The Use of Dispersion Relations to Construct Unified Nucleon Optical Potentials

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Abstract: The dispersion relations provide a simple and accurate way of parametrising the optical potential for a particular nucleus over a range of energies. A method is proposed for obtaining a global nucleon optical potential incorporating the dispersion relations.

1. Introduction

The optical model is now established as a convenient and accurate way of parametrising a wide range of nucleon scattering of data. Global potentials have been obtained that give good fits to differential elastic scattering cross sections and polarisations for all but the lightest nuclei over a range of energies and these potentials have parameters that are almost the same for all nuclei and have a known energy dependence.

The quality of fit obtained with such global potentials does however vary through the periodic table, due to the effects of nuclear structure. For optimum fits to the data these effects are rather small for the real part of the potential but may be appreciable for the imaginary part of the potential. In particular, if the scattering nucleus has a strongly collective character the coupling between the elastic and inelastic channels affects the elastic scattering and renders inadequate the predictions of a global potential.

There are several ways of tackling this difficulty. One is to use the coupled channels formalism for such nuclei and thus include explicitly the effects of the coupling to inelastic channels. This restores the global quality of the fits but requires a knowledge of the coupling parameters. A further disadvantage is that coupled calculations are lengthy and so most of the simplicity of the model is lost.

Another response is to develop a new global potential valid for nuclei of similar structures, as has been done for the actinide nuclei by Madland and Young (1978). This retains the computational simplicity of the optical model but is useful only for a particular set of nuclei. It is thus necessary to develop a new parametrisation for each set of nuclei with different level structures. For lighter nuclei it may even be necessary to have a different set of parameters for each to achieve sufficient accuracy.

This paper emphasises the usefulness of a third method that makes use of the dispersion relations that connect the real and imaginary parts of the optical potential. As described in recent reviews (Hodgson, 1988, 1989) these dispersion relations automatically take into account the coupling between the elastic and inelastic channels. If we know the imaginary potential then the dispersion relations enable the correction to the real part due to the coupling to be evaluated. It is still necessary, of course, to insert

into the calculation some information concerning the structure of the target nucleus, but hopefully this can be done by a single normalising constant instead of by the many numbers necessary to specify the collectivity in coupled-channels calculations.

It is usual in precise parametrisations of the optical potential to fix the form factor parameters and to allow the real and imaginary potential depths to vary in a way that optimises the fit to the data. However recent work has shown (Finlay *et al*, 1985, Su Zong Di and Hodgson, 1988) that the radius parameter must also be allowed to vary with energy if an optimum fit is to be obtained. This could of course be included in the parametrisation at the cost of more adjustable parameters. The advantage of the dispersion relation method is that the energy variation of the radius of the potential is included automatically, **without** additional parameters. Since it is physically based, the dispersion potentials are likely to retain their validity when the accuracy of the data increases, whereas this is not necessarily the case for an ad hoc parametrisation, however elaborate. Furthermore, it seems possible that the interaction can be described by a global potential, valid for many nuclei, in which the structure of each individual nucleus is represented by just one adjustable parameter.

If these assumptions are correct, then it should be possible to define a global optical potential that when inserted into an optical model code modified to include the dispersion correction gives the elastic scattering cross-section at all energies for all nuclei. All the parameters of this potential are fixed and known, except a single parameter that normalises the imaginary potential and thus takes account of the nuclear structure effects. This parameter has to be determined for each nucleus by an optical model analysis at any one energy; once found the values of this parameter can be tabulated for future use. It is to be expected that the values will be the same for nuclei with similar collective structure like the actinides, and it may prove possible to develop simple rules connecting this parameter with say the deformation parameter for the lowest collective state. Such rules would make possible the prediction of cross-sections for nuclei for which no scattering measurements are available, provided something is known of their low-lying band structures.

An advantage of the dispersion relation analysis is that it unifies the data over a range of energies so that once the parameters are determined it is possible to calculate cross-sections at any required energy.

In this paper, the dispersion relations are described in Section 2, together with the results of some recent analyses. In Section 3 a global method of analysing nucleon data is proposed, and some conclusions are presented in Section 4.

2. Dispersion Relations Analysis

The essential idea is to use the dispersion relations to connect the real and imaginary parts of the potential, instead of treating them as independent of each other. The most useful form is

$$V(E) = V_{HF}(E) + \frac{P}{\pi}\int_{-\infty}^{\infty} \frac{W(E')}{E' - E} dE' \tag{2.1}$$

where $V_{HF}(E)$ is the Hartree-Fock potential that varies linearly with energy and can be represented by a Saxon-Woods radial form

$$V_{HF}(r) = \frac{V_{HF}}{1 = \exp\left(\frac{r-R}{a}\right)} \tag{2.2}$$

The imaginary potential $W(E)$ has volume and surface-peaked components. The volume component gives a volume contribution and so may be absorbed in the Hartree-Fock field, while the surface-peaked component gives a surface-peaked addition to the Hartree-Fock field, thus automatically giving the required energy-dependent radius.

Many analyses of experimental data have now been made using the dispersion relations, and here we present a selection of recent results.

One of the advantages of dispersion relations analyses is that they unify the optical potential over the whole range of energies. This is shown for ^{209}Bi in Figure 1, which

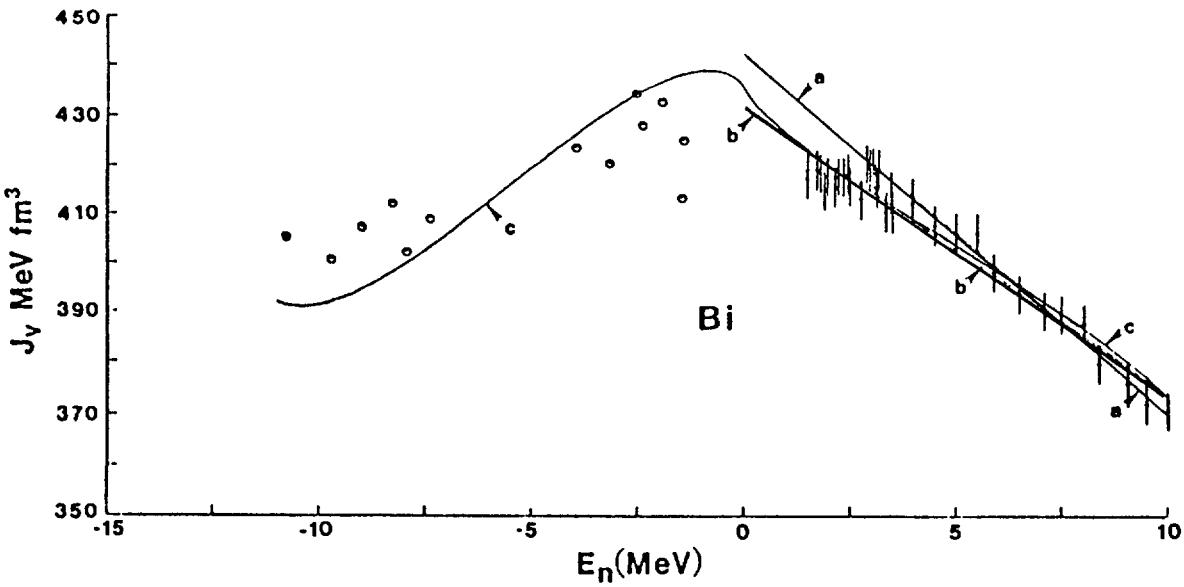

Fig.1 Volume integral per nucleon of the real neutron potential for ^{209}Bi for bound and unbound energies. The points at positive energies are obtained from optical model analyses of elastic scattering data and those at negative energies from the binding energies of particle and hole states. The curves show: a, a linear fit to the scattering data from 4.5 to 10 MeV; b, a linear fit to all scattering data; c, calculated from the dispersion relation (Lawson, Guenther and Smith, 1987).

compares the data for bound and scattering states with the extrapolation made using the dispersion relations. The potentials giving the bound states at their measured energies are given much more accurately than by a simple linear extrapolation of potentials obtained from analyses of scattering data. This is also shown by the energy spectra in Figure 2, where the experimental energies are in much better agreement with those found from a potential with the dispersion correction.

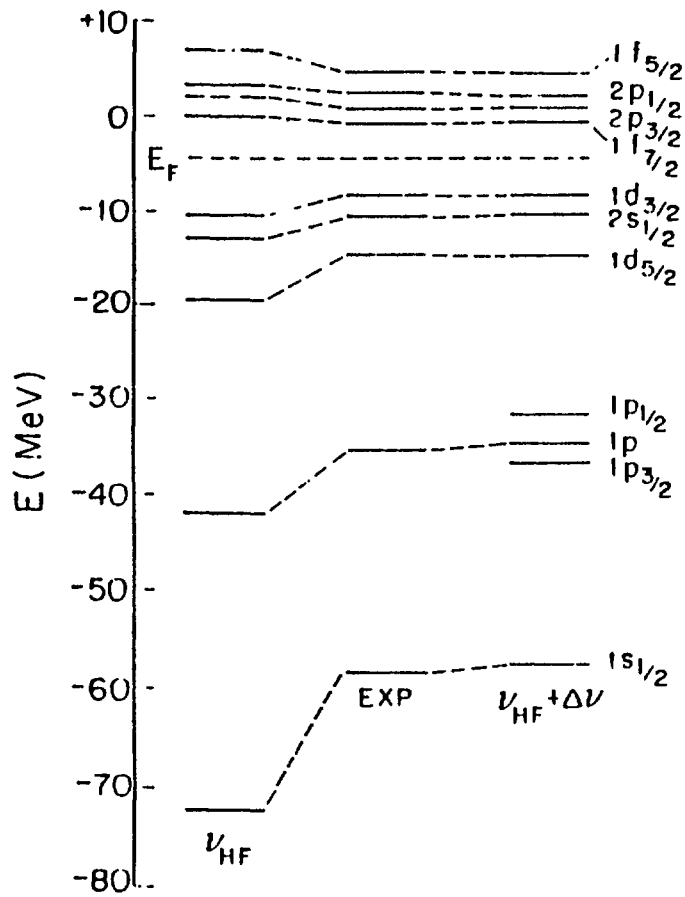

Fig.2 Proton single-particle energies E_{nlj} in ^{40}Ca. The column labelled EXP represents the experimental values. The columns labelled ν_{HF} and $\nu_{HF}+\Delta\nu$ give the energies obtained from the Hartree-Fock potential and from the real part of the full mean field, respectively (Tornow, Chen and Delaroche, 1989).

At positive energies, the dispersion relations have now been used to obtain the analysing powers as well as the differential cross-sections, and a recent example is shown in Figure 3. A very precise analysis of differential cross-sections for the elastic scattering of 4 to 10 MeV neutrons by ^{208}Pb has shown that the differences between the data and the dispersion analyses found by Johnson *et al* (1987) may be attributed to overlapping resonances in that energy range. This shows a limitation on the accuracy of dispersion analyses.

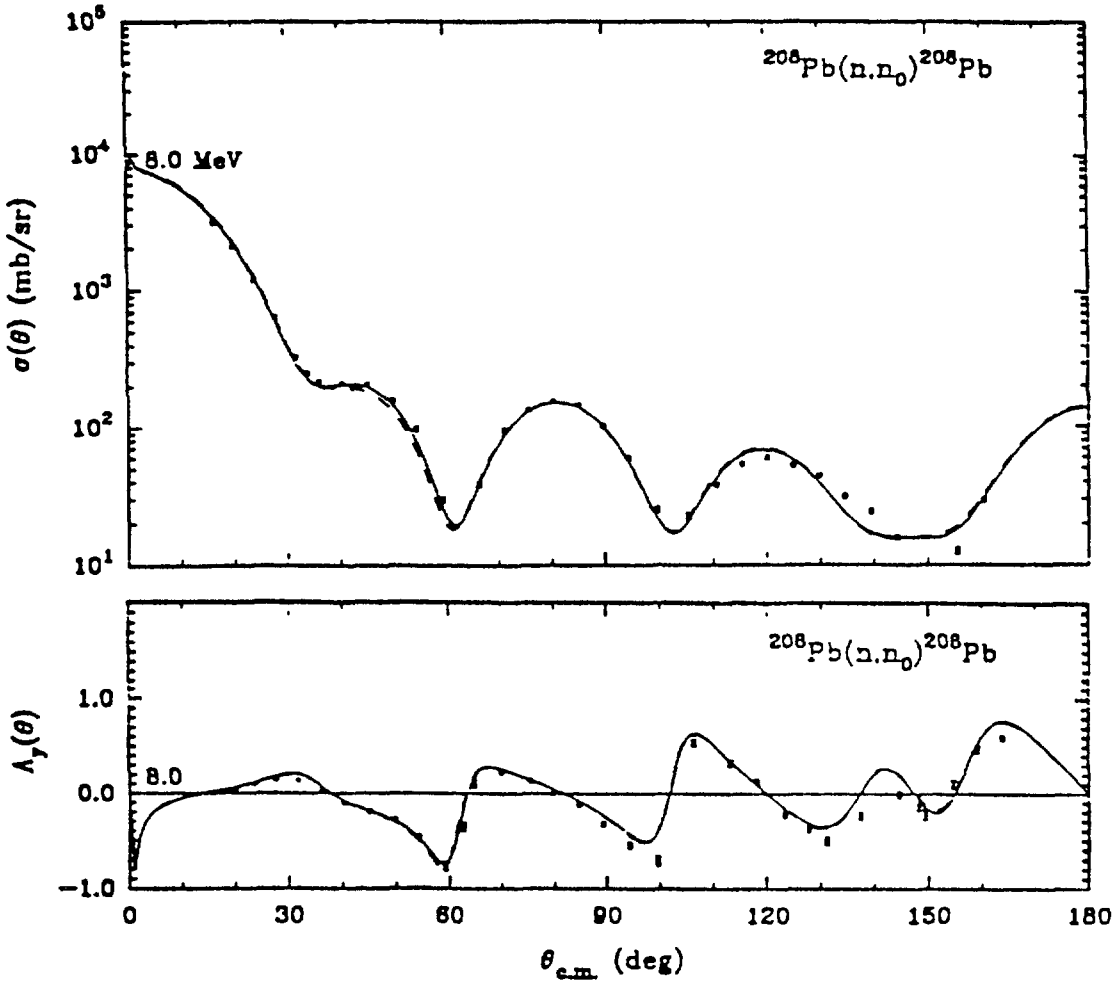

Fig.3 Differential cross-sections and analysing powers for the elastic scattering of 8 MeV neutrons by ^{208}Pb compared with dispersion relation calculations (Roberts *et al*, 1989).

In the case of analyses of neutron data, a particular feature is that the dispersion relations give automatically the increase in radius of the potential at low energies, which produces notable effects on the differential cross-sections (Finlay *et al*, 1985) and on the total cross-sections (Zong Di and Hodgson, 1988). In the past the effects of the increase in radius has been parametrised by using different potentials in different energy regions. The dispersion relations enable the same potential to be used over the whole energy range.

3. Determination of Global Optical Potentials

The success of the dispersion relations analyses of neutron and proton data suggests that it would be useful to use them to obtain global potentials that represent the data to good accuracy over a wide range of nuclei and energies. A method of doing this is described in this section.

The optical model analysis of a set of data for the interaction of nucleons with nuclei over a substantial range of energies can be carried out by a global fitting procedure that differs in several important respects from a standard optical model analysis. The form factor parameters may be fixed to standard average values, and V_{HF} allowed to vary linearly with energy. The main difficulty is that the dispersion relation (2.1) requires the imaginary potential over the whole energy range. The optimum form is suggested by those successfully used in some recent detailed analyses of neutron scattering (Johnson *et al*, 1987; Hicks and McEllistrem, 1988) and specified below. These give simple parametrised expressions for the energy variation of both the real and the imaginary parts of the potential. The same energy variations can be used for other nuclei, with a normalising factor applied to the imaginary part to take account of the structures of the different nuclei. This does however assume that the relative contributions of the volume and surface absorption are the same for all nuclei. The particular feature of these potentials is that the surface imaginary potential falls to zero above a certain energy so that the integral in (2.1) converges.

The two parametrised forms of the imaginary potential already mentioned are:

1. The straight line segment potentials of Johnson *et al* (1987) for ^{208}Pb

$$W_v(E) \begin{cases} = 0 & \text{for } E < 10 \text{ MeV} \\ = 0.17(E-10) & \text{for } 10 < E < 50 \text{ MeV} \\ = 6.8 & \text{for } E > 50 \text{ MeV} \end{cases} \tag{3.1}$$

$$\text{and } W_s(E) \begin{cases} = 0.4(E-E_F) & \text{for } -6 < E < 10 \text{ MeV} \\ = -0.103(E-72) & \text{for } 10 < E < 72 \text{ MeV} \\ = 0 & \text{for } E > 72 \text{ MeV} \end{cases} \tag{3.2}$$

(and symmetric expressions for $E < E_f$).

2. The potentials used by Hicks and McEllistrem (1988) for osmium and platinum

$$W_v(E) \begin{cases} = 0 & \text{for } E < 8 \text{ MeV} \\ = 2.33(E^{\frac{1}{3}} - 8^{\frac{1}{3}}) & 8 < E < 40 \text{ MeV} \\ = 2.33(40^{\frac{1}{3}} - 8^{\frac{1}{3}}) \equiv\approx 8.1 \text{ MeV} & \text{for } E > 40 \text{ MeV} \end{cases} \tag{3.3}$$

$$\text{and } W_s(E) = \frac{a_0 + a_2(E-E_F)^2}{\{E_0^2 + (E-E_F)^2\}^2}$$

(and symmetric expressions for $E < E_F$)

The peak strengths of these potentials are:

	Surface	Volume
Pb	6.4	6.8
Os — Pt	9.2	8.1

It is encouraging that these potentials are very similar in overall shape and absolute and relative magnitudes, with the OsPt potential slightly stronger, as it should be. The difference is quite small, encouraging the hope that all nuclei can be fitted with multiplying factors for W in quite a narrow range. To verify this, it is necessary to fit a wide range of data to find the optimum values of these factors for many nuclei.

4. Conclusions

The dispersion relations potential has several clear advantages over the standard parametrisation. It includes the physically-necessary connection between the real and imaginary parts of the potential and thus automatically includes without additional parameters the energy dependence of the radius that is required by precision analyses. Furthermore, it holds out the hope that it will prove possible to represent the effects of nuclear structure by a single parameter that has a characteristic value for each nucleus.

The extent to which these hopes can be realised can only be evaluated by a series of careful analyses of extensive data sets for several nuclei. The work that has already been done is sufficiently encouraging to suggest that precision optical model analyses should in future be made with the dispersion relations potential.

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