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Quantum Effects in Accelerator Physics

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Abstract

Quantum effects for electrons in a storage ring are discussed, in particular the polarization effect due to spin flip synchrotron radiation. The electrons are treated as a simple quantum mechanical two-level system coupled to the orbital motion and the radiation field. The excitations of the spin system then are related to the Unruh effect, *i.e.* the effect that an accelerated radiation detector is thermally excited by vacuum fluctuations. The talk reviews an earlier work which was done in collaboration with John Bell.

*Invited talk at the Symposium on Quantum Physics at CERN May 2nd and 3rd, 1991, in memory of John S. Bell

1 Introduction

I would like to begin this talk by reminding you about John Bell's strong connection to the field of accelerator physics. In fact he started out his physics career in this field, and although his main contributions have been within other parts of theoretical physics, he has through the years written a number of papers related to this subject. Most of these have been written together with his wife, Mary Bell. Let me just mention as key words: from his early period, works on linear accelerators and the physics of strong focussing machines; from later years, papers on electron cooling in storage rings [1] and on radiation damping [2], on spin and polarization effects of stored electrons [3,4,5] and on the phenomenon of quantum beamstrahlung [6]. On the last of these subjects he wrote, with Mary Bell, several papers in the last few years of his life.

My own contact with this field is only through the work I did with John some years back [4,5]. This started when I was here at CERN as a fellow almost ten years ago. I was then interested in something quite different from accelerator physics, namely in the rather exotic theoretical effect often referred to as the Unruh effect. As shown by Unruh [7], an idealized radiation detector which is accelerated through ordinary Minkowski vacuum gets heated due to interactions with the vacuum fluctuations of the radiation field. For uniform linear acceleration the excitation spectrum has a universal, thermal form, independent of details of the detector. This effect, that vacuum seems hot, as measured in an accelerated system, has by Unruh and others been related to the phenomenon of Hawking radiation from black holes [8].

I discussed with John Bell, whom I had got to know at that time, whether it would be possible to see this effect in real experiments, or whether the strong accelerating forces needed would make it impossible, even in principle, to make a sufficiently robust detector. I was inclined to think so and had some arguments in that direction. But John then got the idea that a depolarization effect which was known to exist for electrons in a storage ring could have something to do with the Unruh heating. We investigated that and found that the effect indeed was related, although there were important complications due to the fact that the electrons were following a circular orbit rather than being linearly accelerated [4]. Motivated by these complications we later examined more carefully the effects of quantum fluctuations for an electron moving in a circular orbit [5].

In the present talk I will mainly talk about the polarization effects for circulating electrons, in the way discussed in our two papers. However since I have been invited to talk about quantum effects in accelerator physics, rather than on the Unruh effect, I will not begin with discussing the latter, but rather make contact with this subject later in my talk. Instead I will focus attention on the quantum description of electrons in a storage ring. I must then admit that I feel a bit uneasy to talk about such a subject here at CERN, with many experts around. But my intention is to describe these effects under simple, idealized conditions and to avoid all details from accelerator physics which I am not so familiar with.

I would like to add in this introduction, that for me it was a highly inspiring experience to work with John Bell. He had a way of reaching the essence of the problem under discussion, and of avoiding all unnecessary complications, which I found both remarkable and challenging. But more generally than this I was much attracted by John Bell's way of understanding and of doing physics. In addition he was a very pleasant person. I always liked very much to see him, as I have done from time to time also in recent years, both to discuss physics with him and also to hear his views on other matters.

2 Quantum effects for accelerated electrons

The motion of particles in accelerators can mostly be understood and described in classical terms. But there are some quantum effects which are non-negligible and which even may be important. These mainly have to do with radiation phenomena and with the radiation reaction on the accelerated particles. Therefore they are much more important for the light electron than for the much heavier proton. For this reason I shall restrict myself to discuss only quantum effects for accelerated electrons.

The accelerated electrons emit radiation, synchrotron radiation, as it is known for particles in a magnetic field. Even for high energy electrons this process is well described by the classical radiation formula. This was explicitly demonstrated by Schwinger [9] who calculated the lowest order quantum correction to the radiated power. Only for extremely high energetic electrons the quantum corrections become important. The condition for this being small can be written as

$$\gamma \ll \gamma_c = \sqrt{\frac{mc\rho}{\hbar}},\ \gamma = \frac{1}{\sqrt{1-(\frac{v}{c})^2}} \tag{1}$$

with m as the electron mass, v its velocity and ρ the radius of curvature of the particle orbit. The important ratio then is

$$\Upsilon = (\frac{\gamma}{\gamma_c})^2 \approx \frac{a}{a_m},\ (\gamma \gg 1) \tag{2}$$

where a is the acceleration of the particle in an inertial rest frame, and a_m is an acceleration parameter determined by the particle mass,

$$a = \frac{\gamma^2 v^2}{\rho},\ a_m = \frac{mc^3}{\hbar} \tag{3}$$

Thus the important physical quantity is the acceleration a rather than the energy of the electron. A typical value for the parameter Υ in cyclic accelerators is 10^{-6}, which shows that quantum effects indeed are very small.

However, recently there has been some interest in the quantum regime $\Upsilon > 1$ in connection with linear accelerators at very high energy [10,11,12]. In linear colliders with energies much above those of machines of today such high accelerations may be produced when electrons pass through a bunch of positrons, and vice versa. A characteristic feature of the radiation process then is that the emitted photon takes a large fraction of the electron energy. This phenomenon has been referred to as quantum beamstrahlung. As already mentioned, some of the latest papers of John Bell were dealing with the description of this effect [6]. However, in my talk I will not discuss this effect, but rather consider quantum effects for much smaller accelerations, $a \ll a_m$.

Even if synchrotron radiation is essentially a classical phenomenon for $\gamma \ll \gamma_c$, this does not mean that all quantum effects associated with this phenomenon are unimportant. The radiation reaction will excite orbital oscillations even for much smaller energies [13]. The radiation field then acts in two ways on the particle. Quantum fluctuations excite the oscillations, while radiation damping tends to reduce the oscillation amplitudes. Balance between these two tendencies defines a minimal, quantum limit to the beam size.

However, a perhaps even clearer demonstration of quantum effects for electrons in a storage ring is associated with the phenomenon of spin-flip radiation [14,15]. The asymmetry between up and down flips in the magnetic field leads to a gradual build up of transverse polarization of the electrons. Under ideal conditions the polarization approaches equilibrium as

$$P(t) = P_0(1 - e^{-t/t_0}), \tag{4}$$

with a maximum polarization

$$P_0 = \frac{8}{5\sqrt{3}} = 0.924 \tag{5}$$

and a build up time

$$t_0 = \frac{8}{5\sqrt{3}} \frac{m^2 c^2 \rho^3}{e^2 \hbar \gamma^5} \tag{6}$$

For existing accelerators this build up time is of the order of minutes to hours.

The phenomenon of spontaneous polarization of electrons circulating in a magnetic field has been analyzed in many publications, both for ideal conditions and for the more realistic situation with particles moving in a variable magnetic field. There also exist review articles on this interesting subject [16,17,18]. In particular the paper by J.D. Jackson focuses attention on the aspects of this phenomenon which can be described in elementary terms. My approach will be along the same lines. But whereas Jackson rejects the idea of a simple description of the effect as a transition between spin energy levels caused by radiation effects, which then would lead eventually to all particles in the lowest energy level, this is exactly the picture I will use. The electrons can be treated as a simple one-particle quantum system interacting with the radiation field. But the effect of the radiation field along the accelerated orbit of the electrons is different from the effect on an electron sitting at rest. Transitions also to the upper energy level are induced by the field along this orbit, and that leads to a small, but non-vanishing depolarization of the electron beam. I will only consider the ideal case of electrons moving in a rotationally invariant magnetic field. A stable, classical circular orbit is produced by a radial gradient in the magnetic field. This corresponds to the situation of a weak focussing machine.

3 Quantum mechanics in the accelerated frame

Let me first remind you about some facts concerning the relativistic, classical description of spin motion. When the spin is described as a 3-vector we actually refer to the spin in the rest frame of the particle. But this vector can be included in the lab frame description of the particle by a boost between the two frames. This is the standard approach. Here I would like instead to stay in the rest frame, and to consider also the particle motion in this frame.

Actually there are three different co-moving frames which are characterized by simple properties, in one way or the other. The first one, which I will denote the L-frame is the one which is obtained from the fixed lab frame by a pure boost. This frame is non-rotational as seen from the lab. The other one is the frame which rotates with the frequency of the orbiting particle. In this frame, which I will denote the O-frame, the accelerating field is stationary. Finally there is a frame, denoted the C-frame, which is non-rotational along the particle orbit. The fact that this is different from the L-frame is a relativistic effect and gives rise to Thomas precession of the spin vector. The relative rotational frequencies of these frames are listed in Table 1 for an electron following a circular path in a magnetic field. Both the magnetic field B and the frequencies refer to comoving frames.

frames	*rotational freq.*	*spin precession*
C	0	$g\frac{e}{2mc}B$
O	$\frac{e}{mc}B$	$(g-2)\frac{e}{2mc}B$
L	$(1-\frac{1}{\gamma})\frac{e}{mc}B$	$((g-2)+\frac{2}{\gamma})\frac{e}{2mc}B$

Table 1. Rotation frequencies of three different co-moving frames and the spin precession frequencies in these frames for an electron orbiting in a constant magnetic field B

Also the spin precession frequency in the three different frames are shown in Table 1. This has the simplest in the C-frame. Since this frame is non-rotational along the orbit, all spin precession in this frame is due to coupling between the magnetic moment of the particle and the external magnetic field. Thus, the precession frequency is proportional to the gyromagnetic factor g, as shown in the table. In the O-frame on the other hand the precession frequency is proportional to $g-2$. This demonstrates the well-known fact that for $g=2$ the spin precesses exactly with the frequency of the orbital motion. Finally, in the L-frame there is a further correction due to the relative rotation of the L- and the O-frame. This correction is identical to the rotation frequency of the orbital motion, and this is smaller than the frequency associated with the Thomas precession by a factor of $1/\gamma$.

Even if the spin motion is simplest in the C-frame, I shall in the following apply the O-frame for the quantum description. The reason for this is that the external fields are stationary in this frame and therefore give rise to a time-independent Hamiltonian. This frame will be extended to a local accelerated coordinate system to allow for fluctuations in the particle about the circular path, which then is assumed to be the classical, stable orbit of the electrons. This coordinate system will necessary contain coordinate singularities at some distance from the orbit. But I will assume fluctuations away from the stable orbit to be small, so that linearized equations are sufficient. I will also assume velocities in this frame to be small, so that a non-relativistic approximation can be applied.

The Hamiltonian, which governs the time evolution in the accelerated frame is not identical to that of the inertial rest frame, but it can be expressed in a simple way in terms of observables from this frame,

$$H' = H - \frac{a}{v}J_z + \frac{a}{c}K_x \tag{7}$$

Here H is the Dirac Hamiltonian, J_z is the generator of rotations in the plane of the electron orbit and K_x is a boost operator. The coordinates in the O-frame then are chosen with the particle acceleration in the negative x-direction and with the orbit velocity in the positive y-direction. The additional terms in the expression for H' are fairly easy to understand. The presence of the generator of rotations is related to the fact that the coordinate axes of the O-frame rotate along the orbit and the presence of the boost operator accounts for the continuous jumping between inertial frames when the particle is accelerated. For the three operators included in H' we have the following expressions,

$$H = c\vec{\alpha}\cdot\vec{\pi} + \beta mc^2 + e\phi + \kappa\frac{e\hbar}{2mc}(i\beta\vec{\alpha}\cdot\vec{E} - \beta\vec{\sigma}\cdot\vec{B}) \tag{8}$$

$$J_z = (\vec{r}\times\vec{p})_z + \frac{1}{2}\hbar\sigma_z \tag{9}$$

$$K_x = -\frac{1}{2c}(xH + Hx) \tag{10}$$

where a term for the anomalous magnetic moment, $\kappa = \frac{1}{2}(g-2)$, has been introduced in the expression for H. All the potentials and fields in these expressions refer to the inertial rest frame of the classical orbit. The notation $\vec{\pi} = \vec{p} - \frac{e}{c}\vec{A}$ has been used for the mechanical moment of the electron.

When the fluctuations around the classically stable orbit are assumed to be small, then a non-relativistic approximation makes sense. A Foldy-Wouthuysen transformation, where we keep only the leading terms, gives a Hamiltonian which then can be split into a spin-independent and a spin dependent term in the following way,

$$H = H_{orb} + H_{spin} \tag{11}$$

$$H_{orb} = (1 - \frac{ax}{c^2})(\frac{\vec{\pi}^2}{2m} + e\phi) - axm + \frac{ia\hbar}{2mc^2}\pi_x - \frac{a}{v}(\vec{r}\times\vec{p})_z + \ldots \tag{12}$$

$$H_{spin} = -\frac{e\hbar}{4mc}\vec{\sigma}\cdot[(1-\frac{ax}{c^2})g\vec{B} + \frac{g-1}{mc}\vec{E}\times\vec{\pi}] - \frac{1}{2}\frac{a\hbar}{v}\sigma_z + \frac{a\hbar}{4mc^2}(\vec{\sigma}\times\vec{\pi})_x + \ldots \tag{13}$$

The Hamiltonian in the accelerated frame includes some complications compared to that of the inertial frame. However, if we consider the orbital motion only to linear order in the deviation from the stable orbit, the main difference in the expressions for H_{orb} is the presence of the centrifugal and Coriolis terms. For the orbital motion the spin effects only give rise to a small perturbation. But also for the spin motion the effect of the fluctuations in the orbit is small, since the spin precession mainly is determined by the strong magnetic field along the classical orbit. In principle one could then determine the particle motion in the following way: One first solves for the orbital motion, neglecting the spin, and then one determines the spin motion, treating the orbital fluctuations and the quantum fields as perturbations. However, when calculating the polarization of the particle beam, one can simplify this approach somewhat, since it is only the fluctuations in the particle orbit which are driven by the coupling to the radiation field which are important.

4 Spin transitions

The spin Hamiltonian can be written in the form

$$H_{spin} = \frac{1}{2}\hbar\vec{\omega}\cdot\vec{\sigma} \tag{14}$$

$$\vec{\omega} = \vec{\omega}_0 + \delta\vec{\omega} \tag{15}$$

with $\vec{\omega}_0$ giving rise to the classical part of the precession,

$$\vec{\omega}_0 = -\frac{e}{2mc}g\vec{B}_0 - \frac{a}{v}\vec{k} = -\frac{e}{2mc}(g-2)B_0\vec{k} \tag{16}$$

and $\delta\vec{\omega}$ as the fluctuation part,

$$\delta\vec{\omega} = -\frac{e}{2mc}[g\delta\vec{B} - \frac{ax}{c^2}g\vec{B}_0 - (g-2)\frac{a}{ec}\vec{i}\times\vec{\pi}] \tag{17}$$

In the last two expressions $\vec{k}$ is the unit vector orthogonal to the plane of motion and $\vec{B}_0 = B_0\vec{k}$ is the external magnetic field along the classical orbit. $\delta\vec{B}$ accounts for the fluctuations in the magnetic field. This can be separated into two parts,

$$\delta\vec{B} = \vec{B}_q + \delta\vec{B}_c \tag{18}$$

where $\vec{B}_q$ denotes the quantum field along the classical orbit and $\delta\vec{B}_c$ is the variation in the external field due to fluctuations in the orbit.

The spin motion now can be determined by time dependent perturbation theory. $\vec{\omega}_0$ then defines the unperturbed part of the spin Hamiltonian and $\delta\vec{\omega}$ the perturbation. To first order, the transition probabilities per unit time between the levels of the unperturbed Hamiltonian are given by

$$\begin{aligned}\Gamma_\pm &= \lim_{T\to\infty}\frac{1}{4T}\left|\int_{-T/2}^{T/2} e^{\pm i\omega_0\tau}\delta\omega_\mp(\tau)|0>\right|^2 \\ &= \frac{1}{4}\int_{-\infty}^{+\infty} d\tau e^{\mp i\omega_0\tau} < 0|\delta\omega_\pm(\tau/2)\delta\omega_\mp(-\tau/2)|0>\end{aligned} \tag{19}$$

$|0>$ in this equation denotes the state of the combined system of radiation field and orbit variables, unperturbed by the spin. $\delta\omega_\pm$ is a linear combination of the x- and y-component of $\delta\vec{\omega}$,

$$\delta\omega_\pm = \delta\omega_x \pm i\delta\omega_y \tag{20}$$

The same notation will be used for other variables in the following.

A useful substitution rule which can be used in the expression for $\delta\omega_\pm$ is the following one,

$$\frac{d}{d\tau}F \rightarrow \pm i\omega_0 F \tag{21}$$

The difference between these two expressions only gives rise to end effects in the integral for the transition amplitude, and for large T this is suppressed in $\Gamma_\pm$ due to the prefactor $1/T$. This substitution rule now can be used to eliminate the orbital variables in the expression for $\delta\omega_\pm$, which can be written in the form

$$\delta\omega_\pm = -\frac{e}{2mc}[gB_{q\pm} + g\delta B_{c\pm} \pm 2iv\omega_0\frac{m}{ec}\dot{z}] \tag{22}$$

With the stable orbit in the symmetry plane of the magnetic field, $\delta B_{c\pm}$ only gets contribution only from the gradient in the z-direction. This implies that (to lowest order) $\delta\omega_\pm$ only depends on the vertical fluctuations in the particle orbit. These fluctuations in turn are determined by coupling to the radiation field in the following way,

$$\ddot{z} - \frac{2e^2}{3mc^3}(\dddot{z} - \frac{a^2}{c^2}\dot{z}) + \Omega^2 z = \frac{e}{m}E_{qz} \tag{23}$$

where a radiation reaction term has been introduced and where non-linear terms have been neglected. The restoring electric force in the z direction can be related to the gradient in the magnetic field,

$$\Omega^2 = \frac{a}{\rho}n \tag{24}$$

$$n = \frac{\rho}{B_0}\frac{\partial B_z}{\partial x} = \frac{\rho}{B_0}\frac{\partial B_x}{\partial z} \tag{25}$$

ρ is the radius of the (classical) electron orbit, and n the fall-off parameter of the magnetic field.

By use of the substitution rule eq.(21) now can be solved for z,

$$z = [\Omega^2 - \omega_0^2 \pm i\Delta\omega_0]^{-1}\frac{e}{m}E_{qz} + \Lambda \tag{26}$$

$$\Delta = \frac{2e^2}{3mc^3}(\frac{a^2}{c^2} + \omega_0^2) \tag{27}$$

Λ here denotes a term which is suppressed fo large T. When the expression for z is inserted in eq.(22), this gives (for $v \approx c$),

$$\delta\omega_\pm = -\frac{e}{2mc}[gB_{q\pm} + (2 + f_\pm(g))E_{qz}] \tag{28}$$

with $f_\pm(g)$ as a resonance term,

$$f_\pm(g) = \frac{(g-2)\Omega^2}{\Omega^2 - \omega_0^2 \pm i\Delta\omega_0} \tag{29}$$

This term blows up when the frequency of the free oscillations in the z-direction is close to the classical spin precession frequency, but it tends rapidly to zero away from the resonance.

The new expression for $\delta\omega_\pm$ (28) now only depends on the free quantum fields, and the transition probabilities can be expressed in terms of correlation functions of these fields along the particle orbit,

$$\Gamma_\pm = \frac{1}{4}\int_{-\infty}^{+\infty} d\tau e^{\mp i\omega_0\tau} < 0|\delta\omega_\mp(\tau/2)\delta\omega_\pm(-\tau/2)|0 > \tag{30}$$

(A correct treatment of the singularity at $\tau = 0$ corresponds to a small shift $\tau \to \tau - i\epsilon$.) $|0>$ then refers to the vacuum state of the radiation field. To calculate these probabilities now is straightforward. The fields in the comoving frame most conveniently are expressed in terms of lab frame fields and the correlation functions of these are found by expressing the field operators in terms of creation and annihilation operators. To leading order in $1/\gamma$ the relevant fourier integrals can be calculated analytically. I will not discuss details about this here, but only show the result for the stationary value of the polarization, fig.(1). The polarization is determined by the population of the two spin levels, and this in turn is found by the standard argument of equilibrium between transitions up and down. We have,

$$P = \frac{\Gamma_+ - \Gamma_-}{\Gamma_+ + \Gamma_-} \tag{31}$$

In fig.(1) the polarization is shown as a function of γ. Except for values close to the resonance with the vertical motion, the standard result for the polarization is found, $P = 0.924$. The effect of the resonance is mainly to depolarize the beam, but an interesting detail is the coherent effect which gives a maximum value of $P = 0.992$ close to the resonance. Thus, at least in principle it is possible to exceed the limiting value of 0.924.

5 The Unruh effect

The expression for the transition probabilities $\Gamma_\pm$ now makes it possible to see the close relation between the polarization effect and the Unruh effect [7]. Let me rewrite it in the form

$$\Gamma_\pm = \int_{-\infty}^{+\infty} e^{i\omega_0\tau} C_\pm(\tau) \tag{32}$$

with

$$C_+(\tau) =< D^\dagger(0)D(\tau) > \tag{33}$$

$$C_-(\tau) =< D(\tau)D^\dagger(0) > \tag{34}$$

I have here introduced the new notation $D = (1/2)\delta\omega_+, D^\dagger = (1/2)\delta\omega_-$. The operator D then is a linear combination of electric and magnetic fields in the comoving frame,

$$D(\tau) = \vec{\alpha}\cdot\vec{E}(x(\tau)) + \vec{\beta}\cdot\vec{B}(x(\tau)), \tag{35}$$

where $x(\tau)$ is the particle orbit. The expression (32) is similar to that which defines transitions in a point detector in the case of the Unruh effect. The main difference is that the world line of the detector then corresponds to linear acceleration rather than to circular motion as in the present case. However the excitations of the accelerated systems in the two cases can be understood qualitatively in the same way. The correlation functions $C_\pm$ gives a measure of vacuum fluctuations of the electromagnetic field along the orbit $x(\tau)$, and these fluctuations give rise to excitations in the detector when C_+ includes a spectral component which coincides with the excitation energy.

To see this more clearly let me first discuss the simplest case, namely with a two level detector at rest. The transitions between the levels are given by the same set of equations, (32-35), but now simply with

$$x(\tau) = (\tau, 0) \tag{36}$$

Since $D(\tau)$ is a linear combination of electromagnetic fields it can be decomposed in the form

$$D(\tau) = \int d^3k \sum_r [c_r(\vec{k}) e^{-ikx} a_r(\vec{k}) + d_r(\vec{k}) e^{+ikx} a_r^\dagger(\vec{k})] \tag{37}$$

where $a_r(\vec{k})$ and $a_r^\dagger(\vec{k})$ are photon annihilation and creation operators and $c_r(\vec{k})$ and $d_r(\vec{k})$ fourier coefficients. According to eq.(32) it is only the positive frequency parts of this operator which is relevant for the transitions. Positive frequency then is measured relative to the proper time along the orbit $x(\tau)$. But with the detector at rest this coincides with positive frquencies measured in the lab frame. And, as is well known, the positive frequency part of lab frame fields only contains annihilation operators. This is simply because all excitations in the lab frame have positive energy. So the relevant component of $D(\tau)$ is

$$\int_{-\infty}^{+\infty} d\tau e^{i\omega_0 \tau} D(\tau) = 2\pi \int d\Omega_k \omega_0^2 \sum_r c_r(\vec{k}) e^{i\vec{k}\cdot\vec{x}} a_r(\vec{k}) \tag{38}$$

which only anihilates photons with the same energy as the energy splitting of the two-level system. As a consequence of this the probability for transitions up in energy is zero, since the D operator then acts on the vacuum state. However, transitions down may be different from zero, since in this case it is instead $D^\dagger$ which acts on the vacuum state. Thus, the vacuum fluctuations only induce transitions to lower energies. This clearly is related to energy conservation in the combined system of detector and radiation field.

If the two-level system moves with constant velocity the picture is the same, since the sign of the zero component of the photon momentum k is the same in all inertial frames. The only way to have a non-zero probability for excitations to higher energies is to include other states than the vacuum state in the one which the operator D acts on. In particular the probability is non-zero for states with temperature $T \neq 0$.

However, for accelerated motion this is no longer the case. $x(\tau)$ then is no longer a linear function of τ and both functions $e^{-ikx(\tau)}$ and $e^{+ikx(\tau)}$

will in general have positive frequency parts in terms of the variable τ. In addition, for the electromagnetic field, there will be a τ-dependent Lorentz transformation connecting the fields in the comoving frame with the lab frame fields. The net effect is to introduce a mixing between the positive and negative frequency parts, so that both the anihilation and the creation parts of the operator D will have positive frequency components in terms of the time variable τ. As a consequence of this there will be in general non-vanishing probabilities for excitations both up and down in energy for the accelerated system, even with the quantum field in the vacuum state.

For uniform linear acceleration along the z-axis, the accelerated path $x(\tau)$ is described by

$$t = \frac{a}{c}\sinh(\frac{a}{c}\tau),\ z = \frac{a}{c}\cosh(\frac{a}{c}\tau),\ x = y = 0, \tag{39}$$

The trajectory $x(\tau)$ in this case depends only on one free parameter, which is the rest frame acceleration a. An interesting symmetry which is present for this motion corresponds to a shift in the τ-parameter in the imaginary direction,

$$x(\tau) = x(\tau + i\frac{2\pi c}{a}) \tag{40}$$

This symmetry, together with general symmetries from field theory, related to PCT-invariance ([19,20,21]), gives a simple relation between the correlation functions corresponding to transitions up and down in energy,

$$C_+(\tau) = C_-(\tau - i\frac{2\pi c}{a}) \tag{41}$$

This relation is similar to one which is present for correlation functions at non-zero temperature, and it leads to a similar result for the ratio between probabilities for transitions up and down,

$$\begin{aligned}
\Gamma_+ &= \int_{-\infty}^{+\infty} d\tau e^{i\omega_0\tau} C_-(\tau - i\frac{2\pi c}{a}) && (42)\\
&= \int_{-\infty}^{+\infty} d\tau e^{i\omega_0(\tau + i\frac{2\pi c}{a})} C_-(\tau - i\frac{2\pi c}{a}) && (43)\\
&= \exp(-\frac{\hbar\omega_0}{a\hbar/2\pi c})\Gamma_- && (44)
\end{aligned}$$

If the ratio between the two transition probabilities now is interpreted as a Boltzmann factor, then there is a simple linear relation between the temperature associated with this factor and the acceleration a,

$$kT_U = \frac{a\hbar}{2\pi c} \tag{45}$$

T_U then is the Unruh temperature for the accelerated system and k the Boltzmann constant. The derivation shows that the thermal property of the excitation spectrum depends only on general properties of the quantum fields and on special properties of the accelerated trajectory $x(\tau)$. Details of the accelerated system is not important. Returning to the case of accelerated electrons, one then might be tempted to consider the case of linear acceleration as the best case for seeing the Unruh effect in a real experiment. An additional magnetic field along the electron path could provide the necessary splitting of the spin energy levels. However, as discussed in ref.[4] the time needed to reach equilibrium is far too long to make this relevant for the motion of electrons in linear accelerators. For electrons in cyclic accelerators much larger accelerations can be obtained and correspondingly much smaller time constants for the approach to equilibrium.

Let me now briefly return to the case of the circulating electrons to look at the electron polarization from the point of view of the Unruh effect. One main difference between circular motion and linear acceleration is that the former depends on two independent parameters, which we may take to be a and γ. If we disregard this complication and naively assume that the acceleration a is the important one, in the same way as for the linear Unruh effect, then we find the following. The energy splitting of the spin system is for $\gamma \gg 1$,

$$\Delta E = \hbar\omega_0 = (g-2)\frac{\hbar a}{2c} \tag{46}$$

This gives a Boltzmann factor

$$\exp(-\frac{\Delta E}{kT_U}) = \exp[-\pi(g-2)] \tag{47}$$

where T_U is the Unruh temperature given by (45). Numerically the factor is close to one, 0.99, as compared to the correct value for the ratio between the population of the two spin levels (away from the resonance), which is close to zero, 0.04. So a naive application of the Unruh temperature formula to the case of electrons accelerated in circular orbit does not give the correct value for the population of the two spin levels, and therefore for the polarization.

However, it may still be of interest to make a closer comparison of the two effects. Instead of considering only the physical value of g we may consider the functional dependence of g for the relative population of the two spin levels. We first note that the correct ratio $R(g)$ in fact can be written as a function of $\Delta E/kT_U$, but it is not a simple exponential fuuction. Nevertheless, a plot of $R(g)$ and the Boltzmann factor (47) shows a clear resemblance between these two functions (Fig.2). The main difference is a relative shift along the g-axis. In fact, such a complication is not totally unexpected, since the Boltzmann factor (47) refers to the rotating O-frame. If the excitation spectrum should have a thermal form it might be more natural to assume this to be the case in the non-rotational C-frame. That would give a shifted exponential curve, but shifted with two units along the g-axis. The correct curve is located somewhere between these two exponential curves, and I have no simple explanation for the exact position of this curve. For large g-factors the rotation of frames is less important, and the ratio $R(g)$ then in fact is exponentialy damped,

$$R(g) \approx \frac{0.016}{g} \exp[-\sqrt{3}(g-2)] \tag{48}$$

This corresponds to a temperature which is somewhat higher than the Unruh temperature

$$T_{eff} \approx \frac{\pi}{\sqrt{3}} T_U \approx 1.8 T_U \tag{49}$$

I will now leave the question of polarization of the accelerated electrons and as a final point briefly return to the orbital excitations. The (Heisenberg) equation of motion for the vertical oscillations can be written as

$$\ddot{z} + 2\Gamma\dot{z} + \Omega^2 z = \frac{e}{m} E_{qz} \tag{50}$$

In this equation $\Gamma = (e^2a^2)/(3mc^5)$ and only the most important part of the radiation damping term has been kept. Making use of the fact that the damping is small, $\Gamma \ll \Omega$, one can solve the equation to find an (approximate) expression for $z(\tau)$ in terms of the quantum field E_{qz}. For the fluctuation in the z-coordinate one finds the following expression

$$< z^2 >= \frac{1}{2\Gamma}(\frac{e}{m\Omega})^2 \int_{-\infty}^{+\infty} d\tau e^{-\Gamma|\tau|} \cos \Omega\tau < E_{qz}(\tau/2) E_{qz}(-\tau/2) > \quad (51)$$

This shows that the fluctuations in the vertical direction are determined by the correlation function of the z-component of the electric field along the classical orbit. The vertical fluctuations in fact can be interpreted as being due to the "circular Unruh effect" in a similar way as the polarization effect. The mean energy associated with the fluctuations is for large γ,

$$< E >_{vert} = m\Omega^2 < z^2 > = \frac{13}{96}\sqrt{3}\frac{a\hbar}{c} \quad (52)$$

It is proportional to the acceleration a, but with a different prefactor as compared with the linear Unruh effect. It corresponds to a somewhat higher temperature

$$T_{eff} \approx 1.5 T_U \quad (53)$$

To linear order the excitation spectrum in this case in fact has a thermal form, and there is no complication with rotating frames. So in this respect the vertical orbit excitations give a simpler demonstration of the Unruh heating in the circular case than the depolarization of the electrons do. But the fluctuations are small and to measure them may be much more difficult task. As a final comment let me just mention that the horizontal fluctuations are different. They are larger than the vertical fluctuations essentially by a factor γ^2. These fluctuations then depend not only on the acceleration a of the electrons (for $\gamma \gg 1$), but on both the parameters a and γ which characterize the circular orbit.

6 Concluding remarks

In this talk I have discussed quantum effects within a simple idealized model of a cyclic accelerator. In a more realistic case there will be several modifications of this picture. In the case of a strong focussing machine the magnetic field will no longer be uniform along the orbit. The unperturbed part of the spin Hamiltonian then will be time dependent, and as a consequence of this the perturbations cannot be described in terms of transitions between *stationary* spin levels. But the effect of the perturbations instead can be described as giving rise to transitions between *periodic orbits* in the spin variable.

In addition to this there may be other perturbations in the magnetic field that cause a coupling between vertical and horizontal oscillations. Also non-linear effects may be important. This will in general lead to a much richer structure of spin-orbit resonances than in the idealized model where only one resonance is present. All these effects certainly have to be taken into account when one wants to model the spin behaviour in a real accelerator. (For some recent references where complications of this kind are included, see [22,23,24]). Nevertheless, to understand the main aspects of the quantum effects for the accelerated electrons, such a simple, idealized model may be of interest. Let me therefore end my talk by pointing to some of the features of this model which I want to stress.

The effect of spontanous polarization of the circulating electrons, and the departure from full polarization, can be understood and described within a simple two-level model for the electrons. The external magnetic field along the orbit defines the (unperturbed) spin levels of the electrons, and the radiation field causes transitions between these two levels. The radiation field acts both directly on the spin, through the coupling to the magnetic moment, and also indirectly, through the fluctuations it introduces in the particle orbit.

Transition probabilities are determined by vacuum correlation functions of the electromagnetic fields along the classical orbit. No explicit reference to the radiation process is needed in this description . The effect then is similar to the Unruh effect for a linearly accelerated two-level system coupled to the radiation field. But there are complications due to the rotations of frames along the orbit.

The fluctuations in the orbital motion can be determined in a similar way. Vacuum fluctuations in the electromagnetic field along the classical orbit introduce orbital excitations. The vertical fluctuations have a thermal excitation spectrum, but with a slightly higher temperature than the Unruh temperature for linear acceleration.

Finally, in the simple model considered here, there is one resonance between spin and vertical oscillations. The main effect of the resonance is to depolarize the electron beam, but a detail which has been noted is the coherent effect which causes the polarization to pass the "maximum" value of 0.92 close to the resonance.

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FIGURE CAPTIONS

FIGURE 1. The equilibrium polarization P as a function of γ close to the resonance with the vertical oscillations. The dashed line corresponds to the limiting value $P = 0.92$, away from the resonance. The scale for γ is relative to the resonance value γ_r. Positive polarization corresponds to the direction opposite to the magnetic field.

FIGURE 2. The ratio $R(g)$ between the equilibrium population of the two spin levels as a function of the gyromagnetic factor g. A is the result of the detailed calculation. B follows from assuming a thermal distribution over the levels in the O-frame, with temperature determined by the Unruh formula.

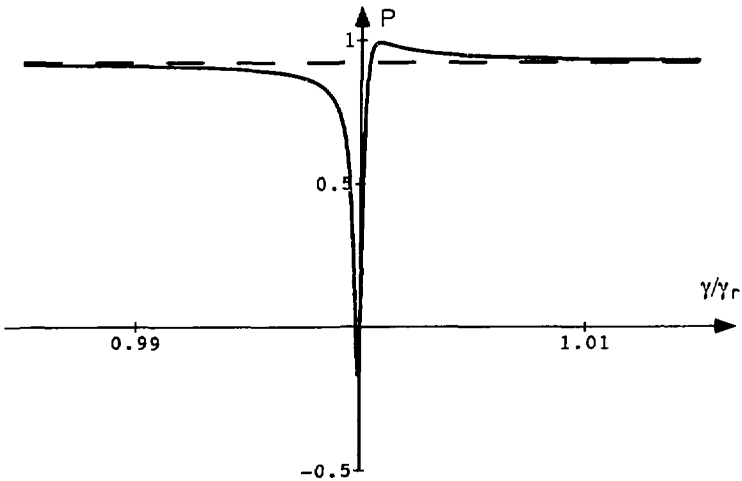

FIGURE 1

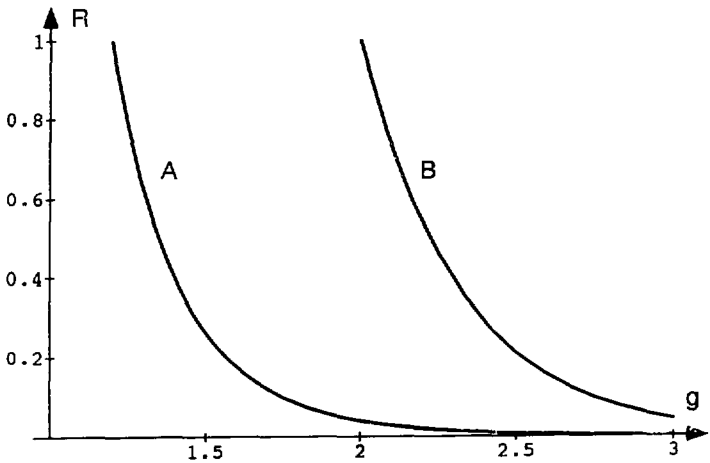

FIGURE 2

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