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The Aharonov-Bohm Effect as a Particle Accelerator

John Costella
School of Physics, University of Melbourne, Parkville, Vic. 3052, Australia.

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Abstract:

It is shown how the Aharonov-Bohm (AB) effect, as displayed conclusively in the experiment of Tonomura *et. al.*, is calculated in classical electrodynamics. Comparison with the quantum prediction yields several new fundamental principles. It is also suggested that the properties of the AB effect may allow its use as a versatile particle accelerator.

1. Introduction

The phase shift predicted by Aharonov and Bohm [1] has been hotly debated for over thirty years. The experiments of Tonomura *et. al.* [2] have convincingly shown that the effect does, in fact, exist physically.

However, Aharonov and Bohm also conjectured [1] that the effect is not predicted by classical electrodynamics. It will be shown that this conjecture cannot be defended in the toroidal geometry. The air-cored toroidal solenoid will be considered first, to clearly illustrate the physical principles being used. This is then generalised to the solid-core toroidal solenoid, the hollow superconducting torus, and the superconductor-coated solid-core toroidal solenoid of Tonomura *et. al.* In each case it will be shown that the classical calculation gives the precise result of Aharonov and Bohm, and moreover that it is a result of classical forces acting on the electron.

A comparison of the quantum and classical predictions of the AB effect yields some unexpected relations of a fundamental nature, which are shown to be related to similar work carried out before the AB effect was postulated.

The energy considerations of the AB effect are also shown to parallel those of the domestic electronic amplifier, and it is suggested that this may be used to build a particle accelerator of great versatility and simplicity.

Notation follows that of Jackson [3], except that SI units are used to allow an assessment of the proposed experiments. The electron is assumed to be a classical point charge, and at *no* time is required to pass through the toroidal magnetic flux or onto the superconducting surface. It assumed that there is *zero* leakage flux from the toroidal space.

2. The Air-Cored Toroidal Solenoid

The simplest geometry of the AB experiment to analyse is that of Lyoboshits and Smorodinskii [4], in which one passes one half of an electron through the "hole" in a toroidal air-cored solenoid. We idealise the solenoid as being a (completely contained) tube of flux travelling around in a circle, in free space (more realistic experimental configurations are considered in later sections).

We first consider a classical electron travelling through the hole of the torus. Calculations are simplified if the electron is chosen to travel down the toroidal axis (the same results obtain if it is off-axis, but considerably more vector integral manipulation is required, and the physical principles are somewhat hidden).

The magnetic flux travelling around the toroidal space is taken to be Φ webers, circulating in an anticlockwise direction as seen by the approaching electron. The "radius" of the torus is R metres, and the cross-sectional area of the flux is S square metres. We assume that $R >> \sqrt{S}$, so that all parts of the flux are at roughly the same distance from the electron (one may take the limit $S \to 0$ if desired).

The electron is fired towards the torus, from a distance much greater than R, at v_0 metres per second. In its own rest frame, the electron has an electrostatic field; in the lab (torus) frame some of this is boosted into a magnetic field. We find that, at any position inside the toroidal space, the magnetic field is parallel to Φ, and its magnitude given by (see [3], p. 554)

$$B_{e,\,torus} = \frac{\mu_0 \gamma_0 v_0 e R}{4\pi (R^2 + \gamma_0^2 z^2)^{3/2}} \tag{1}$$

where $\gamma_0 = (1 - v_0^2/c^2)^{-1/2}$, and z is the perpendicular distance from the electron to the plane of the torus.

We now note that, since we are in free space, the magnetic field inside the torus will be the sum of Φ / S and $B_{e,\,torus}$, since classical electrodynamics is linear. The electromagnetic energy contained in the toroidal space is thus

$$E_{torus} = \frac{1}{2\mu_0} \int_{\text{TORUS}} \left(\frac{\Phi}{S} + B_{e,\,torus} \right)^2 d^3x \tag{2}$$

The energy in the toroidal space before the electron came along was

$$E_{lone\ torus} = \frac{1}{2\mu_0} \int_{\text{TORUS}} \frac{\Phi^2}{S^2}\, d^3x$$

The energy of that part of the electron's field now contained in the torus was, when it was far away,

$$E_{lone\ electron} = \frac{1}{2\mu_0} \int_{\text{TORUS}} B^2_{e,\,torus}\, d^3x$$

(where "lone electron" refers to only the "toroidal part" of the free electron energy). We have therefore increased the electromagnetic energy in the toroidal space, by an amount

$$\Delta E_{emag} = E_{torus} - E_{lone\ torus} - E_{lone\ electron} = \frac{1}{\mu_0 S} \int_{TORUS} \Phi\, B_{e,\,torus}\, d^3x$$

Now, our physical system consists only of a tube of flux and a free electron. Since energy is conserved in classical electrodynamics, the increase in energy of the electromagnetic field must be accompanied by an equal decrease in the kinetic energy of the electron.

At this point, it should be noted that we can differentiate ΔE to obtain the classical force on the electron, which is obviously non-zero (except in the plane of the torus). *The AB conjecture is thus founded on the false premise that a vanishing initial field at the position of the electron means a vanishing force on the electron,* as emphasised recently by Namiot [5]. It ignores the fact that an electric charge, by definition, is surrounded by an electrostatic field that is no less "real" than the particle itself. Indeed, if the interaction between an enclosed current and a toroidal solenoid *were* zero, the common domestic earth-leakage circuit breaker would not work.

To quantitatively obtain the AB phase shift it is simpler to deal directly with the electron energy, instead of calculating the force on the electron. For simplicity, we assume that the electron is non-relativistic, and that its velocity does not significantly depart from its initial value during its flight through the torus. For a small change in the electron energy, we then have

$$\Delta E_{electron} = m\, v_0\, \Delta v$$

where Δv is the resulting change in the electron velocity, and $\Delta v << v_0$. We then refer the electron's motion to a frame travelling towards the torus at a velocity v_0. The position of the electron relative to this frame, $\Delta z'$, after it has travelled through the torus is then given by

$$\Delta z' = \int_{-\infty}^{\infty} \Delta v(t)\, dt$$

Noting that $dt = dz / v_0$, that the volume of the torus is $2\pi S R$, and using the energy conservation relation $\Delta E_{electron} + \Delta E_{emag} = 0$, we immediately obtain

$$\Delta z' = \frac{-e\,\Phi}{2\, m\, v_0\, R} \int_{-\infty}^{\infty} (1 + z^2 / R^2)^{-3/2}\, dz$$

Noting that

$$\int_{-\infty}^{\infty} (1 + z^2 / R^2)^{-3/2}\, dz = 2\, R$$

(the substitution $z = R \tan\theta$ is helpful here), we then find that

$$\Delta z' = -\frac{e\,\Phi}{m\,v_0} \qquad (3)$$

We thus find that the electron has been delayed slightly by the toroidal flux distribution.

The part of the electron that travels outside the torus suffers no delay. The mathematical proof of this is straightforward but algebraically lengthy, and cannot be given in this letter. However, in the limit that the electron passes at a great distance from the torus, it can be seen quite easily. Assume that the electron passes far to the right of the torus. Its magnetic field will be almost vertical across the whole torus. We then note that

$$\Delta E_{emag} = \frac{1}{\mu_0} \int_{\text{TORUS}} \mathbf{\Phi} \cdot \mathbf{B}_{e2,\,\text{torus}}\, d^3x$$

which vanishes because the direction of $\mathbf{\Phi}$ changes around the torus, while the direction of $\mathbf{B}_{e2,\,\text{torus}}$ remains vertical.

We therefore come to the conclusion that an electron passing through a flux torus is delayed relative to one that passes outside the torus. This result is not new; for example, it has been proven for the cylindrical solenoid by Boyer [6] where he (rightly, in the author's opinion) describes the AB effect as a classical electromagnetic lag effect, as well as the analogous proof [7] for the Aharonov-Casher [8] effect. However, the ingenious Lyoboshits-Smorodinskii geometry allows a much simpler and transparent analysis of the AB effect, without the imposition of the pathological boundary conditions accompanying an infinite cylindrical solenoid.

It should be pointed out that the kinetic energy lost by the electron in approaching the torus is regained again as it departs. To relate (3) to the experimental phase shift, one must then only note that in the non-relativistic limit de Broglie's relation reads

$$\lambda = \frac{h}{m\,v_0}$$

and so the phase difference between the two electron waves is given by

$$\Delta\varphi = 2\pi\frac{\Delta z}{\lambda} = -\frac{e\,\Phi}{\hbar}$$

3. The Solid-Cored Toroidal Solenoid

The analysis of section 2 can be easily modified to include the more realistic situation of a magnetic medium inside the toroidal space. For simplicity, assume that the medium is linear, of permeability μ, and completely fills the torus. For realism, wrap a few turns of superconducting wire around the left "arm" of the torus, and set up a supercurrent to create a magnetic flux Φ (the circuit is closed, and not connected to any external source).

In this setup we have the added complication of "matching equations" for the electron magnetic field at the surface of the torus. For an electron travelling along the toroidal axis, the magnetic field is parallel to the torus boundary, and thus **H** is continuous across that boundary, whereas **B** is not (see [3], p. 190). Equation (1) must then be replaced by

$$H_{e,\,torus} = \frac{\gamma_0 \, v_0 \, e \, R}{4 \pi \, (R^2 + \gamma_0^2 \, z^2)^{3/2}}$$

$$B_{r,\,torus} = \mu \, H_{e,\,torus}$$

The torus flux, Φ, must by definition give a B of Φ / S; and therefore an H of Φ / S μ. Noting that the energy density in a linear magnetic medium is given by $\frac{1}{2}$ (**B** · **H** + **D** · **E**), equation (2) becomes

$$E_{torus} = \frac{1}{2} \int_{TORUS} \left(\frac{\Phi}{S} + B_{e,\,torus} \right) \left(\frac{\Phi}{\mu S} + H_{e,\,torus} \right) d^3x$$

$$= \frac{1}{2} \int_{TORUS} \left(\frac{\Phi^2}{\mu S^2} + \frac{\mu}{\mu_0^2} B_{e0}^2 + 2 \frac{\Phi B_{e0}}{\mu_0 S} \right) d^3x$$

where B_{e0} is the free space value of $B_{e,\,torus}$, as given by equation (1). Subtracting off the "lone torus" and "lone electron" energies, we obtain

$$\Delta E_{emag} = \Delta E_{scatt} + \frac{1}{\mu_0 S} \int_{TORUS} \Phi \, B_{e0} \, d^3x$$

where

$$\Delta E_{scatt} = \frac{1}{2 \mu_0} \int_{TORUS} \left(\frac{\mu}{\mu_0} - 1 \right) B_{e0}^2 \, d^3x$$

It can be seen that, apart from the term ΔE_{scatt}, the interaction energy is identical to the air-core case, and thus the AB effect will follow by the same analysis as section 2.

The additional interaction energy ΔE_{scatt} does not depend on the toroidal flux Φ, and would be present if there was just a lump of magnetic material present without any flux at all. Its strength depends on the incoming path of the electron; if the electron were travelling far outside the torus, the magnetic field would be mainly perpendicular to the toroidal boundary, and the subsequent different boundary condition (B continuous instead of H) leads to the interaction energy changing sign, and the electron being attracted rather than repelled.

This "scattering" is another example of the fact that a charged particle can be subjected to a force even if no field is initially present, and that the AB premise to the contrary is false. Conventionally, this effect is "explained" as follows: the moving electron is seen by the lump of magnetic material as a magnet; this magnet induces a magnetisation in the lump, which acts back on the magnet to repel or attract it. Anyone who has tried to pull a horseshoe magnet off their refrigerator can attest to the reality of this force.

It is also easily seen that (for off-axis electrons) the different forms of the matching conditions for tangential and perpendicular components of the electron field do not affect the AB interaction energy, since $\mathbf{\Phi} \cdot \mathbf{B} \equiv \mathbf{\Phi} \cdot \mathbf{B}_{\perp}$, and thus only the tangential component is selected. This was also true for the air-cored torus, so that once one has convinced oneself that the AB effect is independent of trajectory for the air-core case, the same results can be used for the solid-cored case.

It is evident that the "lump scattering" term in the interaction energy reduces the "purity" of the AB experiment somewhat. A full analysis of the geometry suggested of Greenberger *et. al.* [9], where a ferromagnetic core is employed, would require consideration of further "impurities" that have nothing to do with the AB effect. Of particular concern would be the hysteretic nature of ferromagnets, whereby energy is consumed in cycling the magnetic field and lost as heat. Since the solenoid is assumed to be free-running, this energy would be at the expense of the electron. Contrary to the air-core case, it would emerge from the torus with a lower energy (and hence a longer wavelength) than it had when it entered; worse still, the "external" electron wave would not necessarily lose the same amount of energy. The hysteresis loss in a ferromagnet also depends on the torus flux Φ. The extraction of a phase change solely attributable to the AB effect could validly be subject to reasonable doubt. It will be shown that these problems, and the accompanying doubts, are completely eliminated in superconductor-shielded geometries.

4. The Superconducting Torus

Another geometry for the AB effect was proposed by Kuper [10]: a hollow superconducting torus with magnetic flux "trapped" inside. This serves the dual purpose of keeping the flux Φ *in*, and keeping the electron electric and magnetic fields *out*.

The Kuper geometry has the beneficial feature that, once the flux has been established, all sources can be removed. This improves on the idealisation of section 2, where the "windings" of the confining solenoid were not analysed (see, however, the discussion by Boyer [6] for the validity of this omission). In this section *all* physical elements are analysed.

One might be concerned that, for the Kuper torus, the magnetic field of the approaching electron cannot "overlap" the flux Φ, as it did in section 2. The superconducting shell ensures that the flux inside remains equal to Φ at all times; the electron magnetic field now has a toroidal "chunk" missing. Since the internal flux Φ is unchanged, it would then appear that

$$\Delta E_{emag} \overset{?}{=} -\frac{1}{2\mu_0} \int_{\text{TORUS}} B^2_{e,\,torus}\, d^3x \tag{4}$$

This interaction energy would result in an attractive force towards the torus, that did not depend on Φ. We have lost the AB effect!

Fortunately, equation (4) is incorrect: the energy stored in the supercurrent-superflux system was not carefully considered. To perform this analysis, we first note that the boundary condition on **H** gives (see [3], p. 190)

$$B_{outside} - B_{inside} = \mu_0 K \qquad (5)$$

where K is the magnitude of the surface current density on the superconducting shell. To obtain the energy of the (currents + flux) system, consider a superconducting torus with a solenoid inside, initially unenergised. Instead of using **B** and **H** to calculate the stored energy, we use **J** and **A** (see [3], p. 215):

$$\delta E_{supercurrent} = \frac{1}{\mu_0} \int_{TORUS} \mathbf{H} \cdot \delta \mathbf{B} \; d^3x$$

$$\equiv \int_{TORUS} \mathbf{J} \cdot \delta \mathbf{A} \; d^3x$$

$$\equiv \int_{SURFACE} \mathbf{K} \cdot \delta \mathbf{A} \; d^2x$$

We consider slowly energising the torus from an initial enclosed flux of $\phi = 0$, up to its final value $\phi = \Phi$. The toroidal "tube" has a circular cross-section; we denote its radius by ρ, i.e. $S = \pi \rho^2$. The integral of A around the tube is the enclosed flux, ϕ, so that

$$A_{at\ surface} = \frac{\phi}{2 \pi \rho}$$

The boundary condition (5) lets us relate K and ϕ:

$$K = \frac{\phi}{\mu_0 S} = \frac{2 \pi \rho A_{surf}}{\mu_0 S}$$

The surface area of the torus is $(2 \pi \rho) \times (2 \pi R) = 4 \pi^2 \rho R$, so we obtain

$$E_{supercurrent} = 2 \pi R S \mu_0 \int_0^{K_{final}} K \, dK$$

$$= \tfrac{1}{2} \mu_0 V K_{final}^2 \qquad (6)$$

which is the form we require (V is the volume of the torus, $2 \pi R S$). The torus can be seen to have the same energy equation as an inductor — it is a "superinductor". To check that we have, in fact, the correct form for the energy of the torus before the electron is fired, we can substitute

$$K_{final} = \frac{\Phi}{\mu_0 S} \equiv K_0$$

to obtain

$$E_{supercurrent} = \frac{V}{\mu_0} \frac{\Phi^2}{S^2} \qquad (7)$$

which is, indeed, the energy that we would associate with the flux Φ.

We now consider the application of a magnetic field, $B_{applied}$, to the outside of the superinductor, parallel to Φ. It can be seen from (5) that the surface current now *decreases*, since it does not have to maintain as great a magnetic field differential across the surface. Equation (6) then shows that the energy associated with the supercurrent also decreases, namely

$$\Delta E_{s.c.} = \tfrac{1}{2}\mu_0 V \left\{ \left[K_0 - \frac{B_{app.}}{\mu_0}\right]^2 - K_0^2 \right\}$$

$$= \frac{V}{\mu_0}\left[-\frac{\Phi}{S} B_{app.} + \tfrac{1}{2} B_{app.}^2\right]$$

To get the total change in electromagnetic energy for the (torus + field) system, we must subtract the toroidal "chunk" missing from the applied field, as described in the lead-up to equation (4); we then find

$$\Delta E_{emag} = -\frac{V}{\mu_0 S}\,\Phi\, B_{applied}$$

We thus find that the change of electromagnetic energy for an applied field is the *negative* of that derived in section 2. This can be traced to the strange properties of a superinductor: an increase in the applied field leads to a *decrease* in its supercurrent-superflux energy, whereas for the air-core flux we found the exact opposite.

A point of confusion is possible over the "choice" of energy equation for the superinductor. If one decided to use (7) instead of (6) for the energy, one would find that the application of an external field would result in *no* decrease in energy for the superinductor, despite the fact that the supercurrents had decreased. That (6) is, in fact, the correct equation can be seen by reversing the order of application of the fields, i.e. bring the "applied" field up first and then the "internal" flux Φ, and then applying the same analysis as used above. One then only needs to note that, if the final energy depended on the order in which the fields were raised, it would be possible to extract this energy and build a perpetual motion machine.

It should also be noted that, since the internal flux Φ in the superinductor does not change, and our interaction energy depends only on Φ, we can insert a ferromagnetic core into the centre of the torus without any change in the equations. We are therefore not only analysing the Kuper geometry, but also the superconductor-coated toroidal one of Tonomura *et. al.* [2].

At this point of the analysis of the AB effect we still appear to be in deep trouble. With the interaction energy changing sign, the electron "lag" turns into an electron "advance"; it is now attracted to the torus. We have

$$\Delta z' = +\frac{c\,\Phi}{m\,v_0} \tag{8}$$

which is the negative of (3). This difference can (in principle) be measured classically: an electron going through the centre of some free-space counter-clockwise flux is repelled; on approaching the same flux contained in a superconductor it is attracted.

That the problem is indeed serious is seen when the analysis of Aharonov and Bohm is considered. The magnetic vector potential is the *same* for both tori; the phase change should be the same. It would appear that classical electrodynamics and quantum mechanics are incompatible.

Before performing such a monumental write-off, however, it would be advisable to study the predictions of both theories, to see if *something* can be salvaged. Referring back to the results of section 2, equation (8) for the superinductor leads to the prediction that

$$\Delta\varphi = 2\pi\frac{\Delta z}{\lambda} = \frac{e\,\Phi}{\hbar}$$

The Aharonov and Bohm analysis, on the other hand, leads to the prediction that

$$\Delta\varphi = -\frac{e\,\Phi}{\hbar}$$

For these to agree, we come to the apparently nonsensical requirement

$$\frac{e\,\Phi}{\hbar} \stackrel{?}{=} -\frac{e\,\Phi}{\hbar}$$

However, φ is *not* an observable physical quantity. In any experimental test of the AB effect, only the quantity $\varphi\ (\mathrm{mod}\ 2\pi)$ is observable. For the two predictions to agree, we only require that

$$\frac{e\,\Phi}{\hbar} = -\frac{e\,\Phi}{\hbar}\ (\mathrm{mod}\ 2\pi)$$

We can satisfy this equation if and only if

$$\Phi = \frac{n\,h}{2\,e} \qquad \text{(n integral)} \qquad (9)$$

We then come to a remarkable conclusion: *classical electrodynamics and quantum mechanics are incompatible if the flux in a superconducting torus is not an integral multiple of $h/2e$.* Fortunately, Nature is not unkind; such a quantisation of flux is indeed observed [11][12][2].

However, we are out of the frying pan, but not yet out of the fire. So far, we have only considered an *electron* travelling through the Tonomura apparatus. What would happen if we sent around a particle with a different charge, say Q e (Q a real number)? In the theoretical analysis, we would obtain same equations as above, but with e replaced by Q e. Equation (9) would then read

$$\Phi = \frac{n\,h}{2\,Q\,e} \qquad \text{(n integral)}$$

Now, the poor torus may be superconducting, but it is definitely not psychic; *it* does not know what sort of particle we will send through it. The only way it can satisfy this equation in general is if *all* the particles we can fire at it have charges that are multiples of some basic charge. We thus come to the second remarkable conclusion: *classical electrodynamics and quantum mechanics are incompatible if all of the free particles in the universe do not have charges that are integrally related.* Again, we are fortunate that the values of all charges observed to date have, in fact, been integrally related.

We can obtain yet another remarkable result from the AB effect, this time of an experimental nature. From the preceding analysis, it is clear that the minimum non-zero flux quantum attainable in the Tonomura apparatus, Φ_0, is related to the minimum electric charge of *any* free particle in the universe, q_0, via

$$q_0 = \frac{h}{2\,\Phi_0}$$

We now use the experimental value of Φ_0 as determined by Tonomura *et. al.* [2]:

$$\Phi_0 = \frac{h}{2\,e}$$

to find that $q_0 = e$. We also note that if some experiment managed to find extra flux quanta at, say, half of the Tonomura result, this would actually *double* q_0. (The experimental results would doubtless be questioned, considering that we know that particles with $q = e$ *do* exist!) We can therefore state with confidence that *no free charged particle can have a charge smaller in magnitude than e.*

A few comments are in order regarding the above results. The whole analysis of the classical AB effect, and the required agreement with quantum mechanics, may seem familiar. In fact, it is directly related to an argument given by P. A. M. Dirac sixty years ago [13][14]. The difference is that a Dirac monopole has not been observed in the laboratory; the Tonomura superinductor has been (well, at least in Tonomura's laboratory).

The quantisation of flux in a superconductor is also seen to be required by fundamental principles. It does not matter *how* a material manages to exhibit the Meissner effect; if it does, flux *must* be quantised in multiples of $h / 2\,e$. The conventional explanation for flux quantisation involves such things as Cooper pairs; it would appear that this connection is unnecessary. It may, nevertheless, be possible to turn existing arguments regarding superconductivity in reverse, taking the phenomenon of flux quantisation as input rather than output.

In the discussion above, the minimality of the electron charge above has been stated carefully. The AB effect, coupled with the observed value of the minimum flux quantum, only requires that *free* particles have charges that are multiples of e: one must be able to send them through a Tonomura apparatus. This does *not* mean that fractionally-charged particles, such as quarks, cannot exist; it just means that you can never have a *free* quark. It is an interesting observation that quark confinement can be proven using classical electrodynamics and the Schrodinger equation alone. The author, who has not yet grasped the intricacies of QCD, is somewhat relieved by this observation.

5. The AB Accelerator

It may be (rightly) argued that the previous sections have told us nothing we didn't already know about: the AB effect, flux quantisation, charge quantisation and quark confinement. In this section, it is proposed that the Tonomura apparatus could be modified to yield a useful particle accelerator. We consider only a single free electron, passing through a macroscopic superinductor. The flux is assumed to be oriented such that the electron is attracted to, rather than repelled from, the torus. Although we will refer to the superconducting geometry, one could equally use the air- or solid- core geometry, as the interaction is the same (up to a sign).

To an electrical engineer, the most striking thing about the interaction energy derived in section 2,

$$\Delta E_{emag} = \frac{1}{\mu_0 S} \int_{\text{TORUS}} \Phi \, B_{e,\text{torus}} \, d^3x$$

is that it is formally equivalent to that of any electronic amplifier. It represents the coupling of a large "dc" power source, Φ, to the small electron magnetic field energy, $B_{e,\text{torus}}$. It is, literally, an "electronic amplifier", albeit for only one electron!

It was seen in section 2 that the electron gains energy on approaching the torus, and loses the same amount on departing. In practice, it gains this extra energy only for a brief time as it passes through the torus, and the effects of this extra energy have, to date, been so small that only those experiments looking for the AB effect have noticed it. Nevertheless, it may be possible to take advantage of this extra energy while inside the apparatus.

We examine the electron's induced magnetic field as it approaches the torus; assume that it is already relativistic so that $v \sim c$. We obtain γ_0 in terms of the initial energy, E_0:

$$E_0 = m \gamma_0 c^2 \tag{10}$$

In the following analysis, it is assumed that the factor γ_0 in equation (1) remains at its initial value throughout it's flight, even though its energy is being increased, and thus γ is really a function of time. This assumption is partially justified by the fact that the induced magnetic field is actually a retarded one, and so even if the electron is accelerated while in the toroidal hole, the corresponding increase in γ will take time to propagate through to an increased magnetic "self-energy". However, a more acceptable justification for this assumption will be given when a practical accelerator is analysed.

We obtain the extra kinetic energy "given" to the electron at the centre of the torus from the equations of section 2, with $z = 0$, to obtain

$$E = E_0 \left(1 + \frac{e \Phi}{2 m c R} \right) \tag{11}$$

The similarity with a domestic electronic amplifier is again evident; here the initial electron energy E_0 has been "amplified".

Although it would be possible to insert "feasible" parameters at this stage for Φ and R, to gauge how well this accelerator will work, we will first modify the proposed apparatus slightly to make it more practicable. With a thin torus accelerator, the particle barely gets its quota of energy before it leaves the torus again and gives it all back. If we consider instead a number of tori laid on top of each other, like a stack of doughnuts, it is obvious that the electron will stay at its amplified energy for the height of the "stack", and then give it back as it leaves the other side.

We also note that, in practice, there are limits on the magnetic fields obtainable in the laboratory, that we wish to "freeze into" the superinductor. We therefore consider a torus with a finite radial width.

With these enhancements, we must re-assess the theoretical aspects of the AB effect. For a finite radial width, we may consider slicing the superconductors into radial "rings" and integrate (11) over this width.

The finite "stack height" of the apparatus, however, requires a deeper assessment. As is shown in [3], p. 554, the passing electron's magnetic field is radial, but confined to only a small angular width. At a distance R, the lateral "spread" of the pulse is $\sim R / \gamma_0$. For large γ_0, one can see that a finite torus will only have a fraction of its flux "overlapped" by the magnetic pulse. However, the factor of R in R / γ_0 favours the flux being far out from the axis, and in fact compensates the $1 / R$ factor in (11). On the other hand, the factor of γ_0 in R / γ_0 will cancel out the factor of γ_0 in E_0.

For the "extended" torus, we thus consider only that part of the flux that is "overlapped". We take the magnetic field strength in the toroidal strips to be B teslas. In the "middle" of the apparatus (away from the ends) we then find

$$E \sim E_0 + c\,B\,(R_2 - R_1)\ \text{eV} \qquad (12)$$

where a factor of e has gone into converting joules to electron volts. B is in teslas, R_2 and R_1, the outer and inner "radii" of the now-cylindrical-shaped superinductor, are in metres, and c is in metres per second.

We can now start to put in some rough numbers. A reasonable laboratory magnetic field would be 1 tesla, and to fit the experiment on a desk-top we would need $R_2 \sim 1$ metre. We can set R_1 to essentially zero — we only need a small hole to fire the electron in. Inserting these numbers, we find that $E = E_0 +$ 300 MeV.

It may be remarked that this seems to be quite a decent energy for such a simple piece of apparatus. However, from the torus's point of view, it is only 48 picojoules — quite an unsubstantial sum for such a large inductor. The form of the interaction energy ensures that the amplified energy is roughly "mid-way" between the characteristic energies of the two separate components.

The reader may have been wondering what use this extra electron energy is, when it is destined to be lost again when the electron emerges from the other end of the apparatus. To make practical use of it, we note that a *positron*, fired in from the opposite side, will also be accelerated to the same energy, since it sees the approaching toroidal flux rotating the opposite way, but also has the opposite charge. If we aim the two beams at each other, we have built ourselves a colliding accelerator.

To obtain such a useful machine for so little effort must be viewed with scepticism. We have not considered radiation losses by the electron, although these may be reduced in practice by "tapering" the ends of the apparatus so that the electron does not get accelerated too quickly. We assumed that γ_0 remained at its initial value: what happens if we take into account the fact that it changes? The result (12) is seen to be oblivious to γ: although the magnetic field increases as γ increases, the amount of flux "overlapped" decreases by the same amount. Perhaps the machine would work?

If the above analysis is, in fact, not flawed, the opportunities for this accelerator should not be overlooked. A wide variety of charged particles can be collided, as long as one remembers to put negative ones in one end and positive ones in the other. We can increase the maximum energy of the accelerator to whatever value we like, as long as someone winds enough "toroidal flux sausage" around and around the beampipe. With a maximum magnetic field of 10 teslas, and an "outer radius" of 300 metres, (12) predicts an energy boost of 1 TeV, if that equation can be believed. One can also control the energy of the colliding particles by simply turning the magnetic field up or down (a remnant of the "volume control" present in the analogous domestic amplifier).

A curious aspect of the accelerator is also revealed when we consider the high-energy products of our "collider". Charged particles will be decelerated as they leave the apparatus, but any neutral particles created will be oblivious to it, and will emerge with their accelerated energies (therefore, do not perform this experiment on the kitchen table). The energy for these particles, as well as the mass-energy of any charged particles created, will be taken from the superinductor's stored energy. However, even a particle of mass 1 TeV (160 nanojoules) would be merely a flicker to the superinductor required to create it (unless we created them at a rate of billions per second!).

What happens, however, if we put the particles in the wrong end? Equation (12) gives the nonsensical result that the kinetic energy could easily be made *negative*. The answer lies in the assumption that the particles are relativistic, by which we set $v = c$. Equation (12) may be roughly corrected, for a non-relativistic particle, by replacing c by v. It is then seen that, as the particle decelerates, the factor v / c comes into play and the particle "stalls". It would most probably end up stationary, if the accelerator beampipe were long enough: the particle then has no "induced" magnetic field any more, and so it subsequently ignores the apparatus. Whether this would be of use in "cooling" charged particles and atoms would require a full electrodynamical analysis.

Conclusions

It is evident that the AB effect has many intriguing physical properties, and is well worth the attention it has attracted over the last three decades. It would, however, be desirable to experimentally check whether the "classical" AB effects derived in this letter, relating to particle acceleration and deceleration rather than phase shifts, *do* in fact exist in nature. If they do not, the analysis of this letter will be shown to be flawed, and the conclusions reached erroneous. One can only hope that this will not be the case.

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