MAGNETIC TURBULENCE AND ANOMALOUS TRANSPORT

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ABSTRACT

The self consistency conditions for magnetic turbulence are reviewed. The principal features of a magnetic topology involving stochastic flux lines - transport, ambipolarity, basic equations - are summarized. Two driving sources are considered : thermal effects which require large scale residual islands and electron diamagnetism which involves fluctuation scales smaller than the ion Larmor radius and a β_p threshold of order one. Detailed stability criteria and transport coefficients are given in each case.

KEYWORDS : turbulence, transport, self consistency.

I - INTRODUCTION

Though there exists no direct measurements of fluctuating magnetic fields, their existence is suggested by some experimental facts in Tokamaks. The most striking are the large value of the electron thermal diffusivity compared to the particle diffusion [GONDHALEKAR, 1989], the degradation of the confinement with additional power [JET Team, 1989], the loss of runaway electrons during the transition from ohmic to L regime (KWON et al, 1988). Magnetic turbulence is an attractive candidate to explain these effects, because it affects the confining topology and thereby the electron motion. Such a situation is reached when the typical width $2\delta_I$ of the islands created by the various modes exceeds the distance d between resonant surfaces, i.e., when the Chirikov parameter $\sigma = 2\delta_I/d$ is larger than a threshold value σ_c of order one. Diffusion coefficients reach standard values of order 1 m^2s^{-1} for rather low values of the fluctuating magnetic fields, typically 10^{-4}+10^{-3}.

As any turbulence in a confined plasma, the fluctuating magnetic fields must be self consistent. This requires the computation of the microcurrents induced by that fields - these currents are essentially carried by electrons - and the check of their coherence through the Ampère equation. Moreover, the associated electric potential fluctuations cannot be neglected so that the ion dynamics must be taken into account. This program can be systematically performed in the linear regime and has led to the microtearing modes (HAZELTINE et al, 1975), which are unstable in collisional regimes (DRAKE and LEE, 1977 ; MAHAJAN et al, 1979 ; DRAKE et al, 1980 ; FARENGO et al, 1983) and stable in collisionless regimes (TSANG et al, 1978). However, these features are dramatically changed in non linear regimes where the induced microcurrents are determined by the stochasticity of the trajectories rather than by the linear resonances (SAMAIN, 1984 ; REBUT et al, 1986 ; GARBET et al, 1988 and 1990). Indeed, a weak particle diffusion is able to influence the mode stability and a non linear study is necessary to compute the actual thresholds and the associated heat transport. Two limits in the non linear regime are of interest : in the first one, corresponding to a Chirikov parameter σ near the stochasticity threshold σ_c, $\sigma \approx \sigma_c$, the fluctuating magnetic fields create residual islands surrounded by an ergodic sea ; the second case corresponds to a fully stochastic situation, $\sigma \gg \sigma_c$. In both cases, a radial electric field, linked to the electron density and temperature gradients, confines the electrons so as to ensure the ambipolarity. The instability may be driven by two basic mechanisms. First, the inductive voltage can drive microcurrents sustaining the unstable magnetic perturbations through thermal effects (REBUT and SAMAIN, 1972 ; REBUT and HUGON, 1984 ; WHITE and ROMANELLI, 1989). These effects are destabilizing when there exists residual magnetic islands ($\sigma \approx \sigma_c$), where current channels are localized. Secondly, the radial electric field confining the electrons can play a destabilizing role through the electric drift of electrons (SAMAIN, 1984 ; GARBET et al, 1988 and 1990). This mechanism related to diamagnetism is relevant in both non linear regimes ($\sigma \approx \sigma_c$ and $\sigma \gg \sigma_c$), but involves β_p and temperature gradient thresholds. It requires also small island scales to avoid the stabilization by the fluctuating electric fields.

The remainder of this paper is the following. The general features of a magnetic turbulence are reviewed in chapter II. In chapter III, the basic equations for the thermal mechanisms are investigated. Finally, the modes driven unstable by the electron diamagnetism are studied in chapter IV.

II - PROPERTIES OF A MAGNETIC TURBULENCE

II.1 - Magnetic topology

We consider a cylindrical equilibrium field

$$\mathbf{B} = B_0 \left(\mathbf{e}_\varphi + \frac{r}{q(r) R_0} \mathbf{e}_\theta \right)$$

and a potential vector fluctuation directed along $\mathbf{B}$

$$\delta\mathbf{A} = \mathbf{e}_{//} \sum A_{nm\omega} \exp i(m\theta + n\varphi - \omega t) + c.c. \tag{1}$$

The typical frequencies ω are of order of the electron diamagnetic frequencies

$$(\omega^*_{ne}, \omega^*_{Te}) = -\frac{n}{R_o}\frac{T}{eB_\theta}\left(\frac{1}{n_e}\frac{dn_e}{dr}, \frac{1}{T}\frac{dT}{dr}\right)$$

which are themselves smaller than the electron transit frequency $v_e/\mathcal{L}_c$, where $\mathcal{L}_c$ is the correlation length along the perturbed flux lines and $v_e = \sqrt{2T_e/m_e}$ is the electron thermal velocity. We will assume that $\frac{\omega}{n}$ is a constant so that there exists a frame of reference where the turbulence is static, in presence of a radial electric field $-\frac{dU}{dr}$ which is linked to the ratio $\frac{\omega}{n}$

$$\frac{\omega}{n/R_0} = \frac{1}{B_\theta}\frac{dU}{dr} \tag{2}$$

This assumption is not essential but greatly simplifies the analysis and will be used in the following. To each Fourier component $A_{nm} \neq 0$ is associated a resonant surface $r = r_{mn}$ determined by

$$n + \frac{m}{q(r_{mn})} = 0 \tag{3}$$

and an island half width $\delta_{Imn} = \sqrt{\frac{8A_{mn}L_s}{B_0}}$, where $L_s = -\frac{qR_0}{s}\left(s = \frac{r\,dq}{q\,dr}\right)$ is the shear length. In the vicinity of a radius r_0, one may use

$$r_{mn} = \frac{n}{m}\delta_0 + cte \tag{4}$$

where

$$\delta_0 = \left.\frac{rq}{s}\right|_{r=r_0}$$

Defining the Chirikov parameter by $\sigma = \frac{\delta_{Imn} + \delta_{Im'n'}}{|r_{m'n'} - r_{mn}|}$, where $r=r_{mn}$ and $r=r_{m'n'}$ are two successive resonant surfaces, the flux lines are stochastic if σ is larger than a threshold value σ_c of order 1. If $\sigma \gg \sigma_c$, the field lines diffuse radially with a quasilinear diffusion coefficient $D_M = \langle\delta r^2\rangle/2\mathcal{L}$ ($\mathcal{L}$ is the field line length) given by

$$D_M = 2\pi\sum_{m,n}\left|\frac{A_{mn}}{B_0}\right|^2 |K_\theta L_s|\,\delta(r - r_{mn}) \tag{5}$$

Assuming that A_{mn} only depends on m, then the expression (5) reads

$$D_M = \frac{\pi}{128\ R_0}\sum_{m>0}\sigma_m^2\,\delta_{Im}^2 \tag{6}$$

where $\sigma_m = \frac{2m\,\delta_{Im}}{\delta_0}$. That diffusion may be seen as a random walk, with basic steps δ_D after each Kolmogorov length $\mathcal{L}_K$ along the flux lines (RECHESTER et al, 1978 and 1979)

$$\mathcal{L}_K = \frac{L_s}{K_\theta\delta_D} \tag{7}$$

where K_θ is an average poloidal number and

$$\delta_D = \left(\frac{D_M\ L_s}{K_\theta}\right)^{1/3} \tag{8}$$

The diffusion coefficient for particles with a parallel velocity $v_{//}$ is simply

$$D = D_M\,|v_{//}| \tag{9}$$

in the collisionless regime. The distribution function of particles in a stochastic field experiences a cascade process generating components with smaller and smaller transverse scales. The smallest scale δ_s is determined by the residual dissipative processes, a small diffusion for instance. For an associated diffusion coefficient D_s, assumed much smaller than the coefficient D due to the magnetic turbulence, the scale δ_s is

$$\delta_s = \sqrt{\frac{D_s \mathcal{L}_c}{|v_{//}|}} \tag{10}$$

where $\mathcal{L}_c$ is the parallel correlation length of the distribution function, linked to the Kolmogorov length through the relation

$$\mathcal{L}_c = \mathcal{L}_K \operatorname{Log}\left(\frac{D}{D_s}\right) \tag{11}$$

II.2 - The question of ambipolary

Since the diffusion coefficient given by (9) is proportional to the velocity, electrons diffuse faster than ions and the question of ambipolarity arises. Moreover, this asymmetry is reinforced by the ion Larmor radius effects, which reduce the value given by (9) when the island widths are smaller than the ion Larmor radius $\rho_i = m_i v_i / e_i B$, where $v_i = \sqrt{2T_i/m_i}$ is the ion thermal velocity. The electron particle flux must therefore nearly vanish, a condition which of course does not imply the vanishing of the heat flux. In the frame of reference where the islands are static, this cancellation is insured by a radial electric field confining electrons. Following the quasilinear theory, the electron flux is given by

$$\phi_e = -\, n_e\, D_M \left\langle \left(\frac{dn_e}{n_e dr} + \frac{e}{T}\frac{dU}{dr} + \frac{dT}{Tdr}\left(\frac{E}{T} - \frac{3}{2}\right)\right) |v_{//}| \right\rangle \tag{12}$$

and vanishes when

$$-\frac{e}{T}\frac{dU}{dr} = \frac{dn_e}{n_e dr} + \frac{1}{2}\frac{dT}{Tdr} \tag{13}$$

in the collisionless regime. The expression (12) is valid in any frame of reference by changing $\frac{e}{T}\frac{dU}{dr}$ into $\frac{e}{T}\left(\frac{dU'}{dr} + \frac{B_\theta R_0}{n}\omega'\right)$, where $-\frac{dU'}{dr}$ and ω' are the radial electric field and the frequency in the new frame. The condition (13) implies therefore that the turbulence rotates at the electron diamagnetic frequency in the frame where the radial electric field vanishes. Once this condition is fulfilled, the thermal flux ϕ_{Te}, which is given by an average over the energy E similar to (12), reduces to

$$\phi_{Te} = -\frac{3}{\sqrt{\pi}}\, n_e\, D_M\, v_e \frac{dT}{dr} \tag{14}$$

II.3 - Self consistency and basic equations

The potential vector fluctuations must be sustained by microcurrents, through the Ampère equation

$$\frac{d^2 A_{mn}}{dr^2} - \left(\frac{m}{r}\right)^2 A_{mn} = -\,\mu_0\, J_{//mn}$$

In the simplest case, $J_{//mn}$ is localized around the m,n resonant surface. Outside the current layer, $A_{mn}(r)$ decreases as $\exp - |K_\theta (r - r_{mn})|$ and a standard matching procedure provides the constraint

$$2|K_\theta|\ A_{mn} = \mu_0 \int J_{//mn}\ dr \tag{15}$$

The problem is then to calculate $J_{//mn}$ in term of A_{mn}. Generally, the current is derived from a Fokker-Planck equation

$$\left\{v_{//}\nabla_{//}. + \mathbf{v_E}.\nabla. + \frac{e}{m_e}\left(E_{ind} - \nabla_{//}(U + \delta U)\right)\frac{\partial}{\partial v_{//}}. - D_s\Delta. - \mathbf{C}\right\}(F + \delta F) = 0 \tag{16}$$

$$\delta J_{//} = e \int d_3\vec{v}\ v_{//}\ \delta F = \sum_{m,n} J_{//mn}(r)\ \exp i(m\theta + n\varphi) + C.C.$$

where $(F + \delta F)(x,\mu,v_{//})$ is the electron distribution function, $\nabla_{//}.$ is the gradient along the perturbed flux lines, D_s is a small diffusion coefficient which solves small scale divergences, E_{ind} is the inductive field, $\mathbf{C}$ is a collision operator that will be restricted to an electron ion friction term. The fluctuating electric potential δU, which is involved in the electric drift velocity $\mathbf{v_E} = \mathbf{B} \times \nabla(U + \delta U)/B^2$ depends on the ion behaviour and is determined by the electro-neutrality constraint. The calculation of $\delta J_{//}$ and δU is easily performed in the linear collisionless and collisional regimes, leading respectively to stable and unstable situations described in §IV. Two kinds of non linear situations occur according to the value of the Chirikov parameter.

i) Well beyond the stochastic threshold, all flux lines are stochastic and the only source of instability is the diamagnetism. The integration of equation (16) is a difficult task since one has to deal with components of δF at the very small scale δ_s given by equation (10). However, the components of δF at larger scales of order δ_D (see eq. (8)), which determine the response $\delta J_{//}$ coherent with δA, can be obtained by simulating the non linear effects in equation (16) by diffusion operators applied to the non adiabatic part of the distribution function (HIRSCHMAN AND MOLVIG, 1979 ; DIAMOND and ROSENBLUTH, 1981). For instance, discarding the drift and voltage terms in eq. (16) leads to the set of equations

$$i\ K_{//}v_{//}\ F_{mn} - \sum_{m'n'} D_{mn}^{m'n'}\Delta F_{m'n'} = -\ i\ v_{//}\ \frac{m}{r}\frac{\partial F}{\partial r}\frac{A_{mn}}{B} \tag{17}$$

where $K_{//} = \frac{1}{R_0}\left(n + \frac{m}{q(r)}\right)$ and the $D_{mn}^{m'n'}$ are proper diffusion coefficients scaling as the quasilinear value $D_M |v_{//}|$. A simple case corresponds to a diagonal matrix $D_{mn}^{m'n'}$, i.e.

$$D_{mn}^{m'n'} = D_M |v_{//}|\ \delta_{mm'}\ \delta_{nn'} \tag{18}$$

This procedure has been successfully checked by a numerical computation (GARBET et al., this conference).

ii) For a Chirikov parameter near the threshold value, there exists residual islands where the flux lines are integrable over magnetic surfaces $\psi(r,\theta,\varphi) = cte$, with an helicity corresponding to the associated vector potential. Inside these islands, and neglecting all diamagnetic effects, the integration of eq. (16) leads to a temperature T and a current $J_{//}$ which are flux functions $T(\psi)$ and $J_{//}(\psi)$ and are related through the Ohm law

$$\eta(T)\ J_{//} = E_{ind} \tag{19}$$

and the averaged heat equation

$$\left\langle -\ n_e\ \chi_s \Delta T\right\rangle_\psi = P(T) = \frac{E_{ind}^2}{\eta(T)} - P_{rad}(T) \tag{20}$$

where $\chi_s = \frac{3}{2}D_s$ is the thermal diffusivity in the island, $\eta(T)$ is the resistivity, $P_{rad}(T)$ the radiation power, and the bracket means a space averaging between neighbouring surfaces ψ and $\psi + d\psi$.

III - THERMAL EFFECTS

The basic idea of these effects is the filamentation of the equilibrium current $J_{//}$ by residual islands through eq. (19), (20), providing microcurrents which can sustain the corresponding perturbations [REBUT and SAMAIN, 1972 ; REBUT and HUGON, 1984 ; REBUT et al., 1986 ; WHITE and ROMANELLI, 1989]. Such a regime corresponds to a Chirikov parameter near the threshold value and is difficult to study accurately for it is intermediate between a single island integrable topology and a fully stochastic case. The self consistency involves the ratio $\delta J_{//}/J_{//}$ which must be calculated. By using the Ampère equation for the equilibrium current $J_{//}$

$$\frac{1}{r}\frac{d}{dr}\left(\frac{r^2\ B_0}{q(r)\ R_0}\right) = \mu_0\ J_{//}(r)$$

one obtains from eq. (15) the constraint

$$|K_d|\ \delta_I = 4\left(\frac{2}{s} - 1\right) C \tag{21}$$

where the correlation C is defined by

$$C = \int_{-\infty}^{+\infty} d\rho \int_{-\pi}^{+\pi} \frac{du}{2\pi}\ \cos u\ \frac{\delta J_{//}(\rho, u)}{\bar{J}_{//}} \tag{22}$$

with

$$\rho = \frac{\ \ }{r_{mn}}\ ,\ u = m\ \theta + n\ \varphi + \begin{cases} \arg(A_{mn}) & \text{if } B_0\frac{s}{q} > 0 \\ \arg(A_{mn}) + \pi & \text{if } B_0\frac{s}{q} < 0 \end{cases}$$

and $\bar{J}_{//} = J_{//}(r_{mn})$.

III.1 - Island cooling

This first class of mechanism (REBUT and SAMAIN, 1972 ; REBUT and HUGON, 1984) is related to the heat balance within the residual islands. Inside such an island, the total current $\bar{J}_{//} + \delta J_{//}(\psi)$ is related to the temperature $\bar{T} + \delta T(\psi)$ through the Ohm law (19) which reads

$$\frac{\delta J_{//}}{\bar{J}_{//}} = \left(1 + \frac{\delta T}{\bar{T}}\right)^{3/2} - 1$$

where the bars refer to the unperturbed values on the resonant surface. The temperature $\bar{T} + \delta T(\psi)$ is solution of the heat equation (20) where the heat source $P(T) = E_{ind}^2/\eta(T) - P_{rad}(T)$ will be restricted to its unperturbed value $P(\bar{T})$. We will then admit that the magnetic topology can be represented by isolated islands surrounded by a turbulent sea. The equation of a m,n island chain is

$$\psi = 2\rho^2 + \cos u \tag{23}$$

where $-1 \leq \psi \leq 1$, $\psi = 1$ corresponds to the separatrix. Note that in fact, the size of islands is reduced by the ergodic sea. The solution of the heat equation is then

$$\delta T(\psi) = \delta_I^2 \frac{P(\bar{T})}{4\ \bar{n}_e\ \chi_s} (1 - \psi) \tag{24}$$

As expected, δT is positive when $P(\bar{T}) > 0$, i.e., when the Ohmic heating dominates. Conversely, δT is negative when the radiation cooling overcomes the Ohmic input, a situation which can occur at the edge. The contribution of this temperature fluctuation in the correlation (22) is

$$C_1 = \frac{3}{2} \int_{|\psi|<1} d\rho \frac{du}{2\pi} \cos u \frac{\delta T(\psi)}{\bar{T}}$$

which gives, using (24),

$$C_1 = -\frac{4}{5\pi} \frac{P(\bar{T})}{\chi_s\ \bar{n}_e \bar{T}} \delta_I^2 \tag{25}$$

In view of eq. (21), an island heating, $P(\bar{T}) > 0$, is stabilizing ($C<0$) whereas an island cooling is destabilizing ($C>0$). The actual threshold on the island size can be obtained by solving equation (21). An order of magnitude is available by noting that the power P is linked to the equilibrium temperature gradient length L_T and the thermal diffusivity χ in the ergodic sea

$$P \sim \bar{n}_e\ \chi \frac{\bar{T}}{L_T^2}$$

so that (21) becomes

$$s\ |K_d|\ \delta_I \sim \frac{\chi}{\chi_s} \left(\frac{\delta_I}{L_T}\right)^2 \tag{26}$$

where the left side quantity scales as the Chirikov parameter and is therefore of order 1. It is interesting to remark (DUBOIS, 1990) that the diffusivity χ_S inside the island can be much smaller than the diffusivity χ in the ergodic sea. In the case of the island cooling, reasonable island widths $\delta_I \sim \sqrt{\chi_S/\chi}\ L_T$ can therefore be obtained. For such island scales, of the order of the centimeter, the diffusion coefficient given by (6) and (9) is fairly large, i.e., several $m^2\ s^{-1}$.

III.2 - Current profile effects

The second step in the study of thermal effects should consist of calculating the current fluctuations due to the inductive field in the ergodic sea. This necessitates in principle to solve the Fokker Planck equation (16) in the stochastic magnetic field. A crude approximation consists of restricting the temperature and the current to their θ,φ averaged values. The temperature can be then approximated by

$$T = \overline{T} + \delta_I \left(\frac{\partial T}{\partial r}\right) \rho + \frac{1}{2} \delta_I^2 \left(\frac{\partial^2 T}{\partial r^2}\right) \rho^2$$

outside and near an island chain. Using again the Ohm law and noting that the correlation (22) vanishes for a function depending on ρ over all the space (inside and outside the island), one finds that the ergodic sea contributes to $\mathbf{C}$ by

$$\mathbf{C}_2 = K \delta_I^2 \left\{ \left(\frac{\partial T}{\overline{T}\ \partial r}\right)^2 + \frac{2}{\overline{T}} \frac{\partial^2 T}{\partial r^2} \right\} \qquad (27)$$

where

$$K = -\frac{3}{8} \int_{|\psi|<1} d\rho \frac{du}{2\pi} \rho^2 \cos u = \frac{1}{5\pi}$$

Note that the first order contribution in δ_I/L_T vanishes since it corresponds to an odd current with respect to ρ. Consequently, only the terms of order δ_I^2/L_T^2 remain and they are destabilizing if $\frac{d^2 J_{//}}{J_{//} dr^2}$ is positive [WHITE and ROMANELLI, 1989]. The self consistency constraint (21) provides the scaling

$$| K_\theta | \ s\ \delta_I \sim \frac{\delta_I^2}{L_T^2} \sim \sigma_c$$

This last condition can hardly be satisfied since it involves an island width of the order of the temperature gradient length.

IV - DIAMAGNETIC DESTABILIZATION OF MICROTEARING MODES

These modes were initially found to be unstable in the linear collisional regime (HAZELTINE et al, 1975), with a maximum growth rate when the electron-ion collision frequency ν_c is of order of the diamagnetic frequency ω_e^*, while it was later shown that they are linearly stable in the collisionless regime (TSANG et al., 1978). The trapped electrons have been shown to be weakly stabilizing (CONNOR et al, 1988). However, this classical picture is drastically changed in the non linear regime. First, a β_p threshold of order 1 is involved in both regimes. Secondly, the induced transport coefficients in collisional regimes remain less than the actual experimental values, except at the edge of tokamaks. Finally, microtearing modes can be unstable in collisionless non linear regimes, because the parallel impedance of electrons is modified by the induced diffusion. This picture would not be comprehensive without stating the stabilizing effect of the fluctuating electric potential, except if the island widths are much smaller than the ion Larmor radius. This determines indeed an upper bound for the diffusivity.

IV.1 - Summary of the linear results

The linearization of the Vlasov equation (16) provides the following expression for the current harmonics in collisionless regime

$$J_{//mn} = -\frac{n_e e^2}{T}\left\langle \frac{\omega - \omega_e^*}{\omega - K_{//} v_{//} + i\varepsilon} v_{//}^2 \right\rangle A_{mn} \qquad (28)$$

where the inductive field has been neglected, the brackets indicate a velocity average, $\omega = \frac{n}{R_0 B_\theta}\frac{dU}{dr}$ is the mode frequency in the frame where the radial electric field vanishes, $\omega_e^* = \omega_{ne}^* + \omega_{Te}^*\left(\frac{mv^2}{2T} - \frac{3}{2}\right)$ is the velocity dependent diamagnetic frequency and $K_{//} = \frac{1}{R_0}\left(n + \frac{m}{q(r)}\right)$ is the parallel wave number. The width δ_e of the $J_{//mn}(r)$ profile is given by $\delta_e \sim |\omega L_s / K_\theta v_e|$. In the frame of a constant A_{nm} approximation, the self consistency constraint (15) then reads

$$2\,|K_\theta|\,A_{mn}(0) = i\pi\mu_0 \frac{n_e e^2}{T}\int dr \left\langle (\omega - \omega_e^*) v_{//}^2\, \delta(\omega - K_{//} v_{//}) A_{mn}\right\rangle \qquad (29)$$

The current integral is in quadrature of phase with A_{mn} and must vanish. This occurs when

$$\omega = \omega_{ne}^* + \frac{1}{2}\omega_{Te}^* \qquad (30)$$

which is equivalent to the condition (13). Once this condition is fulfilled, no current integral in phase with A_{mn} remains. This means that a microtearing mode is stable within the constant A_{mn} approximation. Moreover, it can be numerically checked that this property is true whatever the A_{mn} profile. In the collisional regime, the response (28) is replaced by

$$J_{//mn} = -\frac{n_e e^2}{T}\left\langle \frac{\omega - \omega_e^*(v^2)}{\omega + i\nu_c \frac{v_e^3}{v^3} - \frac{K_{//}^2 v^2}{3\omega}} \frac{v^2}{3} \right\rangle A_{mn} \qquad (31)$$

where ν_c is the thermal electron-ion collision frequency. The width of the current layer is now given by $\delta_e \sqrt{\nu_c/\omega}$. After an integration of $J_{//mn}$ over r, the self consistency constraint (15) leads to

$$\omega = \omega_{ne}^* + \alpha\,\omega_{Te}^* \qquad (32)$$

$$\frac{\beta_p^*}{|K_\theta \delta_e|} = \frac{1}{\mathrm{Re}(\Lambda)} \qquad (33)$$

where equation (32) is similar to eq. (30) and (13), with α determined by $\mathrm{Im}(\Lambda) = 0$. The function Λ is defined by

$$\Lambda = 4i\sqrt{\frac{\pi}{3}}\int_0^{+\infty} dy\, y^3 \exp - y^2 \,\frac{\alpha + \frac{3}{2} - y^2}{\left(1 + i\frac{\nu_c}{\omega y^3}\right)^{\frac{1}{2}}}$$

and depends only on $\frac{\nu_c}{\omega}$. The parameter β_p^* is

$$\beta_p^* = \frac{n_e T}{B^2/2\mu_0}\left(\frac{L_s}{L_n}\right)^2 (1 + \alpha\eta_e)\,\frac{\eta_e}{4} \tag{34}$$

with $\eta_e = \frac{L_n}{L_T}$, and scales as the equilibrium β_p. The values of α and $\beta_p^*/K_\theta\delta_e$ with respect to the collision parameter ν_c/ω are given on figure 1. It is verified that α varies from 1/2 to 5/4 when the collisionality increases and that the lowest threshold $\beta_p^*/K_\theta\delta_e$ corresponds to $\nu_c/\omega \sim 1$. An important feature of the linear regime is that the width δ_e or $\delta_e\sqrt{\nu_c/\omega}$ of the current layer is typically much smaller than the ion Larmor radius ρ_i. This allows to neglect the influence of the electric potential δU.

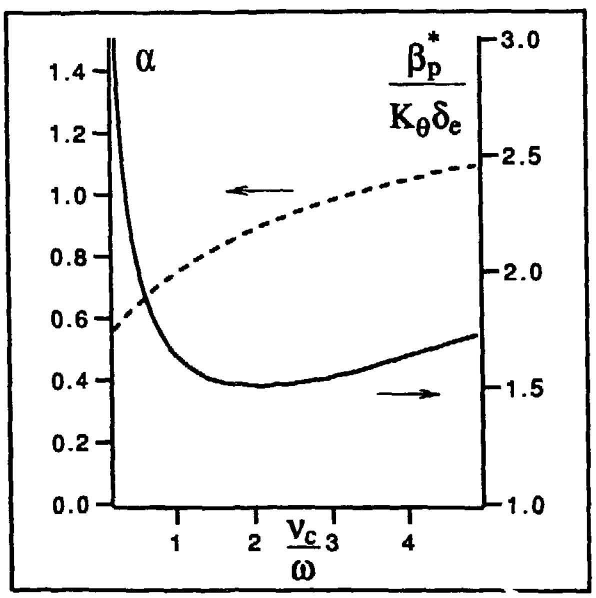


Fig. 1: Values of $\alpha = (\omega - \omega_{ne}^*)/\omega_{Te}^*$ (dashed line) and marginal treshold $\beta_p^*/K_\theta\delta_e$ (solid line) as functions of the collision parameter ν_c/ω.

IV - Non linear regime

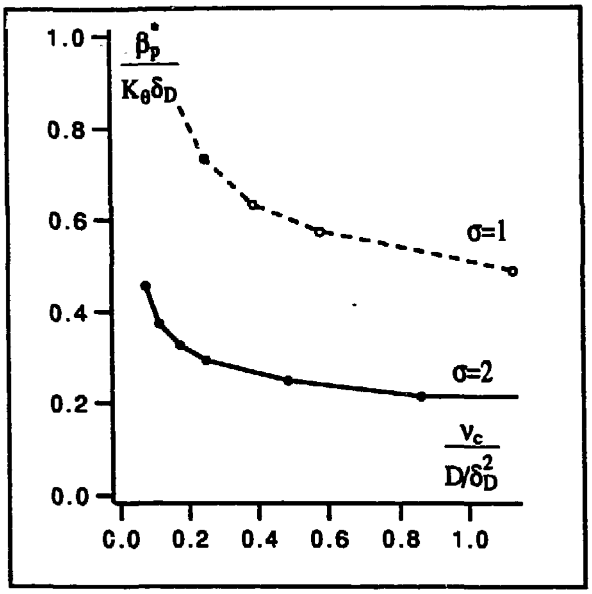
The non linear regime is reached when an electron actually experiences the stochasticity of flux lines. This is realized when the diffusion scale δ_D given by (8) is larger than the linear current width δ_e in collisionless regime or $\delta_e\sqrt{\nu_c/\omega}$ in collisional regime. This allows the resolution of the Fokker-Planck equation (16) with a perturbative procedure where the small parameter is $\delta_e/\delta_D \,\mathrm{Max}\left(1, \sqrt{\nu_c/\omega}\right)$. It can be shown (GARBET et al., 1988) that the zeroth order currents $J_{//mn}(r)$ are in quadrature of phase with the A_{mn}'s. Cancelling these currents is equivalent to cancelling the electron flux $\left\langle \frac{\delta J_{//}\delta B_r}{eB} \right\rangle$ and, as expected, this determines the static electric field $-\frac{dU}{dr}$ through a relation of the type (13). The first order currents are in phase with the A_{mn}'s. However, it can be proved that the integrals $\int dr\, J_{//mn}(r)$

do not vanish in two situations only : in collisional regime or in the case where the A_{mn}'s are not constant within the current layers. This last condition imposes a threshold $\beta_P = 1$ for the instability to occur.

IV.1. <u>Collisional regime</u>

The non linear condition $\delta_D > \delta_e \sqrt{\nu_c/\omega}$ imposes that $D/\delta_D^2 > \sqrt{\omega \nu_c}$. On the other band, the collisional impedance insuring instability is maintained if the collision time ν_c^{-1} is smaller than the diffusion time δ_D^2/D necessary for an electron to diffuse accross a current width. This imposes an upper bound to the induced diffusion coefficient

$$D < \nu_c \, \delta_D^2 \tag{35}$$



<u>Fig.2</u>: Marginal threshold $\beta_p^*/K_\theta\delta_D$ as a function of the collision parameter $\nu_c\delta_D^2/D$. The dashed line corresponds to a Chirikov parameter σ equal to 1 and the solid line corresponds to σ=2.

This situation has been examined [GARBET et al., 1988] by replacing the Fokker Planck equation (16) by the system

$$\left| \begin{array}{l} \nabla_{//} J - D_s \, \Delta N = - e \, \mathbf{v_E} . \nabla N \\ \\ \frac{eT}{m_e} \nabla_{//} N - D_s \, \nabla J + \nu_c J = - \mathbf{v_E} . \nabla J \end{array} \right. \tag{36}$$

where $N(\mathbf{x}, E) - n_e \frac{eU(\mathbf{x})}{T}$ and $J(\mathbf{x}, E)$ represent the density and current of electrons at a given energy E. Following the perturbative procedure described above, this system has been solved by a direct Fourier inversion. The threshold $\beta_p^*/K_\theta\delta_D$ is given in figure 2 as a function of the collision parameter $\nu_c\delta_D^2/D$ for a given Chirikov parameter. Note that the turbulent field must produce overlapping magnetic islands ($\sigma >> \sigma_C$) so that the actual values of $|K_\theta\delta_D|$ cannot be much smaller than 1.

Consequently, it is verified from figure 2 that for reasonable values of $\beta_p^* \sim 1$, the diffusion coefficient D must satisfy the constraint (35). Moreover, if the electric potential fluctuations are taken into account, it appears that the modes are stable except if $\delta_D \leq \rho_i$, so that

$$D \leq v_c\, \rho_i^2 \tag{37}$$

which provides coefficients D in agreement with experimental values only at the edge of tokamaks.

IV.2. Collisionless regime

A diamagnetic driving mechanism basically involves the electron drift which appears in the frame of reference where the islands are static. Indeed, assuming that the scale δ_D is much smaller than the ion Larmor radius, the electric potential fluctuations can be neglected and the ion electric drift is compensated by the polarization drift so that the charge balance equation is

$$\nabla_{//}\, \delta J_{//} + e\ \nabla.(\delta n_e \mathbf{v_z}) = 0$$

Using eq. (13), the perturbed parallel current scales as

$$\delta J_{//} \sim K_\theta\ \frac{n_e T}{B} \left(\frac{\mathcal{L}_c}{L_n}\right) \frac{\delta n_e}{n_e} \tag{38}$$

where $\mathcal{L}_c \sim L_s/K_\theta\delta_D$ is the correlation length given by eq. (11). The density fluctuations, which tend to satisfy $\nabla_{//}(n_e + \delta n_e) = 0$, scale as

$$\frac{\delta n_e}{n_e} \sim K_\theta \left(\frac{\mathcal{L}_c}{L_n}\right) \frac{\delta A}{B} \tag{39}$$

Within the frame of the constant A_{mn} approximation, the self consistency constraint (15), eq. (38) and (39) provide the order of magnitude for $\beta_p^*/K_\theta\delta_D$

$$\frac{\beta_p^*}{|K_\theta\delta_D|} \sim \frac{n_e T}{B^2/2\mu_0} \left(\frac{L_s}{L_n}\right)^2 \frac{1}{|K_\theta\delta_D|} \sim 1 \tag{40}$$

Since $|K_\theta\delta_D|$ scales as the Chirikov parameter, i.e, is of order 1, the β_p^* threshold must also be of order 1. This is also a consequence of the general result quoted above, which states that in non linear collisionless regime, unstable modes correspond to $A_{mn}(r)$ components varying significantly over a scale δ_D.

A calculation of this threshold may be performed by replacing the Vlasov equation (16) by a set of equations of type (17), determining each Fourier harmonic F_{mn} at the scale δ_D. For reference, figure 3 shows the actual F_{mn} deduced from a direct computation of the Vlasov equation in a simple case $V_E = 0$, A_{mn} = cte and the modelized values deduced from (17) (GARBET et al., this conference).

This modelization allows an easier treatment of the non linear problem and a direct insight in the involved mechanisms (GARBET et al., 1990). Restricting in a first approach the matrix $D_{mn}^{m'n'}$ to a diagonal term $D = D_M |v_{//}|$ (cf. eq. (18)) and assuming

This is believed to be due to wall pumping, which is triggered by an increase in the area seen by the plasma when the ergodic divertor is activated. Th' that the operator $- D_M \Delta$ may be replaced by a constant $\delta K_{//} = |K_\theta \delta_D / L_s|$, the current response is simply

$$J_{//mn} = - \frac{n_e e^2}{T} \left\langle \frac{(\omega - \omega_e^*)\, v_{//}^2}{\omega - K_{//} v_{//} + i\, \delta K_{//} |v_{//}|} \right\rangle A_{mn} \qquad (41)$$

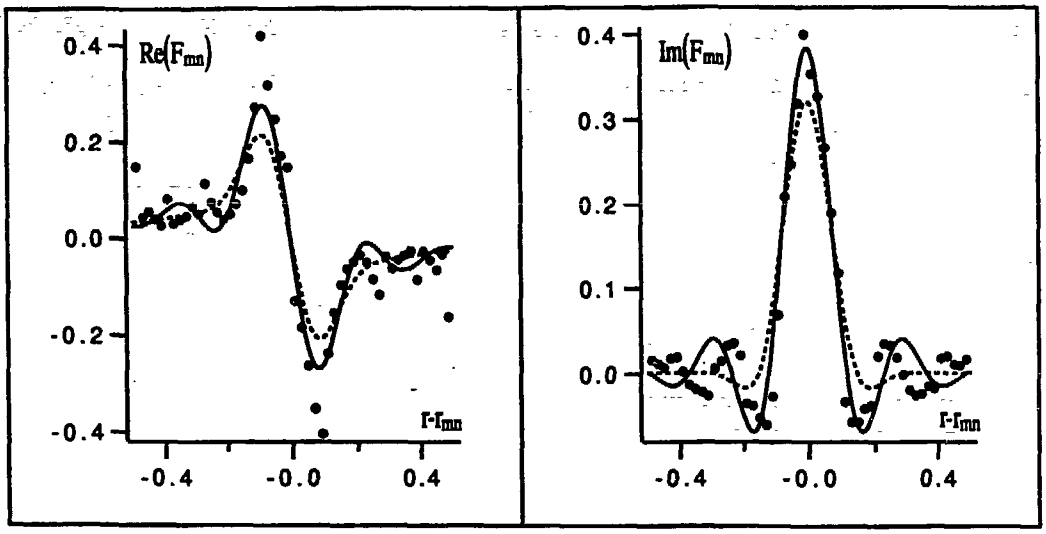


Fig. 3: Comparison of the Fourier harmonics m,n of the electron distribution function given by a numerical code (points) and the values derived from eq. (17) for a diagonal matrix $D_{m,n}^{m',n'}$ (dashed line) and for the case of coupling to the neighbouring harmonics (solid lines). In the numerical computation, 11 island chains ($m=1, n=5,..,-5$, $\arg(A_{mn})=0$) were implemented, corresponding to a Chirikov parameter $\sigma=2$. The scales are $\delta_{Imn} = 0.2$ and $\delta_D = \left(\frac{\pi\sigma}{64}\right)^{1/3} \delta_I \approx 0.09$.

Performing the expansion in powers of $\delta_e/\delta_D = \omega/\delta K_{//} v_e$, the current at zeroth order is, as expected, in quadrature of phase with A_{mn} and vanishes if $\omega = \omega_{ne}^* + \frac{1}{2}\omega_{Te}^*$ whatever $A_{mn}(r)$. The first order current, once introduced in the Ampère law leads to

$$- \frac{d^2 A_{mn}}{dr^2} + \frac{\beta_p^*}{\delta_D^2} V(r - r_{mn})\, A_{mn} = - K_\theta^2 A_{mn} \qquad (42)$$

where

$$\delta_D = \left(\frac{D_M L_s}{K_\theta}\right)^{1/3}$$

$$V(x) = \frac{\delta_D^2 \left(\delta_D^2 - x^2\right)}{\left(x^2 + \delta_D^2\right)^2}$$

$$\beta_p^* = \frac{n_e T}{B^2/2\mu_0} \left(\frac{L_s}{L_n}\right)^2 \left(1 + \frac{\eta_e}{2}\right) \frac{\eta_e}{4}$$

A very strong effect is observed
for $I_{ED} > 22.5$ kA. It consists

The equation (42) admits solution for β_p^* values which lie approximatively along the line

$$\beta_p^* = 0.25 + 12\,|K_\theta\delta_D| \qquad (43)$$

The stabilizing role of the electric potential fluctuations is evaluated by including their effects in the electron and ion density and current responses and by solving the neutrality condition. Figure 4 shows the variation of $(\delta_D/\rho_i)^2$ with β_p^* Since

$$D = K_\theta\delta_D \left(\frac{\delta_D}{\rho_i}\right)^2 \frac{\rho_i^2\,|v_{//}|}{L_s} \qquad (44)$$

this curve provides an upper bound for D as a function of β_p^*. Typically, for the parameters of figure 4, an upper bound for the thermal diffusivity is obtained

$$\chi \leq 0.019\,\frac{\rho_i^2 v_e}{L_s}\left(\beta_p^* - \beta_{pcr}^*\right) = 7.4\,\frac{A\,T^{3/2}}{B^2 L_s}\left(\beta_p^* - \beta_{pcr}^*\right)$$

where β_{pcr}^* is given by (43) for $K_\theta\delta_D$=0.14, the temperature is in keV, other quantities are in MKSA units and A is the mass number.

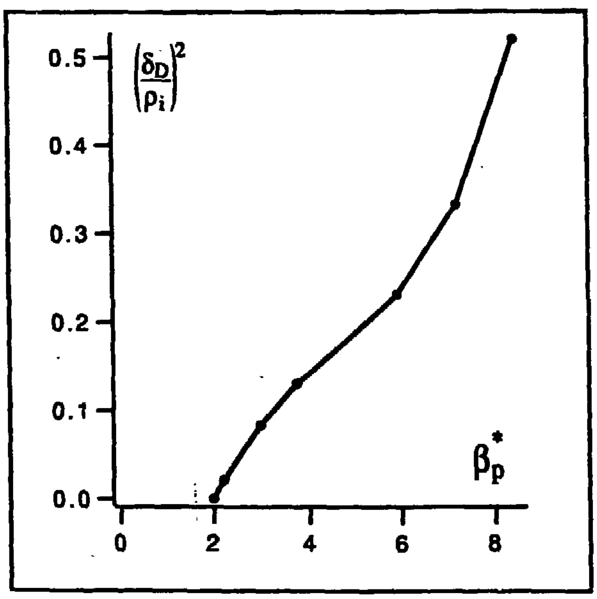


Fig. 4: Curve $(\delta_D/\rho_i)^2 \propto D$ (see eq.(44)) as a function of β_p^*. The parameters are $K_\theta\delta_D$ = 0.14, η_e=2, δ_e/δ_D = 0.2, q=2.

V - CONCLUSION

While the magnetic turbulence is an attractive candidate to explain the anomalous heat transport, it exists in situations which are not necessarily reached in tokamaks. Two basic mechanisms have been quoted : thermal effects and diamagnetic destabilization. Concerning the first one, the filamentation of the equilibrium current demands large residual islands, with a scale comparable to the temperature gradient length. This constraint is somewhat released if the thermal insulation

of composition: on the inner wall, the metal is always not very far radially from the graphite tile (1 cm at most) whereas the outboard limiter is mainly carbon, the metallic structures being located at least 5 cm awa radiall . Seco 1 inside these islands is better than in the plasma bulk. These modes do not involve a β threshold. Conversely, an essential characteristic of the diamagnetic mechanism is that the fluctuation large scales must be small compared to the ion Larmor radius, since the behaviours of electrons and ions with respect to the turbulent electric field must be different. Moreover, such a turbulence requires a poloidal β_p of order 1. It then produces a large transport which could be involved in the β limitation of tokamaks.

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Figure 15 shows that in deuterium shots, the radiation power reduction is nearly proportional to the density reduction. The inverted profiles show that an increase

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