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GENERAL SUPER VIRASORO CONSTRUCTION ON AFFINE $\mathcal{G}$ *

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ABSTRACT

We consider a bosonic current algebra and a theory of free fermions and construct a general $N = 1$ super Virasoro current algebra. We obtain a master-set of equations which comprises the bosonic master equation for general Virasoro construction on affine $\mathcal{G}$. As an illustration we study the case of the group $SU(2)$.

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Until not far ago, the most common approach for going from an affine Lie algebra to a conformal field theory was through the Sugawara and coset constructions. More recently, however, a generalization of this method was given [1,2]. The basic idea consists in taking a current algebra of level k

$$J^a(z)J^b(w) = \frac{k}{2}\frac{\eta^{ab}}{(z-w)^2} + if^{ab}{}_c\frac{J^c(w)}{(z-w)} \quad , \tag{1}$$

where J^a are the (1,0) currents of an affine $\mathcal{G}$, and $f^{ab}{}_c$ and η^{ab} are, respectively, its structure constants and Killing metric. The energy-momentum tensor is then written as a general quadratic sum of the affine currents

$$T(z) = R_{ab} : J^a J^b : (z) \quad , \tag{2}$$

with R_{ab} a symmetric matrix. The requirement that this stress tensor satisfies

$$T(z)T(w) = \frac{c}{2(z-w)^4} + \frac{2T(w)}{(z-w)^2} + \frac{\partial T(w)}{(z-w)} \tag{3}$$

results in the so-called master equation and its central charge [1]

$$\begin{aligned} R_{mn} &= kR_{mc}\eta^{cd}R_{dn} - f^{ab}{}_m f^{cd}{}_n R_{ac}R_{bd} - f^{ac}{}_m f^{bd}{}_c R_{ab}R_{dn} - f^{ac}{}_n f^{bd}{}_c R_{ab}R_{dm} \\ c &= k\eta^{ab}R_{ab} \quad . \end{aligned} \tag{4}$$

This equation has a rich and interesting structure. It accounts for the familiar affine-Sugawara constructions, coset constructions and the linear conformal deformations. The first solution, different from the Sugawara and coset constructions,was found for the group $SU(2)$ at level 4 using the diagonal ansatz $R_{ab} = \lambda_a\delta_{ab}$ [3]. A large number of new solutions, based on different approaches, for the group $SU(3)$ were also reported [4]. Geometrically, the master equation is identified in [5] as an Einstein-like system on the group manifold where the central charge is $c = dim\mathcal{G} - 4\mathcal{R}$, and $\mathcal{R}$ is the curvature scalar. A conformal action for the generic high k affine-Virasoro construction is also obtained [6]. Recent developments include a connection between the master equation and graph theory [7], a perturbative technique for solving the master equation [8], and an extension to non-compact groups in connection with W_3-gravity [9].

The purpose of this note is to supersymmetrize the general Virasoro construction and to find the supersymmetric version of the Virasoro master equation.

We consider the case of $N = 1$ supersymmetry and we rely heavily on the equivalence between the $N = 1$ supercurrent algebra and the direct product of a bosonic current algebra and a theory of free fermions. We find a master-set of equations among which we identify the bosonic master equation. This set would be the first stone in any supersymmetric extension of the analyses that were dedicated to the bosonic master equation.

In what follows we briefly review the essential properties of the supersymmetric current algebra and fix our notation. The Sugawara form of the super energy-momentum tensor is given by [10]

$$\mathbf{T}(Z) = \frac{1}{k}\eta_{ab} : D\mathbf{J}^a\mathbf{J}^b : (Z) + \frac{2i}{3k^2} f_{abc} : \mathbf{J}^a : \mathbf{J}^b\mathbf{J}^c :: (Z) \quad , \tag{5}$$

where $\mathbf{J}^a$ are supercurrents obeying a supercurrent algebra in the form of the operator product expansion [11]

$$\mathbf{J}^a(Z_1)\mathbf{J}^b(Z_2) = \frac{k}{2}\eta^{ab}\frac{1}{Z_{12}} + i\frac{\theta_{12}}{Z_{12}} f^{ab}{}_c \mathbf{J}^c(Z_2) \quad . \tag{6}$$

In these expressions $Z = (z, \theta)$ denotes the holomorphic coordinates of two-dimensional superspace. The supercovariant derivative is written as $D = \frac{\partial}{\partial\theta} + \theta\frac{\partial}{\partial z}$ and $Z_{12} \equiv z_1 - z_2 - \theta_1\theta_2$, $\theta_{12} \equiv \theta_1 - \theta_2$. The super Virasoro current algebra is represented by

$$\mathbf{T}(Z_1)\mathbf{T}(Z_2) = \frac{c}{6Z_{12}^3} + \frac{3\theta_{12}}{2Z_{12}^2}\mathbf{T}(Z_2) + \frac{1}{2Z_{12}}D\mathbf{T}(Z_2) + \frac{\theta_{12}}{Z_{12}}\partial\mathbf{T}(Z_2) \quad . \tag{7}$$

The central charge is given by

$$c = \frac{(3k - 2g)}{2k} dim\mathcal{G} \quad , \tag{8}$$

where g is the eigenvalue of the quadratic Casimir operator of $\mathcal{G}$ in the adjoint representation,

$$f^{ab}{}_c f^{cd}{}_b = g\eta^{ad} \quad . \tag{9}$$

The superspace fields $\mathbf{J}^a(Z)$ and $\mathbf{T}(Z)$ can be decomposed into components as

$$\begin{aligned} \mathbf{J}^a(Z) &= \psi^a(z) + \theta J^a(z) \\ \mathbf{T}(Z) &= \frac{1}{2}G(z) + \theta T(z) \quad , \end{aligned} \tag{10}$$

where ψ^a are fermionic currents in the adjoint representation of the group $\mathcal{G}$, and G is the supersymmetric partner of the bosonic stress tensor $T(z)$. In terms of these components the supercurrent algebra in (6) yields

$$\begin{aligned} \psi^a(z_1)\psi^b(z_2) &= \frac{k}{2}\eta^{ab}\frac{1}{(z_1 - z_2)} \\ J^a(z_1)\psi^b(z_2) &= if^{ab}{}_c\frac{\psi^c(z_2)}{(z_1 - z_2)} \\ J^a(z_1)J^b(z_2) &= \frac{k}{2}\frac{\eta^{ab}}{(z_1 - z_2)^2} + if^{ab}{}_c\frac{J^c(z_2)}{(z_1 - z_2)} \quad , \end{aligned} \tag{11}$$

while the super Virasoro algebra is equivalent to

$$\begin{aligned} T(z_1)T(z_2) &= \frac{c}{2(z_1 - z_2)^4} + \frac{2T(z_2)}{(z_1 - z_2)^2} + \frac{\partial T(z_2)}{(z_1 - z_2)} \\ T(z_1)G(z_2) &= \frac{3G(z_2)}{2(z_1 - z_2)^2} + \frac{\partial G(z_2)}{(z_1 - z_2)} \\ G(z_1)G(z_2) &= \frac{2c}{3(z_1 - z_2)^3} + \frac{2T(z_2)}{(z_1 - z_2)} \quad . \end{aligned} \tag{12}$$

This supercurrent algebra enjoys an interesting property by being equivalent to a direct sum of a bosonic current algebra of level $(k - g)$ and an algebra of free Majorana fermions [10,11]. This can be seen by introducing a modified current $\hat{J}^a(z)$ defined by

$$\hat{J}^a(z) = J^a(z) + \frac{i}{k} f^a{}_{bc} : \psi^b\psi^c : (z) \quad . \tag{13}$$

The operator product expansions in (11) reduce, then, to the following relations

$$\begin{aligned} \psi^a(z_1)\psi^b(z_2) &= \frac{k}{2}\eta^{ab}\frac{1}{(z_1 - z_2)} \\ \hat{J}^a(z_1)\psi^b(z_2) &= 0 \\ \hat{J}^a(z_1)\hat{J}^b(z_2) &= \frac{\hat{k}}{2}\frac{\eta^{ab}}{(z_1 - z_2)^2} + if^{ab}{}_c\frac{\hat{J}^c(z_2)}{(z_1 - z_2)} \quad , \end{aligned} \tag{14}$$

where

$$\hat{k} \equiv k - g \quad . \tag{15}$$

Therefore, the modified currents $\hat{J}^a$ form a level $\hat{k}$ bosonic current algebra and commute with the fermionic fields $\psi^a(z)$. The components of the super stress tensor are written in terms of these modified currents in the form

$$\begin{aligned} T(z) &= \frac{1}{k}\eta_{ab} : \hat{J}^a\hat{J}^b : (z) - \frac{1}{k}\eta_{ab} : \psi^a\partial\psi^b : (z) \\ G(z) &= \frac{2}{k}\eta_{ab} : \psi^a\hat{J}^b : (z) - \frac{2i}{3k^2} f_{abc} : \psi^a : \psi^b\psi^c :: (z) \quad . \end{aligned} \tag{16}$$

As it should be, the central charge (8) is rewritten as the sum of the central charge for the level $\hat{k}$ bosonic current algebra and the $(dim\mathcal{G})$ free fermions

$$c = \frac{\hat{k}}{\hat{k}+g} dim\mathcal{G} + \frac{1}{2} dim\mathcal{G} \quad . \tag{17}$$

This equivalence between the $N = 1$ supercurrent algebra and the direct product of a free fermion algebra and a bosonic current algebra will be at the centre of our construction of a generalized super Virasoro algebra. Moreover, this equivalence allows for the explicit occurrence of the bosonic master equation among the set of equations that we found. As in the bosonic case, we start from a general form for the stress tensor $T(z)$ and its supersymmetric partner $G(z)$,

$$\begin{aligned} T(z) &= R_{ab} : \hat{J}^a \hat{J}^b : (z) + L_{ab} : \psi^a \partial \psi^b : (z) \\ &\equiv T_1(z) + T_2(z) \\ G(z) &= M_{ab} : \psi^a \hat{J}^b : (z) + D_{abc} : \psi^a : \psi^b \psi^c :: (z) \\ &\equiv G_1(z) + G_2(z) \quad , \end{aligned} \tag{18}$$

where R_{ab}, L_{ab}, M_{ab} are symmetric matrices and D_{abc} is totally antisymmetric. The currents $\hat{J}^a$ and ψ^a obey the operator product expansion in Eq.(14). In this generalized construction the currents $\hat{J}^a$ and ψ^a are not necessarily related by (13) and $\hat{k}$ is not necessarily equal to $(k-g)$. The requirement that $T(z)$ and $G(z)$ satisfy the $N = 1$ supercurrent algebra in (12) yields a set of equations for R_{ab}, L_{ab}, M_{ab} and D_{abc}

In order to perform the different contractions, we have chosen to define the normal ordered product of two operators $A(z)$ and $B(z)$ at coincident points as [12]

$$: AB : (z) = \frac{1}{2\pi i} \oint_z \frac{dx}{x-z} A(x) B(z) \tag{19}$$

This leads naturally to the expansion

$$A(z)B(w) = \underbrace{A(z)B(w)} + : AB : (w) + O((z-w)) \quad , \tag{20}$$

where the contraction ($\underbrace{\quad}$) is the singular part of the expansion of two operators at distinct points

$$\underbrace{A(z)B(w)} = \sum_{r=r_0}^{\infty} \frac{H^{(r)}(w)}{(z-w)^r} \quad , \qquad r_0 \in \mathrm{N} \quad . \tag{21}$$

A crucial property of the definition (19) is an analogue of Wick's theorem for calculating the operator product expansion of $A(z)$ with a composite field $: BC : (w)$

$$\underbrace{A(z) : BC :}(w) = \frac{1}{2\pi i} \oint_w \frac{dx}{(x-w)} \{ \underbrace{A(z)B(x)} C(w) + (-1)^{BC} \underbrace{A(z)C(w)} B(x) \} \tag{22}$$

where $(-1)^{BC} = -1$ iff both B and C are fermionic fields. The above definition of the normal ordered product is neither commutative nor associative and we have

$$: [A, B\} : (z) \equiv : AB : -(-1)^{AB} : BA := \sum_{r=1}^{r_0} \frac{(-1)^{r+1}}{r!} \partial^r H^{(-r)}(z) \quad . \tag{23}$$

In our computation, the following formulae for multiple normal products are useful

$$\begin{aligned} : A : BC :: &= (-1)^{AB} : B : AC :: + :: [A, B\} : C : \\ :: AB :: CD :: &= : A : B : CD ::: - : A : \{B, : CD :\} :: \\ &- (-1)^{B(C+D)} :: [A, : CD :\} : B : \\ &+ : \{ : AB :, : CD : \} : \quad . \end{aligned} \tag{24}$$

Using Wick's theorem in (22) and the operator product expansion in (14) we find

$$\begin{aligned} T_1(z) \hat{J}^d(w) &= (\hat{k} R_{ab} \eta^{da} - R_{ac} f^{da}{}_e f^{ec}{}_b) \\ &\times \left[\frac{1}{(z-w)^2} + \frac{\partial}{(z-w)} \right] \hat{J}^b(w) \\ &- i(R_{ab} f^{da}{}_e + R_{ae} f^{da}{}_b) \frac{1}{(z-w)} : \hat{J}^e \hat{J}^b : (w) \\ T_2(z) \psi^d(w) &= -\frac{k}{2} L_{ab} \eta^{ad} \left[\frac{1}{(z-w)^2} + \frac{2\partial}{(z-w)} \right] \psi^a(w) \\ G_1(z) \hat{J}^d(w) &= \frac{\hat{k}}{2} M_{ab} \eta^{bd} \left[\frac{1}{(z-w)^2} + \frac{\partial}{(z-w)} \right] \psi^a(w) \\ &+ i M_{ab} f^{bd}{}_c \frac{1}{(z-w)} : \psi^a \hat{J}^c : (w) \\ G_1(z) \psi^d(w) &= \frac{k}{2} M_{ab} \eta^{ad} \frac{1}{(z-w)} \hat{J}^b(w) \\ G_2(z) \psi^d(w) &= \frac{3}{2} k D_{abc} \eta^{cd} \frac{1}{(z-w)} : \psi^a \psi^b : (w) \quad . \end{aligned} \tag{25}$$

After a tedious calculation, using these operator product expansions together with the rearrangement formulae in (24), the requirement that $T(z)$ and $G(z)$ satisfy the $N = 1$ super Virasoro algebra in (12) yields the following master-set of independent equations

$$\begin{aligned}
R_{mn} &= \hat{k} R_{mc}\eta^{cd}R_{dn} - f^{ab}{}_{m}f^{cd}{}_{n}R_{ac}R_{bd} - f^{ac}{}_{m}f^{bd}{}_{c}R_{ab}R_{dn} - f^{ac}{}_{n}f^{bd}{}_{c}R_{ab}R_{dm} \\
L_{cb} &= -kL_{ab}L_{cd}\eta^{ad} \\
R_{db} &= \frac{k}{4}M_{ab}M_{cd}\eta^{ca} \\
L_{ae} &= \frac{9}{4}k^2 D_{abc}D_{efd}\eta^{cd}\eta^{fb} - \frac{\hat{k}}{4}M_{ab}M_{ed}\eta^{db} \\
M_{ac} &= \frac{1}{2}[\hat{k}M_{cd}R_{ab}\eta^{bd} + M_{cd}R_{fb}f^{fd}{}_{e}f^{eb}{}_{a} + a \longleftrightarrow c] \\
M_{ac} &= -\frac{1}{2}[kM_{ad}L_{cb}\eta^{bd} + a \longleftrightarrow c] \\
D_{dab} &= -\frac{k}{3}\eta^{ce}[L_{de}D_{abc} + L_{be}D_{dac} + L_{ae}D_{bdc}] \\
0 &= \eta^{cd}[D_{abc}D_{def} + D_{fbc}D_{dae} + D_{ebc}D_{dfa}] \\
0 &= iM_{cd}[R_{ab}f^{bd}{}_{e} + R_{eb}f^{bd}{}_{a}] \\
0 &= -iM_{ce}M_{da}f^{ae}{}_{b} + 3kM_{ab}D_{cde}\eta^{ea} \quad . \qquad (26)
\end{aligned}$$

As expected, the first equation of this set is just the bosonic master equation. In the actual computation, however, one comes across a larger set of equations which eventually reduces to the above set. With the help of the master-set, the central charge is given by the simple form

$$c = \hat{k}\eta^{ab}R_{ab} - \frac{k}{2}\eta^{ab}L_{ab} \quad . \qquad (27)$$

The Sugawara construction

$$\begin{aligned}
R_{ab} &= \frac{1}{\hat{k}+g}\eta_{ab} \\
L_{ab} &= -\frac{1}{k}\eta_{ab} \\
M_{ab} &= \frac{\pm 2}{\sqrt{k(\hat{k}+g)}}\eta_{ab} \\
D_{abc} &= \frac{\mp 2i}{3k\sqrt{k(\hat{k}+g)}}f_{abc} \\
c &= \frac{3\hat{k}}{\hat{k}+g}dim\mathcal{G} + \frac{1}{2}dim\mathcal{G} \quad , \qquad (28)
\end{aligned}$$

is always a solution, and similarly when η_{ab}, f_{abc} and g are restricted to a subgroup $\mathcal{H}$ of $\mathcal{G}$. We should emphasize the fact that $\hat{k}$ is not necessarilly equal to $(k-g)$.

However, in the case when $\hat{k} = k - g$ the above solution reduces to that in equation (16).

We now consider deformations of this construction by including background charges. A more general form for the energy-momentum tensor $T(z)$ and its super partner $G(z)$ is given by

$$\begin{aligned}
T(z) &= T_1(z) + T_2(z) + Q_a\partial\hat{J}^a(z) + H_{ab}\partial(:\psi^a\psi^b:)(z) \\
G(z) &= G_1(z) + G_2(z) + P_a\partial\psi^a(z) \quad , \qquad (29)
\end{aligned}$$

where H_{ab} is antisymmetric and Q_a and P_a are some background charges. The condition that this new construction satisfies a super Virasoro algebra changes only the first two equations of the master-set in (26) and leads to a set of restrictions on the allowed background charges. The first two equations of the master-set (26), therefore, read

$$\begin{aligned}
R_{mn} &= \hat{k}R_{mc}\eta^{cd}R_{dn} - f^{ab}{}_{m}f^{cd}{}_{n}R_{ac}R_{bd} - f^{ac}{}_{m}f^{bd}{}_{c}R_{ab}R_{dn} - f^{ac}{}_{n}f^{bd}{}_{c}R_{ab}R_{dm} \\
&+ i(R_{mc}f^{ce}{}_{n} + R_{nc}f^{ce}{}_{m})Q_e \\
L_{cb} &= -kL_{ab}L_{cd}\eta^{ad} + k\eta^{da}(H_{ab}L_{cd} + H_{ac}L_{bd}) \quad , \qquad (30)
\end{aligned}$$

while the rest of the equations remain unaltered. The additional conditions are written as

$$\begin{aligned}
Q_e &= Q_a(\hat{k}\eta^{ab}R_{be} + f^{ab}{}_{d}R_{bc}f^{cd}{}_{e}) \\
H_{cb} &= -k\eta^{da}(H_{ab}L_{cd} - H_{ac}L_{bd}) \\
Q_d &= \frac{k}{4}M_{dc}\eta^{ca}P_a \\
H_{cd} &= \frac{3}{4}kD_{cdb}\eta^{ba}P_a \\
P_c &= -kL_{cd}\eta^{da}P_a - \frac{2}{3}kH_{cb}\eta^{ba}P_a \\
P_c &= -kL_{cd}\eta^{da}P_a - \frac{3}{2}k^2 H_{de}D_{cbf}\eta^{fd}\eta^{be} \\
0 &= \eta^{cd}[H_{de}D_{abc} + H_{db}D_{eac} + H_{da}D_{bec}] \\
0 &= -kP_a\eta^{da}(L_{cd} + 2H_{cd}) - \hat{k}M_{cb}\eta^{ba}Q_a - 3k^2 H_{de}D_{cbf}\eta^{fd}\eta^{be} \\
0 &= iM_{db}f^{ba}{}_{c}Q_a - kH_{db}M_{ac}\eta^{ab} \quad . \qquad (31)
\end{aligned}$$

Using the new master-set (30) together with the additional conditions (31), we find a simple expression for the deformed central charge

$$c = \hat{k}\eta^{ab}R_{ab} - \frac{k}{2}\eta^{ab}L_{ab} - \frac{3}{2}k\eta^{ab}P_aP_b \quad . \tag{32}$$

We have examined the case $\mathcal{G} = SU(2)$ for the special ansatz

$$D_{abc} = i(\lambda_a + \lambda_b + \lambda_c)f_{abc} \quad , \tag{33}$$

and we report here only on one solution with a central charge $c = \frac{2}{3}$. This is given by

$$\begin{aligned}
M_{11} &= \frac{s^2}{x} \ , \ M_{12} = s \ , \ M_{13} = \frac{sy}{x} \\
M_{22} &= x \ , \ M_{23} = y \ , \ M_{33} = \frac{y^2}{x} \\
R_{11} &= \frac{s^2}{\hat{k}r} \ , \ R_{12} = \frac{sx}{\hat{k}r} \ , \ R_{13} = \frac{sy}{\hat{k}r} \\
R_{22} &= \frac{x^2}{\hat{k}r} \ , \ R_{23} = \frac{xy}{\hat{k}r} \ , \ R_{33} = \frac{y^2}{\hat{k}r} \\
L_{11} &= -\frac{\hat{k}rs^2}{4x^2} \ , \ L_{12} = -\frac{\hat{k}rs}{4x} \ , \ L_{13} = -\frac{\hat{k}rsy}{4x^2} \\
L_{22} &= -\frac{1}{4}\hat{k}r \ , \ L_{23} = -\frac{\hat{k}ry}{4x} \ , \ L_{33} = -\frac{\hat{k}ry^2}{4x^2} \\
D_{123} &= i(\lambda_1 + \lambda_2 + \lambda_3) = 0 \quad ,
\end{aligned} \tag{34}$$

where x and y are free parameters while s and r are determined by

$$\begin{aligned}
r &= x^2 + y^2 + s^2 \\
r^2 &= \frac{4x^2}{k\hat{k}} \quad .
\end{aligned} \tag{35}$$

The energy-momentum tensor $T(z)$ and its supersymmetric partner $G(z)$ corresponding to this solution are remarkably of the form

$$\begin{aligned}
T(z) &= \frac{1}{\hat{k}} : \tilde{J}\tilde{J} : (z) - \frac{1}{k} : \tilde{\psi}\partial\tilde{\psi} : (z) + \partial\hat{J}(z) \\
G(z) &= \frac{2}{\sqrt{k\hat{k}}} : \tilde{\psi}\tilde{J} : (z) \quad ,
\end{aligned} \tag{36}$$

where

$$\begin{aligned}
\tilde{J}(z) &= \alpha J^1(z) + \beta J^2(z) + \gamma J^3(z) \\
\tilde{\psi}(z) &= \alpha\psi^1(z) + \beta\beta\psi^2(z) + \gamma\psi^3(z) \\
\hat{J}(z) &= \frac{i}{\hat{k}}[\beta\gamma J^1(z) - \alpha\gamma J^2(z) + \alpha\beta J^3(z)] \quad ,
\end{aligned} \tag{37}$$

and the different coefficients are given by

$$\alpha^2 = \frac{s^2}{r} \ , \ \beta^2 = \frac{x^2}{r} \ , \ \gamma^2 = \frac{y^2}{r} \quad . \tag{38}$$

Therefore, the above solution is not totally new but describes a deformed super Sugawara construction for the subgroup $U(1)$. Nevertheless, this does not completely exclude the possibility of finding new solutions for the $SU(2)$ general super Virasoro construction.

In conclusion, we have considered a theory of a bosonic current algebra together with a free fermionic one and constructed an $N = 1$ supersymmetric current algebra. We obtained a master-set of equations which was analysed for the group $SU(2)$. In fact one could have worked directly with superfields and could have constructed a generalized super stress tensor of the form

$$\mathbf{T}(Z) = V_{ab} : D\mathbf{J}^a\mathbf{J}^b : (Z) + W_{abc} : \mathbf{J}^a : \mathbf{J}^b\mathbf{J}^c :: (Z) \quad . \tag{39}$$

The requirement that $\mathbf{T}(Z)$ has the operator product expansion written in (7) results in a different master-set which would not explicitly contain the bosonic master equation.

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