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FUTURE PROSPECTS OF PREGEOMETRY*

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ABSTRACT

Pregeometry is a theory, first suggested by Sakharov in 1967, in which gravity is taken as a quantum effect of matter fields and in which Einstein's theory of general relativity for gravity appears as an approximate and effective theory at long distances. It is shown by reviewing the extensive developments of pregeometric theories of gravity for the last more than a decade that the original idea by Sakharov is really working. Many future prospects of the theories are discussed in some detail.

1. Introduction

In 1967, Sakharov proposed an eminent idea for the theory of gravitation[1]. He said, "In Einstein's theory of gravitation one postulates that the action of space-time depends on the curvature (R is the invariant of the Ricci tensor):

$$S(R) = \frac{1}{16\pi G}\int d^4x\sqrt{-g}R. \tag{1}$$

The presence of the action (1) leads to a 'metrical elasticity' of space, i.e., to generalized forces which oppose the curving of space. Here we consider the hypothesis which identifies the action (1) with the change in the action of quantum fluctuations of the vacuum when space is curved. Thus, we consider the metrical elasticity of space as a sort of level displacement effect." Let us call this picture of gravity "pregeometry" by following Wheeler[2]. Namely, pregeometry is a theory in which gravity is taken as a quantum effect of matter fields and in which Einstein's theory of general relativity for gravity appears as an approximate and effective theory at long distances.

He continued[1], "In present-day quantum field theory it is assumed that the energy momentum tensor of the quantum fluctuations of the vacuum $T^i_{\ k}(0)$ and the corresponding action S(0), formally proportional to a divergent integral of the fourth power over the momenta of the virtual particles of the form $\int k^3dk$, are actually equal to zero. Recently Ya. B. Zel'dovich[3] suggested that gravitational interactions could lead to a

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'small' disturbance of this equilibrium and thus to a finite value of Einstein's cosmological constant, in agreement with the recent interpretation of the astrophysical data. Here we are interested in the dependence of the action of the quantum fluctuations on the curvature of space. Expanding the density of the Lagrange function in a series in power of the curvature, we have (A and B~1)

$$L(R) = L(0) + A\int kdk\cdot R + B\int\frac{dk}{k}R^2 + \cdots . \qquad (2)$$

The first term corresponds to Einstein's cosmological constant. The second term, according to our hypothesis, corresponds to the action (1), i.e.,

$$G = \frac{1}{16\pi A\int kdk}, \quad A \sim 1. \qquad (3)$$

The third term in the expansion, written here in a provisional form, leads to corrections, nonlinear in R, to Einstein's equations."

Sakharov further sketched how to proceed by saying[1], "The divergent integrals over the momenta of the virtual particles in (2) and (3) are written down from dimensional considerations. Knowing the numerical value of the gravitational constant G, we find that the effective integration limit in (3) is

$$k_0 \sim 10^{28}\text{eV} \sim 10^{+33}\ \text{cm}^{-1}.$$

In a gravitational system of units $G = \hbar = c = 1$. In this case $k_0 \sim 1$. According to the suggestion of M. A. Markov, the quantity k_0 determines the mass of the heaviest particles existing in nature and which are called 'maximons' by him. It is natural to suppose also that the quantity k_0 determines the limit of applicability of present-day notions of space and causality. Consideration of the density of the vacuum Lagrange function in a simplified 'model' of the theory for noninteracting free fields with particles $M \sim k_0$ shows that for fixed ratios of the masses of real particles and 'ghost' particles (i.e., hypothetical particles which give an opposite contribution to that of the real particles to the R dependent action) a finite change of action arises that is proportional to M^2R and which we identify with R/G. Thus, the magnitude of the gravitational interaction is determined by the masses and equations of motion of free particles, and also, probably, by the 'momentum cut-off'." In 1978, in a way toward constructing the super-grand unified composite model in which not only quarks and leptons but also Higgs scalars, gauge bosons and even the graviton are all composites of more fundamental particles, the subquarks [4], we eventually found in a simple field theoretical model of pregeometry that this whole idea of pregeometry by Sakharov is indeed working[5].

For simplicity, suppose that the most fundamental matters in nature are N real scalars, ϕ_i $(i=0,1,2,\cdots,N-1)$. Then, the simplest action for the matter fields in scalar pregeometry that is invariant under general coordinate transformation is given by

$$S_0 = F^{-1}\int d^4x(-\det\partial_\mu\phi\cdot\partial_\nu\phi)^{1/2} \qquad (4)$$

where F is an arbitrary constant and a•b denotes $\eta^{ij}a_ib_j$ with η^{ij} = diag(-1,1,1,•••,1). This action is effectively equivalent to another action for the matter fields with space-time metric $g^{\mu\nu}$ as an auxiliary field,

$$S_0' = \int d^4x\sqrt{-g}(\tfrac{1}{2}g^{\mu\nu}\partial_\mu\phi\cdot\partial_\nu\phi-F) \qquad (5)$$

where g = det $g_{\mu\nu}$ since S_0' leads to the following "equation of motion" for the metric:

$$g_{\mu\nu} = F^{-1}\partial_\mu\phi\cdot\partial_\nu\phi. \qquad (6)$$

The effective action for the metric due to quantum fluctuations of the matter fields is given by

$$S_0^{eff} = -i\ell n\{\int\prod_i[d\phi_i]\exp(iS_0')\}. \qquad (7)$$

The path-integration over ϕ_i can be formally performed to yield

$$S_0^{eff} = iNtr\ \ell n(\frac{1}{\sqrt{-g}}\partial_\mu\sqrt{-g}\tfrac{1}{2}g^{\mu\nu}\partial_\nu)-F\int d^4x\sqrt{-g}\ . \qquad (8)$$

This action can be expanded in terms of the curvature scalar and curvature tensors as

$$S_0^{eff} = \int d^4x\sqrt{-g}[\lambda + \frac{1}{16\pi G}R + c(R^2+dR^{\mu\nu}R_{\mu\nu}) + \cdots] \qquad (9)$$

with

$$\lambda = \frac{N\Lambda^4}{8(4\pi)^2} - F,\ \frac{1}{16\pi G} = \frac{N\Lambda^2}{24(4\pi)^2},\ c = \frac{N\ell n\Lambda^2}{240(4\pi)^2}\ \text{and}\ d=2 \qquad (10)$$

where λ, G and Λ are the cosmological constant, the Newtonian gravitational constant and the momentum cut-off of the Pauli-Villars type, respectively.

This completes a simple demonstration of how the remarkable idea of pregeometry as a theory of gravity, which is not alternative to but more fundamental than Einstein's is really working.

2. Super-Grand Unification in Pregauge-Pregeometry

What are the consequences of pregeometry with which one can distinguish between Einstein's theory of gravity and Sakharov's one? Since at low energies (or long distances) an infinite series of the terms higher order in the curvature scalar and curvature tensors in the effective action of gravity as in Eq.(9) can be neglected compared to the Einstein-Hilbert action proportional to R, there is no difference between these two theories as far as the pure gravitational phenomena at low energies are concerned. However, the first remarkable consequence of pregeometry is that the fine structure constant α of electromagnetism and the Newtonian constant G of gravitation are related to each other. This consequence seems to be known to Sakharov already in 1967 as he concluded Ref.1 by saying, "This approach to the theory of gravitation is analogous to the discussion of quantum electrodynamics in Ref.6, where the possibility is mentioned of neglecting the Lagrangian of the free electromagnetic field for the calculation of the renormalization of the elementary electric charge. In the paper of L. D. Landau and I. Ya. Pomeranchuk the magnitude of the elementary charge is expressed in terms of the masses of the particles and the momentum cut-off: For a further development of these ideas see Ref.7, in which the possibility is established of formulating the equations of quantum electrodynamics without the "bare" Lagrangian of the free electromagnetic field." In 1977, in a unified model of the Nambu-Jona-Lasionio type for all elementary-particle forces including gravity[8] we eventually derived a simple relation between α and G, which has turned out to be similar to the relation conjectured by Landau in 1955[9]. We have succeedingly found that such relation as called G-α relation is one of the eminent consequences of any super-grand unified theories in pregeometry[10], which will be illustrated in the following:

Notice that the actions S_0 and S'_0 have the global O(N-1,1) symmetry, which can be taken as an internal symmetry in pregeometry. Let us extend these actions by making this O(N-1,1) symmetry a local one[11]. It can be done by replacing $\partial_\mu\phi_i$ by

$$(D_\mu\phi)_i = \partial_\mu\phi_i - i(\lambda^{ab})_i{}^j\phi_j A_{ab\mu} , \tag{11}$$

where $A_{ab\mu}$ (a,b = 0,1,2,$\cdots$,N-1 and $A_{ab\mu} = -A_{ba\mu}$) are the auxiliary gauge fields and λ^{ab} are the O(N-1,1) generator matrices with the elements of $(\lambda^{ab})_{ij} = i(\delta_i{}^a\delta_j{}^b - \delta_j{}^a\delta_i{}^b)$. Then, the action S_0 becomes

$$S = F^{-1}\int d^4x(-\det D_\mu\phi\cdot D_\nu\phi)^{1/2} \tag{12}$$

while the action S'_0 does

$$S' = \int d^4x\sqrt{-g}\left(\frac{1}{2}g^{\mu\nu}D_\mu\phi\cdot D_\nu\phi - F\right). \qquad (13)$$

These actions S and S' define a model of scalar pregauge-pregeometry, which provides the simplest toy model for super-grand unification of gravitation and gauge interactions such as strong and electroweak interactions. As S_0 and S_0', S and S' are effectively equivalent to each other since S' leads to the following "equation of motion" for the metric:

$$g_{\mu\nu} = F^{-1}D_\mu\phi\cdot D_\nu\phi\ . \qquad (14)$$

Notice that S' also leads to the following "field-current identities" of the gauge fields[12]:

$$A_{ab\mu} = \frac{\phi_a\overleftrightarrow{\partial}_\mu\phi_b}{2(\phi\cdot\phi)}\ . \qquad (15)$$

The effective action for the metric and gauge fields is given by

$$S^{eff} = -i\ln\left\{\int\prod_i[d\phi_i]\exp(iS')\right\}\ , \qquad (16)$$

whose path-integration over ϕ_i can be formerly performed to yield

$$S^{eff} = i\,\mathrm{tr}\,\ln\left(\frac{1}{\sqrt{-g}}\Delta_\mu\sqrt{-g}\frac{1}{2}g^{\mu\nu}\Delta_\nu\right) - F\int d^4x\sqrt{-g} \qquad (17)$$

where

$$(\Delta_\mu)_i{}^j = \delta_i{}^j\partial_\mu - i(\lambda^{ab})_i{}^jA_{ab\mu} \qquad (18)$$

and tr denotes the trace operation not only over the space-time but also over the internal quantum numbers of i and j. This action can be expanded not only in terms of the curvature scalars and tensors but also in terms of the field strength of gauge field as

$$S^{eff} = \int d^4x\sqrt{-g}\left[\lambda + \frac{1}{16\pi G}R + c(R^2 + dR^{\mu\nu}R_{\mu\nu}) - \frac{1}{4e^2}\mathrm{tr}\,F^{\mu\nu}F_{\mu\nu} + \cdots\right] \qquad (19)$$

where

$$(F_{\mu\nu})_{ab} = \partial_\mu A_{ab\nu} - \partial_\nu A_{ab\mu} + f^{cd,ef}{}_{ab} A_{cd\mu} A_{ef\nu} , \tag{20}$$

$$\frac{1}{e^2} = \frac{1}{3(4\pi)^2} \ell n \Lambda^2 \tag{21}$$

and $f^{cd,ef}{}_{ab}$ are the structure constants of O(N-1,1) defined by

$$[\lambda^{ab}, \lambda^{cd}] = if^{ab,cd}{}_{ef} \lambda^{ef} . \tag{22}$$

In the effective action (19), not only the Einstein-Hilbert action of gravity but also the Yang-Mills action of gauge interactions is reproduced as an effective action at low energies (or long distances) due to the quantum fluctuations of matter fields in the vacuum. This illustrates a simple example for "pregauge-pregeometric theories". One of the consequences of such theories is the relation between the Yang-Mills gauge coupling constant and the Newtonian gravitational constant which follows immediately from Eqs. (10) and (21) as

$$e^2 = 3(4\pi)^2 / \ell n(24\pi GN) . \tag{23}$$

It can also be shown[13] that if the mass of fundamental scalar M is as large as the Planck mass ($G^{-1/2} \sim 10^{19}$GeV), the relation would become, instead of (23),

$$e^2 = 32\pi GNM^2 . \tag{24}$$

3. Pregeometric Origin of the Big Bang[14]

Pregeometry has also changed the notion of the space-time metric completely. As indicated in Eq.(6), the space-time metric can be taken as a kind of composite object of the fundamental matters. Therefore, we can even imagine that at high temperature the space-time metric would dissociate into its constituents just as ordinary objects do. Then, the metric would vanish although the fundamental matters still remain in the mathematical manifold of the space-time. Namely, the pregeometric phase is the phase of the space-time in which the metric $g^{\mu\nu}(g_{\mu\nu})$ vanishes (diverges) and, therefore, the distance of $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$ diverges. There, the space-time still exists as a mathematical manifold for the presence of the fundamental matters. Such an extraordinary phase may be realized in such regions as that beyond the space-time singularity i.e., before the big bang and that far inside a black hole where the temperature is extremely high (as high as the Planck mass).

In order to calculate the temperature-dependent effective action for gravity in pregeometry, we introduce the temperature-dependence by the following replacements in our effective action of S_0^{eff} in Eq.(8):

$$p^0 \to i2\pi nT \quad \text{and} \quad \int d^4p \to 2\pi T \int d^3p \sum_{n=-\infty}^{\infty} \tag{25}$$

where p and *T* are the *momentum* and the temperature of the fundamental matters, respectively, and n is an integer. For simplicity, we restrict ourselves to the specific case where the metric is parametrized with two parameters as

$$g^{\mu\nu} = b^2 \mathrm{diag}(-1, \xi^2, \xi^2, \xi^2) \ . \tag{26}$$

After a simple calculation, we finally obtain the following effective potential for the metric at high temperature:

$$V^{eff} \cong \frac{N\xi^3 T}{2\pi^2} \int_0^\infty d\zeta \ \zeta^2 \ln[1 - \exp(-\sqrt{\zeta^2 + m^2 b^2}/T)] \tag{27}$$

where m is the mass of the fundamental matters. The behavior of V^{eff} as a function of b for a fixed ξ is illustrated in Fig.1(a). It looks like a deep well with the minimum at the origin where b=0. The bottom of the well where

$$V^{eff}\Big|_{b=0} \cong -N\pi^2 \xi^3 T^4/90 \tag{28}$$

gets deeper and deeper as T^4 as temperature increases. The pregeometric phase would then be realized at the bottom of this well where b=0 or $g^{\mu\nu}$=0. This might indicate that the only stable phase at finite temperature in pregeometry would be the pregeometric one. This expectation seems, however, too naive since no other interactions among the fundamental matters than the pregeometric one have not yet been taken into account.

In order to obtain a more realistic effective potential for the metric, we phenomenologically assume that due to the other interactions among the fundamental matters there exist some effects that prevent the space-time metric from vanishing at least at low temperature. These effects may add to the effective potential such a contribution as illustrated in Fig.1(b). Furthermore, we assume that these effects of the other interactions on gravity are less dependent of the temperature than those of the pregeometric ones. A typical behavior of the sum of these different contributions is illustrated in Fig.1(c). In this figure, one can then naturally find that although only the pregeometric phase is stable at very high temperature, the geometric phase where the metric is finite and non-vanishing will turn out to be stable as the temperature goes down. This remarkable possibility of phase transitions of the space-time between the geometric and pregeometric phases will exhibit a characteristic feature of pregeometry, if it is found.

Where can we find such phase transition of the space-time? It seems very attractive to interpret the origin of the big bang of our Universe as such a local and spontaneous phase transition of the space-time from the pregeometric phase to the geometric one in the overcooled space-time manifold which had been present in the "pre-big-bang" era for some reason. The enormous energy of the big bang can be taken as the latent heat liberated by the phase transition of the space-time. This interpretation of the big bang also suggests that there may exist thousands of universes created and expanding in the space-time manifold as our Universe. It even predicts that such different universes may collide with each other. Furthermore, even in our present Universe there may exist "pregeometric holes", the local spots in the pregeometric phase with an extremely high temperature where the space-time metric disappears, liberating enormous latent heat, and/or "space-time discontinuities", the local plains where the metric (and, therefore, the light velocity or the Newtonian gravitational constant) discretely changes due to the phase difference of two adjacent space-times (or two colliding universes). I have been strongly urging astronomical and cosmological experimentalists to search for these pregeometric holes and space-time discontinuities, which are much more exotic than black holes. It would be fascinating if the recently observed "Great Wall" of galaxies be caused by such space-time discontinuity.

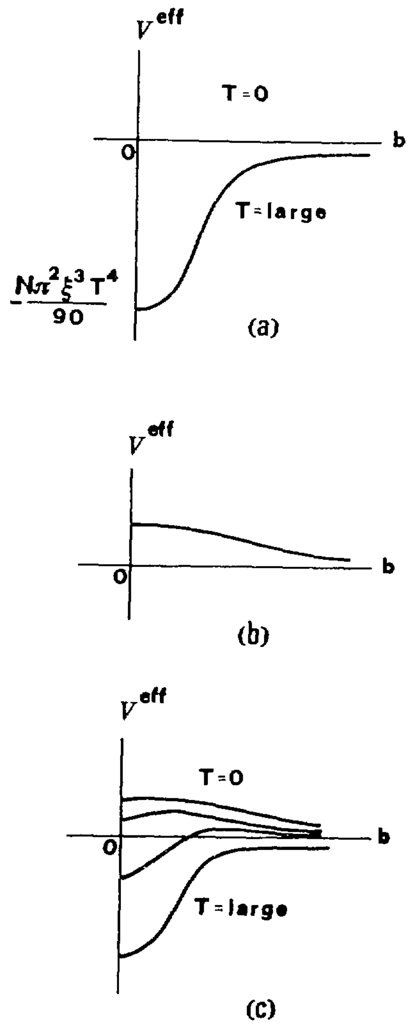

FIG 1

4. Pregeometric Origin of the Space-Time Dimensions

Pregeometry has further provided us a basis on which we can discuss such a "prephysical" problem as how the space-time dimension of our Universe may be determined. I have presented three possible paradigms which may determine the number of space-time dimensions[15]. Let us first define the space-time as a domain where the space-time metric is finite and non-vanishing, i.e.,

$$g_{\mu\nu}(x) \neq 0 \quad \text{and} \quad |g_{\mu\nu}(x)| < \infty \quad \text{for} \quad \mu,\nu=0,1,2,\cdots,N-1 \tag{29}$$

in a mathematical manifold of the space-time coordinates of N dimensions, (x^μ).

The first paradigm is to assert that since there is no reason why the space-time is a priori of a definite number of dimensions, it may be of indefinite dimensions. In other words, the dimension of the space-time, N, may also be a "dynamical" variable. As other ordinary dynamical variables, the number of dimensions may be determined by the "Hamilton principle" of

$$\delta_n S_n = 0 \quad \text{for} \quad n=N \quad \text{or} \quad S_n = \min \quad \text{for} \quad n=N \tag{30}$$

where S_n is the Einstein-Hilbert action integral of general relativity for gravity extended to a n-dimensional space-time. See the details in Ref.15, in which we have demonstrated how N can be determined in a simple model for S_n.

The second paradigm is to assert that the space-time metric may be of the definite dimensions if it is made of some fundamental matters as in pregeometry[11]. In fact, as seen previously, in scalar pregeometry the space-time metric may be taken as the following vacuum expectation value of the operator product of fundamental scalar fields:

$$g_{\mu\nu}(x) = F^{-1}\langle\partial_\mu\phi(x)\cdot\partial_\nu\phi(x)\rangle_0 . \tag{31}$$

The dimension of the space-time is determined by the "condensation" of the fundamental fields as

$$\langle\partial_\mu\phi(x)\cdot\partial_\nu\phi(x)\rangle_0 \neq 0 \quad \text{for} \quad \mu,\nu=0,1,2,\cdots,N-1, \tag{32}$$

and, therefore, dependent on the unknown dynamics of the fundamental fields. However, in general it may be that the more fundamental fields exist, the more space-time dimensions would appear. A similar idea has long been conjectured in the "space-color correspondence"[16]:

$$(x_0\ x_1\ x_2\ x_3) \leftrightarrow (C_0\ C_1\ C_2\ C_3) \tag{33}$$

where C_α (α=0,1,2,3) are "chroms", the subquarks which are the fundamental constituents of quarks and leptons carrying the color quantum number of $SU(4)_c$[4].

The third paradigm is an additional constraint such as asymptotic freedom, no anomaly, renormalizability, finiteness, and so on. It is now well-known that the space-time dimension is required to be twenty-six for the Lorentz algebra to hold in bosonic string theories and that it is to be ten for the super-Poincare algebra to hold in superstring theories[17]. This once fashionable argument for determining the space-time dimension certainly falls into the third paradigm. The ingenious argument by Sakharov[18] in 1984 for selecting a particular series of $N=4\ell$ (ℓ=1,2, 3,$\cdots$) for the space-time dimension from the assumed positivity of the extended Einstein-Hilbert Lagrangian also seems to fall into the third paradigm. It is, however, weaker than the first two since such an additional constraint is rather mundane (or "theoretishe-theoretical") without any direct relation with the origin of the space-time.

Let us now demonstrate how the space-time dimension may be determined in the second paradigm. In scalar pregeometry, the action integral S'_0 in Eq.(5) can be extended into

$$S'_n = \int d^n x\sqrt{-g}\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\cdot\partial_\nu\phi - \frac{n-2}{2}F\right) \tag{34}$$

in the N-dimensional space-time (μ,ν = 0,1,2,$\cdots$,n-1). As before, differentiating the action S'_n with respect to $g^{\mu\nu}$ gives the "equation of motion" for the metric

$$-\frac{1}{2}g_{\mu\nu}\left(\frac{1}{2}g^{\alpha\beta}\partial_\mu\phi\cdot\partial_\nu\phi - \frac{n-2}{2}F\right) + \frac{1}{2}\partial_\mu\phi\cdot\partial_\nu\phi = 0 , \tag{35}$$

from which the "field-current identity" of the space-time metric of

$$g_{\mu\nu} = F^{-1}\partial_\mu\phi\cdot\partial_\nu\phi \tag{36}$$

follows. By using this relation, the auxiliary field of $g_{\mu\nu}$ can be eliminated in the action S'_n, which then becomes an action of the Nambu-Goto type,

$$S_n = F^{-(n-2)/2}\int d^n x[-\det(\partial_\mu\phi\cdot\partial_\nu\phi)]^{1/2}. \tag{37}$$

Therefore, the action S'_n is effectively equivalent to the action S_n at least at the classical level of dynamics.

Now, an important observation is that in deriving the "field-current identity" (36) we have already assumed the existence of $g^{\mu\nu}$ as the inverse matrix of $g_{\mu\nu}$ and, therefore, the existence of the non-vanishing determined of $g_{\mu\nu}$, i.e.,

$$\det(\partial_\mu\phi\cdot\partial_\nu\phi) \neq 0. \tag{38}$$

An elementary theorem on matrix and determinant tells us that this condition requires the number of space-time dimensions, n, cannot be larger than that of the fundamental scalars, N, i.e.,

$$n \leqq N. \tag{39}$$

This completes the demonstration. Hereafter we shall take the case of n=N, which is the simplest and most beautiful.

For n=N, let $e_{i\mu}$ be the N-bein of space-time which is related with the space-time metric as

$$g_{\mu\nu} = e_\mu \cdot e_\nu \ . \tag{40}$$

One can then find that in scalar pregeometry the long hypothesized space-color correspondence is realized in the modified form of

$$e_{i\mu} \leftrightarrow \partial_\mu\phi_i \quad \text{for} \quad \mu, i = 0, 1, 2, \cdots, N-1 \tag{41}$$

if the scalar fields ϕ_i are identified with the chroms C_i, the most fundamental particles carrying the quantum numbers of "O(N-1,1) colors". This scalar pregeometry with the O(N-1,1) color symmetry can be easily extended into the one with the U(N-1,1) color symmetry by converting the real scalar fields into complex ones. It can be also extended into a model of scalar pregauge-pregeometry which is invariant not only under general-coordinate transformation for the space-time but also under local-gauge transformation for the color, as discussed in the last section.

5. Pregeometry and String Theories[19]

Superstring theory once advocated as a candidate for theory of everything is a priori a kind of pregeometry from the beginning since it is supposed to reproduce Einstein's theory of gravity at low energies (or in the zero-slope limit)[17]. Moreover, as a field-theoretical model it has the same structure as pregeometry. In fact, the action S_n of pregeometry in Eq.(37) with N scalars in two dimensions (n=2) is identical with the Nambu-Goto action of the bosonic string model in N dimensions,

$$S_{N-G} = \int d^2\sigma[-\det(g^{\mu\nu}\partial_\alpha X_\mu \partial_\beta X_\nu)]^{1/2}, \quad (42)$$

if we re-interpret the space-time coordinates x_μ (μ=0,1) and the fields $\phi_i(x)$ (i=0,1,2,$\cdots$,N-1) in scalar pregeometry as the world-sheet parameters σ_α (α=0,1) and the space-time coordinates $X_\mu(\sigma)$ (μ=0,1,2,$\cdots$,N-1) of the string, respectively. Let us call this "field-space correspondence",

$$\phi_i(x_\mu) \leftrightarrow X_\mu(\sigma_\alpha) \; . \quad (43)$$

In general, a pregeometry with N scalars in n dimensions corresponds to a model of n-dimensional objects in the N-dimensional space-time. Also, general coordinate invariance in pregeometry corresponds to re-parametrization invariance in string models.

It seems then tempting to extend a string model, a membrance model, etc. by applying the field-space correspondence to pregeometry. The most straightforward extension may be to induce the gravitation-like effects in the extended objects through quantum fluctuations as in pregeometry. It can be called a pregeometric string model. Another type of extension is to make the space-time symmetry local in the parameter space. It can be called a gauge-invariant string model and is an analogy with pregauge-pregeometry which has been discussed in Section 3. A lot of other extensions are possible. It seems to me, however, that pregauge-pregeometry is closer to the "final theory of everything" than superstring theories.

6. Pregeometry and Topological Field Theories

Very recently, topological field theories whose Lagrangians do not depend on the space-time metric and can be taken as total derivatives have been extensively studied[20]. It is partially because they may provide us an insight into what physics looks like in quantum gravity at short distances where the space-time metric is no longer well-defined.

It has been expected that pregeometry and topological field theories are closely related to each other by their definitions. In fact, Floreanini and Percacci[21] have recently made a simple but interesting observation that scalar pregeometry for n=N can be taken as a model in topological field theories since the Lagrangian density in the action (37) is a total derivative as more clearly seen in the transformed form of

$$S_n = F^{-(n-2)/2}\int d^nx\,|\det\partial_\mu\phi_i|. \quad (44)$$

More recently, Akama and Oda[22] have remarkably shown that the scalar pregauge-pregeometric action presented in Ref.11 is a topological invariant if the number of internal gauge groups is equal to that of space-time dimensions.

What is not clear in Ref.11 is whether the space-color correspondence (41) can be derived from any particular action. The simplest candidate for such action is

$$S''_n = F^{1/2}\int d^nx\; e[e^{\mu}\cdot\partial_{\mu}\phi-(n-1)F^{1/2}] \tag{45}$$

where $e = \det e_{i\mu}$. Indeed, differentiating S''_n with respect to $e^{i\mu}$ gives the "equation of motion" for the n-bein,

$$-e_{i\mu}[e^{\nu}\cdot\partial_{\nu}\phi-(n-1)F^{1/2}] + \partial_{\mu}\phi_i = 0, \tag{46}$$

from which the "space-field identity" of the n-bein of

$$e_{i\mu} = F^{-1/2}\partial_{\mu}\phi_i \tag{47}$$

follows. This completes a realization of the space-color correspondence but it turns out to be too restrictive to be practical. See the details in Ref.23.

I wish to emphasize here that this close connection between pregeometry and topological field theories is worth making further detailed investigation.

7. Future Prospects of Pregeometry - Beyond the End of Space-Time

As seen in the previous sections, pregeometry has provided us a new theoretical framework in which we can discuss such basic problems as super-grand unification of all forces in pregauge-pregeometry, the phase transition of the space-time, the origin of the big bang and that of the space-time dimensions. The super-grand unification in pregauge-pregeometry seems to be the most natural and promising as it is completely along the line of composite models which has always been a right track in modern physics[4]. Not only quarks and leptons but also gauge bosons and even the graviton can be taken, in a sense, as composites of subquarks, the most fundamental particles. This line of development toward pregauge-pregeometry (or "pregaugeometry" in short) illustrated in Section 2 as the theory of everything is certainly the most desirable feature to be expected.

As for renormalizability of quantum gravity, pregeometry also indicates a possible direction: the presence of R^2 term in the effective action as seen in Eq. (9) or (19) enforces faster damping of the graviton propapator as $(p^2)^{-2}$ for the large momentum squared of p^2 and may make quantum gravity renormalizable apart from the problem of possible ghosts[24]. This nice feature of effective theories of gravitation has recently been emphasized by Ne'eman[25]. Because of the infinite series of R^n terms for n>2, however, actual demonstration of renormalizability of the effective action for gravity seems to be difficult. Pregeometry tells us that it is more important to find which type of theory may describe gravity at high energies (or short distances) than to find whether a particular theory like Einstein's is renormalizable or not.

In pregeometry, neither the Einstein-Hilbert term of R nor the cosmological term is fundamental. Both of them are induced or effective. Therefore, the question of why the cosmological constant is absent or so small is reduced to the one of why it is not induced. A possible answer is that conformal (or scale) invariance respected by pregeometric dynamics forbids the appearance of the cosmological constant. Instead, the Einstein-Hilbert term appears due to the spontaneous breakdown of scale invariance in the Universe. This situation has been illustrated in details in a simple pregeometric model with spontaneously broken conformal invariance in Ref.26, which will not be repeated here. Another possible answer is that the cosmological constant vanishes due to the exact cancellation between the positive contributions induced by scalar fundamental matter fields and the negative ones induced by spinor fields. This miraculous cancellation would occur if the number of fundamental scalars is twice as large as that of fundamental Dirac spinors[14]. It may not be a mere coincidence that the minimal composite model consists of a color-quartet of scalar subquarks ("chroms") and an iso-doublet of spinor subquarks ("wakems")[4], which realizes the above "primitive supersymmetry".

One of the hottest subjects in quantum gravity and particle physics is to find what happens near, at and beyond the Planck length. Near the Planck length, in pregeometry all the higher order terms in R survive in the effective action for gravity, which is given schematically by

$$S^{eff}_{grav} = \int d^4x\sqrt{-g}f(R) \qquad (48)$$

$$\text{where} \quad f(0) = 0, \quad f'(0) = \frac{1}{16\pi G}, \cdots. \qquad (49)$$

What would happen near the Planck length is hard to imagine from the properties of the lower terms, $f'(0)R + \frac{1}{2}f''(0)R^2$, being influenced strongly by the higher terms and even by the limiting behavior of $f(\infty)$. At and beyond the Planck length, physics can be described not by the effective action but by the unknown pregeometric dynamics. Wheeler said[2], "Is it not likewise hopeless to go from the 'elasticity of geometry' to an understanding of particle physics, and from particle physics to the uncovering of pregeometry?"

What should be emphasized here before conclusion is a fundamental difference between the Einstein's picture of describing gravity and the Sakharov's one. Einstein's theory of gravity consists of the following two equations: One is the field equation for the space-time metric of

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu}, \qquad (50)$$

where $T_{\mu\nu}$ is the energy-momentum tensor of matter fields, and the other is the geodesic equation of motion for a point-particle of

$$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau} = 0, \qquad (51)$$

where τ is the proper time and $\Gamma^{\mu}_{\alpha\beta}$ is the connection coefficient. The field equation can be derived from the stability of the Einstein-Hilbert action of

$$S_E = \int d^4x\sqrt{-g}\left(\frac{1}{16\pi G}R + L_m\right) \qquad \text{with} \quad T_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\partial\sqrt{-g}L_m}{\partial g^{\mu\nu}}, \tag{52}$$

where L_m is the Lagrangian for matter fields, while the geodesic equation from that of the integral of

$$I = \int ds \qquad \text{with} \quad ds^2 = g_{\mu\nu}dx^{\mu}dx^{\nu}. \tag{53}$$

In the Einstein's picture, given the energy-momentum tensor of matter (and given the appropriate boundary conditions), the space-time metric be determined by the field equation (50), and, then, the motion of a point-particle by the geodesic equaion (51).

On the other hand, Sakharov's theory of gravity can be formulated to consist of the single action of

$$S'_s = \int d^4x\sqrt{-g}L_m(g_{\mu\nu}, \phi_i), \tag{54}$$

where ϕ_i represent the matter fields, and the quantization conditon of

$$[\phi_i(x),\ \phi_j(y)] = i\delta_{ij}\Delta(x-y), \tag{55}$$

where Δ is the causal function. As discussed in Section 1, S'_s is effectively equivalent to

$$S_S^0 = -i\ln\{\int[dg_{\mu\nu}]\exp(iS'_s)\} \tag{56}$$

or to

$$S_S^{eff} = -i\ln\{\int\prod_i[d\phi_i]\exp(iS'_s)\}. \tag{57}$$

The stability of S_s^0 requires the field equations for the matter fields such as

$$\frac{1}{\sqrt{-g}}\partial_{\mu}\sqrt{-g}g^{\mu\nu}\partial_{\nu}\phi_i + \frac{\partial F}{\partial\phi^i} = 0 \qquad \text{with} \quad g_{\mu\nu} = F^{-1}\partial_{\mu}\phi\cdot\partial_{\nu}\phi. \tag{58}$$

In the Sakharov's picture, given the quantum state ($|\psi\rangle$) of the matter fields which satisfy both the quantization condition (55) and the field equations (58), the space-time metric be determined by the expectation value such as

$$g_{\mu\nu}(x) = \langle\psi|F^{-1}\partial_\mu\phi(x)\cdot\partial_\nu\phi(x)|\psi\rangle. \tag{59}$$

After the metric is determined, the motion of a particle can be determined by the geodesic equation (51). However, since the particle is made of the fundamental matter fields ϕ_i's, the motion can be determined in principle more directly without using the geodesic equation. Therefore, in pregeometry the geodesic equation (or the Newton's equation of motion in the non-relativistic limit) is unnecessary, in principle, or, in other words, may not be independent of the fundamental action S_s^0. To demonstrate the unnecessity of the geodesic equation in the four-dimentional space-time in pregeometry is difficult and has not yet been successfully made. It may, however, be instructive and suggestive to take the simplest possible case of scalar pregeometry in the one-dimensional space-time. The fundamental action is then given by

$$S_1^0 = \int dx\, \frac{d\phi}{dx}F^{1/2} = \int d\phi\, F^{1/2} \quad \text{or} \quad S_1' = \int dx\sqrt{g}\left[\frac{1}{2}g^{-1}\left(\frac{d\phi}{dx}\right)^2+\frac{1}{2}F\right], \tag{60}$$

from the latter of which follows

$$g = F^{-1}\left(\frac{d\phi}{dx}\right)^2. \tag{61}$$

On the other hand, the geodesic integral is given by

$$I_1 = \int\sqrt{g}dx = \int dx\, F^{-1/2}\frac{d\phi}{dx} = \int d\phi\, F^{-1/2} \tag{62}$$

Therefore, it is trivial to find that $\delta S_1^0 = 0$ and $\delta I_1 = 0$ are not independent of each other since both are identical for constant F.

What pregeometry means can be much wider than Sakharov himself might think it is. This has been emphasized also by Wheeler[2], who tried to formulate pregeometry in the middle of sixties even earlier than Sakharov. Wheeler even suggested that pregeometry is the calculus of propositions. It seems to me that pregeometry is at least a promising theory, framework or machinery for "prephysics", a new line of physics (or philosophy but not metaphysics) in which some basic assumptions taken as sacred ones in ordinary physics such as the four-dimensionality of space-time, the invariance under general coordinate transformation, the microscopic causality, the principle of superposition and so on are to be reasoned. Therefore, I wish to conclude this talk simply by quoting the following

Wheeler's word: Never more than today does one have the incentive to explore pregeometry.

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