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FEW-BARYON SYSTEMS IN THE SU(2)-SKYRME MODEL

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1. Introduction

The success of the Skyrme model[1] in describing nucleons as quantum states of the chiral soliton makes it natural to apply the model to nuclei. In traditional physics nuclei are considered as bound states of nucleons. The first attempt to describe nuclei within the Skyrme model was the potential approach[2]. In such an approach one has to calculate the potential of interaction between the skyrmions at all distances R. Nuclei would then arise as bound states of skyrmions in this potential. There are some difficulties in such way of constructing the nuclei. First of all the distance R between skyrmions is determined absolutely arbitrary and the potential $V(R)$ has some sense only for large R, where the potential must be identical to the one-pion exchange potential. To obey the last condition one usually uses the so-called Product Ansatz for chiral field configurations and has the problem to obtain the intermediate-range attraction in effective nucleon-nucleon potential. The last problem is intensively discussed now, and is to be solved soon.

But this approach is not the only one to describe the nuclei-like states and is unnatural for the Skyrme model. The Skyrme model gives us straight way for constructing a system with an arbitrary baryon charge. We have to look for solitons of classical fields with the corresponding topological charge and then to quantize solitonic degrees of freedom to obtain an object with nuclear quantum numbers. From the investigations of the system with baryon number equal to 2,3 and 4 without vibrations[3]-[7] and including the breathing mode[8]-[9] one may conclude that the intermediate attraction problem is to a great extent artificial or technical one. It is seen from this fact that even the first variational approximations and more precise calculations lead to a high value of the binding energy for a system with $B = 2$. We have to emphasize that the binding energy of light nuclei in such an approach is bigger when we take into account the monopole vibrations. Naturally, the distance parameter R does not appear here in any way. In general we can assume that nucleons must be born only out of nuclei or at their surface. Strictly speaking nucleons do not exist in the interior of nuclei. Indeed, the skyrmion must have a possibility to rotate freely in the space and isospace in order to obtain nucleon quantum numbers. But it is not the case in nuclei because the interaction potential between the skyrmions depends on their relative orientation in space and isospace. As a result, only nuclei as a whole may have right nuclear quantum numbers. (See[10]).

Up to now there are some papers concerning calculations of nuclear states in the Skyrme model. The most impressive results were obtained in the calculations of minimal-energy solitons for configurations with topological charge two by numerical methods[4], [5]. Recently a variational ansatz was proposed independently in[11], [12]. This ansatz obeys the symmetry conditions formulated in[5], [6] and, being very simple, gives the possibility to do one more step in analytical analysis of the problem and to take vibrational modes, for example, the monopole one,into account in a simple way. This analyses gives a natural explanation of the origin of the ansatz from[3] and also gives some new solutions. Among them we have solutions that we may interprete as compound nuclear states including antibaryons and meson-like states composed of skyrmion-antiskyrmion pairs. We have to note that our analysis is very similar to the one in[13] in the part concerning to $B = 2$ states. But in[13] possible contributions to the skyrmion-antiskyrmion pairs in the nuclear structure, that take place for $l \neq 0$ states, were not discussed. (In our case number l has in general the same sense as in[13] and will be introduced later).

2. The Properties of Static Field Configurations

In view of the above consideration, it seems to be very important to express such solutions in an analytical form. It is useful to investigate them analytically step by step and to perform numerical calculations at the end.

2.1 Generalized Ansatz for the Static Solutions

It is well known that historically the first stationary solution of the Skyrme model in a sector with topological charge N was the hedgehog configuration or the so-called Skyrme-Witten solution

$$U_{SW}(\vec{r}) = cos(F(r)) + i(\vec{\tau} \cdot \vec{N}) sin(F(r)). \tag{1}$$

Here $\vec{N}$ determines a definite direction in the isotopic space, and hedgehog configuration is specified by the vector $\vec{N} = \vec{r}/r$,and τ^i are the Pauli matrices. In (1) $F(r)$ is the chiral angle describing the absolute value of the pion field. The function $F(r)$ obeys the following boundary conditions $F(0) = n \cdot \pi$, $F(\infty) = 0$. These conditions ensure finiteness of the energy for a soliton with a topological number n, which is equal to a baryon number B. It was shown in[7] that the only configuration that provides minimum energy of the soliton with $n = 1$ is that given by (1). However, for other sectors such a form is not obligatory. For example, in[3] the solutions defined by the "$k\phi$" configuration:

$$\vec{N} = \{cos(k\phi) \cdot sin(\theta),\ sin(k\phi) \cdot sin(\theta),\ cos(\theta)\}, \tag{2}$$

(θ, ϕ) being the angles of vector $\vec{r}$ in the spherical coordinate system, have been considered. In (2), k is an integer determining also the topological charge. Some interesting properties of the states generated by these solutions were described in [3], [14]. In the sector with baryon charge $B = 2$, this form of the solution gives us low mass states in the range of about two nucleon masses. Quantization procedure generates rich spectra of rotational bands[14].

In the present paper, we use a new form of the solution given by the next vector[11],[12]

$$\vec{N} = \{cos(\Phi(\phi)) \cdot sin(T(\theta)),\ sin(\Phi(\phi)) \cdot sin(T(\theta)),\ cos(T(\theta))\}, \quad (3)$$

where $\Phi(\phi), T(\theta)$ are some arbitrary functions.

It will be shown that this ansatz is a generalization of the hedgehog and "$k\phi$"-configurations. In some sense the present ansatz gives an explanation of the origin and approximate character of the last. As it will be seen, (3) leads to a series of new solutions in baryon and topologically trivial sectors. Some of these new states are classically stable.

2.2 Mass Functional and Solutions for Static Equations

Let us consider the Lagrangian density $\mathcal{L}$ for the stationary solution:

$$\mathcal{L} = \frac{F_\pi^2}{16} \cdot Tr(L_k L_k) + \frac{1}{32e^2} \cdot Tr\Big[L_k, L_i\Big]^2. \quad (4)$$

Here $L_k = U^+ \partial_k U$ are the left currents. After some tedious algebra, (1), (3), (4) lead to the expression

$$\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_4, \quad (5)$$

where

$$\mathcal{L}_2 = -\frac{F_\pi^2}{8} \cdot \Big\{(F')^2 + \Big[\frac{sin^2 T}{sin^2\theta} \cdot (\Phi')^2 + (T')^2\Big] \cdot \frac{sin^2 F}{r^2}\Big\} \quad (6)$$

and

$$\mathcal{L}_4 = -\frac{sin^2 F}{2e^2 r^2}\Big\{\frac{sin^2 T}{sin^2\theta}(T'\Phi')^2 \frac{sin^2 F}{r^2} + \Big[\frac{sin^2 T}{sin^2\theta}(\Phi')^2 + (T')^2\Big](F')^2\Big\}. \quad (7)$$

In (6), (7) we use the symbol prime to denote the following derivatives

$$\Phi' = \frac{d\Phi}{d\phi}\ ;\ T' = \frac{dT}{d\theta}\ ;\ F' = \frac{dF}{dr}\ . \quad (8)$$

The variation of (5) with respect to $\Phi(\phi)$ gives us

$$\Phi'' = 0\ ,\ that\ is\ \Phi(\phi) = k \cdot \phi + Const. \quad (9)$$

We consider only solutions with a vanishing value of this constant. The number k must be an integer in order to obtain a single-valued solution $U(\vec{r})$ in the whole $\vec{r}$-space.

Now we have the following expression for the mass of the soliton

$$M = M_2 + M_4, \tag{10}$$

$$M_2 = \frac{\gamma}{4}\int_0^\infty dx x^2 \int_0^\pi d\theta sin\theta \left\{(F')^2 + \left[\frac{sin^2 T}{sin^2\theta}k^2 + (T')^2\right]\frac{sin^2 F}{x^2}\right\}, \tag{11}$$

$$M_4 = \gamma \cdot \int_0^\infty dx \cdot x^2 \int_0^\pi d\theta \cdot sin\theta \cdot$$
$$\cdot\left\{\left[\frac{sin^2 T}{sin^2\theta}k^2 + (T')^2\right](F')^2 + \frac{sin^2 F}{x^2}\frac{sin^2 T}{sin^2\theta}k^2(T')^2\right\}\frac{sin^2 F}{x^2}, \tag{12}$$

where $\gamma = \pi \cdot F_\pi / e$ and $x = F_\pi \cdot e \cdot r$.

In order to minimize the functional M, the functions $T(\theta)$ and $F(x)$ have to obey the following equation

$$\frac{\delta M}{\delta T} = 0 \ , \ \frac{\delta M}{\delta F} = 0, \tag{13}$$

or more strictly

$$\left[x^2 + 2a sin^2 F\right]F'' + 2xF' + \left[a(F')^2 - \frac{a}{4} - 2b\frac{sin^2 F}{x^2}\right]sin(2F) = 0, \tag{14}$$

$$2 \cdot \left[A + k^2 \cdot B \cdot \frac{sin^2 T}{sin^2\theta}\right] \cdot T'' - k^2 \cdot A \cdot \frac{sin(2T)}{sin^2\theta} +$$
$$+ k^2 B\frac{sin(2T)}{sin^2\theta}(T')^2 + 2T' ctg\theta\left[A - k^2 B\frac{sin^2 T}{sin^2\theta}\right] = 0. \tag{15}$$

The coefficients a, b and A, B in (14), (15) are the following integrals:

$$a = \int_0^\pi \left[k^2\frac{sin^2 T}{sin^2\theta} + (T')^2\right] sin\theta \, d\theta, \ b = k^2\int_0^\pi \frac{sin^2 T}{sin^2\theta}(T')^2 sin \ \theta d\theta, \tag{16}$$

$$A = \int_0^\infty sin^2 F \cdot \left[\frac{1}{4} + (F')^2\right] dx, \quad B = \int_0^\infty \frac{sin^4 F}{x^2} dx. \tag{17}$$

From (10)-(12) and (16),(17) we conclude that the function $T(\theta)$ has to be some integer factor of π for $\theta = 0$ and $\theta = \pi$. We consider only the configurations with finite masses; that is why we have $F(0) = n \cdot \pi$ with integer n. Without loss of generality we take $F(\infty) = 0$. It is not difficult to prove that the asymptotic behaviour of F is presented by

$$F(x) \to \frac{1}{x^{p+1}}, \; for \; x \to \infty, \quad with \; p = \frac{\sqrt{1+2a} - 1}{2}. \tag{18}$$

In the vicinity of the coordinate system origin

$$F(x) \to \pi \cdot n - \alpha \cdot x^p, \tag{19}$$

where α is some numerical factor.

It is clear that $T(\theta)$ has the following behaviour near the boundary of the domain of its definition

$$T(\theta) \to \theta^k, \; for \; \theta \to 0; \quad T(\theta) \to \pi \cdot l - (\pi - \theta)^k, \; for \; \theta \to \pi. \tag{20}$$

Here l is an integer number.

Now all solutions U_{nkl} are classified by the set of integer numbers n, k and l. The solutions of (14)-(15) are graphically represented in Figs.1,2 for some values of k and l.

2.3 Baryon Charge Distribution and the Soliton Structure

Now we consider more carefully the structure of solitons. For that purpose let us calculate the baryon charge density

$$J_0^B(\vec{r}) = -\frac{1}{24\pi^2} \cdot \epsilon_{0\mu\nu\rho} Tr(L_\mu L_\nu L_\rho). \tag{21}$$

The straightforward calculation gives

$$J_0^B(r,\theta) = \frac{-1}{2\pi^2} \cdot \frac{sin^2 F}{r^2} \cdot \frac{dF}{dr} \cdot \frac{sinT}{sin\theta} \cdot \frac{dT}{d\theta} \cdot \frac{d\Phi}{d\phi}\,. \tag{22}$$

Here we have used (1) and (3). The expression for the topological charge density, (22), is the generalization of the one for "$k\phi$" ansatz from[3] and the Skyrme-Witten ansatz.

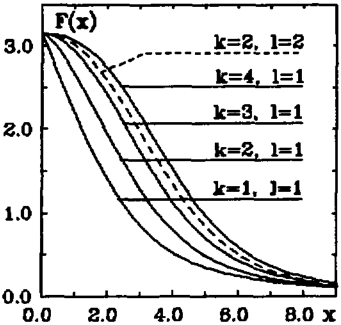
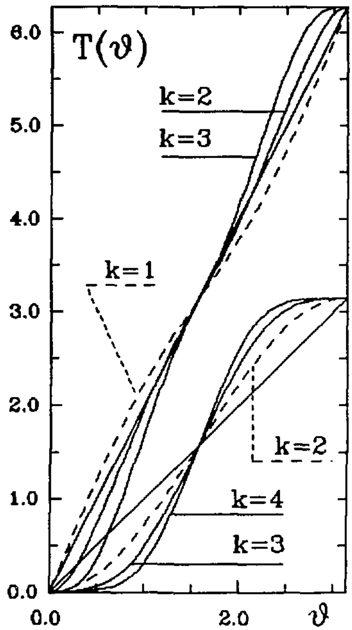


Fig.1 Solution F(x) of eq.(14) for some values of l and k.

Fig.2 Solution T(ϑ) of eq.(15) for some values of l and k. ▶



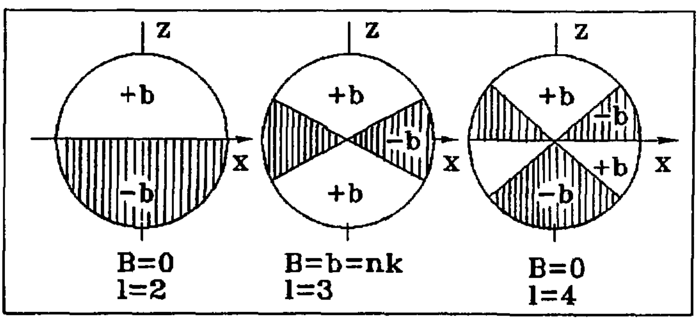


Fig.3 Baryon charge distribution on the (x,z)-plane for solutions characterized by number n,k,l.

Equation (22) immediately results in the expression for the corresponding topological charge

$$B = \frac{k \cdot n}{2} \cdot \left[1 - cos(\pi \cdot l)\right] . \tag{23}$$

One can see now that for even l we have meson- like solitons. In Fig.3 the baryon charge distributions are schematically presented in the (X, Z) - plane for solitons characterized by the number k, n ($F(0) = \pi \cdot n$, $F(\infty) = 0$) and boundary conditions $T(0) = 0$, $T(\pi) = \pi \cdot l$, $l = 2, 3, 4$.

In Table 1 we symbolically present the structure of solitons, the total baryon charge and the values of the mean square radius for some number of k and l. For example, we point out $2S2\bar{S}$ structure for $k = 2$, $l = 2$ solution when the baryon charge distribution divides the whole space into four axially symmetric regions. One unity of a positive baryon charge is concentrated in two of them and one unity of a negative charge in two others.

Table 1 Structure of states ($n = 1$, k, l) and the mean square radii of baryon charge distributions

k / l	1	2	3
1	$B = 1$ (S) $r_x^2 = r_y^2 = r_z^2 = 1.49$	$B = 0$ $(S - \bar{S})$ $r_x^2 = r_y^2 = 0$ $r_z^2 = 0$	$B = 1$ $(S - \bar{S} - S)$ $r_x^2 = r_y^2 = -4.1$ $r_z^2 = 22.8$
2	$B = 2$ (2S) $r_x^2 = r_y^2 = 6.5$ $r_z^2 = 2.9$	$B = 0$ $(2S - 2\bar{S})$ $r_x^2 = r_y^2 = 0$ $r_z^2 = 0$	$B = 1$ $(2S - 2\bar{S} - 2S)$ $r_x^2 = r_y^2 = -8.7$ $r_z^2 = 62$
3	$B = 3$ (3S) $r_x^2 = r_y^2 = 16.2$ $r_z^2 = 4.2$	$B = 0$ $(3S - 3\bar{S})$ $r_x^2 = r_y^2 = 0$ $r_z^2 = 0$	$B = 3$ $(3S - 3\bar{S} - 3S)$ $r_x^2 = r_y^2 = -9.9$ $r_z^2 = 114.3$

The mean square radii demonstrate a very different form of the obtained stationary configurations. Some of the mean square radii are negative. Evidently only the negative (antisoliton) baryon charge distribution may lead to such values.

In Figs.4,5 we present our more detailed results for the baryon density distribution in the (X, Z) plane and "3-dimensioned picture" of the same distribution for a dibaryon. It is easy to obtain from Fig.4 that the peak in the baryon-number density is near $\rho = \sqrt{x^2 + y^2} \simeq 1.5/F_\pi e$, $z = 0$. The solution with $k = 2$, $l = 1$ has the toroidal structure, as was pointed out in[5].

In Fig.6,7 one sees the contour plots for baryon density distribution for $S - \bar{S} - S$ skyrmions that give us concrete knowledge about such a

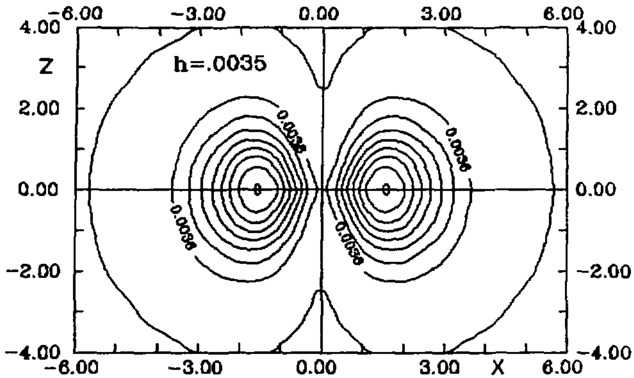


Fig.4 Contour plot for baryon-number density distribution in the (X, Z)-plane for a two-skyrmion configuration. Contour interval h=0.0035.

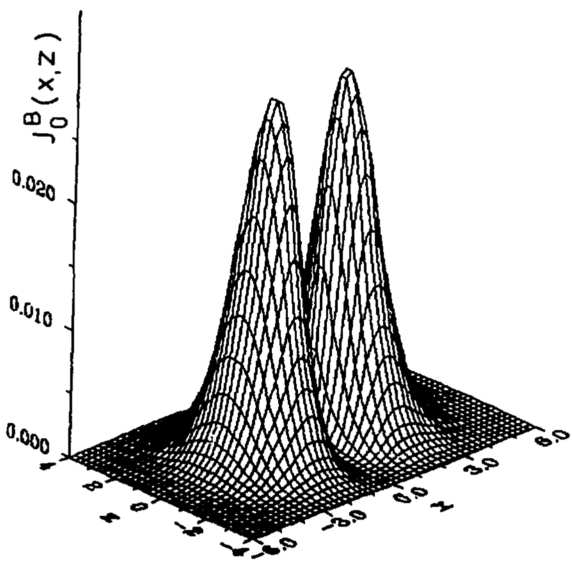


Fig.5 "Three-dimensional" picture for baryon-number density distribution for a two-skyrmion configuration.

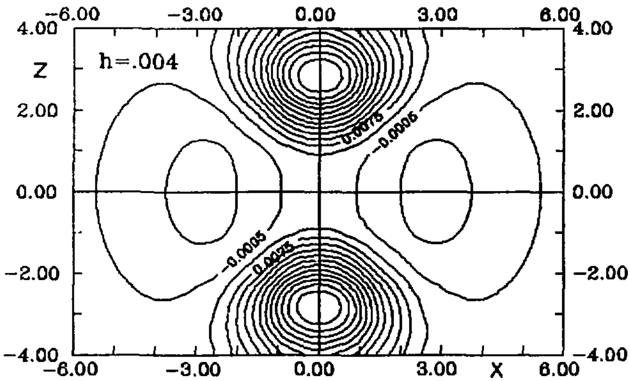


Fig.6 The same as in Fig.4 for S-$\bar{S}$-S skyrmion configuration. Contour interval h=0.004.

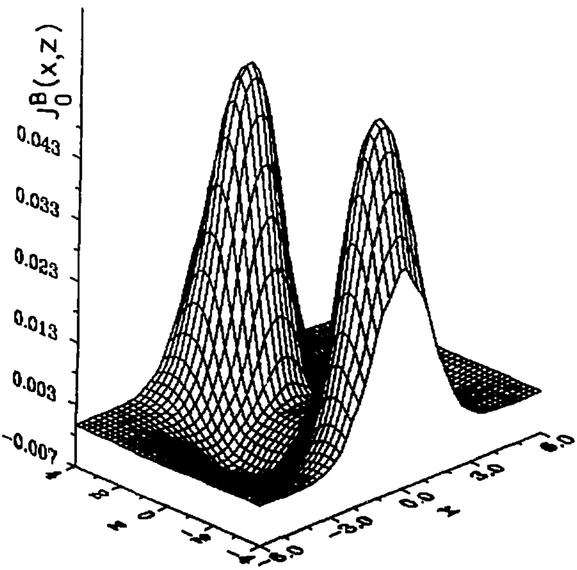


Fig.7 The same as in Fig.5 for S-$\bar{S}$-S skyrmion configuration.

complicated object. Evidently this soliton has not the simple toroidal structure. Only one antiskyrmion (with $B = -1$) has the toroidal structure and two others have not. One may conclude that $B = 0$ and $l = 4$ solitons (see Fig.3) consists of one toroidal skyrmion, one toroidal antiskyrmion and a skyrmion-antiskyrmion pair of the nontoroidal form localized near z-axis.

We have to note here that the quantum states of the $S - \bar{S} - S$ type ($k = 1$, $l = 3$) should experimentally appear as compound nuclear states in the interactions of a stopped antiproton with a deuteron. So we have unusual possibility to include antinucleons in the compound state structure in the same manner as nucleons.

2.4 The Masses of Classical Solitons

When we discuss multiskyrmion configurations we search for and investigate not only classically stable configurations.(The decay in two or more skyrmions is forbidden energetically). Nonstable configurations are also in our attention because they may become stable after the quantization procedure.

The numbers presented in Table 2 correspond to our calculations in the chiral symmetry limit (pion mass m_π is taken to be zero). The variational procedure with the chiral symmetry breaking term

$$\mathcal{L}_\pi = -\frac{F_\pi^2 m_\pi^2}{8} \cdot Tr(1 - U) , \tag{24}$$

which takes into account nonvanishing pion mass, gives us a possibility to compare our numerical results for some of the solutions with those from [15]. For this aim we choose the constants $F_\pi = 108$ MeV and $e = 4.84$ which used in[15]. Our results for diskyrmion mass is 1670 MeV and the skyrmion mass from[15] obtained by the so-called "hat"-method is 1660 MeV. Our results and those from[15] for more weight multi-skyrmion are in the following correspondence: 3-skyrmion - 2580 MeV and 2530 MeV; 4-skyrmion - 3572 Mev and 3452 MeV; 5-skyrmion - 4635 Mev and 4420 MeV. Some descrepances in the calculated mass values of the multi-skyrmions give us a possibility to estimate the errors, probably introduced by the variational ansatz. The errors are less than 5 percent for $B \leq 5$. Here, we have to present the virial theorem that allows us to estimate our numerical errors in the framework of the ansatz. The following quantity

$$\Delta = \gamma \cdot \Big\{ \{A_2 - A_1\} \cdot a + B \cdot b - C \Big\} \tag{25}$$

must be zero. Here A_1, A_2, C are given by

$$A_1 = \frac{1}{4}\int_0^\infty sin^2 F, \; A_2 = \int_0^\infty (F')^2 \cdot sin^2 F \, dx, \; C = \frac{1}{2}\int_0^\infty (F' \cdot x)^2 \, dx \, . \tag{26}$$

For the quantity Δ we obtain $\Delta \simeq 1$ MeV for the most "difficult" case (k=4, l=3).

Table 2 The classical masses calculated in the present paper with the generalized ansatz

k / l	1	2	3	4
1	11.605	26.358	46.332	71.169
2	**22.458**	**45.536**	73.533	106.609
3	**34.585**	**66.701**	**103.081**	144.321
4	47.675	**89.310**	**134.450**	
5	61.569	113.119		

The calculated soliton masses for $n = 1$ and some values of k, l are presented in Table 2 in the units of $(\pi F_\pi/e)$. So we extended the soliton spectrum up to *$n \cdot k$ multi-baryon configurations* for odd l. For example, a three-baryon state corresponds to the $k = 3$, $n = 1$, $l = 1$ member of Table 2 with the binding energy of about 5.4 MeV per baryon. Moreover, we also have the spectrum meson-like ($N \cdot k/2$ -baryon - $n \cdot k/2$ - antibaryon) configurations for even l. (See, for example, the $k = 2$, $n = 1$, $l = 2$ case that corresponds to a two-baryon - two-antibaryon meson-like configuration with the mass about 3192 MeV). Some of the obtained configurations are classically stable objects which are seen from Table 2 (they are marked by the boldface letters). The mass of such object is less than the sum of the masses of their baryon components. The classical "binding energy" of these states may easily be obtained by using Table 2 for arbitrary values of F_π and e.

From Table 2 one sees almost linear dependence of the classical masses on the baryon charge. Such a dependence dramatically differs from $M \sim B(B+1)$ that may be obtained for the hedgehog ansatz[16].

3. Spectra of Quantum States

The purpose of this part is to obtain the quantum mechanical effective Hamiltonian in the framework of the collective coordinate method. We use the breathing and rotational degrees of freedom as collective coordinates and calculate the masses and binding energies of the lowest states in this method.

3.1 Effective Hamiltonian in Terms of Collective Variables

Let us describe the important steps necessary to obtain the effective Hamiltonian. Now the chiral fields are considered to be time-dependent and of the form:

$$U(\vec{r},t) = exp\Big\{i\tau^i \cdot \Gamma^{ij}(t) \cdot N^j(R^{-1}_{nk}x_k) \cdot F(xe^{-\lambda})\Big\}, \qquad (27)$$

where $R(t)$ and $I(t)$ are the spatial and isospin rotation 3x3 matrices, and $\lambda(t)$ is the time-dependent parameter of the dilatational vibrations. Inserting (27) into the Lagrangian in which the time components L_0 of the currents now play their impotant role, we have

$$L = -M + \frac{F_\pi^2}{16} \cdot \int Tr(L_0 L_0) d^3r + \frac{1}{16e^2} \cdot \int Tr\Big[L_0, L_k\Big]^2 d^3r . \quad (28)$$

Performing the canonical transformation and determining canonically conjugate variables

$$p = \frac{\partial L}{\partial \dot{\lambda}}, \quad T^i = \frac{\partial L}{\partial \omega^i}, \quad S_i = \frac{\partial L}{\partial \Omega_i}, \quad (29)$$

where the angular velocities Ω_i and ω^i for the rotation and isorotation are given by

$$R_{ik}^{-1} \dot{R}_{kj} = \epsilon_{ijk} \Omega_k, \quad \dot{I}^{ik} (I^{-1})^{kj} = \epsilon^{ijk} \omega^k , \quad (30)$$

we obtain the Hamiltonian for $k \neq 1$

$$\begin{aligned} \hat{H} &= M(\lambda) + \frac{\hat{p}^2}{2m(\lambda)} + \frac{\hat{T}^2}{2Q_T(\lambda)} + \frac{\hat{S}^2}{2Q_S(\lambda)} - \\ &- \frac{1}{2} \left\{ \frac{1}{Q_T(\lambda)} + \frac{k^2}{Q_S(\lambda)} - \frac{1}{Q(\lambda)} \right\} \cdot \hat{T}_3^2 . \end{aligned} \quad (31)$$

Here the symbols p, T and S are interpreted now as follows: impulse p corresponds to the vibrational operator, T and S are the isospin and spin operators. The vibrational potential $M(\lambda)$ is given by the following expression

$$M(\lambda) = M_2 \cdot exp(-\lambda) + M_4 \cdot exp(\lambda) . \quad (32)$$

For the inertial values $m(\lambda)$, $Q_T(\lambda)$, $Q_S(\lambda)$, $Q(\lambda)$ we have :

$$\begin{aligned} m(\lambda) &= \frac{2\pi}{F_\pi e^3} \int_0^\infty (F')^2 \Big\{ \frac{e^{-3\lambda}}{2} + \\ &+ e^{-\lambda} \frac{sin^2 F}{x^2} \int_0^\pi \Big[k^2 \frac{sin^2 T}{sin^2 \theta} + (T')^2 \Big] sin\theta d\theta \Big\} x^4 dx, \end{aligned} \quad (33)$$

$$Q_T(\lambda) = \frac{\pi}{F_\pi e^3} \int_0^\infty x^2 dx \int_0^\pi sin\theta d\theta \Big\{ -e^{-\lambda} \frac{sin^4 F}{x^2} (k^2 \frac{sin^2 T}{sin^2 \theta} cos^2 T + (T')^2) +$$

$$+sin^2F\Big[\frac{e^{-3\lambda}}{4}+e^{-\lambda}\Big((F')^2+[k^2\frac{sin^2T}{sin^2\theta}+(T')^2]\frac{sin^2F}{x^2}\Big)\Big](1+cos^2T)\Big\} \quad (34)$$

$$Q_S(\lambda)=\frac{\pi}{F_\pi e^3}\int_0^\infty x^2dx\int_0^\pi sin\theta d\theta\Big\{sin^2F\cdot\Big[\frac{e^{-3\lambda}}{4}+$$

$$+e^{-\lambda}\Big((F')^2+[k^2\frac{sin^2T}{sin^2\theta}+(T')^2]\frac{sin^2F}{x^2}\Big)\Big](k^2\frac{sin^2T}{sin^2\theta}cos^2\theta+(T')^2)$$

$$-e^{-\lambda}\frac{sin^4F}{x^2}(k^4\frac{sin^4T}{sin^4\theta}cos^2\theta+(T')^4)\Big\}, \quad (35)$$

$$Q(\lambda)=\frac{2\pi}{F_\pi e^3}\int_0^\infty x^2dx\int_0^\pi sin\theta d\theta\Big\{-e^{-\lambda}\frac{sin^4F}{x^2}k^2\frac{sin^4T}{sin^2\theta}+$$

$$+sin^2F\Big[\frac{e^{-3\lambda}}{4}+e^{-\lambda}\Big((F')^2+[k^2\frac{sin^2T}{sin^2\theta}+(T')^2]\frac{sin^2F}{x^2}\Big)\Big]sin^2T\Big\}. \quad (36)$$

3.2 Spectrum of Few-Baryon States

It shoud be noted that $S_3^{b.f.}-k\cdot T^3_{b.f.}=0$. It is a constraint for the wave function of the quantized Skyrmion. More strictly, the wave function is given by

$$<I,R|TK,SM,L>=\frac{\sqrt{(2T+1)(2S+1)}}{8\pi^2}\cdot D^T_{KL}(I)\cdot D^S_{M-k\cdot L}(R)\ , \quad (37)$$

as in[3], and its parity is given as $P=(-1)^L$. If we neglect the vibrational degrees of freedom, we obtain the expression for the mass spectrum for $B=2$ ($k=2,\ l=1$):

$$E_{S,T,T_3}=\frac{F_\pi}{e}\cdot\Big\{70.55+\frac{e^4}{2}\cdot\Big[\frac{S(S+1)}{272.4}+\frac{T(T+1)}{183.0}-\frac{1}{83.2}T_3^2\Big]\Big\} \quad (38)$$

(for an arbitrary value of F_π and e).

Now we present some numerical results for the calculated soliton states with $B=2$ (see Table 3.) and lowest multibaryon states with $B=3,4$ (Table 4.). The calculations were performed in the harmonic approximation with the values of the constants: $e=4.84$ and $F_\pi=108$ MeV ($M_{nucl}=931$ MeV).

Table 3. The calculated energies for the B=2 (k=2, l=1) soliton states with isospin T, spin parity S^P and quantum number n=0 corresponding to the vibrational mode

T	0	0	1	1	1
S^P	0^+	1^+	0^+	1^+	2^-
$E - 2M_{nucl}$ (MeV)	-214	-172	-154	-118	-53

Table 4. The lowest multibaryon states ($k = 2, 3,\ l = 1$)

B	3	3	4	4	4
T	1/2	3/2	0	0	1
S	3/2	3/2	0	1	0
T_3	1/2	1/2	0	0	0
$E - BM_{nucl}$ (MeV)	-268.0	-210.5	-324.0	-312.7	-294.5

One can see that all candidates to the light nuclear states have very high binding energy values. But nuclear state-like configurations should not be identified with nuclei since a lot of quantum corrections is not yet taken into account.

The calculation shows that the classically nonstable state $k = 4$, $l = 1$ has the binding energy +88 MeV and becomes stable when the quantum correction is taken into account (see Table 4). This may have a more general sense.

At the end of this section, we give some remarks of technical nature about the numerical procedure. One can see that $\lambda = 0$ is not the stable point (minimum of the effective potential) when we take into account the rotations of a skyrmion. So λ_{min} is to be obtained before the solution of the Schrödinger equation is performed. In our calculations of the quantum spectra of masses the role of such a procedure was not essential except for the nucleon case. The contributions of vibrational degrees of freedom as well as rotational ones are seen from Table 5 where we give our results for λ_{min}, rotational energy E_{rot} (this value includes classical mass), and energy of the vibrational phonon $h\omega_{vibr}$ for F_π = 108 MeV and e = 4,84. The values λ_{min} = 0 correspond to the cases when the procedure of λ-minimizing the effective potential has not been performed.

Table 5. Spectrum of Tribaryons

λ_{min}	T	S	T_3	E_{rot} (MeV)	$h\omega_{vibr}$ (MeV)
0.0	1/2	3/2	1/2	2461	
-.027	1/2	3/2	1/2	2460	127
0.0	3/2	3/2	1/2	2526	
-0.071	3/2	3/2	1/2	2519	125
0.0	5/2	1/2	1/2	2601	
-0.117	5/2	1/2	1/2	2582	123
0.0	5/2	3/2	1/2	2633	
-0.133	5/2	3/2	1/2	2607	122

3.3 Few Remarks about the Existence of the Nucleon- Antinucleon States

The $l = 2$ solutions correspond to $B = 0$ states. Some of these states, being correctly quantized should be considered as nucleon-antinucleon bound states. One can see that the "classical" mass of these states is of the order of two nucleon masses ($k = 2$). So one hopes that if these states will be stable after inclusion of the quantum corrections, the binding energy will be small compared with two nucleon masses. The same is true for the other cases with $l = 2$.

We have calculated the $T = S = 0$ states taking into account only the breathing mode. In accordance to our numerical results, the states which were stable before quantization remain stable and after inclusion of the breathing mode. The state $k = 1$, $l = 2$ that was unstable before quantization procedure ($M_{cl}(k{=}1,l{=}2)$-$2\cdot M_{cl}(k{=}1,l{=}1)$ =220.7 MeV) performed is yet unstable (56.7 MeV). The states with $k = 2, 3, 4$, $l = 2$ are stable. Some of these last states may appear as compound states in the reactions with stopped antinucleons.

4. Conclusions

The bound states with baryon number $B = 2, 3, 4$ with the toroidal structure have been investigated in the framework of the very general assumption about the form of the solution of the Skyrme model equations. The meson-like states with baryon number $B = 0$ have been obtained. They are not of the toroidal structure. More complicated baryon states consisting of toroidal solitons and nontoroidal substructures have been obtained as well. Some of these last states may appear as compound states in the reactions with stopped antinucleons. The searches for such states are very desirable to confirm the chiral soliton picture of strong interacting system.

We have constructed the effective quantum Hamiltonian taking into account the breathing mode and rotational degrees of freedom. We have shown that all the candidates to the light nuclear states have very high binding energy values. But nuclear state-like configurations should not be identified with nuclei since a lot of quantum corrections is not yet taken into account.

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References

[1] Skyrme T.H.R. Nucl.Phys. **31**, 556 (1962)

[2] Jakson A., Jakson A.D., Pasquier V., Nucl.Phys. **A432**, 567 (1985); Vinh Mau R., Lacombe M., Loiseau B., Cottingham W.N., Lisboi P., Phys. Lett. **B150**, 259 (1985)

[3] Weigel H., Schwesinger B., Holzwarth G.: Phys. Lett. **B168**, 556 (1986)
[4] Kopeliovich V.B., Stern B.E., Pisma v ZhETF, **45**, 165 (1987)
[5] Verbaarshot J.J.M., Phys.Lett. **B195**, 235 (1987)
[6] Manton N.S., Phys.Lett., **B192**, 177 (1987) vspace-0.3cm
[7] Kozhevnikov I.R., Rybakov Yu.P., Fomin M.B, T.M.F., **75**, 353 (1989)
[8] Nikolaev V.A., Tkachev O.G.: JINR Communications **P4-86-515**, Dubna, 1986; Nikolaev V.A., in "Particles and Nuclei" **vol. 20 N2**, p.403, Moscow, Atomizdat, 1989.
[9] Nikolaev V.A., Tkachev O.G., JINR Rapid Communications **N1[34]-89**, p.28, Dubna, 1989; **N4[37]-89**, p.18, Dubna, 1989
[10] Dyakonov D.I.: Sov.J.Nucl.Phys. **45**, 1592 (1987)
[11] Nikolaev V.A., Tkachev O.G.: JINR Preprint **E4-89-56** (1989); TRIUMF (FEW BODY XII), **TRI-89-2**, p.F25, Vancouver, 1989
[12] Kurihara T., Kanada H., Otofuji T., Saito S., TRIUMF (FEW BODY XII), **TRI-89-2**, p.F27, Vancouver, 1989; Progr. of Theor. Phys. **81**, N4, 858 (1989)
[13] Braaten E., Carson L., Phys.Rev. **D38**, 3525 (1988)
[14] Nikolaev V.A., Tkachev O.G. JINR Communications **P4-86-515**, Dubna, 1986; Nikolaev V.A., in Proceedings of the VIII International Seminar on Hight Energy Physics Problems, Dubna, 1986, D1, 2-86-668, vol.1, p.78
[15] Zenkin C.B., Kopeliovich V.B., Stern B.E. in, Proceedings of the Simposium on Nucleon-Nucleon and -Nuclei Interactions, p.415, LNPhI, Leningrad, 1986
[16] Bogomolny E.B., Fateev V.A. Sov.J.Nucl.Phys. **37**, 228 (1983)

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Николаев В.А., Ткачев О.Г. E4-89-848
Малобарионные системы в SU(2)-модели Скирма

В рамках очень общего предположения относительно формы решений уравнений модели Скирма исследованы классически стабильные солитоны с барионными числами 1,2,3,4. Некоторые из этих солитонов имеют тороидальную структуру, другие же - более сложную. Получен эффективный квантовомеханический гамильтониан и его спектр в методе коллективных переменных. Все полученные состояния с квантовыми числами легчайших ядер имеют энергию связи больше экспериментально наблюдаемой. Некоторые из полученных состояний, содержащих антибарионы как свои структурные единицы, могут проявиться в реакциях с остановившимися антибарионами как ядерные компаунд-состояния.

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Nikolaev V.A., Tkachev O.G. E4-89-848
Few-Baryon Systems in the SU(2)-Skyrme Model

The classically stable solitons with baryon number 1, 2,3,4 have been investigated in the framework of the very general assumption about the form of the solutions for the Skyrme model equations. Some of the solitons have the toroidal structure and some of them are more complicated. The effective quantum-mechanical Hamiltonian and its spectrum are obtained by using the collective variable method. All the states with quantum numbers of light nuclei have the binding energy greater than the experimental one. Some of the calculated states containing antibaryons as substructure units should appear in the experiments with stopped antibaryons as compound nuclear states.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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