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NEUTRAL PION PHOTOPRODUCTION OFF NUCLEONS AND NUCLEI NEAR THRESHOLD

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I. Introduction

For a long time the $\gamma N \rightarrow N\pi$ reactions have been an active area of research in particle physics. A great variety of elementary amplitudes have been elaborated with success in the framework of the dispersive methods[1,2], the relevant low-energy theorems (LET)[3] and the phenomenological Lagrangian method[4,5]. Interest in the π^0-photoproduction has revived because of the complete review of the experimental data on the proton near threshold[6,7]. The consistent set of the very near threshold experimental results from SACLAY and MAINZ are in strong disagreement with the firm statements of the theoretical predictions based on the LET and PCAC hypothesis: the value of the E_{0+} dipole dominant in the low-energies region is not well described but is in accordance with the dispersive BDW amplitude[1]. However, as it will be shown further, in this version of an elementary amplitude certain corrections connected with the vector meson exchange have not been included. These corrections lead to the same result as LET and PCAC.

Recently, much attention has been paid to the experimental investigation of the coherent π^0 photoproduction off nuclei at the threshold [8,9,10]. The main task of these investigations has been an extraction of additional information about the elementary amplitude, particularly, about the values of the M_{1+} and M_{1-} -multipoles. As has been already shown in Refs.[10,11] and will be discussed in the present paper, the (γ,π^0) reaction on nuclei is very sensitive to different model predictions of these multipoles. On the other hand, by means of this process one can get a unique tool to examine the properties of the πA -

interaction at very low energies and obtain information about the behaviour of a pion inside nuclei. The intensive experimental investigations in MAINZ, SACLAY and Tomsk gave an opportunity to solve the above-mentioned problems.

One of the methods widely used for theoretical description of the coherent pion photoproduction off nuclei is the distorted wave impulse approximation (DWIA). At present time there are two versions of the DWIA: 1) the traditional DWIA in the coordinate space[12]) and 2) the DWIA in the momentum space[11]). Both give similar results but differ significantly in the evaluation of the effects of the pion wave distortion in the final state: 50–100% and $\lesssim 14\%$, respectively. The main reason of this discrepancy is connected with different description of the pion behaviour inside a nucleus and consequently with various proposals for the off-shell extrapolation of the pion-nucleon scattering and photoproduction amplitudes. In the present paper we use the momentum space DWIA because in this method one can consistently take into account the nonlocality of the photoproduction operator and off-shell effects.

In Sec. II we shall discuss the general properties of the elementary amplitudes. Note that in contrast with Ref.[11]) we use here the full version of the BL-amplitude where, for example, the Δ-isobar width is $\vec{q}$-dependent. In Sec. III we consider the problems of the nuclear photoproduction amplitude (the Fermi-motion, the off-shell effects). The results are summarized and compared with the available experimental data for ^{4}He, ^{6}Li, ^{12}C, ^{16}O, ^{40}Ca and ^{208}Pb in Sec. IV.

I. The $\gamma N \to N\pi$ reaction

A great amount of the experimental information on the photoproduction on a proton has recently been completed with the unique data in the threshold region[6,7]). These measurements make it possible to make thorough theoretical analyses of the elementary amplitude. Different prescriptions were used to construct the elementary amplitude [1-5]). Of a number of versions we choose two: 1) the BL-amplitude [4,5]) obtained by the phenomenological Lagrangians and 2) the BDW-amplitude [1]) developed in terms of the dispersive relation method.

A. The BDW-amplitude

The pion photoproduction amplitude can be written in the πN c.m. frame as

$$\mathcal{F} = i\mathcal{F}_1 \vec{\sigma}\cdot\vec{\epsilon} + \mathcal{F}_2 \vec{\sigma}\cdot\vec{\eta}\,\vec{\sigma}\cdot[\vec{\varkappa}\times\vec{\epsilon}] + i\mathcal{F}_3 \vec{\sigma}\cdot\vec{\varkappa}\,\vec{\eta}\cdot\vec{\epsilon} + i\mathcal{F}_4 \vec{\sigma}\cdot\vec{\eta}\,\vec{\eta}\cdot\vec{\epsilon}\,, \qquad (1)$$

where $\vec{\varkappa} = \vec{k}/|\vec{k}|$ and $\vec{\eta} = \vec{q}/|\vec{q}|$ are the unit vectors of the photon and pion momenta in the πN c.m. frame; $\vec{\epsilon}$ is the photon polarization vector. In the isospin space, the functions $\mathcal{F}_\alpha$ are usually represented in terms of the isoscalar $\mathcal{F}_\alpha^{(0)}$ and isovector $\mathcal{F}_\alpha^{(\pm)}$ or $\mathcal{F}_\alpha^{(I)}$ parts where I is the isospin of the πN system. In the multipole analyses they are decomposed over the electric $E^I_{\ell\pm}$ and magnetic $M^I_{\ell\pm}$ multipoles.

In the case of the BDW-amplitude the fixed-t dispersion relations were used to evaluate the s-, p- and d-wave multipole amplitudes by using conformal mapping techniques for solving the integral equations. The imaginary parts of the multipole amplitudes were obtained with the help of the Fermi-Watson theorem

$$Im\, f^I_{\ell\pm}(W) = Re\, f^I_{\ell\pm}(W)\cdot tg\,\delta^I_{\ell\pm}(W), \qquad (2)$$

where $f^I_{\ell\pm}(W) \equiv \{E^I_{\ell\pm}(W), M^I_{\ell\pm}(W)\}$; $W = E_\pi(\vec{\tilde{q}}) + E_N(\vec{\tilde{q}})$ is the total energy of the πN system; $\delta^I_{\ell\pm}$ is the πN scattering phase shift. In Ref.[1]) the values of the real parts of the multipoles and the phase shift $\delta^I_{\ell\pm}$ are tabulated for the photon energies from 160 to 500 MeV.

In the threshold region ($E_\gamma <$ 160 MeV) we use the long-wavelength approximation[13]) and the following expression for $f^I_{\ell\pm}$:

$$f^I_{\ell\pm}[W(\vec{\tilde{q}})] = |\vec{\tilde{q}}|^\ell \left(a^I_{\ell\pm} + b^I_{\ell\pm}\, |\vec{\tilde{q}}|^2\right) . \qquad (3)$$

The coefficients $a^I_{\ell\pm}$ and $b^I_{\ell\pm}$ were determined from the equivalence of the logarithmic derivatives at $E^{LAB}_\gamma =$ 160 MeV and interpolation of the multipoles with the help of cubic splines in the $E^{LAB}_\gamma <$ 160 MeV region. The values of the multipoles dominant at the threshold,obtained in this manner, are summarized in Ref.[14]).

The BDW amplitude gives good description of the experimental data in the case of the charged pion photoproduction. However, for the π^o mesons a serious discrepancy arises especially in the low energy region ($E_\gamma \lesssim$ 200 MeV). This result can be due to the absence of the vector meson exchange contribution in the original BDW amplitude. In accordance with Ref.[15]) we have used the pole model to insert this exchange contribution. With this procedure and neglecting the terms of an order of $|\vec{\tilde{q}}|^2/m^2$, for example, in the case of the ω^0 meson exchange we have

$$\mathcal{F}^\omega_1 = \left[(W-m)^2 - |\vec{\tilde{k}}|(E_\pi - |\vec{\tilde{q}}|\cdot x) - R_\omega m^2_\omega (W-m)/2m\right]\cdot D,$$

$$\mathcal{F}^\omega_2 = |\vec{\tilde{q}}||\vec{\tilde{k}}|\cdot\left[1 + R_\omega m^2_\omega/(W+m)/2m\right] W\cdot D/m,$$

$$\mathcal{F}_3^{\omega} = -(W-m)|\vec{q}|\left[1 - R_{\omega}(W-m)/2m\right]\cdot D,$$
$$\mathcal{F}_4^{\omega} = -(W-m)|\vec{q}|^2\left[1 + R_{\omega}(W+m)/2m\right]\cdot D/2m, \qquad (4)$$

where m and m_{ω} are the masses of the proton and ω^0-meson, respectively; $E_{\pi} = \sqrt{m_{\pi^0}^2 + \vec{q}^2}$ is the pion energy in the πN c.m. frame; $R_{\omega} = G_{\omega}^T / G_{\omega}^V$; $G_{\omega}^V(G_{\omega}^T)$ is the vector (tensor) constant of the ωNN vertex;

$$D = \frac{m}{4\pi W}\,\frac{1}{m_{\pi^0}}\,\frac{G_{\omega}^V g_{\omega\pi\gamma}\sqrt{4\pi/137}}{m_{\omega}^2 - m_{\pi^0}^2 + 2|\vec{k}|(E_{\pi} - |\vec{q}|\cdot x)}\,. \qquad (5)$$

We use $g_{\omega\pi\gamma} = 0.374$ for the radiative constant of the ω-meson. This value corresponds to the $\omega\to\pi\gamma$ width $\Gamma_{\omega\to\pi\gamma} = 0.89$ MeV [16]).

The values of the vector and tensor constants are determined not well enough[17,18]):

$$7 \lesssim G_{\omega}^V \lesssim 20 \qquad \text{and} \qquad 0 \lesssim G_{\omega}^T \lesssim 4.5\,.$$

For this reason, one can use them as free parameters in the π^0-photoproduction on a nucleon.

In the case of the ρ^c-meson exchange one can obtain the same expressions by the following substitutions in eqs.(4-5): $m_{\omega}\to m_{\rho}$, $g_{\omega\pi\gamma}\to g_{\rho\pi\gamma} = 0.103$ MeV ($\Gamma_{\rho\pi\gamma} = 0.067$ MeV[19])), $G_{\omega}^{V(T)} \to G_{\rho}^{V(T)}$. The values of the constant at the ρNN vertex are determined better than for the ω-meson[18]):

$$\left(G_{\rho}^V\right)^2/4\pi = 0.55\pm0.06\,, \qquad R_{\rho} = G_{\rho}^T/G_{\rho}^V = 6.1\pm0.6 \qquad (6)$$

B. The BL-amplitude

In terms of the effective Lagrangian method the elementary amplitude is developed on the basis of few relevant Feynman diagrams depicted in Fig. 1, which correspond to the Born terms

(a, b), S-channel Δ-isobar term (c) and ω^c-exchange (d). The last two are phenomenological: the coupling constants and the Δ-isobar width are regarded as free parameters and are evaluated from the analyses of the experimental data in the resonance region.

For our purposes we choose the most popular version of the BL-amplitude. It is the unitary version in which the contributions of the order $(m_\pi/m)^2$ to the Born terms are included[5])

$$\mathcal{F}^{Born}_{p\pi^0} = -\frac{eg}{8\pi W}\left\{\frac{\mu_p}{2E_a(p_a^c-E_a)}\vec{\sigma}\cdot\left[\vec{q}-\frac{E_\pi}{2m}(\vec{q}+2\vec{p}_i)\right]\vec{\sigma}\left[\vec{k}\times\vec{\epsilon}\right]\right.$$

$$+\frac{\mu_p}{2E_b(p_b^o-E_b)}\vec{\sigma}\cdot[\vec{k}\times\vec{\epsilon}]\,\vec{\sigma}\cdot\left[\vec{q}-\frac{E_\pi}{2m}(2\vec{p}_i-\vec{q})\right]$$

$$-i\frac{\vec{p}_i\cdot\vec{\epsilon}}{E_a(p_a^c-E_a)}\sigma\left[\vec{q}-\frac{E_\pi}{2m}(\vec{q}+2\vec{p}_f)\right]-i\frac{\vec{p}_f\cdot\vec{\epsilon}}{E_b(p_b^c-E_b)}\vec{\sigma}\left[\vec{q}-\frac{E_\pi}{2m}(2\vec{p}_i-\vec{q})\right]$$

$$+i\vec{\sigma}\cdot\vec{\epsilon}\left[\frac{m}{E_b(p_b^c+E_b)}\left(E_\pi-\frac{\vec{\sigma}(\vec{p}_i-\vec{q})\vec{\sigma}\cdot\vec{q}}{2m}-\frac{\vec{\sigma}\vec{q}\,\vec{\sigma}\cdot\vec{p}_i}{2m}\right)\left(1+\frac{|\vec{k}|}{2m}k_p\right)\right.$$

$$\left.\left.+\frac{m}{E_a(p_a^o+E_a)}\left(E_\pi-\frac{\vec{\sigma}\cdot\vec{p}_f\,\vec{\sigma}\cdot\vec{q}}{2m}-\frac{\vec{\sigma}\cdot\vec{q}\,\vec{\sigma}\cdot(\vec{q}+\vec{p}_f)}{2m}\right)\left(1-\frac{|\vec{k}|}{2m}k_p\right)\right]\right\}. \quad (7)$$

In eq.(7) we use the same notation as in Ref.[5]**). The Born term (7) is written in the arbitrary frame. One must make the following substitution:** $\vec{p}_i \to -\vec{k}$ **and** $\vec{p}_f \to -\vec{q}$ **to get the expression in the** πN **c.m. frame.**

In the case of the π^0**-photoproduction the** Δ**-isobar term contributes to the amplitudes** $\mathcal{F}_\alpha$ **(in the** πN **c.m. frame) as follows:**

$$\mathcal{F}_1^\Delta = 3x\,M_{1+}^\Delta\ ;\ \mathcal{F}_2^\Delta = 2M_{1+}^\Delta\ ;\ \mathcal{F}_3^\Delta = -3M_{1+}^\Delta\ ;\ \mathcal{F}_4^\Delta = 0\,, \quad (8)$$

where

$$M_{1+}^{\Delta}=-\frac{M_{\Delta}(M_{\Delta}+m)\,\tilde{G}_1\tilde{G}_3}{18\pi W(W^2-M_{\Delta}^2+iM_{\Delta}\Gamma_{\Delta})}\,\frac{|\tilde{\vec{k}}|\,|\tilde{\vec{q}}|}{m_{\pi^+}^3}\,e^{i\Psi_1}, \qquad (9)$$

$$\Gamma_{\Delta}=110\left(\frac{|\tilde{\vec{q}}|}{|\tilde{\vec{q}}_{\Delta}|}\right)^3\frac{M_{\Delta}}{W}\,\frac{1+(R|\tilde{\vec{q}}_{\Delta}|)^2}{1+(R|\tilde{\vec{q}}|)^2}\ \mathrm{MeV}. \qquad (10)$$

The constants in (8-10) have the following values:

$M_{\Delta}=1225$ MeV; $\tilde{G}_1=0.282\sqrt{4\pi/137}$; $\tilde{G}_3=2.18$,
$R=0.007\ \mathrm{MeV}^{-1}$, $|\tilde{\vec{q}}_{\Delta}|=222.9$ MeV.

Inclusion of the ω^0-exchange Born-diagram makes it possible to obtain a good fit to low and Δ_{33} resonance energy data. The corresponding expressions are the same as in eq.(4) with the ωNN-vertex constants $G_{\omega}^{V}=10$ and $G_{\omega}^{T}=0$.

C. The analyses of $E_{0+}^{p\pi^0}$, $M_{1+}^{p\pi^0}$ and $M_{1-}^{p\pi^0}$

At the present time, the reliable experimental information on the threshold π^0-photoproduction off a proton has been obtained in SACLAY[8]) and MAINZ[9]). The values of the leading in this region multipoles E_{0+}, M_{1+} and M_{1-} were extracted from the set of data on the cross-sections, differential in the photon energies ($E_{\gamma}=146.5\div169.2$ MeV [8]) and $E_{\gamma}=132\div157$ MeV [9])), and from the angular distribution on neutral pions for $E_{\gamma}=158$ MeV[9]).

The experimental data and the results of different theoretical analyses are summarized in Table 1. One can see the discrepancy of the $E_{0+}^{p\pi^0}$ (BL) with the experimental data. It is the striking disagreement of the experimental value with one predicted by the low-energy theorems and PCAC hypothesis (which coincides with the BL result).

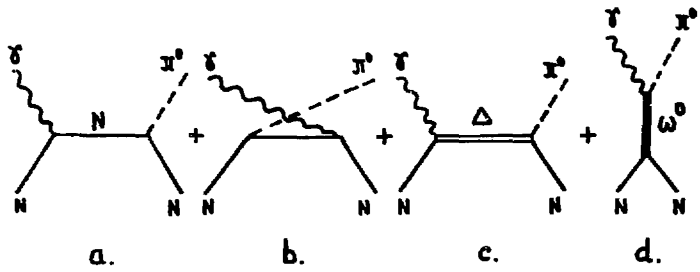

Fig. 1. Different contributions to the BL π^0-photoproduction amplitude [5] . (a) and (b) are the Born terms, (c) is the s-channel Δ formation, (d) is the ω^0 exchange term.

Table 1. Values of the $E_{0+}^{p\pi^0}$ (in $10^{-3}/m_{\pi+}$) and $M_{1\pm}^{p\pi^0}$ (in $10^{-3}\check{q}\check{k}/m_{\pi+}^3$) multipoles at threshold.

	Experiment	BL(PV) [5]	BDW [1]	BDW + ω^0 [14]
E_{0+}	-1.8 ± 0.6 [20] -0.5 ± 0.3 [8]	-2.4	-0.13	-2.4
M_{1+}	8.0 ± 0.3 [8]	7.8	3.5	4.8
M_{1-}	-2.0 ± 1.5 [8]	-4.8	-3.3	-1.4

The only amplitude which agrees with the experimental value for $E_{0+}^{p\pi^0}$ is the BDW one. However, in this case one cannot obtain satisfactory description of the differential cross-sections in the Δ_{33} resonance region and the total cross-sections in the threshold region (see Fig. 2 and 3). As has been pointed out earlier, the reason can be connected with the absense of the vector meson exchange contributions in the BDW amplitude. Regarding the coupling constants in the ωNN -vertex as free parameters, one can get the description of the above-mentioned cross sections in the Δ_{33} region and at the threshold when

$$G_\omega^V = 15.0 \qquad \text{and} \qquad G_\omega^T = 4.39. \tag{11}$$

The value of the $E_{0+}^{p\pi^0}$ dipole in this case is in good agreement with $E_{0+}^{p\pi^0}$ (PCAC):

$$E_{0+}^{p\pi^0}(BDW+\omega^c) = E_{0+}^{p\pi^0}(BDW) - \frac{e}{8\pi W}\frac{g_{\omega\pi\gamma}}{m_\omega^2+m_{\pi^0}^2}\left[m_\omega^2 G_\omega^T - m_{\pi^0}^2 G_\omega^V\right] = -2.38\,\frac{10^{-3}}{m_{\pi^+}}. \tag{12}$$

There is one more interpretation of this result. If one assumes the pole model to be correct in the case of the ρ^0-exchange, then the following procedure can be applied:

$$G_\omega^T \to G_\omega^T + \frac{g_{\rho\pi\gamma}}{g_{\omega\pi\gamma}} G_\rho^T = 4.39. \tag{13}$$

Then, with the following values for the width of the $\rho\to\pi\gamma$ decay[19]) and the constants of the ρNN vertex[18])

$$(G_\rho^V)^2/4\pi = 0.55\pm0.06; \qquad G_\rho^T/G_\rho^V = 6.1\pm0.6\ ;\ \Gamma_{\rho\to\pi\gamma} = 0.067\ MeV,$$

it is easy to estimate the value of the tensor constant of the ωNN -vertex

$$G_\omega^T = -0.02\pm0.67 \tag{14}$$

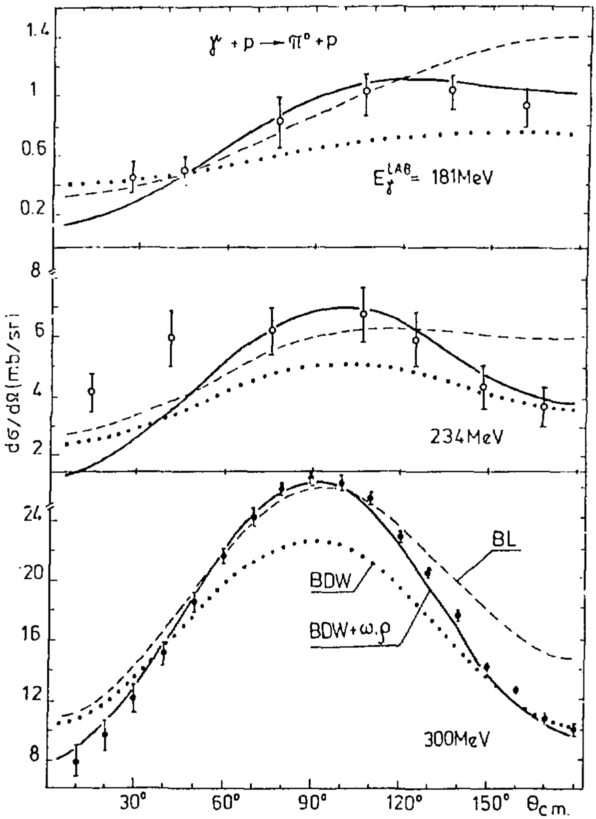

Fig. 2. The differential cross-sections for the $\gamma p \to p\pi^{o}$ reaction. The solid and dotted lines are the *BDW* results with and without ω^{o} exchange contribution, respectively. The dash-dotted line is the BL results. The experimental data are taken from Ref. [22] ϕ , Ref. [23] ϕ.

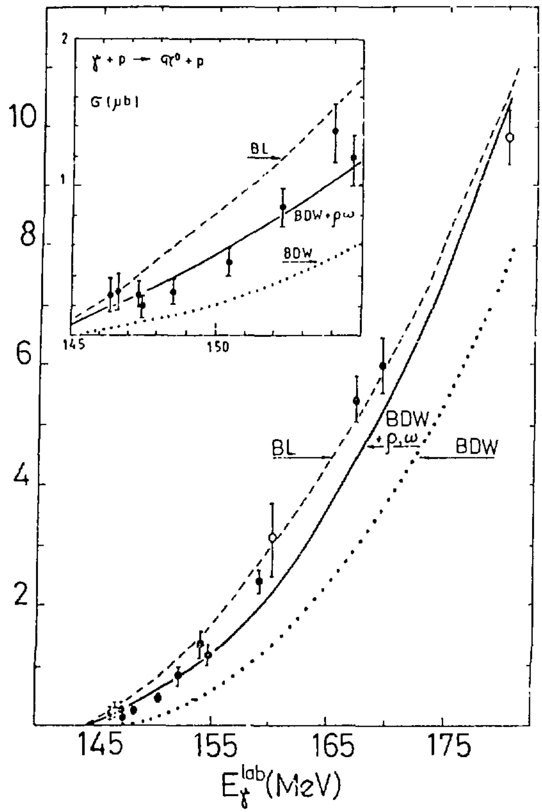

Fig. 3. The total cross-section for the $\gamma p \rightarrow p\pi^0$ reaction. See caption to Fig. 2 for the meaning of the curves. The experimental points are taken from Ref. [6] ϕ and Ref. [21] ϕ .

The result is stable when the vector constant G_ρ^V is taken into account because $g_{\rho\pi\gamma} G_\rho^V \ll g_{\omega\pi\gamma} G_\omega^V$.

One can see from Table 1 that a certain discrepancy exists in the case of the magnetic multipoles as well. These multipoles are represented as

$$M_{1\pm}(W) = \tilde{M}_{1\pm}(W) \times 10^{-3} \frac{\tilde{q}\tilde{k}}{m_{\pi^+}^3} , \qquad (15)$$

where

$$\tilde{q}^2 = \frac{(W^2 - m^2 + m_{\pi^0}^2)^2}{4W^2} - m_{\pi^0}^2 ; \quad \tilde{k}^2 = \frac{(W^2 - m^2)^2}{4W^2} . \qquad (16)$$

In the case of the BL -amplitude the analysis of the magnetic quadrupoles M_{1+} and M_{1-} is based on the experimental data for the $\gamma p \to p\pi^0$ reaction in the Δ_{33}-region. The reasonable choice of the model parameters (the Δ isobar width Γ_Δ , the resonance position M_Δ and the electromagnetic constant $G_1 e^{i\varphi_1}$) leads to the nice agreement with the experimental data for M_{1+} but slight overestimation of M_{1-} .

Note that the depicted above disagreements can manifest themselves in the coherent π^0-photoproduction off the zero-spin nuclei (^{4}He, ^{12}C and ^{40}Ca, for example) where only the nonspin-flip part of the elementary amplitude is required. This part at the threshold is determined by the magnetic quadrupoles $\mathcal{F}_2^{(+)} = 2M_{1+}^{(+)} + M_{1-}^{(+)}$. Further, we shall examine the sensitivity of the (γ,π^0) reaction on the nucleus to the value of the magnetic quadrupoles.

II. Operator of the pion photoproduction off nuclei in the DWIA method

We start with the DWIA method [24] to realise the problems which arise in the description of the pion photoproduction off complex nuclei. We shall not consider the details of including the strong πA interaction in the final state because this question has already been examined in Refs.[11,24]. Let us only note that for taking this interaction into account we use the pion-nuclear amplitudes obtained by the numerical evaluation of the Lippman-Shwinger equation in the momentum space. The corresponding pion-nuclear optical potential was constructed from the analysis of a great amount of empirical data for the elastic pion scattering differential cross-sections and for the total cross-sections of the pion-nuclear interaction for the nuclei with A = 4÷40[25].

A. General expressions

In terms of the DWIA in the momentum space the $T_{\pi\gamma}$-matrix of the coherent pion photoproduction process can be expressed as

$$\langle \vec{q}_0 | T^{\lambda}_{\pi\gamma}(E) | \vec{k} \rangle = \langle \vec{q}_0 | U^{\lambda}_{\pi\gamma} | \vec{k} \rangle + \int \frac{d\vec{q}}{(2\pi)^3} \frac{\langle \vec{q}_0 | T'_{\pi\pi}(E) | \vec{q} \rangle \langle \vec{q} | U^{\lambda}_{\pi\gamma} | \vec{k} \rangle}{E - E(\vec{q}) + i\varepsilon} , \qquad (17)$$

where $\vec{q}$ and $\vec{k}$ are the momentum of a pion and a photon in the πA c.m. frame, respectively; $\lambda = \pm 1$ is the photon polarisation; $T'_{\pi\pi} = (A-1) T_{\pi\pi} / A$ is the T-matrix of the pion-nuclear scattering; $E(\vec{q}) = E_{\pi}(\vec{q}) + E_A(\vec{q})$ and $E \equiv E(\vec{q}_0)$ are the energies of the pion-nuclear systems in the intermediate and final states; $E_{\pi}(\vec{q}) = \sqrt{m_{\pi}^2 + \vec{q}^{\,2}}$ and $E_A(q) = \sqrt{M_A^2 + \vec{q}^{\,2}}$ are the energies of a pion and a nucleus.

In the framework of the impulse approximation the plane-wave part $U^{\lambda}_{\pi\gamma}$ of the nuclear amplitude is expressed in terms of the $t^{\lambda}_{\pi\gamma}$ matrix of the photoproduction off a free nucleon and the nuclear transition density $\rho(\vec{p}\,', \vec{p})$

$$\langle \vec{q} | U_{\pi\gamma}^{\lambda} | \vec{k} \rangle = \int \rho(\vec{p}\,', \vec{p}) \langle \vec{q}, \vec{p}\,' | t_{\pi\gamma}^{\lambda}(W) | \vec{k}, \vec{p} \rangle \, d\vec{p} \, d\vec{p}\,', \tag{18}$$

where $\vec{p}\,(\vec{p}\,')$ is the nucleon momenta in the initial (final) state. The variable W is the energy of the free pion-nucleon system.

In the specific applications it is convenient to use the amplitudes $V_{\pi\gamma}$ and $F_{\pi\gamma}$ connected with the $U_{\pi\gamma}^{\lambda}$- and $T_{\pi\gamma}^{\lambda}$-matrices as follows:

$$\langle \vec{q} | T_{\pi\gamma}^{\lambda}(E) | \vec{k} \rangle = -2\pi \sqrt{\frac{E(\vec{q})E(\vec{k})}{E_{\pi}(\vec{q})E_{A}(\vec{q})E_{\gamma}E_{A}(\vec{k})}} \, F_{\pi\gamma}(E; \vec{q}, \vec{k}\lambda), \tag{19}$$

$$\langle \vec{q} | U_{\pi\gamma}^{\lambda} | \vec{k} \rangle = -2\pi \sqrt{\frac{E(\vec{q})E(\vec{k})}{E_{\pi}(\vec{q})E_{A}(\vec{q})E_{\gamma}E_{A}(\vec{k})}} \, V_{\pi\gamma}(\vec{q}; \vec{k}\lambda). \tag{20}$$

Then, one has for the differential cross sections

i) in the plane- wave approximation:

$$\frac{d\sigma}{d\Omega}(PWIA) = \frac{|\vec{q}_0|}{|\vec{k}|} \overline{\sum} \left| V_{\pi\gamma}(\vec{q}_0, \vec{k}\lambda) \right|^2 ; \tag{21a}$$

ii) in the distorted wave approximation

$$\frac{d\sigma}{d\Omega}(DWIA) = \frac{|q_0|}{|k|} \overline{\sum} \left| F_{\pi\gamma}(E; \vec{q}_0, \vec{k}\lambda) \right|^2 . \tag{21b}$$

where $\overline{\sum}$ means the summation over the final state and averaging over the spin of the initial state.

B. Fermi motion of the nucleons

The first important question arising even when computing the plane wave part $V_{\pi\gamma}$ of the nuclear amplitude (18) is the allowance for the nucleon motion. In other words, the problem consists in the necessity of integrating over the nucleon momenta in (18). The traditional way of solving this problem, which is widely adopted in other processes, is the nonrelativistic reduction, i.e. the emanation off the elementary t-matrix linear over $\vec{p}$ terms of an order of $1/m$

$$\langle \vec{q},\vec{p}'|t(W)|\vec{k},\vec{p}\rangle=\delta(\vec{p}'+\vec{q}-\vec{p}-\vec{k})\left[t(W:\vec{k},\vec{q},\vec{p})\Big|_{\vec{p}=0}+\frac{\partial t(W,\vec{k},\vec{q},\vec{p})}{\partial \vec{p}}\Big|_{\vec{p}=0}\cdot\vec{p}+\dots\right], \quad (22)$$

where the nucleon momentum $\vec{p}$ acts as $-i\vec{\nabla}_r$ on the nuclear initial wave function, when one passes to the coordinate space over the nucleon variables. However, this prescription can hardly be adopted in the case of the π^0-photoproduction, especially in the Δ_{33} resonance region. In this case the leading multipole, namely $M_{1+}(W)$ (where W is $\vec{p}$-dependent), has a sharp energy dependence like $[W^2-M_\Delta^2+iM_\Delta\Gamma_\Delta]^{-1}$.

There are other ways. The direct one is the numerical evaluation of the integrals over $\vec{p}$. It leads to the 5-dimensional integrals (with integrating over $\vec{q}$ in (17)) and necessitates the exact nuclear wave functions to be known.

The main idea of the third approach that was used for the pion-nuclear scattering[25-27] and the pion photoproduction[11,24] is in the substitution of $\vec{p}$ and $\vec{p}'$ in the elementary t-matrix with their effective values (the factorisation approximation):

$$\vec{p}\to\vec{p}_{eff}=-\frac{\vec{k}}{A}-\frac{A-1}{2A}(\vec{k}-\vec{q})\ ;\ \vec{p}'\to\vec{p}'_{eff}=\vec{p}_{eff}+\vec{q}-\vec{k}. \quad (23)$$

In order to evaluate the accuracy of this approximation let us represent the nuclear transition density $\rho(\vec{p}',\vec{p})$ in the form

$$\rho(\vec{p}',\vec{p})=4\varphi^*_{00}(\vec{p}')\varphi_{00}(\vec{p})+\frac{A-4}{3}\sum_m\varphi^*_{1m}(\vec{p}')\varphi_{1m}(\vec{p}), \quad (24)$$

where $\varphi_{\ell m}(\vec{p})$ is the harmonic oscillator function. Then, it is possible to make the numerical integration over nucleon momenta $\vec{p}$ in expression (18) with the help of the procedure developed for example in Ref.[28,29]).

To compare the exact results obtained in that manner with the results based on the factorisation approximation (23) and found without Fermi motion we introduce the following quantities:

$$dR_n = 1 - \left(\frac{d\sigma_n}{d\Omega}\right) \Big/ \left(\frac{d\sigma}{d\Omega}\right) \text{ and } R_n = 1 - \sigma_n / \sigma , \qquad (25)$$

where $d\sigma/d\Omega$ and σ are the differential and total cross-sections obtained by the numerical integration over p ; $d\sigma_n/d\Omega$ and σ_n are the corresponding values with

$$n = 0 \quad \text{for} \quad \vec{p} = 0 \quad \text{and} \quad \vec{p}\,' = \vec{k} - \vec{q} \qquad (26)$$

$$n = 1 \quad \text{for} \quad \vec{p} = \vec{p}_{eff} \quad \text{and} \quad \vec{p}\,' = \vec{p}\,'_{eff} \qquad (27)$$

The results of this analysis are plotted in Fig. 4 for the $^{12}C(\gamma, \pi^0)$ reaction. One can note the good accuracy of the factorisation approximation ($n = 1$) at the threshold ($\sim$ 2-3%). In the resonance region ($E_\gamma^{LAB} \sim 290$ MeV) it is of 10%. At the same time the "frozen nucleon" approximation ($n = 0$) gives the results which differ from the exact ones by more than 10%. In this case the discrepancy is strong, especially for the angular distributions. In the angular region where the differential cross-section decreases 10 times, the maximum and minimum values of dR_0 are about 20-30% and dR_1 are about 2-3%. So the factorisation approximation provides one with the high accuracy of integration. Morover, the substitution (23) leads to the simultaneous fulfillment of the energy and momenta conservation for the πA

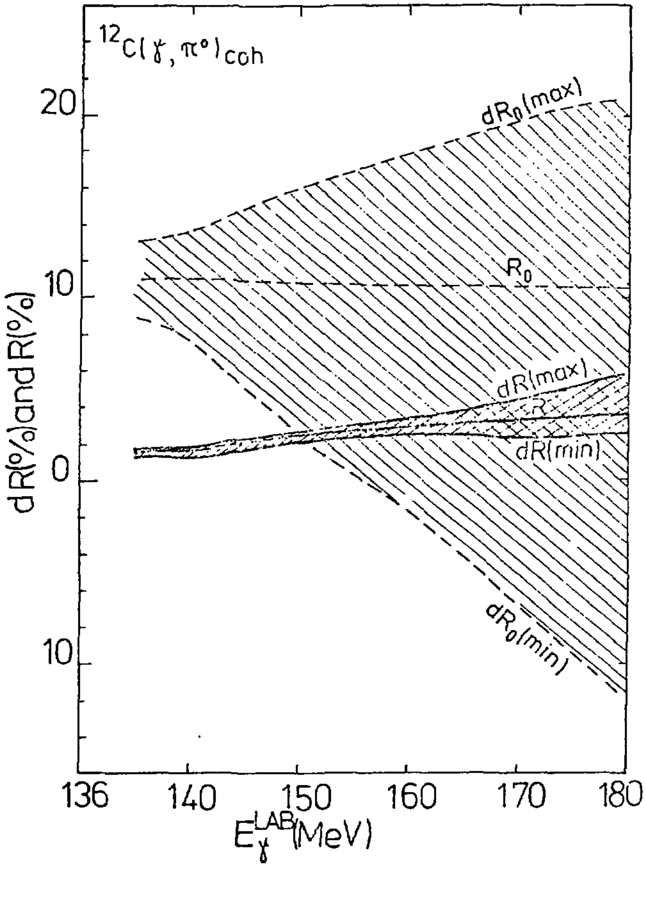

Fig. 4. The accuracy of the factorization approximation. See the text for the meaning of the curves (expressions (25) – (27)).

and πN systems, which is of great importance for the threshold analysis of the pion photoproduction. To complete the discussion of this approximation let us note that the substitution (23) makes it possible to express $V_{\pi\gamma}$ through the well-known electromagnetic nuclear form-factors and thus to avoid the problems of the nuclear structure description.

C. "Off-shell" effects

The next problem in the determination of the nuclear amplitude is to find the connection between the $\langle \vec{q},\vec{p}'|t^{\lambda}_{\pi\gamma}(W)|\vec{k},\vec{p}\rangle$ -matrix in the πA c.m. frame and the amplitude $\langle \tilde{\vec{q}}\,|f^{\lambda}_{\pi\gamma}(W)|\tilde{\vec{k}}\rangle$ usually determined in the πN c.m. frame. From the Lorentz-invariance of $f^{\lambda}_{\pi\gamma}$ one has:

$$\langle \vec{q},\vec{p}'|t^{\lambda}_{\pi\gamma}(W)|\vec{k},\vec{p}\rangle = -2\pi\delta(\vec{p}'+\vec{q}-\vec{p}-\vec{k})\sqrt{\frac{W_i W_f}{E_N E_{N'} E_\gamma E_\pi}}\,\langle \tilde{\vec{q}}|f^{\lambda}_{\pi\gamma}(W)|\tilde{\vec{k}}\rangle, \quad (28)$$

where W_i and W_f are the invariants determined as

$$W_i=\left[\left(E_\gamma+E_N(\vec{p})\right)^2-\vec{\mathcal{P}}^2\right]^{1/2};\quad W_f=\left[\left(E_\pi(\vec{q})+E_{N'}(\vec{p}')\right)^2-\vec{\mathcal{P}}^2\right]^{1/2}, \quad (29)$$

$\vec{\mathcal{P}}=\vec{k}+\vec{p}=\vec{q}+\vec{p}'$ is the total momentum of the γN or πN systems; $E_N(\vec{p})=\sqrt{m^2+\vec{p}^2}$ is the nucleon energy. The pion and photon momenta in the πN c.m. frame are obtained by the Lorentz transformation for the vectors $\vec{q}$ and $\vec{k}$

$$\tilde{\vec{q}}=\vec{q}+\left[\frac{\vec{\mathcal{P}}\cdot\vec{q}}{E_\pi(\vec{q})+E_{N'}(\vec{p}')+W_f}-E_\pi(\vec{q})\right]\frac{\vec{\mathcal{P}}}{W_f};\quad \tilde{\vec{k}}=\vec{k}+\left[\frac{\vec{\mathcal{P}}\cdot\vec{k}}{E_\gamma+E_N(\vec{p})+W_i}-E_\gamma\right]\frac{\vec{\mathcal{P}}}{W_i}. \quad (30)$$

Expression (28) is correct only on the energy shell when $W=W_i(\vec{k},\vec{p})=W_f(\vec{q},\vec{p}')$. As has already been mentioned, this equality is fulfilled for the plane wave part $U_{\pi\gamma}$ of the nuclear amplitude (17) with the high degree of accuracy after the substitution (23). However, one covers the off-energy-shell region when taking into account the πA interaction by the second term in (17) because of integrating over $\vec{q}$ from 0 to ∞. The problem of definition of the $t_{\pi\gamma}$-matrix arises in the off-shell region where $|\vec{q}| \neq |\vec{q}_0|$, and consequently, $W_i(\vec{k},\vec{p}) \neq W_f(\vec{q},\vec{p}\,')$.

In the theory of the pion scattering expression (28) is generalized to the off-shell region and the elementary amplitude behaviour is parametrized in accordance with the results of the separable model of the πN interaction[30]. When applied to the photoproduction process, this approach provides the following expression for the multipole amplitudes $f_{\ell\pm}(W)=\{E_{\ell\pm}(W); M_{\ell\pm}(W)\}$ in the off-energy-shell region

$$f_{\ell\pm}(W) = f_{\ell\pm}(Z)\, g^{(\ell)}_{\pi N}(\tilde{\vec{q}}) / g^{(\ell)}_{\pi N}(\tilde{\vec{q}}_Z), \qquad (31)$$

where $\tilde{\vec{q}}$ and $\tilde{\vec{q}}_Z$ are the moments of the pions in the πN c.m. frame which correspond to the energies $W_f(\vec{q},\vec{p}\,')$ and Z;

$$g^{(\ell)}_{\pi N}(\tilde{\vec{q}}) = \tilde{\vec{q}}^{(\ell)} / (1+\alpha \tilde{\vec{q}}^2)^2, \qquad (32)$$

with α =0.224 fm^2 is the πN form-factor. This approach leads to introduction of the new free parameter $Z(\vec{q}_Z)$ - the energy of the πN system at which the partial amplitudes are calculated. What is the connection between it and the energy of the pion-nuclear system $E(\vec{q}_0)$? There are different prescriptions which coincide on the energy shell in contrast with the off-energy-shell region

$$Z = W_i = \left[(E_\gamma + E_N(\vec{p}))^2 - (\vec{k}+\vec{p})^2\right]^{1/2};$$

$$\xi=\left[m_\pi^2+m^2+2E_\pi(\vec{q}_0)E_N(\vec{p})-2\vec{q}\cdot\vec{p}'\right]^{1/2};$$
$$\xi\equiv W_f=\left[\left(E_\pi(\vec{q})+E_N(\vec{p}')\right)^2-\left(\vec{q}+\vec{p}'\right)^2\right]^{1/2}. \qquad (33)$$

It is also possible to determine ξ as $\xi=(W_i+W_f)/2$ or $\xi=\sqrt{W_i W_f}$ [31].

Due to the sharp energy dependence of M_{1+}, the final result in the π^0-photoproduction is very sensitive to the choice of ξ, and consequently, to different off-shell extrapolations. This circumstance can be considered as a tool for the definition of ξ when the π^0-photoproduction off the nuclei (for example off ^{12}C) is investigated in the Δ_{33} resonance region. Different results corresponding to various ξ are plotted in Fig. 5 at $E_\gamma^{LAB}=290$ MeV. The choice $\xi=W_f(\vec{q},\vec{p}')$ is preferable , in our opinion, not only because it provides one with the nice description of the empirical data but also because it is consistent with the consequence of the Relativistic Potential theory [33]. Finally, this choice leads to the nonlocal photoproduction operator which depends only on the particle momenta (but not the energies). This point guarantees the relativistic (and in the nonrelativistic limit - Galileo) invariance of the result [31].

III. The coherent π^0-photoproduction at threshold. Results and discussion

We proceed to discuss the application of the procedure given above to the threshold π^0-photoproduction off the zero-spin nuclei ($N=\xi$). In this case, of 12 amplitudes $\mathcal{F}_\alpha^{(\pm,0)}(\alpha=1\div4)$ only a nonspin-flip amplitude contributes

$$\mathcal{F}_2^{(+)}(W)=2M_{1+}^{(+)}(W)+M_{1-}^{(+)}(W). \qquad (34)$$

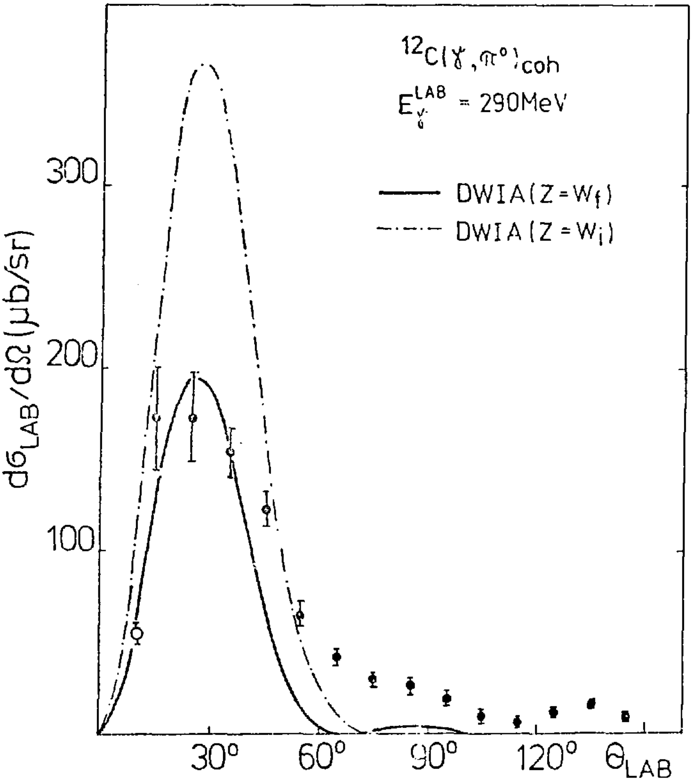

Fig. 5. The differential cross-section for the $^{12}C(\gamma,\pi^0)$ coh. reaction in the resonance region. The solid line is the DWIA with $z = W_f$. The dotted line is the DWIA with $z = W_i$. The experimental points are taken from Ref. [32].

As has already been mentioned, the factorization approximation makes it possible to reduce the accounting of the nuclear structure to the experimentally determined nuclear form-factor

$$F^{ch}_{exp}(\vec{Q}) = F_0(\vec{Q}) f^{ch}_p(\vec{Q}), \qquad (35)$$

where $f^{ch}_p(\vec{Q}) = (1 + \alpha_p \vec{Q}^2)^{-2}$ is the charge photon form factor; $\alpha_p = 0.0548\ fm^2$. We shall define the nuclear form factor $F_0(\vec{Q})$ in terms of the symmetrized Fermi density model[34,35], as it has been done in Ref.[11]).

A. The invariance of the BL-amplitude

There are two ways of constructing the plane-wave part of the nuclear photoproduction amplitude $V_{\pi\gamma}$. The first has been discussed previously. Here, with the help of expression (18) and the factorization approximation one has

$$V^{coh}_{\pi^0\gamma}(\vec{q},\vec{k}\lambda) = \frac{A}{\sqrt{2}} W_A(\vec{q},\vec{k}) \mathcal{F}^{(+)}_2(W) \sin\tilde{\Theta}_\pi e^{i\lambda\tilde{\varphi}_\pi} F_0(\vec{Q}), \qquad (36)$$

where $\tilde{\Theta}_\pi$ and $\tilde{\varphi}_\pi$ are the polar and azimuthal angles of an outgoing pion in the πN c.m. frame; $\vec{Q} = \vec{k} - \vec{q}$ is the transferred momentum. The kinematic factor is determined as

$$W_A(\vec{q},\vec{k}) = \left[\frac{E_A(\vec{k}) E_A(\vec{q}) W_i(\vec{k},\vec{p}) W_f(\vec{q},\vec{p}\,')}{E(\vec{q}) E(\vec{k}) E_N(\vec{p}) E_N(\vec{p}\,')}\right]^{1/2}, \qquad (37)$$

where the values of $\vec{p}$ and $\vec{p}\,'$ are found in (23).

When expression (36) is rewritten in the πA c.m. frame with the Lorentz transformation (30) one has

$$V^{coh}_{\pi\gamma}(\vec{q},\vec{k}) = \frac{A}{\sqrt{2}} W_A(\vec{q},\vec{k}) f_0(W,\vec{q},\vec{k}) \sin\Theta_\pi e^{i\lambda\varphi_\pi} F_0(\vec{Q}), \qquad (38)$$

where

$$f_0(W;\vec{q},\vec{k})=\tilde{F}_2^{(+)}(W)\frac{|\vec{k}||\vec{q}|}{|\tilde{\vec{k}}||\tilde{\vec{q}}|}\left[1-\frac{E_\pi}{W_i}-\frac{E_\pi}{W_f}+O\left(\frac{\vec{p}^2}{W^2}\right)\right]. \qquad (39)$$

The second way (see in Ref.[11]) is based on the fact that the BL-amplitude is developed in the arbitrary frame. That is why there is no need to use the intermediate πN c.m. frame in the calculations. By comparing the results obtained by these two ways one can examine the relativistic invariance of the BL-amplitude.

We should like to notice some modifications that took place in the present paper in contrast with Ref.[11]).

1) The Δ -isobar width is $\vec{q}$ -dependent according to (10);

2) in order to restore the unitarity of the amplitude, the complex factor $e^{i\psi_1}$ is introduced in the coupling constant G_1. This constant and G_3 are normalized to the charged pion mass (not on the π^0 meson mass).

In Table 2 we compare the results of different ways of describing the plane wave operator. Here, we use the representation of $f_0(W;\vec{q},\vec{k})$ as the sum of the Born, Δ and ω^0-exchange contributions

$$f_0(W;\vec{q},\vec{k})=\frac{m}{4\pi W}(t_B+t_\Delta+t_\omega). \qquad (40)$$

One can clearly see that the difference between two results corresponding to the above described procedures of the transition to the πA c.m. frame is about 10%. It is the accuracy with which the BL amplitude satisfies the relativistic invariance.

To our mind, it is more expedient to use the first approach (which is used thereafter), because it permits one to connect the photoproduction of pions off nuclei with the multipoles $M_{1\pm}(W)$ on a free nucleon.

Table 2. Numerical values for different components of the BL-amplitude [in fm^{-3}] ; Θ_π =60°.

E_γ^{LAB} [MeV]	140		150		160	
	Ac.m.	2cm → Ac.m.	Ac.m.	2c.m. → Ac.m.	A c.m.	2c.m. → Ac.m.
$t_B \times 10^3$	2.79	2.53	4.78	4.35	5.92	5.36
$\mathrm{Im}\, t_\Delta \times 10^2$	-0.283	-0.298	-1.11	-1.16	-2.28	-2.38
$\mathrm{Re}\, t_\Delta$	-0.274	-0.288	-0.288	-0.302	-0.305	-0.319
t_ω	-0.102	-0.104	-0.102	-0.104	-0.102	-0.104
$d\sigma/d\Omega$ (60°)	0.755	0.821	4.99	5.81	12.98	14.10
σ_{tot} (PWIA)	6.96	7.55	40.5	45.5	90.9	98.3

B. The sensitivity to different values of $M^{(+)}_{1\pm}(W)$

One can see from Fig. 6 the importance of taking the ω^0-exchange contribution into account by comparing the BDW-(point curve) and BDW $+\omega^0$ results (solid curve). This 50%-contribution is determined practically by the vector component of the ωNN-vertex. That is why it is very important to know the precise value of G^V_ω in the coherent π^0-photoproduction. The determined above value $G^V_\omega = 15$ has been based on the analysis of the experimental data for the $\gamma p \to p\pi^0$ process in the Δ_{33}-region and at the threshold. Moreover, with this value we get the PCAC value for $E^{p\pi^0}_{0+}$. However, if this value is wrong, then we are to change the obtained result for G^V_ω.

The best agreement with the experimental data on the free proton in the case of the BL-amplitude is obtained with $G^V_\omega = 10$. In this case the best agreement with nuclear data is achieved too. However, any conclusions are prematurely because $E^{p\pi^0}_{0+}(BL)/E^{p\pi^0}_{0+}(exp) \sim 5$ and $M^{p\pi^0}_{1-}(BL)/M^{p\pi^0}_{1-}(exp) \sim 2.4$.

With the experimentally determined values of M_{1+} and M_{1-} one can easily get the maximum ($\mathcal{F}^{max}_2$) and minimum ($\mathcal{F}^{min}_2$) of the nonspin-flip amplitude

$$\mathcal{F}^{max}_2(exp) = 16.1\times 10^{-3}\,\frac{\tilde{q}\cdot\tilde{k}}{m^3_{\pi+}}\ ;\quad \mathcal{F}^{min}_2(exp) = 11.9\times 10^{-3}\,\frac{\tilde{q}\cdot\tilde{k}}{m^3_{\pi+}}, \tag{41}$$

where the momenta $\tilde{\vec{q}}$ and $\tilde{\vec{k}}$ are expressed in terms of W in eq.(16).

Note that we have the experimental values for the magnetic quadrupoles only on a proton

$$M^{p\pi^0}_{1\pm} = M^{(0)}_{1\pm} + M^{(+)}_{1\pm}, \tag{42}$$

while for the π^0 photoproduction off a nucleus one needs only $M_{1\pm}^{(+)}$. The estimation of the $M_{1\pm}^{(0)}$ amplitudes shows that their contribution to $\mathcal{F}_2^{max}$ and $\mathcal{F}_2^{min}$ is small $\left(M_{1\pm}^{(0)}/M_{1\pm}^{(+)} \sim 0.1\right)$ The results obtained with $\mathcal{F}_2(exp)$ are plotted as a strip in Fig. 6.

To our mind, the reason for the discrepancy with the data which arises in this case may be connected with the value of $M_{1-}(W)$. Moreover, the experiment on a proton has been performed for $E_\gamma^{LAB}(\gamma p) \lesssim 155$ MeV which corresponds to $E_\gamma^{LAB}(\gamma A) \lesssim 142$ MeV. At higher energies it is necessary to make more thorough analysis taking into account not only the E_{0+} and $M_{1\pm}$ multipoles but other multipoles (for example, E_{1+}) and their energy dependence.

C. The role of the pion wave distortion

The role of the strong pion-nuclear interaction has been investigated for the coherent pion photoproduction in Refs.[11,12]. In the present paper, we shall continue this research by comparing the DWIA results with different off-shell extrapolations of the elementary amplitude with the experimental data obtained recently in SACLAY, MAINZ and Tomsk. The theoretical calculations have been performed with the BL amplitude.

Using the ^{40}Ca nucleus as an example, let us discuss some features of the angular distributions. From Fig. 7 it is clearly seen that the inclusion of the pion-nuclear interaction enlarges the differential cross-section (because of the attractive behaviour of the πN-potential p-wave), especially in the second maximum. At the same time, the minimum is shifted to the region of small transferred momenta. An analogous effect (but not so strong) takes place in the electron scattering off nuclei where

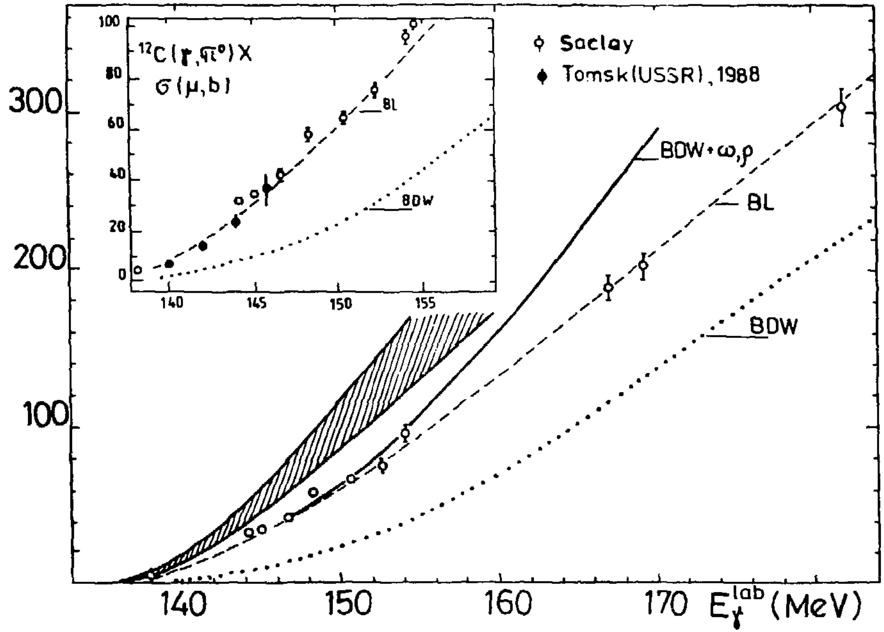

Fig. 6. The total cross-section for the $^{12}C(\gamma,\pi^0)_{coh.}$ reaction. The DWIA calculations with $\chi = W_f$ are made with the BL, BDW and (BDW+ ω^0) amplitudes. The strip is the DWIA results with the experimental values of the $M_{1\pm}$ multipoles. The experimental points are from Ref. [8] ϕ

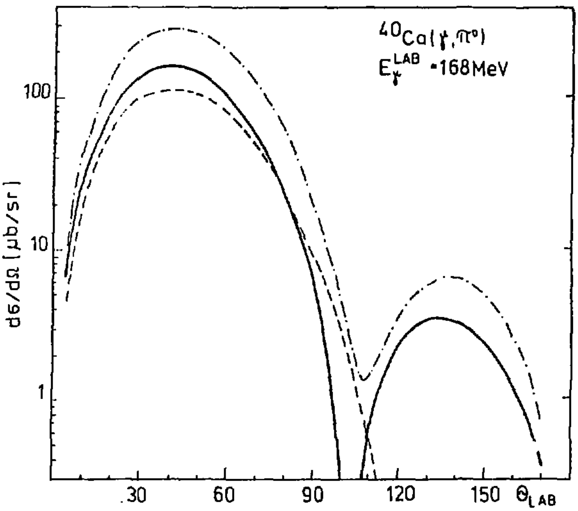

Fig. 7. The differential cross-section for the $^{40}Ca(\gamma,\pi^0)_{coh}$ reaction. The dashed line is the PWIA calculations with the BL-amplitude. The solid and dash-dotted lines are the DWIA ($\not{z} = W_f$) calculations with and without pion absorption, respectively.

the distortion can be included effectively by shifting the transferred momentum by the value that is proportional to the Coulomb potential inside a nucleus. In our case, this shift seems to be proportional to the depth of the attractive πA p-wave potential.

The next interesting effect is connected with the pion absorption. It is taken into account with the phenomenological second order optical potential U_2 which is proportional to the Fourier-transformation of the square of the nuclear density $\rho^2(\vec{r})$. Switching off this term in the total optical potential leads to the enlarging of the differential cross-section of the pion photoproduction and vanishing of the clear minimum. So the pion absorption makes the nucleus to be opaque at certain angles of outgoing pions. This angle region is of particular interest for the investigation of the slow-pion real absorption.

In the case of the ^{12}C nucleus (see Fig. 8), the photon energies of the experiment are not sufficient to cover the region of the second maximum. That is the reason for the difference from the angular distributions for the ^{40}Ca nucleus. In general, our DWIA calculations are in good agreement with the experimental data, except for the small-angle ($\theta_\pi < 20°$) region. According to the extraction rules for the nuclei with $J_0 = 0$ the angular distributions are determined by the spherical function $Y_{L_\pi \lambda}(\Omega_{\vec{q}_0}) \sim \sin\theta_\pi \quad (L_\pi \geq 1, \lambda = \pm 1)$ [11], and consequently $d\sigma/d\Omega(\mathrm{Coh}) \sim \sin^2\theta_\pi \xrightarrow[\theta_\pi \to 0]{} \theta_\pi^2$ independently of the approximation. The experimental deviation from this behaviour seems to be explained by the noncoherent contributions.

The total cross-sections for ^4He , ^{12}C, ^{16}O, ^{40}Ca and ^{208}Pb are plotted in Figs. 9-11. Here, we find that in the threshold region the difference between two versions of the off-shell extrapolation is not as strong ($\sim$ 10%) as the sensitivity of the results to the value of the magnetic quadrupoles. As for the discrepancy in the case of ^4He and ^{16}O, we are looking forward to getting an additional experimental information about the angular distributions.

To complete the discussion we note two assumptions which have been made when dealing with ^{208}Pb ($Z \neq N$) where the contribution of the $\mathcal{F}_\lambda^{(0)}$ amplitude exists. We suppose that 1) the proton and neutron distributions (normalized to unity) are equal and 2) one can neglect the contribution of the ρ^0-exchange.

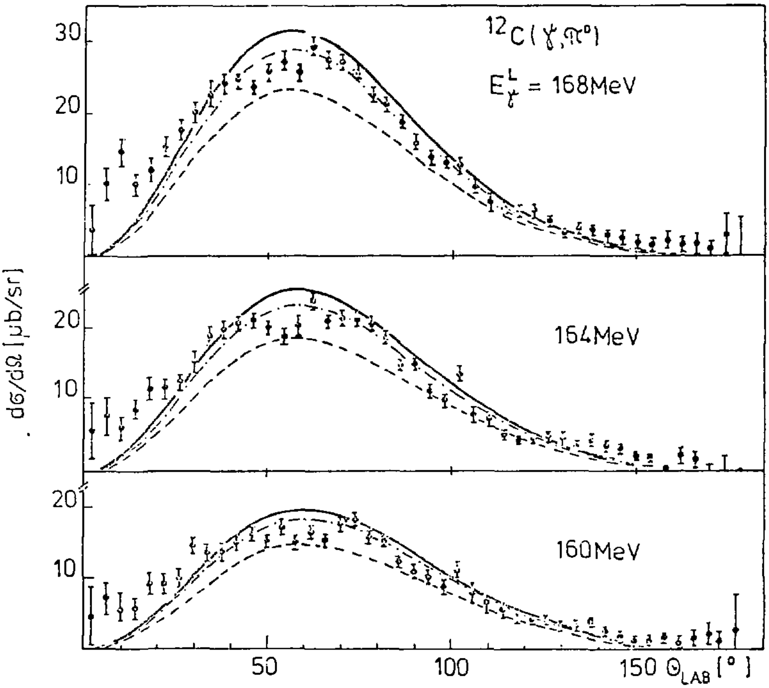

Fig. 8. The differential cross-section for the $^{12}C(\gamma,\pi^0)_{coh}$ reaction near threshold. The calculations are made with the BL amplitude. The solid line is the DWIA with $\xi = W_f$. The dash-dotted line is the DWIA with $\xi = W_i$. The dashed line is the DWIA calculations. Data are from [9].

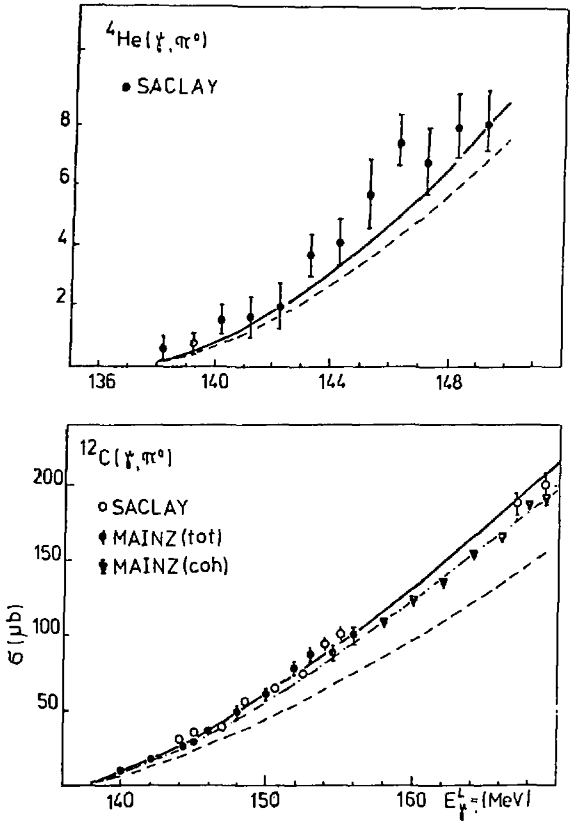

Fig. 9. The total cross-section in the threshold region for the coherent (a) $^4He(\gamma,\pi^o)$ and (b) $^{12}C(\gamma,\pi^o)$ reactions. The meaning of the curves is the same as in Fig. 8. The experimental points are taken from Ref. [36] for ^4He and Refs. [7÷9] for ^{12}C.

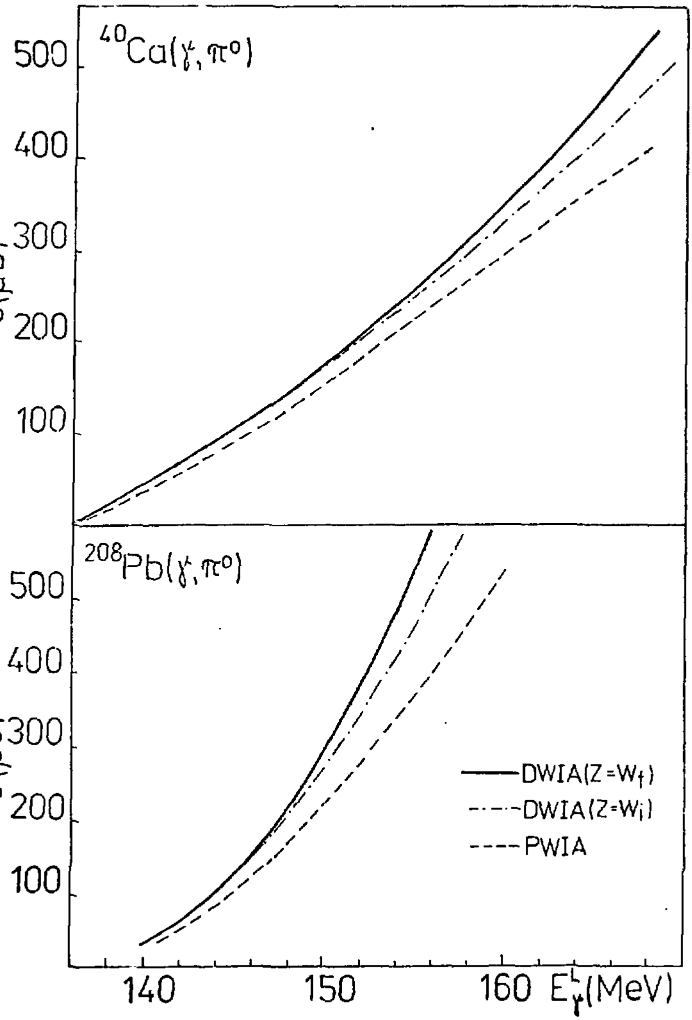

Fig. 10. The same as in Fig. 9 for the $^{40}Ca(\gamma,\pi^o)$ and $^{208}Pb(\gamma,\pi^o)$ reactions.

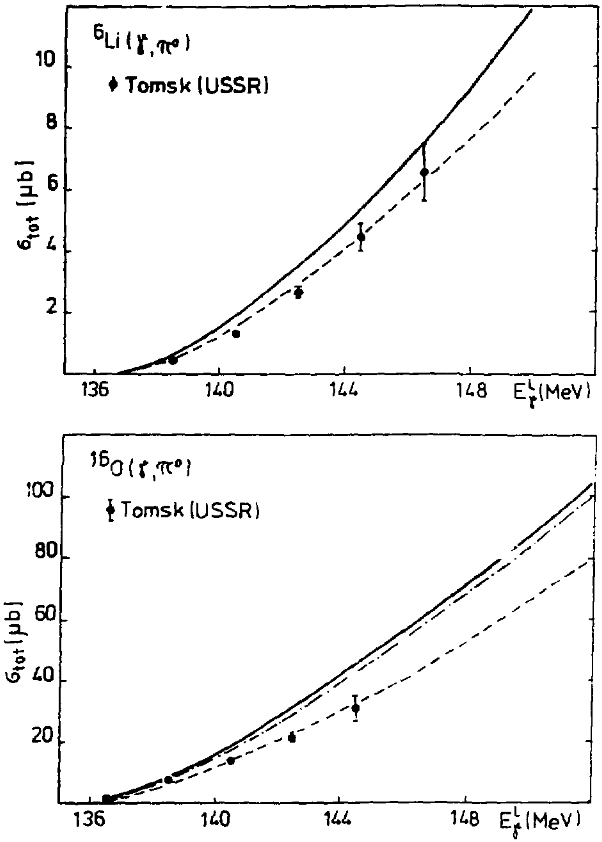

Fig. 11. The same as in Fig. 9 for the ^{6}Li(γ, π^0) and ^{16}O(γ, π^0) reactions. The experimental data are taken from Ref. [10].

The last assumption is demanded by the approximations of the BL amplitude and by the small value for the $(N-Z)/A \sim 0.2$.

Conclusion

To our mind, the results of the present calculations give a possibility to make the following firm statement: in terms of the DWIA method with the unitary BL model one can obtain nice description of the coherent π^0 photoproduction off nuclei near threshold. To get this goal it is necessary to take correctly into account the nonlocality of the elementary operator and the off-energy-shell effects. All these conclusions are valid in the Δ_{33}-resonance region as well.

Consequently, from the agreement of the theoretical calculations with the available experimental data one can conclude that the BL model gives correct values for the $M_{1\pm}$ quadrupoles as they are dominant in the threshold region for the considered process. However, as has already been pointed out, the predictions of this model for the electric dipole and magnetic quadrupole $E_{0+}(BL) = -2.38 \times 10^{-3}/m_{\pi^+}$ and $M_{1-}(BL) = -4.8\,\tilde{q}\tilde{k} \times 10^{-3}/m_{\pi^+}^3$ strongly contradict the results of the phase analysis of the modern experimental data for the $\gamma p \to p\pi^0$ reaction total cross section: $E_{0+}(exp) = (-0.5 \pm 0.3) \times 10^{-3}/m_{\pi^+}$ and $M_{1-}(exp) = (-2.0 \pm 1.5)\tilde{q}\tilde{k} \times 10^{-3}/m_{\pi^+}^3$. The reason for this discrepancy is not clear. Note that the calculations with the experimental values of M_{1+} and M_{1-} overestimate the experimental points for the π^0 photoproduction off nuclei.

The situation near threshold can be significantly improved by using the results of the multipole analysis in the dispersive method (BDW amplitude) instead of the BL model. The modification of the BDW amplitude by taking into account the contribution of the vector meson (ω^{0} and ρ^{0}) exchange with the reasonable values of the strong constants at the ωNN and ρNN vertices makes it possible to:

i) get the value of $E_{0+}^{p\pi^0}$ (BDW) $= E_{0+}^{p\pi^0}$ (PCAC),

ii) describe the total cross sections of the $\gamma p \to p\pi^0$ process near threshold and;

iii) get agreement with the nuclear experimental data at $E_\gamma \lesssim 155$ MeV with the following values of the magnetic quadrupoles: $M_{1+}(\mathrm{BDW}+\omega^0) = 4.8\, qk\, 10^{-3}/m_{\pi^+}^3$ and $M_{1-}(\mathrm{BDW}+\omega^0) = -1.4\,\tilde{q}\tilde{k} \times 10^{-3}/m_{\pi^+}^3$. However, the DWIA results obtained with this amplitude overestimate the experimental data at high energies.

It seems not possible to obtain a self-consistent description of the π^0 photoproduction on nucleons and nuclei. To solve this problem one needs the common experimental investigation not only of the total cross sections, but also the angular distributions for the π^0 photoproduction on nucleons and nuclei.

On the other hand, using the $^{40}Ca(\gamma,\pi^0)$ reaction as an example we have shown the great sensitivity of the results for the angular distribution to the effect of the pion wave distortion. This fact is the starting point for extraction of information about the properties of the pion-nuclear optical potential at low energies and about the role of the pion real absorption by nuclei.

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Фоторождение π^0-мезонов на нуклонах и ядрах
возле порога

В рамках метода DWIA в импульсном представлении проведен анализ современных экспериментальных данных процесса когерентного фоторождения пионов на ядрах ^{4}He, ^{6}Li, ^{16}O, ^{12}C, ^{40}Ca и ^{208}Pb. Показано, что имеющиеся в настоящее время амплитуды процесса на свободном нуклоне не позволяют взаимосогласованно описывать данные для процесса на ядре и на свободном нуклоне.

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Neutral Pion Photoproduction off Nucleons
and Nuclei Near Threshold

The modern experimental data on the coherent photoproduction off ^{4}He, ^{6}Li, ^{12}C, ^{16}O, ^{40}Ca and ^{208}Pb are analysed in terms of the DWIA in the momentum space. It is shown that the elaborated elementary amplitudes do not give a possibility of the simultaneous self-consistent description of the process on nucleons and nuclei.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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