IC/90/409
INTERNAL REPORT
(Limited Distribution)

International Atomic Energy Agency
and
United Nations Educational Scientific and Cultural Organization
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

AN ANALYTICAL MODEL FOR CLIMATIC PREDICTIONS *

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MIRAMARE -- TRIESTE
December 1990

* To be submitted for publication.

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ABSTRACT

A climatic model based upon analytical expressions is presented. This model is capable of making long-range predictions of heat energy variations on a regional or global scales. These variations can then be transformed into corresponding variations of some other key climatic parameters since weather and climatic changes are basically driven by differential heating and cooling around the earth. On the basis of the mathematical expressions upon which the model is based, it is shown that the global heat energy structure (and hence the associated climatic system) are characterized by zonally as well as latitudinally propagating fluctuations at frequencies downward of 0.5 day $^{-1}$. We have calculated the propagation speeds for those particular frequencies that are well documented in the literature. The calculated speeds are in excellent agreement with the measured speeds.

1. INTRODUCTION

Analytical expressions have already been developed for representing, quite generally, variations of different climatic parameters with respect to both time t and latitude ϕ [1-4]. These expressions have been shown [1,5,6] to be capable of simulating all the previously observed climatic periodicities. Besides, it has recently been shown that the expressions are also capable of simulating the physical conditions that set up and control the global pressure belts and the associated general circulation system [6]. It is implicit, therefore, that the above-mentioned expressions may constitute an analytical model for climatic variations if they are sufficiently synthesized and made a function of longitude θ as well. In this paper we have done this synthesis and also made θ one of the variables (in addition to t and ϕ) within the synthesized versions of the above-named analytical expressions. In addition, we have taken advantage of the new expressions so achieved to pinpoint and verify physical causes of some short-period weather variations whose causes and characteristics were not fully understood hitherto.

2. ANALYSIS

Since it is the differences in heating and cooling around the earth that are the fundamental causes of weather and climate [7], we shall concentrate out attention on variations in the heat energy $F(t,\phi,\theta,T)$ at an arbitrary location, where T denotes the overall time-length involved. Effectively this means that $F(t,\phi,\theta,T)$ is equal to $F(t,\phi,\theta)$ whose values are valid or exist at any time between $t = 0$ and $t = T$. The lower limit placed on T is 2 days and the upper limit is infinity.

Detailed descriptions of $F(t,\phi,T)$ have been given in Refs. [1] to[6]. We can transform $F(t,\phi,T)$ into the synthesized form $F(t,\phi,\theta,T)$ through the following steps. Firstly we synthesize the components in $F(t,\phi,T)$ which represent variations in the two polar regions, and in doing so we introduce another latitude variable $\phi' = \frac{\pi}{2} + \phi$. Secondly we introduce into $F(t,\phi,T)$ a longitude-dependent time-shift process which will take into account longitude differences in F due to the earth's rotation motion. After these two steps, an expression for $F(t,\phi,\theta,T)$ is arrived at which takes the following form:

$$F(t,\phi,\theta,T) = \sum_{m=1}^{\infty} a_m \left\{ \left[1 + \frac{1}{\alpha} \sum_{k=1}^{N} f_k \left(t + \frac{\chi\theta}{2\pi}, \phi \right) \right] \alpha D_0 \left(t + \frac{\chi\theta}{2\pi}, \phi \right) \right\}^m \tag{1}$$

for $\phi \leq 65°$ north or south of the equator, and

$$F(t,\phi,\theta,T) = \sum_{m=1}^{\infty} b_m \left\{ \left[1 + \frac{1}{\alpha} \sum_{k=1}^{N} f_k \left(t + \frac{\chi\theta}{2\pi} + \frac{e\phi'}{\pi}, \phi \right) \right] \alpha D_1 \left(t + \frac{\chi\theta}{2\pi} + \frac{e\phi'}{\pi}, \phi \right) \right\}^m \tag{2}$$

for $\phi > 65°$ north or south of the equator, such that the terms used in Eqs.(1) and (2) are defined or given as follows. For $\phi = 0$, $D_0(t,\phi)$ is described in detail in Ref. [1], and for non-zero $\phi \leq 65°$

north or south of the equator, the appropriate form of $D_0(t,\phi)$ is given as

$$D_0(t,\phi) = \frac{2}{\pi}\left\{\frac{1}{2}Ag + A\sum_{n=1}^{\infty}\frac{\sin(ng\pi)}{n\pi[1-(ng)^2]}\cos n\omega_{01}(t-\frac{B}{2})\right\}Y(t,T) \qquad (3)$$

where $B = 24$ hours, $\omega_{01} = 2\pi$ radians per day, A is a latitude-dependent term, $g = \frac{1}{2} + \frac{A_s}{B}\sin\omega_s t$, A_s and ω_s denote the amplitude and angular frequency, respectively, of the seasonal influence on the sunrise-sunset duration,

$$Y(t,T) = \sin(\omega_{02}t) + \sin(2\omega_{02}t) + \frac{1}{3}(3\omega_{02}t) + \frac{1}{5}\sin(5\omega_{02}t) + \ldots\ldots\ldots \qquad (4)$$

where $\omega_{02} = \frac{\pi}{2T}$ radians per day, α = constant component of incident (extraterrestrial) solar energy, $f_k = k^{th}$ variable component of incident (extraterrestrial) solar energy, a_m or b_m (for $m \geq 1$) is a location-dependent parameter, $\chi = (24)(60)(60)$, N is the number of variable components in extraterrestrial solar energy,

$$e = (365.25)(12)(60)(60)$$

and

$$D_1(t,\phi) = \frac{2}{\pi}\left\{\frac{1}{2}Ag + A\sum_{n=1}^{\infty}\frac{\sin(ng\pi)}{n\pi[1-(ng)^2]}\cos n\omega_{01}(t-\frac{B}{2}) + \frac{A_s}{\pi}\left[-\frac{\sin H - H\cos H}{1-\cos H} + \frac{2}{1-\cos H}\sum_{n=1}^{\infty}\frac{\sin(nH)\cos H - n\sin H\cos nH}{n(n^2-1)}\cos\left(n\omega_s t - \frac{n\pi}{2}\right)\right]\right\}Y(t,T) \qquad (5)$$

where

$$H = \frac{2\pi k_\phi(\phi-\phi_0)}{T_s}, \quad T_s = \frac{2\pi}{\omega_s}, \; k_\phi$$

is a latitude-dependent parameter, and ϕ_0 is the value of ϕ at which $A_s = \frac{1}{2}B$. Note that in Eq.(2), (6), (7) and (8), the latitude terms ϕ' and ϕ are applied under the condition that ϕ is negative south of the equator and positive north of the equator. This, however, does not apply to Eq.(1). Clearly parameter a_1 or b_1 represent {1 − (corresponding albedo)}. In order to take into consideration the poleward transport of energy, a_1 and b_1 are replaced in Eqs.(1) and (2), respectively, by a_{1R} and b_{1R} such that:

$$\text{For} \quad 45^\circ \leq \phi \leq 135^\circ, \quad a_{1R} = a_1[1 + Q\sin 2(\phi' + \frac{\pi}{4})] \qquad (6)$$

$$\text{and for} \quad 45^\circ < \phi \leq 65^\circ, \quad a_{1R} = a_1 + \in \sin 2(\phi' + \frac{\pi}{4}) \qquad (7)$$

where

$$Q = \frac{\text{Total energy transported poleward from an equatorial location}}{\text{Total solar energy correspondingly absorbed at that location}}$$

and

$$\in = \frac{Q\{\text{ maximum solar energy absorbed at latitude } 0^\circ\}}{\text{maximum solar energy absorbed at latitude } \phi'}$$

The parameter b_{1R} is given as follows:

$$b_{1R} = b_1 + \in \sin 2(\phi' + \frac{\pi}{4}) \tag{8}$$

According to some observations [7], the order of magnitudes for parameters Q and $\in$ are approximately as follows: $Q \approx 0.2$, $\in \approx 1.8$ at $\phi' = 0$, and $\in \approx 1.3$ at $\phi' = \pi$. For any given location, parameters a_m and b_m for $m \geq 2$ may be evaluated in accordance with the procedure given in reference [2]. Parameters A and k_ϕ are numerically adjusted until $D_1\left(t + \frac{\chi\theta}{2\pi} + \frac{\epsilon\phi'}{\pi},\ \phi\right)$ mimics as accurately as possible the daily and seasonal sequences by which the location involved samples out incoming solar energy. In order to obtain a realistic form of f_k for any value of $k \leq N$, one should bear in mind that the transmission coefficient of the upper atmospheric optical window is proportional to the square root of the intensity I of overhead extraterrestrial radiation [8],[9]. It can, therefore, be shown mathematically that a change ΔI in I would result into a corresponding change ΔX in the solar energy intensity X just below the ionosphere such that

$$\frac{\Delta X}{X} = \left(1 + \frac{\Delta T}{I}\right)^{3/2} - 1 \tag{9}$$

Eq.(9) represents some nonlinear amplification in which $\frac{\Delta X}{X}$ is a nonlinearly amplified form of $\frac{\Delta I}{I}$. It is interesting to note that for sufficiently small variations ΔI from a mean non-zero value I_0 of I, then the latter is related to X through a "three-halves power law" given as

$$X \approx p I^{3/2} \tag{10}$$

where $p = \frac{X_0}{I_0^{3/2}}$ and X_0 is the corresponding mean value of X. In fact Eqs.(9) and (10) apply generally to any earthward electromagnetic radiation provided that it is in the visible spectrum.

Eq.(1) and (2) may together be used to simulate or predict short-range or long-range heat energy and related climatic variations. Corresponding variations in a number of meteorological parameters may be derived from predicted variations of $F(t, \phi, \theta, T)$ as shown in Ref.[2]. If a proper temperature height profile or lapse rate is assumed, the resultant circulation system may be constructed from the horizontal gradients in F as well as the (thermally driven) air stability criteria. The practical validity of $F(t, \phi, T)$ has been ascertained in Refs.[1]-[6]. What then remains for $F(t, \phi, \theta, T)$ is to show that the ϕ'-dependent and θ-dependent time-shifting operations introduced therein are realistically acceptable. In fulfilling this, we show below that not only do these operations give rise to variations which are in good agreement with observations but they also lead to explanations of some features of short-term weather variations that were hitherto poorly understood.

Physically the time-shift of $\frac{\chi\theta}{2\pi}$ in equations (1) and (2) introduces zonally aligned fluctuations in $F(t, \phi, \theta, T)$ with a phase change β that is proportional to angular frequency ω and given as $\beta = \frac{\chi\theta}{2\pi}\omega$. All the fluctuations having periods greater than 2 days undergo phase shifts of less than 180° per each complete rotation of the earth. Weather fluctuations at periods upward of 2 days, therefore, exist and continuously propagate eastward at various speeds. If V_θ represents such eastward propagation speed (in longitude degrees per day) then it can be shown that

$$V_\theta = \frac{180\,\chi\omega}{\pi} \quad (11)$$

where ω is in radians per second. According to Eq.(11), fluctuations at periods of 3 to 120 days have eastward propagation speeds ranging from 154.4 to 3.9 m/s. In particular, the dominant intraseasonal tropical variability at periods from 30 to 60 days have eastward propagation speeds from 15.4 to 7.7 m/s. The latter agrees quite well with measured eastward propagation speed of approximately 10 m/s for these 30 to 60-day fluctuations. Similar analysis on the time- shift of $\frac{e\phi'}{\pi}$ in equation (2) leads to the establishment of latitudinally aligned fluctuations with northward or southward propagation speed V_ϕ given (in latitude degrees per year) as

$$V_\phi = \frac{360\,ew}{\pi} \quad (12)$$

Note that the latitudinal propagations implied by Eq.(12) are also represented (in a different manner) in Eq.(1) through

$$D_0\left(t + \frac{\chi\theta}{2\pi}, \phi\right)$$

Owing to the nature of Eqs.(1) and (2) as well as the period of the Sun's latitudinal motions, fluctuations at a period of 1 year are expected to display dominance in the latitudinal motions represented by Eq.(12). Now according to the latter equation, $V_\phi = 1.3$ m/s for fluctuations at a period of 1 year. Measured latitudinal propagation speeds associated with such fluctuations are between 1 and 2 m/s [10], quite in agreement with the speed of 1.3 m/s calculated from Eq.(12).

3. CONCLUSION

We have presented equations (i.e. Eqs.(1) and (2)) which can be used to simulate heat energy variations at individual locations or over the whole earth simultaneously. Since changes in weather and climate are basically driven by differences in heating and cooling around the earth, predicted global variations in $F(t,\phi,\theta,T)$ can be transformed into corresponding variations of some other key meteorological parameters [2]. An interesting original contribution of the formulations in the text is the finding that the global heat energy structure (and hence that of the associated climatic system) is characterized by eastward zonal motions as well as some meridional motions whose frequencies are given analytically in the text and whose propagation speeds are given by Eqs.(11) and (12). For those specific frequencies which are well documented in the literature [10-13], we have calculated the propagation speeds involved in the motions, and the calculated speeds agree very well with the observed ones as indicated in the text.

Acknowledgments

The author would like to thank Professor Abdus Salam, the International Atomic Energy Agency and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste.

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