^\mu \Delta q_i = \langle P, S | j^\mu_{5i} | P, S \rangle, \tag{4}$$

where $j^\mu_{5i} = \overline{\psi}_i \gamma^\mu \gamma_5 \psi_i$. Using (4), one can obtain a relationship between $\Delta u - \Delta d$ and g_A/g_V, which is known as the Bjorken sum rule,[2] and a relationship between the combination $\Delta u + \Delta d - 2\Delta s$ and the F/D ratio[3] from hyperon decay. By making use of these relationships and the measured value of the first moment of g_1, one can determine the value of the singlet combination $\Delta\Sigma = \Delta u + \Delta d + \Delta s$. The result is that $\Delta\Sigma$ differs significantly from unity and is consistent with zero, which seems to imply that quarks carry very little of the proton's total spin. On the basis of the static quark model, one would expect $\Delta\Sigma$ to be of order unity. This apparent conflict between the EMC measurement and intuition from the static quark model has come to be known as the "spin crisis."

Efremov and Teryaev[4] (ET); Altarelli and Ross[5] (AR); and Carlitz, Collins, and Mueller[6] (CCM) have suggested that the first moment of g_1 contains, in addition to the usual quark contributions, a contribution proportional to the first moment of spin-dependent gluon distribution $\Delta g(x)$. ET, AR, and CCM conjecture that this gluonic contribution could bring the value deduced for $\Delta\Sigma$ into agreement with intuition from the static quark model. Since a gluon has no electric charge, it first contributes to deep-inelastic scattering at one-loop order ($O(\alpha_S)$). However, to leading order, the combination $\alpha_S(Q^2)\Delta g(Q^2)$ does not evolve with Q^2, so $\alpha_S \Delta g$ could be large even at large Q^2.

IDENTIFYING THE HARD GLUONIC CONTRIBUTION: GENERALITIES

Fig. 1 depicts one of the $O(\alpha_S)$ diagrams that yield a gluonic contribution to the deep-inelastic scattering cross section. In this order there are, in addition, three diagrams obtained by permuting the photon and gluon attachments to the quark line and a set diagrams obtained by reversing the arrows on the quark lines (quark goes to antiquark). We work in a frame in which the proton is moving in the plus z direction and the virtual photon has no transverse components of momentum.

A given Feynman diagram does not, in general, have a single interpretation in terms of partonic distributions and cross sections. Rather, the interpretation depends on the values of loop momenta under consideration. In order to obtain a contribution from the diagram of Fig. 1 that is proportional to $\Delta g(x)$, we focus on momenta p such that the gluon is nearly on its mass shell.

Even in this restricted region of momentum space, the diagram has more than one interpretation. When k_T is large compared with any hadronic scale, then the q and $\overline{q}$ produced by the gluon are at large angles. The quark propagator is far off the mass shell, and, consequently, asymptotic freedom allows one to compute the box

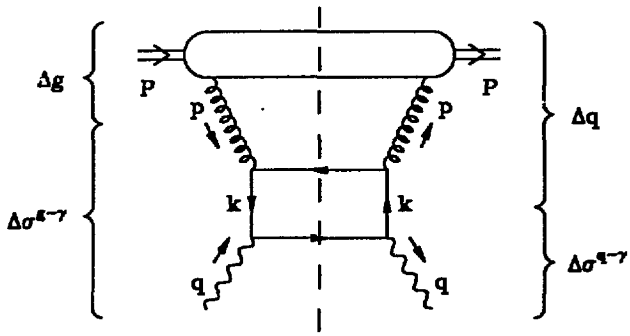

Fig. 1. An order α_S process that yields a gluonic contribution to deep-inelastic scattering.

subdiagram perturbatively. This contribution has the following interpretation: the proton emits a nearly-collinear, nearly-on-shell gluon with spin-dependent probability $\Delta g(x)$; the gluon then undergoes a hard interaction with the virtual photon, via the quark box subdiagram, with spin-dependent probability $\Delta\sigma^{g-\gamma}(x)$. Hence, we have a contribution to $\Delta\sigma^{g-\gamma}(x)\otimes\Delta g(x)$, as indicated by the left-hand set of braces in Fig. 1. The symbol $\otimes$ denotes the convolution $B(x)\otimes C(x) = \int_x^1 (dy/y)\, B(x/y)C(y)$, which has the property that the nth moment of the convolution is the product of the nth moments: $(B\otimes C)_n = \int_0^1 dx\, x^{n-1} B(x)\otimes C(x) = B_n C_n$. We call $\Delta\sigma^{g-\gamma}(x)\otimes\Delta g(x)$ the hard gluonic contribution to $g_1(x)$, and we call $\Delta\sigma^{g-\gamma}(x)$ the hard gluonic coefficient.

When k_T is small, the q and $\overline{q}$ are nearly collinear to each other and, hence, to the proton. The quark propagator is near the mass shell. In fact, when k_T is zero, the quark propagator becomes singular. In this region, because k^2 is small, the box diagram cannot be calculated perturbatively. Rather, one must absorb such soft contributions, including the collinear singularity, into the definition of $\Delta q(x)$. That is, the diagram of Fig. 1 must be given the following interpretation: the proton emits a nearly-collinear, nearly-on-shell quark with spin-dependent probability $\Delta q(x)$; the quark then undergoes a hard interaction with the virtual photon with spin-dependent probability $\Delta\sigma^{q-\gamma}(x)$. That is, we have a contribution to $\Delta\sigma^{q-\gamma}(x)\otimes\Delta q(x)$, as indicated by the right-hand set of braces in Fig. 1.

The central issue in determining the size of the hard gluonic coefficient $\Delta^{g-\gamma}(x)$ is the partitioning of the complete proton–virtual-photon spin-dependent cross section $\Delta\sigma^{p-\gamma}(x)$ into a quark piece plus a gluon piece:

$$\Delta\sigma^{p-\gamma}(x) = \sum_i \Delta\sigma_i^{q-\gamma}(x)\otimes\Delta q_i(x) + \Delta\sigma^{g-\gamma}(x)\otimes\Delta g(x). \qquad (5)$$

Of course, $\Delta\sigma^{p-\gamma}(x)$, being a physical (measured) quantity, is independent of the partitioning. The precise method of partitioning the contributions on the RHS of (5) is known as the factorization scheme. A change of factorization scheme merely shifts contributions from $\Delta\sigma^{g-\gamma}(x)$ to $\Delta q(x)$ in such a way that $\Delta\sigma^{p-\gamma}(x)$ is unchanged.

IDENTIFYING THE HARD GLUONIC CONTRIBUTION: SPECIFICS

Now let us define the hard gluonic contribution more precisely. First we define $\Delta\sigma^{q-\gamma}(x)\otimes\Delta q(x)$. Then we subtract this quantity from the diagram of Fig. 1 plus its crossed-box and reversed-arrow partners. The remainder is the hard gluonic contribution $\Delta\sigma^{g-\gamma}(x) \otimes \Delta g(x)$. In order to simplify the discussion, we specialize to the light-cone gauge $n \cdot A = 0$, where n is a unit vector in the minus light-cone direction. We have given a manifestly gauge-invariant discussion of these issues previously.[7]

In order to identify $\Delta\sigma^{q-\gamma}(x) \otimes \Delta q(x)$, we first approximate the box and crossed-box subdiagrams by expressions that are correct when the q and $\overline{q}$ are collinear: we drop k_T in the q-γ vertices and in the lower final-state–mass-shell δ function. Then, the k_T integration becomes UV divergent, so we must impose a regulator to ensure that $k_T^2 \lesssim \mu_{fact}^2$, where μ_{fact}^2 is the "factorization scale", with $\mu_{fact}^2 \gg \Lambda_{QCD}^2$. The choice of regulator specifies the factorization convention. In light-cone gauge, the crossed-box subdiagrams vanish in the collinear approximation, so only the box subdiagram contributes to $\Delta\sigma^{q-\gamma}(x) \otimes \Delta q(x)$. Owing to the collinear approximation, this contribution factors, as shown in Fig. 2, into piece corresponding to $\Delta q(x)$, which contains a cut-triangle subdiagram with a cut vertex[8] $\gamma^+\gamma^5\delta[x-(k^+/P^+)]$, and a piece corresponding to $\sigma^{q-\gamma}(x) = e_q^2\delta[x - (Q^2/2P \cdot q)]$.

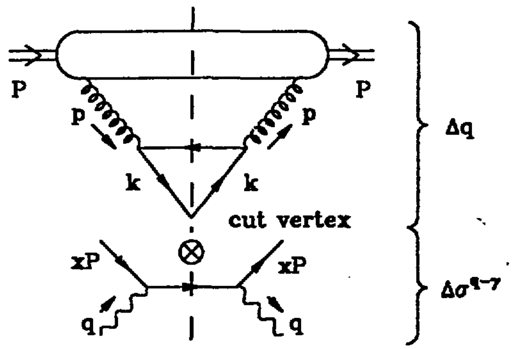

Fig. 2. The factored form that arises from applying the collinear approximation to the diagram of Fig. 1.

It was shown in Ref. 7 that, if one chooses conventional UV regulators for $\Delta q(x)$, such as $\overline{MS}$ or Pauli-Villars regulators, the first moment of the hard gluonic contribution vanishes because the first moment of the hard coefficient vanishes: $\Delta\sigma^{g-\gamma} \equiv \int_0^1 dx\, \Delta\sigma^{g-\gamma}(x) = 0$.

GENERAL THEOREM ON THE VANISHING OF $\Delta\sigma^{g-\gamma}$

The vanishing of the first moment of the hard gluonic coefficient is actually quite general. In Ref. 7 it was shown that, if the UV regulator for Δq respects the analyticity of the unregulated graph (parton-level analyticity) and respects the gauge invariance of Green's functions (allows shifts of loop momenta), then Δq is given by a gauge-invariant matrix element of j_5^μ and $\Delta\sigma^{g-\gamma}$ vanishes. The main features of the proof are as follows. The analyticity condition allows one to use contour methods to convert the cut-triangle subgraph to an uncut triangle. Then the Δq_i are given by matrix elements of the axial-vector current according to (4). Gauge invariance of the regulator implies that the value of the uncut triangle is uniquely determined.[9] Thus, Δq and, implicitly, $\Delta\sigma^{g-\gamma}$, are independent of the choice of regulator, provided that the regulator satisfies the unitarity and gauge-invariance requirements. But we already know at least one such regulator, namely $\overline{MS}$, for which $\Delta\sigma^{g-\gamma}$ vanishes. Hence, $\Delta\sigma^{g-\gamma}$ vanishes for all such regulators.

SIDESTEPPING THE THEOREM

It is possible to sidestep the theorem on the vanishing of $\Delta\sigma^{g-\gamma}$ by relaxing the requirement that the UV regulator for $\Delta q(x)$ respect shifts of loop momenta. Specifically, one can regulate with a direct cutoff on the k_T integration:

$$\Delta q^{cutoff}(x,\mu_{fact}^2) = \int_0^{\mu_{fact}^2} dk_T^2\, \Delta\mathcal{P}(x,k_T) \neq \Delta q^{\overline{MS}}(x,\mu_{fact}^2). \tag{6}$$

Here, $\Delta\mathcal{P}(x,k_T)$ is the spin-, x-, *and* k_T-dependent distribution to find a quark in the proton. One can measure $\Delta\mathcal{P}(x,k_T)$ by measuring the k_T of q and $\overline{q}$ jets accompanying deep-inelastic scattering. Indeed, CCM have suggested such a measurement as a means for identifying the hard gluonic contribution. The quantity $\Delta\mathcal{P}(x,k_T)$ also appears in the expression for the k_T distribution of lepton pairs in spin-dependent Drell-Yan production. With the k_T-cutoff definition of $\Delta q(x)$, the first moment of the hard gluonic coefficient is nonvanishing: $\int_0^1 dx\, \Delta\sigma^{g-\gamma}(x) = -\alpha_S N_F/(2\pi)$, as advocated by ET, AR, and CCM. The k_T-cutoff regulator looks Lorentz variant because it singles out the transverse plane. However, P and q already define a transverse plane, so the cross section can be written in terms of invariants. This definition amounts to defining $\Delta q(x)$ in terms of the triangle graph, but with the "anomaly part" omitted.

RELATIONSHIP TO THE OPERATOR-PRODUCT EXPANSION

Since the k_T-cutoff regulator does not allow shifts of loop momenta, it does not respect the gauge invariance of Green's functions. Consequently, the singlet quark distributions are gauge-variant matrix elements of j_5^μ:

$$2M_P S^+ \sum_i \Delta q_i^{cutoff} = \langle P,S|\sum_i j_{5i}^+|P,S\rangle_{cutoff-l.c.g.}. \tag{7}$$

Here, the subscript on the matrix element indicates that the current is to be regulated using the k_T-cutoff method and that the matrix element is to be evaluated in the light-cone gauge. If we express the RHS of (7) in terms of a dimensionally-regulated (gauge-invariant) matrix element of j_{5i}^+, then a gauge-variant gluon operator appears:

$$\langle P,S|\sum_i j_{5i}^+|P,S\rangle_{cutoff-l.c.g.} = \langle P,S|\sum_i j_{5i}^+|P,S\rangle_{\overline{MS}} - \langle P,S|K^+|P,S\rangle_{l.c.g.}, \tag{8}$$

where $K^+ = [\alpha_S N_F/(2\pi)]\epsilon^{+\mu\nu\rho}A_\mu^a\partial_\nu A_\rho^a$ is the winding-number current. The matrix element of K^+ is proportional to Δg.

In the manifestly gauge-invariant formalism for Δq presented in Ref. 7, gauge invariance is maintained through the appearance nonlocal path integrals of the gauge field. These path integrals lead to matrix elements of a nonlocal gluonic operator:

$$2M_P S^+ \sum_i \Delta q_i^{cutoff} = \langle P,S|\sum_i j_{5i}^+ - \tilde{K}^+ + K^+|P,S\rangle_{cutoff}, \tag{9}$$

where $\tilde{K}^+ = [\alpha_S N_F/(4\pi)]\,\mathrm{Tr}\,\epsilon^{+\mu\nu\rho}F_{\mu\sigma}(D\cdot n)^{-1}F_{\nu\rho}n^\sigma$. In the light-cone gauge, matrix elements of $\tilde{K}^+$ reduce to matrix elements of K^+. The inverse derivative is defined by

$(\partial_y)^{-1}f(y) = \int_{-\infty}^{y} dy'\, f(y')$, and the inverse covariant derivative $(D)^{-1}$ is defined by its Taylor-series expansion in powers of the gauge field. Factors of the inverse derivative correspond to "eikonal" propagators that arise from the Feynman rules for the path integral. Comparing (9) with (7) and (8), we see that the cutoff matrix element of j_{5i}^{+} can be expressed in terms a dimensionally-regulated matrix element of j_{5i}^{+} and a matrix element of the nonlocal gluon operator $\tilde{K}^{+}$:

$$\langle P,S|\sum_i j_{5i}^{+}|P,S\rangle_{cutoff-l.c.g.} = \langle P,S|\sum_i j_{5i}^{+}|P,S\rangle_{\overline{MS}} - \langle P,S|\tilde{K}^{+}|P,S\rangle. \qquad (10)$$

We conclude that the matrix element corresponding to Δq^{cutoff} is either gauge variant or involves a nonlocal operator. Hence, it does not correspond to the any of the matrix elements that appear in a conventional operator-product expansion.[11]

WHY USE THE k_T-CUTOFF REGULATOR TO DEFINE Δq?

The k_T-cutoff method for defining the quark distributions seems somewhat awkward, given that Δq^{cutoff} does not correspond to a gauge-invariant matrix element of a local operator. In fact, if one wishes to preserve the Bjorken sum rule and the standard relationship of the quark distributions to hyperon decay, it is necessary to use a more conventional UV regulator for the flavor-nonsinglet combinations of the quark distributions. On the other hand, $\Delta q^{cutoff}(x, \mu_{fact}^2)$ is directly related to a measurable quantity, namely, $\Delta\mathcal{P}(x, k_T)$. Perhaps a more important point is that, unlike conventional regulators, the k_T-cutoff method respects quark helicity conservation. For example, in dimensional regularization $\{\gamma_5, \gamma_\mu\} \neq 0$ for the higher-dimensional γ_μ's that appear in quark propagators in loops. Thus, the combination of a gluon-quark vertex and a quark propagator can change the Dirac matrices $\frac{1}{2}(1 \pm \gamma_5)$, which correspond in four dimensions to helicity projectors, to $\frac{1}{2}(1 \mp \gamma_5)$. The k_T-cutoff method produces no such helicity flip and, hence, may lead to a definition of Δq that is closer to intuition about the quark's spin.

CONCLUSIONS AND DISCUSSION

We have seen that, if $\Delta q(x)$ is defined using a UV regulator that respects the optical theorem at the parton level and respects gauge invariance of Green's functions (allows shifts of loop momenta), then the hard gluonic contribution to the first moment of $g_1(x)$ vanishes. By relaxing the requirement that the UV regulator allow shifts, one can obtain a nonvanishing hard gluonic contribution to the first moment of $g_1(x)$, as advocated by ET, AR, and CCM. However, in this case, Δq is no longer given by a gauge-invariant matrix element of j_5^{μ}: either the matrix element is gauge variant, or the operator is nonlocal.

The presence or absence of a hard gluonic contribution to $\int_0^1 dx\, g_1(x)$ is purely a matter of the factorization convention chosen in defining $\Delta q(x)$ and, implicitly, $\Delta\sigma^{g-\gamma}(x)$. The choice of factorization scheme has no effect on predictions for high-energy cross sections: it merely shifts contributions from $\Delta\sigma^{g-\gamma}(x)$ to $\Delta q(x)$ in (5). Of course, one must take care to apply a given factorization scheme consistently for all p-QCD processes and sum rules. Although $\Delta\sigma^{g-\gamma}(x)$ is arbitrary, we wish to emphasize that $\Delta g(x)$ is well-defined at this order in α_S and measurable.

The original motivation for the suggestion that there is a hard gluonic contribution to the first moment of $g_1(x)$ was the desire to bring $\Delta\Sigma$ into agreement with intuition from the static quark model. In fact, unlike dimensional regulators, the k_T-

cutoff UV regulator that we have discussed does not flip quark helicity. Thus, the k_T-cutoff–regulated Δq might correspond more closely with intuition than does the $\overline{MS}$-regulated Δq. On the other hand, if one could carry out a renormalization-group integration from the soft scale of the constituent-quark model to the hard scale of deep-inelastic scattering, then it is clear that one would encounter nonperturbative helicity-flipping phenomena associated with the breaking of chiral symmetry. Thus, it seems likely that the quark-helicity information contained in the static quark model would be lost anyway.

Suppose one takes the point of view that, for reasons not as yet understood, the nonrelativistic quark model is approximately correct at deep-inelastic scattering scales. In a nonrelativistic bound state, it seems likely that Δg would vanish as v/c. In positronium, for example, the spin-dependent photon distribution is suppressed by at least one power of v/c (although the spin-independent distribution is not). Hence, in the static limit, it seems that the various definitions of Δq would all lead to a vanishing hard gluonic contribution to the first moment of g_1.

So far, there is no reason to believe that one definition of Δq is any closer to intuition from the static quark model than another. Measurement of Δg might shed some light on this issue. Lattice simulations[10] already suggest that Δg is too small for the presence of a hard gluonic contribution to "explain" the spin crisis.

Of course, the spins of the quarks and gluons are not the whole story: there is also orbital angular momentum. Unfortunately, there is no leading-twist operator that corresponds to orbital angular momentum.[11] The twist expansion is essentially an expansion in powers of the parton transverse momentum. Since the quark angular momentum operator $\overline{\psi}(\vec{r}\times\vec{p})\psi$ involves the operator $\vec{r}$, it cannot be written in terms of a finite number of powers of the transverse momentum. In this sense, the quark angular momentum is actually an infinite-twist quantity. Consequently, there is no experimental test in hard scattering of the sum rule $(1/2)\sum_i \Delta q_i + \Delta g + \langle L_z\rangle = 1/2$. It is clear, then, that the EMC result does not represent a fundamental disagreement between theory and experiment: there is a spin crisis only if one believes that the static quark model has something to do with spin phenomena at deep-inelastic scales.

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