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TOWARDS A TENSOR CALCULUS FOR κ-SUPERSYMMETRY *

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Abstract

We present a new manifestly space-time and world-volume supersymmetric formulation of the simplest super p-branes, massive $d = 2$, $N = 1$ superparticle and $d = 4$, $N = 1$ superstring, in terms of properly constrained world-line and world-sheet superfields. We identify the relevant κ-supersymmetries with a kind of local supersymmetry in the world-volume superspaces and, based on this, develop a tensor calculus for constructing higher-order supersymmetric and κ-invariant corrections to the corresponding minimal super p-brane actions. The latter are represented by pure Wess-Zumino terms in the world-volume superspaces. A "double analyticity" principle for extending this superfield approach to other super p-branes is suggested.

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1. Recently, the construction of super p-branes with "rigidity" (with the actions modified by corrections in external curvatures) became a subject of several studies [1, 2]. One of the basic motivations for considering such models is that the external curvature corrections may appear as possible counterterms in a quantum theory of super p-branes.

A serious problem one faces when trying to construct the relevant actions in the covariant Green-Schwarz formalism is how to maintain the famous local κ-supersymmetry which is necessary for ensuring the correct field content of given super p-brane theory. In [1, 2] the form of κ-invariant extensions of the rigidity terms quadratic in external curvature has been guessed for $d = 10$, $N = 1$ superstring ($p = 1$) and massive $d = 2$, $N = 1$ superparticle ($p = 0$). As was noticed in [1], searching for such extensions could be put on a systematic ground provided one is aware of a kind of "tensor calculus" for κ-symmetry analogous to that existing for ordinary supersymmetries, e.g. in $N = 1$ or $N = 2$ supergravities. There, the "tensor calculus" actually amounts to a superfield formulation, so it is natural to look for an appropriate superfield realization of κ-supersymmetry.

In the present note we develop such a formalism for the massive $d = 2$ superparticle and do basic steps in extending it to the simple case of $d = 4$, $N = 1$ superstring. We also conjecture a general principle of constructing analogous representations for other super p-branes.

Our crucial observation is that the minimal actions of the super p-branes just mentioned (and presumably of all the other ones) can be written in a world-volume superfield form manifestly invariant both under super-Poincare group acting in the target superspace and world-volume supersymmetry. Equivalence to the standard component super p-brane actions is achieved due to imposing certain covariant constraints on the world-volume superfields. The original κ-supersymmetry is recognized as an odd part of appropriate local subgroup of the general diffeomorphism group of the world-volume superspace. This symmetry coincides with the $d = 1$, $N = 1$ superconformal symmetry in the case of $d = 2$ superparticle and a Kac-Moody extension of the left-moving $d = 2$, $N = 2$ superconformal symmetry in the case of $d = 4$ superstring. Having understood κ-symmetry as transformations in the world-volume superspace, it is further straightforward to find out the rules of building tensor objects out of the basic superfields, i.e. the sought tensor calculus for κ-supersymmetry. For the superparticle we in this way deduce a general form of supersymmetric and simultaneously κ-invariant counterterms. As an instructive example we re-derive the particular correction given in [2]. In both super p-brane models we are considering here the minimal super p-brane actions are represented by pure Wess-Zumino terms in the relevant world-volume superspaces. We discuss the relationships with the twistor approach of the Kharkov group [3, 4] and the PBGS (partial breaking of global supersymmetry) superfield formalism of ref. [2, 5, 6, 7, 8].

2. We begin with reformulating the massive $d = 2$, $N = 1$ superparticle. Its covariant action can be written as [2]

$$S_{WZ} = -m \int ds \left(\sqrt{-\pi^n \pi_n} + \frac{i}{2} \bar{\theta} \Gamma^3 \dot{\theta} \right), \tag{1}$$

where $\pi^n = \dot{x}^n + \frac{i}{2}\bar{\theta}\Gamma^n\dot{\theta}$, $n = 0, 1$; $\Gamma^3 = \Gamma^0\Gamma^1$, $\bar{\theta}^\mu \equiv \varepsilon^{\mu\nu}\theta_\nu, \mu, \nu = 1, 2$ and

$\Gamma^0 = -i\sigma_2, \Gamma^1 = -\sigma_3$ are real $d = 2$ Dirac matrices. This action respects a rigid $d = 2$, $N = 1$ space-time supersymmetry (including $d = 2$ Lorentz rotations) which acts in the target superspace (x^n, θ^ν) and also the reparametrization invariance and local fermionic κ-symmetry. The latter is represented by the following transformations

$$\begin{aligned} \delta_k x^m &= -\frac{i}{2}\bar{\theta}\Gamma^m(1+\Gamma)k(s), \\ \delta_k\theta_\mu &= (1+\Gamma)_\mu{}^\nu k_\nu(s), \end{aligned} \tag{2}$$

where $k(s)$ is an arbitrary anticommuting real two-component spinor parameter and

$$\Gamma = \frac{\pi^m\Gamma_m\Gamma^3}{\sqrt{-\pi^2}}, \qquad \Gamma^2 = 1. \tag{3}$$

Let us show that the action (1) admits an elegant reformulation in terms of $d = 1$, $N = 1$ superfields. To this end, we promote the target superspace coordinates - fields to the superfields defined on a world-line $N = 1$ superspace $\xi = (s, \sigma)$

$$\begin{aligned} X^m &= x^m + i\sigma z^m, \\ \Psi^\mu &= \theta^\mu + \sigma\lambda^\mu, \end{aligned} \tag{4}$$

and consider the following superfield action

$$S_{WZ} = -m\int d^2\xi\, \bar{\Psi}\Gamma^3 D\Psi \tag{5}$$

accompanied by the constraint

$$DX^m - \frac{i}{2}\bar{\Psi}\Gamma^m D\Psi = 0, \tag{6}$$

where $D = \partial_\sigma - \frac{i}{2}\sigma\partial_s$ is a $d = 1$ spinor covariant derivative. The constraint (6) amounts to the following component relations

$$\begin{aligned} \pi^m &= -\bar{\lambda}\Gamma^m\lambda \Longrightarrow \bar{\lambda}\Gamma^3\lambda = \sqrt{-\pi^n\pi_n}, \\ z^m &= \frac{1}{2}\bar{\theta}\Gamma^m\lambda. \end{aligned} \tag{7}$$

With taking into account these relations, integration in (5) over σ yields just the original superparticle action (1).

Thus we have found a new world-line superfield representation of the $d = 2$ massive superparticle action. Eqs. (5), (6) are manifestly invariant under $d = 2$ Poincare group. With respect to supertranslations (6) is still covariant while (5) is invariant up to a full spinor derivative in the integrand (supertranslations act as constant shifts of Ψ). Such a behaviour is typical for the Wess-Zumino-type actions, which leads us to interpret the Lagrangian density in (5) as a pure Wess-Zumino superfield term. As distinct from the

superfield Wess-Zumino type action which was deduced in [7, 2] based upon the PBGS interpretation of massive superparticle, (5) respects linearly realized super-Poincare and world-line supersymmetries, as well as linearly realized $d = 2$ Lorentz symmetry. On the other hand, the superfields Ψ^ν in (5) are constrained, so one cannot vary them as independent in order to get the superfield equations of motion. However, for our purposes this fact is not critical. Actually, one may gain the action of [7, 2] by properly fixing a gauge with respect to the superconformal local symmetry in the world-line superspace (see below) and explicitly solving the constraint (6) via an unconstrained superfield.

The most important peculiarity of the superfield representation (5), (6) is that it is invariant not only under the rigid finite-dimensional world-line supersymmetry, but also under infinite-dimensional local $d = 1$ superconformal symmetry

$$\begin{aligned} \delta_E s &= E - \frac{1}{2}\sigma DE \\ \delta_E\sigma &= iDE, \end{aligned} \tag{8}$$

where $E(\xi)$ is an arbitrary even superparameter. The superfields X^n, Ψ^ν are assumed to be scalars with respect to (8). The invariance of (5), (6) under (8) can be easily checked using the transformation laws of D and the integration measure $d^2\xi$

$$\delta_E d^2\xi = \frac{1}{2}\dot{E}d^2\xi, \quad \delta_E D = -\frac{1}{2}\dot{E}D. \tag{9}$$

The bosonic reparametrization symmetry of the action (5) is associated just with the first, even component of the superparameter $E(\xi)$. How could one interpret the odd component? The local transformations of the target superspace coordinates corresponding to the choice $E = -i\sigma\alpha(s)$, $\alpha(s)$ being an arbitrary odd parameter, are given by

$$\begin{aligned} \delta_\alpha x^m &= i\alpha(s)\, z^m = -\frac{i}{2}\bar{\theta}\Gamma^m\lambda\alpha(s), \\ \delta_\alpha\theta^\mu &= -\lambda^\mu\alpha(s). \end{aligned} \tag{10}$$

Comparing (10) with the κ-symmetry transformations (2) one observes that both sets of the transformations coincide upon the identification

$$\alpha = \frac{2\bar{k}\Gamma^3\lambda}{\sqrt{-\pi^2}}. \tag{11}$$

Thus, the κ-symmetry transformations for the massive superparticle are actually identical to the odd $d = 1$, $N = 1$ superconformal ones. Remarkably, the same conclusion has been drawn earlier in [3, 4] for the massless $d = 4$ superparticle in the twistor approach. In fact, the formulation presented in this Section is very reminiscent of the twistor one developed in [3, 4]. The superfield $D\Psi^\mu(\xi)$ can be interpreted as a $d = 2$ supertwistor, the first component of which $\lambda^\mu(s)$ is a $d = 2$ counterpart of the twistor variable employed in [3, 4]. The superfield constraint (6) and its component representation (7) are also direct analogs of those appearing in the cited papers. However, there is an essential difference between the two approaches. Namely, due to specific features of the algebra of $d = 2$ Dirac gamma matrices, one may construct from $\lambda^{\mu}(s)$ the nonvanishing invariant of the Lorentz group $\bar{\lambda}\Gamma^{3}\lambda$ while no such invariants exist in $d = 4$ (and $d = 3$). Just due to this important property the first of the relations (7) implies the second one and, being put together, they entirely express the twistor variable $\lambda^{\mu}(s)$ in terms of $x^{m}(s)$ and $\theta^{\mu}(s)$. Thus, this variable is not independent in the case of massive $d = 2$ superparticle and plays an auxiliary role[1]. One may always get rid of it by making use of the relations (7). However, to consider this entity is useful for exposing the hidden world-line superconformal symmetry of the action (1) and identifying the odd sector of this symmetry with κ-symmetry.

3. Once we have succeeded in a geometric interpretation of κ-supersymmetry of the $d = 2$ superaprticle as the symmetry with respect to superconformal local transformations of the world-line superspace $\{\xi\}$, the problem of constructing general κ-invariants in the superparticle theory is reduced to working out the tensor calculus for the $d = 1$ superconformal group (8). An important simplification stems from the requirement of super-Poincare invariance. With the constraint (6) taken into account, the only basic $d = 2$ super-Poincare invariants we dispose of are $D_{\sigma}\Psi^{\mu}$ and

$$\omega_{s}^{n} \equiv \dot{X}^{n} + \frac{i}{2}\bar{\Psi}\Gamma^{n}\dot{\Psi}. \tag{12}$$

Moreover, exploiting once more (6), one gets

$$\omega_{s}^{m} \;=\; -D\bar{\Psi}\Gamma^{m}D\Psi, \tag{13}$$

so we are left with the objects

$$D\Psi^{\mu},\ DD\Psi^{\mu},\ \ldots(D)^{n}\Psi^{\mu},\ \ldots \tag{14}$$

as the only building-blocks of super-Poincare invariants in the present case.

It is easy to indicate general rules of how to construct the superconformal tensors from these quantities. As a first step, one introduces the density

$$J = \sqrt{D\bar{\Psi}\Gamma^{3}D\Psi}, \quad \delta_{E}J = -\frac{1}{2}\dot{E}J \tag{15}$$

which can be used to compensate superconformal transformations of the supervolume $d^{2}\xi$ and spinor derivative D (note that J has a nonvanishing flat limit that is easy to see, e.g., by passing to the "physical gauge"[2, 7] $X^{0} = s,\ \ \Psi^{1} = \sigma$). Further, one defines super-Poincare and superconformally invariant spinor derivative

$$\begin{aligned} \nabla &= J^{-1}\left(D + J^{-1}DJ\,Z\right) \\ Z\,\Psi &= 0, \quad Z\,D\Psi = -1, \quad etc., \quad \delta_{E}\nabla = 0. \end{aligned} \tag{16}$$

[1]Perhaps, it could be treated as independent after passing to the first order form of the action (36).

Now we are in a position to write down the most general expression for higher-derivative super-Poincare and κ-invariant corrections to the minimal $d = 2$ superparticle action (1) (or, equally, (5))

$$S_{corr} = \int d^{2}\xi JL\left(\nabla\Psi,\ (\nabla)^{2}\Psi,\ \ldots(\nabla)^{n}\Psi,\ \ldots\right), \tag{17}$$

where L is an arbitrary odd function of its arguments. Of course, Ψ^{μ} is related to X^{m} as before via the constraint (6).

As an instructive example we present the superfield form of the correction studied in [2]

$$L_{2} = \mu^{-1}(\nabla)^{2}\bar{\Psi}\Gamma^{3}(\nabla)^{3}\Psi, \quad [\mu] = 1. \tag{18}$$

The corresponding action, after doing some algebra, is written as

$$S_{2} = -\frac{1}{4\mu}\int d^{2}\xi\left[\frac{\dot{\bar{\Psi}}\Gamma^{3}D\dot{\Psi}}{\left(D\bar{\Psi}\Gamma^{3}D\Psi\right)^{2}} - \frac{\left(\dot{\bar{\Psi}}\Gamma^{3}D\Psi\right)\left(D\dot{\bar{\Psi}}\Gamma^{3}D\Psi\right)}{\left(D\bar{\Psi}\Gamma^{3}D\Psi\right)^{3}}\right]. \tag{19}$$

It is straightforward to show that, with employing relations (7), the component action coincides with that given in [2]. For simplicity we quote only a bosonic part of the component action

$$S_{2}^{bos} \sim \int ds\left[\frac{\left(\dot{\bar{\lambda}}\Gamma^{3}\lambda\right)^{2} - \left(\dot{\bar{\lambda}}\Gamma^{3}\dot{\lambda}\right)\left(\bar{\lambda}\Gamma^{3}\lambda\right)}{\left(\bar{\lambda}\Gamma^{3}\lambda\right)^{3}}\right]. \tag{20}$$

Coming back to the original variables with the help of eqs.(7) and using the completeness relation

$$\delta_{\mu}^{\sigma}\delta_{\rho}^{\nu} = \frac{1}{2}\delta_{\mu}^{\nu}\delta_{\rho}^{\sigma} + \frac{1}{2}\Gamma^{m\nu}_{\ \ \mu}\Gamma^{\ \sigma}_{m\rho} + \frac{1}{2}\Gamma^{3\nu}_{\ \ \mu}\Gamma^{3\sigma}_{\ \ \rho}, \tag{21}$$

one reduces (20) to

$$S_{2}^{bosonic} \sim \int ds\frac{B^{n}B_{n}}{\sqrt{-\pi^{2}}}, \tag{22}$$

where

$$B^{n} = \partial_{s}\left(\frac{\pi^{n}}{\sqrt{-\pi^{2}}}\right). \tag{23}$$

This expression is the same as in [2].

We stress that such a simple recipe of constructing higher-order invariants in the superfield formulation presented here is attributed mainly to the fact that different symmetries of the superparticle are realized linearly and explicitly in this formulation. In the PBGS approach [2, 5, 6, 7, 8] $d = 2$ Lorentz and one of two supertranslation invariances are realized nonlinearly and implicitly that makes difficult an algorithmic construction of higher-order invariants. As has been remarked earlier, both superfield formalisms are related via a proper choice of the gauge with respect to the superconformal group, so one can easily rewrite the generic action (17) in terms of the PBGS superfields as well. We do not touch here details of this passing. Note that just the PBGS interpretation of $d = 2$ superparticle, having revealed an adequacy of the $N = 1$ world-line superspace, has prompted to us the above covariant superfield formulation. In the next Section we construct an analogous formulation for $d = 4,\ N = 1$ superstring, employing results of the PBGS analysis of the latter [5].

4. Here we present a new world-sheet superfield form of the covariant action of $d = 4,\ N = 1$ Green-Schwarz superstring. In Sec.5 we suggest a general principle which will hopefully lead to an analogous formulation in the most interesting case of $d = 10$ Green-Schwarz superstring.

We start by choosing an appropriate world-sheet superspace. Based upon the PBGS analysis of ref.[5], we identify it with the $d = 2,\ N = (2,0)$ superspace

$$\begin{aligned} \{\xi\} &= \left\{s^-, \tau, \bar\tau, s^+\right\} \qquad (24) \\ \delta s^- &= i\left(\tau\bar\beta - \beta\bar\tau\right), \quad \delta\tau = \beta, \quad \delta\bar\tau = \bar\beta, \end{aligned}$$

where β is a complex constant parameter of rigid $d = 2,\ N = (2,0)$ supertranslations. The second $d = 2$ light-cone bosonic coordinate s^+ is inert under the world-sheet supersymmetry. Next we define on $\{\xi\}$ the superfields

$$X^m(\xi), \quad \Psi^\mu(\xi), \quad \overline{\Psi}^{\dot\mu}(\xi) = (\Psi^\mu(\xi))^\dagger \qquad (25)$$

which parametrize a target $d = 4,\ N = 1$ superspace and are transformed in the standard way under rigid Poincare supersymmetry

$$\delta X^m = i\left(\Psi\sigma^m\bar\epsilon - \epsilon\sigma^m\overline{\Psi}\right), \quad \delta\Psi^\mu = \epsilon^\mu, \quad \delta\overline{\Psi}^{\dot\mu} = \bar\epsilon^{\dot\mu}. \qquad (26)$$

The most nontrivial problem is to find appropriate superfield constraints analogous to (6) and reducing the field content of (25) to that of $d = 4,\ N = 1$ superstring. It turns out that the correct generalization of (6) has the following suggestive form of the "double analyticity" relations

$$\overline{D}X^m_L = 0, \quad \overline{D}\Psi^\mu = 0 \quad and \quad DX^m_R = 0, \quad D\overline{\Psi}^{\dot\mu} = 0 \qquad (27)$$

where

$$D = \partial_\tau + i\bar\tau\partial_-, \quad \overline{D} = -\partial_{\bar\tau} - i\tau\partial_- \qquad (28)$$

and

$$X^m_L = X^m + i\Psi\sigma^m\overline{\Psi}, \quad X^m_R = X^m - i\Psi\sigma^m\overline{\Psi} \qquad (29)$$

are the bosonic coordinates of two conjugated chiral subspaces in the target $N = 1$ superspace. The constraints (27) have a clear meaning: these require that chiralities (analyticities) in the target and world-sheet superspaces are correlated, in the sense that the coordinates of chiral target superspaces have definite chiralities with respect to their world-sheet superspace arguments.

Before presenting the component solution of (27) we indicate some important consequences of these constraints. They imply that in the super-Poincare invariant 1-forms

$$\omega^m = dX^m + i\Psi\sigma^m d\overline{\Psi} - id\Psi\sigma^m\overline{\Psi}, \quad d\Psi^\mu, \quad d\overline{\Psi}^{\dot\mu} \qquad (30)$$

there survive only the following independent world-volume superspace projections

$$D\Psi^\mu, \quad \partial_-\Psi^\mu \quad and \quad c.c, \quad \omega^m_+ = \partial_+X^m + i\Psi\sigma^m\partial_+\overline{\Psi} - i\partial_+\Psi\sigma^m\overline{\Psi} \qquad (31)$$

while ω^m_- is expressed as

$$\omega^m_- = -D\Psi\sigma^m\overline{D\Psi}. \qquad (32)$$

From the last relation it follows

$$\omega^m_-\omega_{-\,m} = 0, \qquad (33)$$

which reflects the fact that in the present case we obtain the superstring action not in the general Nambu-Goto form (as in the superparticle case) but in a partially fixed gauge with respect to the world-volume reparametrization group (see below).

Let us now quote the general component solution of constraints (27)

$$\begin{aligned} X^m &= x^m + i\tau\left(\lambda\sigma^m\bar\theta\right) + i\bar\tau\left(\theta\sigma^m\bar\lambda\right) - \tau\bar\tau\,\partial_-\left(\theta\sigma^m\bar\theta\right) \qquad (34) \\ \Psi^\mu &= \theta^\mu + \tau\lambda^\mu + i\tau\bar\tau\,\partial_-\theta^\mu \\ \overline{\Psi}^{\dot\mu} &= \bar\theta^{\dot\mu} + \bar\tau\bar\lambda^{\dot\mu} - i\tau\bar\tau\,\partial_-\bar\theta^{\dot\mu} \\ \pi^m_- &\equiv \omega^m_-|_{\tau=0} = \lambda\sigma^m\bar\lambda \Longrightarrow \pi^m_-\pi_{-\,m} = 0. \qquad (35) \end{aligned}$$

Here the fields x^m and θ^μ, $\bar\theta^{\dot\mu}$ are coordinates of the target $d = 4,\ N = 1$ superspace, λ^μ is a commuting spinor, a twistor variable in the terminology of [3, 4]. Eq. (35) will be further interpreted as the gauge-fixing condition. Note that all the superpartners of eq. (35) collected in the superfield constraint (33) are fullfilled automatically as a consequence of the twistor expression for π^m_-. We point out that an essential difference from the $d = 2$ superparticle case is that no nonvanishing Lorentz invariants can be constructed out of the commuting spinors λ^μ , $\overline{\lambda^{\mu}}$ and just this property is the origin of the constraint (35).

It remains to define the superfield action which, together with the constraints (27) would yield the correct component superstring action. We have found that it is given once again by a superfield Wess-Zumino term, this time living in the chiral subspace $\{\xi_L\} \equiv \left\{s^-_L = s^- + i\tau\bar\tau\ ,\ \tau\ , s^+\right\}$ of the world-sheet superspace (24)

$$S = \mu^2 \int d^3\xi_L\, \omega^m_+\left(\Psi\sigma_m\overline{D\Psi}\right) \equiv \mu^2 \int d^3\xi_L\, L. \qquad (36)$$

Using the basic constraints (27) and their consequences (32), (33) we were able to prove that (the computations are a bit more complicated than in the superparticle case)

A. The Wess-Zumino type Lagrangian density in (36) is chiral,

$$\overline{D}L = 0.$$

B. It is shifted by full spatial and spinor derivatives under Poincare supertranslations (26), thereby leaving (36) invariant.

C. It is invariant under $N = (2,0)$ superconformal transformations gauged by the remaining bosonic coordinate s^+

$$\begin{aligned} \delta s_L^- &= E + \bar{\tau}\overline{D}E, \quad \delta\tau = \frac{i}{2}\overline{D}E, \\ \delta\omega_+^m &= -\partial_+ E\omega_-^m, \quad \delta D = -\frac{i}{2}D\overline{D}E\, D, \quad \delta\overline{D} = -\frac{i}{2}\overline{D}DE\,\overline{D} \end{aligned} \tag{37}$$

$$\begin{aligned} \delta d^3\xi_L &= \frac{i}{2}\overline{D}DE\, d^3\xi_L \\ E &= E\left(s_L^-, \tau, \bar{\tau}, s^+\right) \end{aligned} \tag{38}$$

(the superfields X^m, Ψ^μ, $\overline{\Psi}^{\dot\mu}$ are scalars under superconformal transformations, like in the superparticle case).

The action (36) is also manifestly inavriant under $d = 4$ Lorentz transformations and conformal reparametrizations of s^+

$$\delta s^+ = b(s^+).$$

The constraints (27) obviously respect all these symmetries as well.

It is a simple exercise to integrate in (36) over τ, which after employing the relations (34) yields the following component action

$$S = -\mu^2 \int d^2s \left[\pi_+^m \pi_{-\,m} + 2i\pi_-^m\left(\theta\sigma_m\partial_+\bar{\theta}\right) - 2i\pi_+^m\left(\theta\sigma_m\partial_-\bar{\theta}\right)\right] \tag{39}$$

which is recognized as the Green-Schwarz action for $d = 4$, $N = 1$ superstring in the gauge

$$\pi_-^m \pi_{-\,m} = 0. \tag{40}$$

This gauge condition reduces the group of general world-sheet reparametrizations down to its "triangular" subgroup

$$\delta s^- = a(s^-, s^+), \quad \delta s^+ = b(s^+), \quad a(s^-, s^+) \equiv E(\xi)|_{\tau=0} \tag{41}$$

and linearizes the Nambu-Goto piece of the superstring Lagrangian. The form of the component WZ term does not change upon imposing the gauge (40).

Let us clarify the meaning of κ-invariance in the present case. The transformations corresponding to the choice

$$E = \tau\epsilon + \bar{\epsilon}\bar{\tau}, \quad \epsilon = \epsilon(s^+, s^-) \tag{42}$$

in (37) are realized on the fields x^m, θ^μ as

$$\begin{aligned} \delta x^m &= \frac{1}{2}\bar{\epsilon}\left(\lambda\sigma^m\bar{\theta}\right) - \frac{1}{2}\epsilon\left(\theta\sigma^m\bar{\lambda}\right) \\ \delta\theta^\mu &= -\frac{i}{2}\bar{\epsilon}\lambda^\mu, \quad \delta\bar{\theta}^{\dot\mu} = \frac{i}{2}\epsilon\bar{\lambda}^{\dot\mu}. \end{aligned} \tag{43}$$

Passing to a space-time spinor Grassmann parameter $\kappa_{\dot\mu}(s^+, s^-)$

$$\bar{\epsilon}(s) = \overline{\lambda}^{\dot\mu}\kappa_{\dot\mu}\,, \tag{44}$$

one brings (43) into the form

$$\begin{aligned} \delta x^m &= i\delta\theta\sigma^m\bar{\theta} - i\theta\sigma^m\delta\bar{\theta} \\ \delta\theta^\mu &= -\frac{i}{4}\pi_-^m\sigma_m^{\mu\dot\mu}\kappa_{\dot\mu} \end{aligned} \tag{45}$$

which, after a field-dependent redefinition of the parameter $\kappa_{\dot\mu}$, is recognized as the familiar κ-transformation for $d = 4$, $N = 1$ superstring [5] in the particular gauge (40). Note that $\kappa_{\dot\mu}$ is a + component of some $d = 2$ vector, as it should be.

Thus we have proven that in the case of $d = 4$, $N = 1$ superstring κ-supersymmetry can also be identified with the odd sector of some restricted local group acting now in the world-sheet superspace. This group is the Kac-Moody extension (37), (38) of $N = (2,0)$ superconformal transformations. Note a direct parallel with Siegel's description of chiral bosons [9] where the bosonic analog of κ-symmetry transformations can be viewed upon as the result of gauging the left-handed Virasoro symmetry by the right-handed $d = 2$ light-cone coordinate (in the case of right-handed chiral bosons).

Now it is straightforward to establish a tensor calculus for constructing higher-order corrections to the minimal action (36) which respect both $d = 4$ supersymmetry and fermionic κ-symmetry. An essential peculiarity of the case at hand is that one cannot construct out of the super-Poincare covariant objects (31) an appropriate density which would compensate the superconformal (i.e. κ-) transformations of the chiral superspace integration measure $d^3\xi_L$. At the same time, the measure of the full $N = (2,0)$ superspace (24), $d^4\xi = ds^+ds^-d\tau d\bar{\tau}$, can easily be seen to be invariant under the transformations (37), so a natural arena for arranging higher-order tensors in the superstring case is offered just by this superspace. Using the density

$$J = \omega_+^m \omega_{-\,m}, \quad \delta J = -\partial_- E\, J\,,$$

one may define superconformally-covariant derivatives of the basic entities (31) and further form, by simple recipes, invariant superfield Lagrangians. Of course, one may always rewrite them in the chiral superspace $\{\xi_L\}$ integrating over $\bar{\tau}$, however, the manifest superconformal invariance will be lost when doing so. Thus, as opposed to the superparticle case, the mini mal $d = 4$ superstring action and the higher-order corrections live in different world-sheet superspaces.

We confine our presentation here to writing down the action which presumably yields a $d = 4$ version of the correction term guessed in [1]

$$S_2 = \gamma \int d^4\xi\; J^{-1}\,\partial_+\omega_-^m\omega_{+\,m}, \quad [\gamma] = 0. \tag{46}$$

This expression respects all the symmetries listed above and so definitely gives rise to a component action possessing both space-time supersymmetry and κ-symmetry. Its dimensionality and linearizing limit coincide with those of the term just mentioned (in the gauge (40)). Detailed computations will be given elsewhere.

5. To summarize, we have presented new Wess-Zumino type world-volume superfield representations for the minimal actions of massive $d = 2$, $N = 1$ superparticle and $d = 4$, $N = 1$ superstring (in the latter case in a particular gauge), identified the corresponding κ-symmetries as local symmetries acting in the world-volume superspaces and proposed a simple tensor calculus for constructing higher-order supersymmetric and κ-invariant corrections to the minimal actions. Our formalism is very closed to the Kharkov's twistor approach [3, 4], however we proceeded from a practical goal of finding out the tensor calculus for κ-symmetry while the main incentive of ref.[3, 4] was to understand the relation between the spinning particles and strings on the one hand and the superparticles and superstrings on the other. These authors did not consider the super p-branes we dealt with here. Note also a parallel with the spinning super p-brane (actually spinning superparticle) approach of [10]. The crucial difference is that, due to enforcing the superfield constraints (6), (27), we are always left with the standard field representation of a given super p-brane while the field representation of theories of the kind treated in [10] is much wider than that of the original super p-branes.

Besides being suitable for setting up minimal and higher-order κ-invariant super p-brane actions in the flat target superspace, the approach presented promises to be efficient for promoting these actions to the general curved superspace, which corresponds to the super p-branes moving in an arbitrary curved background. This problem amounts now to covariantizing the constraints (6), (27) and the flat superspace objects (14), (31) with respect to the relevant target superspace supergravities and further to composing appropriate world-volume superspace invariants from the space-time superspace covariants. An interplay between the super p-branes, κ-symmetry and the intrinsic superspace geometries of the background supergravities is expected to be visualized in such a representation.

The most intriguing question is, of course, how to generalize all this to other super p-branes with κ-symmetry and, in particular, how to re-interpret the latter as originating from some world-volume superspace symmetries. It seems that the PBGS arguments [6] combined with the double analyticity principle employed in setting up the $d = 4$ superstring constraints (27) may provide a key to answer this question, at least for higher-dimensional superstrings ($d = 6,\ 10$) .

The PBGS reasonings tell us that the Grassmann dimensionality of the world-volume superspace has to be twice as less compared to that of the target superspace. Then the double analyticity principle requires that the coordinates of a proper analytic subspace of the target superspace respect a kind of the world-volume superspace analyticity. These analyticities in the case of $d = 4$ superstring are the $d = 4$, $N = 1$ chirality and the $d = 2$, $N = (2,0)$ $U(1)$ analyticity, respectively. As is known, for $N > 1$ and in higher dimensions another type of analyticity, the harmonic analyticity [11], proves to be adequate. Thus one may conjecture that the analogous formulations of $d = 6$ and $d = 10$ superstrings essentially employ the notions of the harmonic superspace and harmonic analyticity. In particular, it has been argued in [12] and [13] that the crucial role in the geometry of $d = 10$ superparticle and superstrings, as well as in that of $d = 10$ supergravities is played

by the light-cone harmonic analyticity corresponding to an extension of $d = 10$, $N = 1$ superspace by the Lorentz harmonics on the coset $SO(1,9)/SO(8)\times SO(1,1)$. It is remarkable that, identifying the world-sheet superspace in this case with the $d = 2$, $N = (8,0)$ one (it contains just half of odd variables as compared with the target superspace whose odd coordinates form a 16-component $SO(1,9)$ spinor), we gain a natural place for the spinor $SO(1,9)$ harmonics exploited in [14]. Indeed, let us regard 8 internal Grassmann coordinates as a $SO(8)$ spinor τ^i, $i = 1,...8$, and consider the quantity $D_{\tau^i}\Psi^\mu(s^+, s^-, \tau^i)$, where Ψ^μ, $\mu = 1,...16$, denotes the $d = 10$, $N = 1$ target superspace Grassmann coordinate transforming as a 16-component $d = 10$ spinor. Being properly normalized, this object represents just half of the spinor $SO(1,9)/SO(8)\times SO(1,1)$ harmonic variables of [14] and so can be used to introduce a kind of the light-cone harmonic analyticity in the target superspace. Its lowest component is just a $d = 10$ analog of the twistor variable λ^μ! What could be the world-sheet superspace analyticity in this case? One possible answer is that this is the analyticity corresponding to an extension of $N = (8,0)$ superspace by harmonics on the coset $SO(8)/SU(4)\times U(1)$ which were also considered in [14]. In such a harmonic superspace there exists an analytic subspace containing half of the original 8 odd coordinates, in a clear analogy with the above $U(1)$ analyticity of the $d = 4$ superstring.

We intend to address these and other issues related to further generalizations of our approach in the nearest future.

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