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ABSTRACT

A steady two-dimensional laminar boundary layer flow of a viscous incompressible and electrically conducting fluid past a vertical heated plate with variable temperature in the presence of a transverse uniform magnetic field has been investigated by bringing the effect of viscous and Joules heating. The non-dimensional boundary layer equations are solved using the implicit finite difference method along with Newton's approximation for small Prandtl number chosen as typical of coolant liquid metals at operating temperature.

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1. INTRODUCTION

The natural convection boundary layer flow of an electrically conducting fluid up a hot vertical wall in the presence of a strong cross field has been studied by several authors [1–4] because of its application in nuclear engineering in connection with the cooling of reactors. Later, Cramer and Pai [5], studied the above problem with varying surface temperature in the presence of a strong magnetic field. On the other hand, Wilks [6] investigated the problem with uniform heat flux illustrating the problem by formulating in terms of regular and inverse series expansion of a characterizing coordinate that provided a link between the similarity states appropriate at the leading edge and down stream. Recently, Hossain and Ahmed [7] have studied a combined effect of forced and free convection with uniform heat flux in the presence of strong field near the leading edge. In all the above studies the effect of viscous and Joule heating is neglected. So, in the present paper, we propose to study the effect of viscous and Joule heating on the free convection flow of an electrically conducting viscous incompressible fluid past a semi-infinite plate of which temperature varies linearly with the distance from the leading and in the presence of uniform transverse magnetic field. The governing equations are solved numerically using the finite difference method and Newton's linearization approximation. Numerical solutions are obtained for small Prandtl number, appropriate for coolant liquid metal, in the presence of large magnetic fields.

2. BASIC EQUATIONS

The basic equations (1) and (2) describe steady two-dimensional, laminar free convection boundary-layer-type flow of viscous incompressible and conducting fluid through a uniformly distributed transverse magnetic field of strength B_0.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\beta(T - T_\infty) + \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma_0 B_0^2}{\rho}\, u \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} + \frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\sigma_0 B_0^2}{\rho c_p}\, u^2 \tag{3}$$

where u and v are the velocity components associated with the direction of increasing coordinates x and y are measured along and normal to the vertical plate, respectively, T is the temperature of the fluid in the boundary layer, g is the acceleration due to gravity, β the coefficient of thermal expansion, k the thermal conductivity, ρ the density of the fluid, σ_0 the electrically conductivity, ν the kinematic coefficient of viscosity, c_p the specific heat at constant pressure and T_∞ is the temperature of the ambient fluid.

The boundary conditions are

$$\left.\begin{array}{lll} u = v = 0\,, & T = T_w(x) & \text{at} \quad y = 0 \\ u \to 0\,, & T \to T_\infty & \text{as} \quad y \to \infty \end{array}\right\} . \tag{4}$$

In formulating Eqs.(1)–(3), it has been assumed that (i) the ratio of thermal diffusivity to magnetic diffusivity is small compared to unity, (ii) fluid property variations are limited to density variation which is taken into account only in so far as it effects the buoyancy terms only and (iii) the short circuit assumption applies.

To reduce Eqs.(1)–(3) to ordinary differential equations, here we introduce the stream function ψ defined by $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$ throughout Eqs.(1)–(3) to get

$$\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y} - \frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2} = g\beta(T - T_\infty) + \nu\frac{\partial^3\psi}{\partial y^3} - \frac{\sigma_0 B_0^2}{\rho}\frac{\partial\psi}{\partial y} \tag{5}$$

$$\frac{\partial\psi}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} + \frac{\nu}{c_p}\left(\frac{\partial^2\psi}{\partial y^2}\right)^2 + \frac{\sigma_0 B_0^2}{\rho c_p}\left(\frac{\partial\psi}{\partial y}\right)^2 \tag{6}$$

We now introduce the following set of transformations for the dependent and independent variables:

$$\left.\begin{aligned} \psi(\eta,\xi) &= (g\beta N)^{1/4}\nu^{1/2} x F(\eta,\xi), \quad \eta = (g\beta N)^{1/4}\nu^{-1/2} y \\ \theta(\eta,\xi) &= (T - T_\infty)/(T_\omega - T_\infty), \quad \xi = g\beta x/c_p \\ T_\omega - T_\infty &= Nx, \quad M^2 = \sigma_0 B_0^2/\rho(g\beta N)^{1/2} \end{aligned}\right\} \tag{7}$$

into Eqs.(5) and (6) and (4), to find

$$f''' + ff'' - (M^2 + f')f' + \theta = \xi\left\{f''\frac{\partial f'}{\partial\xi} - f''\frac{\partial f}{\partial\xi}\right\} \tag{8}$$

$$\sigma^{-1}\theta'' + f\theta' - f'\theta = \xi\left\{f'\frac{\partial\theta}{\partial\xi} - \theta'\frac{\partial f}{\partial\xi} - M^2(f')^2 - (f'')^2\right\} \tag{9}$$

$$\left.\begin{aligned} f(0,\xi) = f'(0,\xi) = 0, \quad \theta(0,\xi) = 1 \\ f'(0,\infty) = \theta(0,\infty) = 0 \end{aligned}\right\}. \tag{10}$$

The physical quantities of interest are local friction factor τ_ω and local heat transfer. These are defined by

$$\tau_\omega = -\rho\,\nu^{-1/2}(g\beta N)^{3/4} x\, f''(0,\xi) \tag{11}$$

and

$$q = -k\nu^{-1/2}(g\beta N)^{1/4}(T_\omega - T_\infty)\theta'(0,\xi)\,. \tag{12}$$

3. SOLUTIONS AND DISCUSSIONS

Eqs.(8) and (9) along with the boundary condition (10) have been descretized with a simple implicit finite difference scheme, similar to that used by Keller and Cebeci [9]. Before we describe Eqs.(8) and (9), we write them in terms of the first order system of pdes, as given below:

$$f' = U \tag{13a}$$

$$U' = V \tag{13b}$$

$$V' + fV - (M^2 + U)U + \theta = \xi\left(U\frac{\partial U}{\partial\xi} - V\frac{\partial f}{\partial\xi}\right) \tag{13c}$$

$$\theta' = W \tag{14a}$$

$$\sigma^{-1}W' + fW - U\theta = \xi\left(U\frac{\partial\theta}{\partial\xi} - W\frac{\partial f}{\partial\xi} - M^2U^2 = V^2\right) \tag{14b}$$

and the boundary conditions turn into

$$\left.\begin{aligned} f(0,\xi) = U(0,\xi) = 0, \quad \theta(0,\xi) &= 0 \\ U(\eta_\infty,\xi) = \theta(\eta_\infty,\xi) &= 0 \end{aligned}\right\} \tag{15}$$

At the net rectangle, we denote the net points by

$$\left.\begin{aligned} \xi_0 = 0, \quad \xi^n = \xi^{n-1} + k_n \qquad & n = 1,2,\dots N \\ \eta_0 = 0, \quad \eta_j = \eta_{j-1} + h_j \qquad & j = 1,2\dots J, \; \eta_J = \eta_\infty \end{aligned}\right\}. \tag{16}$$

Here n and j are just the sequence number.

Now, we approximate the quantities (f,U,V,θ,W) at points (ξ^n,η_j) η the net by $(f_j^n, U_j^n, V_j^n, \theta_j^n, W_j^n)$. We also employ g_j^n for points and quantities midway between net points and for any net function:

$$\left.\begin{aligned} \xi^{n-1/2} &\equiv \tfrac{1}{2}(\xi^n + \xi^{n-1}), \quad \eta_{j-1/2} \equiv \tfrac{1}{2}(\eta_j + \eta_{j-1}) \\ g_j^{n-1/2} &\equiv \tfrac{1}{2}(g_j^n + g_j^{n-1}), \quad g_{j-1/2}^n \equiv \tfrac{1}{2}(g_j^n + g_{j-1}^n) \end{aligned}\right\}. \tag{17}$$

We now write down, the finite–difference approximation of Eqs.(13) and (14) for the mid–point (ξ^n, η_{j-n}), below:

$$h_j^{-1}(f_j^n - f_{j-1}^n) = U_{j-1/2}^n \tag{18a}$$

$$h_j^{-1}(U_j^n - U_{j-1}^n) = V_{j-1/2}^n \tag{18b}$$

$$h_j^{-1}(\theta_j^n - \theta_{j-1}^n) = W_{j-1/2}^n$$

$$h_j^{-1}(V_j^n - V_{j-1}^n) + (1+\alpha_n)\left\{(fV)_{j-1/2}^n - (U^2)_{j-1/2}^n\right\} - M^2U_{j-1/2}^n + \theta_{j-1/2}^n$$
$$+\alpha_n\left\{V_{j-1/2}^{n-1}f_{j-1/2}^n - f_{j-1/2}^{n-1}V_{j-1/2}^n\right\} = R_{j-1/2}^{n-1} \tag{18c}$$

$$\sigma^{-1}h_j^{-1}(W_j^n - W_{j-1}^n) + (1+\alpha_n)\left\{(fW)_{j-1/2}^n - (U\theta)_{j-1/2}^n\right\}$$
$$+\alpha_n\left\{\theta_{j-1/2}^{n-1}U_{j-1/2}^n - U_{j-1/2}^{n-1}\theta_{j-1/2}^n + W_{j-1/2}^{n-1}f_{j-1/2}^n - f_{j-1/2}^{n-1}W_{j-1/2}^n -\right.$$
$$\left. - M^2(U^2)_{j-1/2}^n - (V^2)_{j-1/2}^n\right\} = T_{j-1/2}^{n-1} \tag{18d}$$

where $\alpha_n = \xi^{n-1/2} / k_n$

$$
\begin{aligned}
R_{j-1/2}^{n-1} &= -L_{j-1/2}^{n-1} + \alpha_n \left\{ (fV)_{j-1/2}^{n-1} - (U^2)_{j-1/2}^{n-1} \right\} \\
L_{j-1/2}^{n-1} &= h_j^{-1} \left(V_{j-1/2}^{n-1} - V_{j-1/2}^{n-1} \right) + (fV)_{j-1/2}^{n-1} - M^2 U_{j-1/2}^{n-1} - (U^2)_{j-1/2}^{n-1} + \theta_{j-1/2}^{n-1} \\
T_{j-1/2}^{n-1} &= -M_{j-1/2}^{n-1} + \alpha_n \left\{ (fW)_{j-1/2}^{n-1} - (U\theta)_{j-1/2}^{n-1} - M^2 (U^2)_{j-1/2}^{n-1} - (V^2)_{j-1/2}^{n-1} \right\} \\
M_{j-1/2}^{n-1} &= \sigma^{-1} h_j^{-1} \left(W_j^{n-1} - W_{j-1}^{n-1} \right) + (fW)_{j-1/2}^{n-1} - (U\theta)_{j-1/2}^{n-1} \,. \qquad (19)
\end{aligned}
$$

The wall and the edge boundary conditions are

$$
f_0^n = 0, \; U_0^n = 0, \; \theta_0^n = 1, \; U_J^n = 0, \; \theta_J^n = 0 \,. \qquad (20)
$$

If we assume $f_j^{n-1}, U_j^{n-1}, V_j^{n-1}, \theta_j^{n-1}$, and W_j^{n-1} to be known for $0 \le j \le J$, Eqs.(18)–(20) are a system of $5J + 5$ equations for the solution of $5J + 5$ unknowns $(f_j^n, U_j^n, V_j^n, \theta_j^n, W_j^n)$, $j = 0, 1, \ldots, J$. These non–linear systems of algebraic equations are linearized by means of Newton's method which then solved in a very efficient manner by using the Keller–box method, discussed by Cebeci and Bradshaw [10] in a simpler way.

The resulting solution for the velocity and temperature functions are shown graphically in Figs.1 and 2 and the numerical values for shear–stress and the heat transfer coefficients are presented in Table 1.

The assumptions used to establish the governing equations are particularly appropriate to liquid metals. Moreover, as liquid metals are currently used as coolants in nuclear engineering (Wilks [6]), we have persued here solutions into lower Prandtl number range, e.g. 0.05 for lithium, 0.01 for mercury and 0.005 for sodium. In fact, detailed numerical solutions have been obtained for $\sigma = 1, 0.72, 0.5, 0.1, 0.05, 0.01, 0.005$. Associated numerical data is available from the author. As confirmation of satsifactory reconciliation between the previous works and the present analysis, we present the solutions only for $\sigma = 0.05, 0.01$ and 0.005. In all the figures from 1 to 2, the solid curves are due to Cramer and Pai [5], in absence of viscous and Joule heating, which qualitatively agrees with the results of these authors. From Fig.1a we may conclude that the presence of viscous dissipation reduces the flow field. This further reduces owing to the increase in the dissipative heating when the fluid is being heated. A similar situation is also observed from Fig.1b, in the case of the temperature field. Figs.2a and 2b represent, respectively, the velocity and temperature field for different values of Prandtl number in the absence, as well as in the presence, of viscous and Joule heating. From Fig.2a we may conclude that dissipative heat reduces the velocity field more in the lower Prandtl number fluid than that of the higher Prandtl number. On the other hand, in the case of temperature field, the dissipative heat reduces it faster in the higher Prandtl number than that in the lower Prandtl number fluid.

We now discuss the effect of viscous and Joule heating on the shear stress and the rate of heat transfer at the surface of the wall heat. From Table 1 it may easily conclude that the presence, as well as an increase in the dissipative, reduces both the skin–friction and the rate of heat transfer

at the surface. This rate of decrease in the skin–friction and the rate of heat transfer due to the presence of viscous and Joule heating slow down with the increase in the magnetic field.

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REFERENCES

[1] K.R. Singh and T.G. Cowling, "Thermal convection in magnetohydrodynamics", *J. Mech. Appl. Math.* **16**, 1–5 (1963).

[2] E.M. Sparrow and R.D. Cess, "Effect of magnetic field on free convection heat transfer", *Int. J. Heat Mass Transfer* **3**, 267–274 (1961).

[3] N. Riley, "Magnetohydrodynamic free convection", *J. Fluid Mech.* **18**, 577–586 (1964).

[4] H.K. Kuiken, "Magnetohydrodynamic free convection in strong cross flow field", *J. Fluid Mech.* **40**, 21–38 (1970).

[5] K.R. Cramer and S.I. Pai, *Magnetofluid Dynamics for Engineers and Applied Physicists* (McGraw–Hill Book Co., New York, 1974) pp.164–172.

[6] G. Wilks, "Magnetohydrodynamic free convection about a semi–infinite vertical plate in a strong cross field", *J. Appl. Math. Phys.* **27**, 621–631 (1976).

[7] M.A. Hossain and M. Ahmed, "MHD forced and free convection boundary layer flow near the leading edge", *Int. J. Heat Mass Transfer* **33**, 571–575 (1990).

[8] H.B. Keller and T. Cebeci, Accurate numerical methods for boundary layer flows, part I Two–dimensional laminar flows, Proceedings of the Second International Conference on Numerical Methods in Fluid Dynamics, (Springer, New York, 1971) p.92.

[9] H.B. Keller and T. Cebeci, "Accurate numerical methods for boundary layer flows", part II Two–dimensional turbulent flows, *AIAAJ* **10**, 1193–1199 (1972).

[10] T. Cebeci and P. Bradshaw, *Physical and Computational Aspects of Convective Heat Transfer* (Springer–Verlag, New York, 1984).

TABLE CAPTIONS

Table 1 Values $f''(0)$ and $\theta'(0)$ for different values of M and ξ when $\sigma = 0.005$.

Table 1

M	ξ	$f''(0)$	$-\theta'(0)$
1	0.000	0.81952	0.06152
	0.005	0.81360	0.07401
	0.010	0.80873	0.08354
2	0.000	0.48258	0.04121
	0.005	0.48078	0.04961
	0.010	0.47918	0.05649
3	0.000	0.32868	0.03218
	0.005	0.32818	0.03737
	0.010	0.32765	0.04178
4	0.000	0.24800	0.02704
	0.005	0.24792	0.03098
	0.010	0.24760	0.03354
5	0.000	0.19886	0.02540
	0.005	0.19883	0.02801
	0.010	0.19874	0.03033

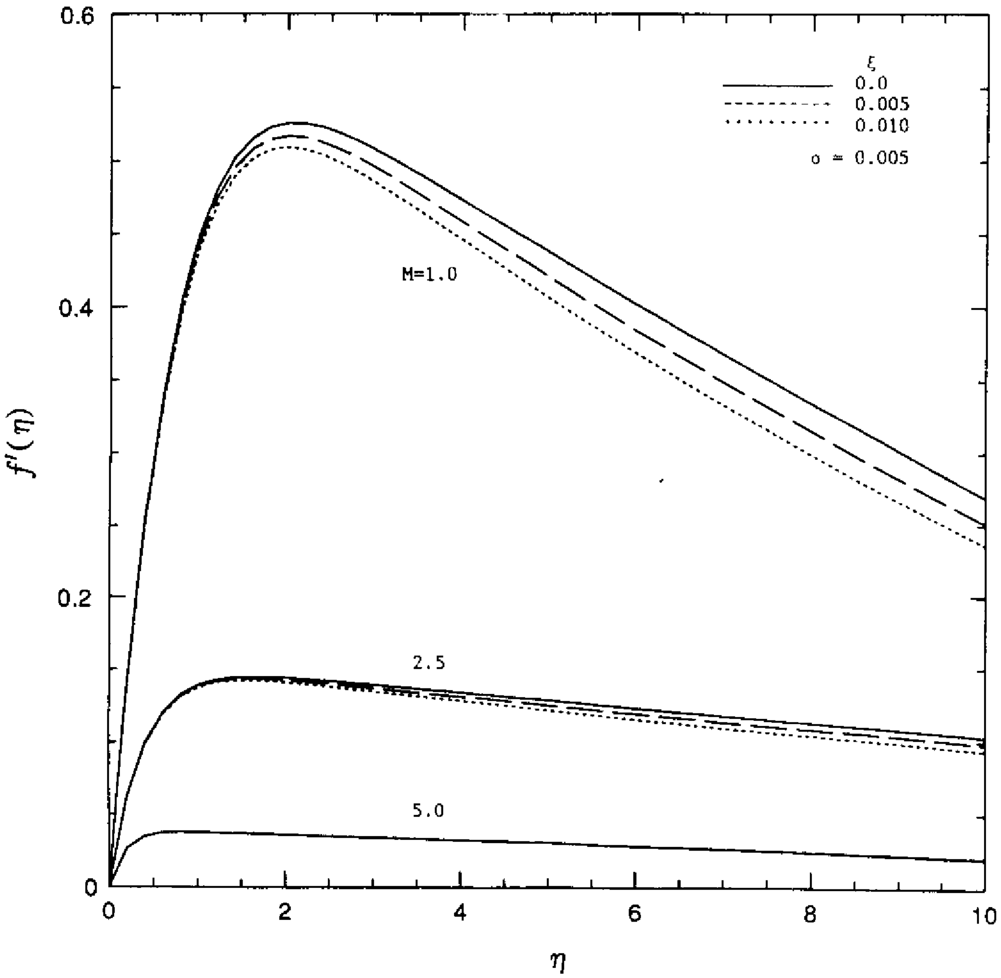
FIGURE CAPTIONS

Fig.1a Velocity profiles against η for different values of ξ and M with $\sigma = 0.005$.

Fig.1b Temperature profiles against η for different values of ξ and M with $\sigma = 0.005$.

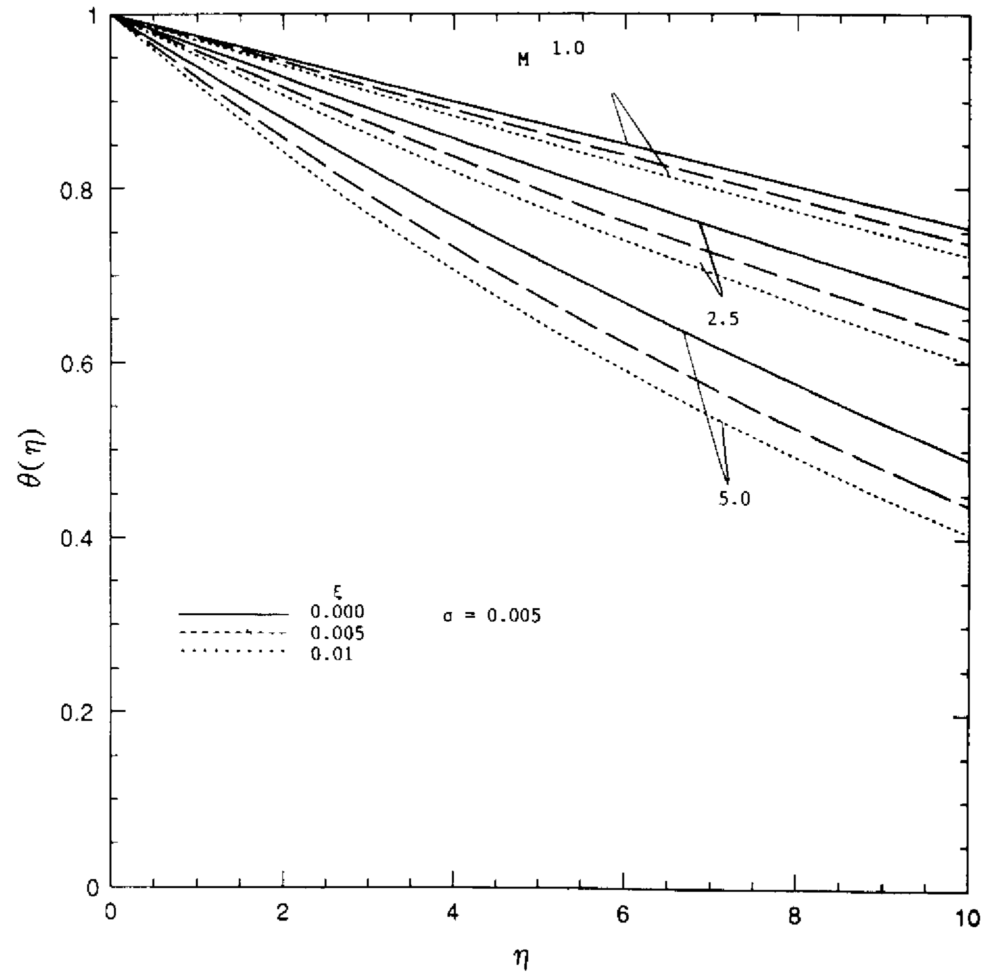
Fig.2a Velocity profiles against η for different values of σ and ξ with $M = 1.0$.

Fig.2b Temperature profiles against η for different values of σ and ξ with $M = 1.0$.



ξ
0.0
0.005
0.010
σ = 0.005
M=1.0
2.5
5.0
$f'(\eta)$
η

Fig.1a



M
1.0
2.5
5.0
ξ
0.000
0.005
0.01
σ = 0.005
$\theta(\eta)$
η

Fig.1b

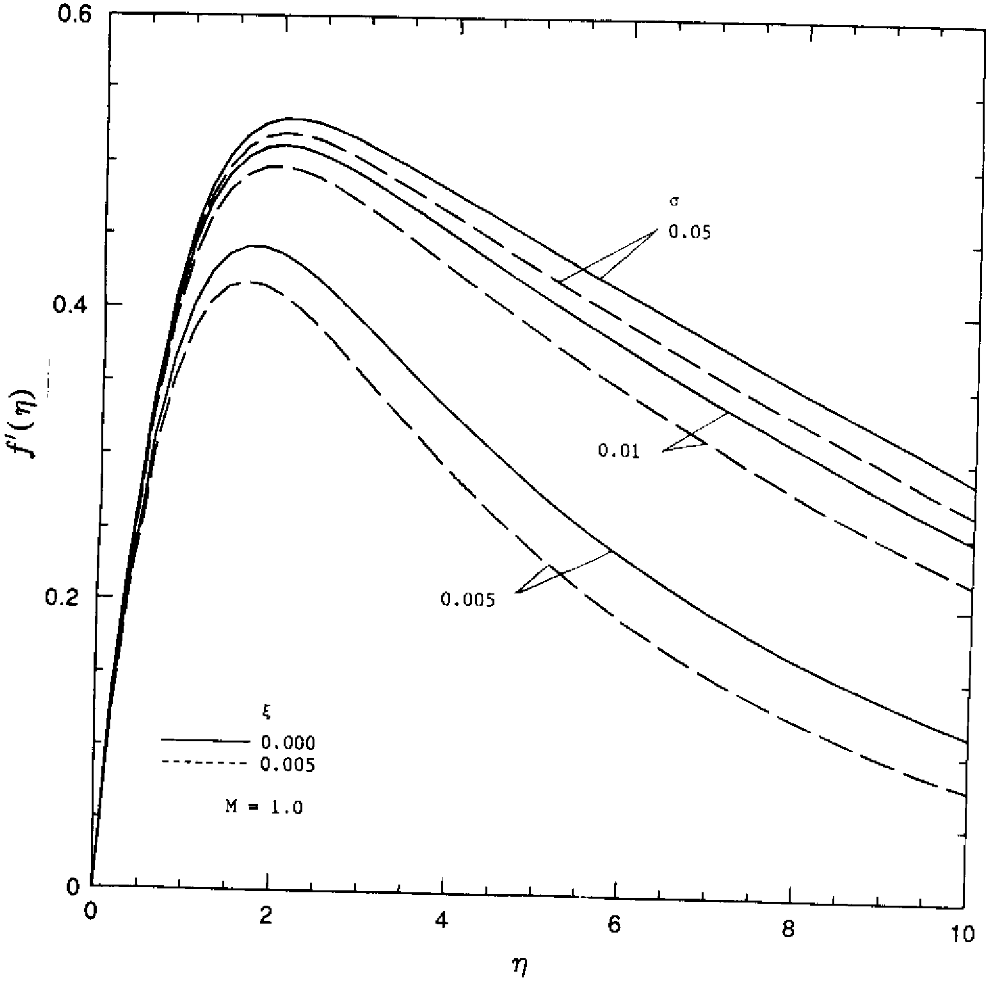

0.6
0.4
0.2
0
$f'(\eta)$
0
2
4
6
8
10
η
σ
0.05
0.01
0.005
ξ
0.000
0.005
M = 1.0

Fig.2a

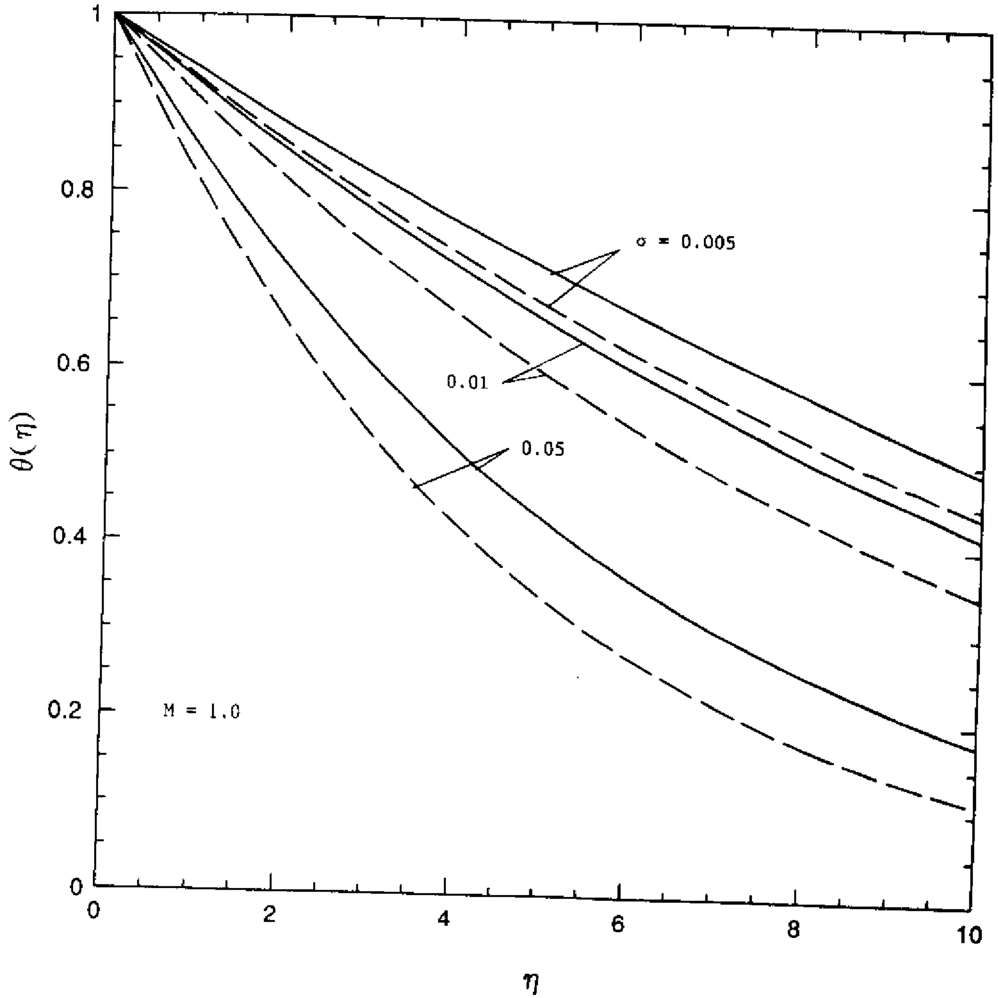

1
0.8
0.6
0.4
0.2
0
$\theta(\eta)$
0
2
4
6
8
10
η
σ = 0.005
0.01
0.05
M = 1.0

Fig.2b