WIS-PH--89/73.

WIS-89/73/November-PH

Dimensional Reduction of 10d Heterotic String Effective Lagrangian with Higher Derivative Terms

Zygmunt Lalak*

Institute for Theoretical Physics, Warsaw University

Hoza 69, 00-681 Warsaw, Poland

Jacek Pawełczyk†

Department of Physics, Weizmann Institute of Science

Rehovot 76100, Israel

ABSTRACT

Dimensional reduction of the 10d SUGRA-YM containing up to four derivatives is described. Unexpected nondiagonal corrections to 4d gauge kinetic function and negative contributions to scalar potential are found. We analyze the general structure of the resulting Lagrangian and discuss possible phenomenological consequences.

* Supported in part by the Ministry of National Education Program CPBP.01.03.

† On leave of absence from Institute for Theoretical Physics, Warsaw University, Warsaw, Poland

Introduction.

Superstring theories are at present the unique candidate to unify all the known fundamental interactions [1]. However, as for today, there exists no satisfactory description of the observable low-energy ($E < 100$ Gev) phenomena stemming from them.

It is strongly believed that at low energies strings should give a kind of supergravity-Yang-Mills unified theory which contains standard model matter and gauge group [1]. In turn, every 4d N=1, ?-derivatives SUGRA-YM is completely described by 3 independent functions i.e. one real Kähler potential K and two holomorphic functions of chiral superfields: superpotential W and gauge kinetic function $f_{\alpha\beta}$ [2]. In particular, these functions determine the correct tree-level scalar potential and the correct classical vacuum of the theory.

There exist many ways of reconstructing K, W, $f_{\alpha\beta}$ for a given string theory. One is the dimensional reduction of the bosonic sector of the 10d SUGRA which is the field-theory limit of the 10d string [3],[4] (see also Appendix.1). In the simplest case considered by Witten, which corresponds to 1-generation Calabi-Yau compactification [3], it was possible to write down the lowest order (in the number of fields) expressions for K, W, f and, consequently, the full 4d SUGRA-YM Lagrangian. In more general case [5] the effect of this procedure describes the untwisted sector of the orbifold-like compactified heterotic string.

Another possibility, which allows one to treat the twisted orbifold sector as well, is the stringy reconstruction of Kähler function using 4d string interactions. An recent example is the work by Dixon, Kaplunovsky and Louis [7], where K is constructed as a Taylor-like expansion around the manifold of neutral moduli N for given compactification in powers of the ratio $\varphi/\sqrt{N}$; here φ is any other field

- charged moduli, twisted or untwisted matter. This conclusion has its support in results concerning the perturbation calculations for 2d σ-models [15].

In this work we would like to go a step further in employing the first method. We perform dimensional reduction of the 10d SUGRA-YM Lagrangian containing up to four derivatives [6]. As a result we are given corrections to the 4d Lagrangian obtained by Witten. They turn out to appear as powers of the ratio $\varphi/\sqrt{N}$; in our case there is only one neutral modulus, the T field, so the order parameter is $C/\sqrt{T}$. In the paper we will use Witten definitions of S, C, T fields, and one can guess that in order to bring our results into standard 4d N=1 SUGRA form redefinitions of the fields are needed. This point will be alluded to in the final part of our work.

The outline of the paper is as follows: section I appendices are devoted to setting conventions and defining the procedure of truncation; section II contains all the corrections to the Witten's Lagrangian obtained in course of reduction of higher derivative terms; section III deals with the general structure of the enriched Lagrangian; in section IV we suggest the simple generalization of the standard Kähler potential and in section V we discuss the corrected scalar potential and its phenomenology. In section VI we present our comments and conclusions. Appendices 1 and 2 contain some formulas and notation we use throughout the work. Appendix 3 collects all corrections to the kinetic energy terms.

I. Truncation procedure

In this part of the work we describe dimensional reduction of the effective 10d Lagrangian for the heterotic string obtained in the work of Gross and Sloan [6]. This Lagrangian contains all bosonic interaction terms up to 4 derivatives. It is the next-to-leading order corrected effective Lagrangian for string interactions - the role of the coupling constant is played by the $(\alpha')^{1/2}$ times the number of fields [1],[16]. Dimensional reduction yields effective bosonic lagrangian of a string theory compactified on a Calabi-Yau manifold (to be more correct one should speak about orbifold-like compactification). We would like to emphasize that we shall keep only terms that contain at most 2 derivatives and are of the order at most $(\varphi/\sqrt{T})^6$ compare to those written in [3]. Following Witten [3] we impose SU(3) holonomy condition. Witten's procedure yields theory with one fermion family and one T field. Generalization to more complicated Z_N orbifold compactifications should be straightforward (see for example [5]). Our conventions and some useful formulae have been written down in Appendix 1.

We begin with writing down the relevant terms of Lagrangian obtained in [6]. Other terms contained in [6] do not contribute to the 2-derivative 4d effective Lagrangian of string theory. We also suppress terms previously considered in [3]. In formula (1) we used the following notations: $H_{MNR} = \partial_M B_{NR} +$ cyclic permutations, g is 10d gauge coupling constant, κ is gravitational coupling constant and eight index tensor t is defined in [1]. Antisymmerization is understood in the following sense

$$H^{TM[N}H_T^{RS]} \equiv H^{TMN}H_T^{RS} + H^{TMR}H_T^{SN} + H^{TMS}H_T^{NR} \qquad (1a)$$

The definition of the modified curvature tensor $\bar{R}$ is given below

$$\bar{R}^{RS}_{MN} \equiv R^{RS}_{MN} + \kappa\phi^{-3/4}\nabla_{[M}H^{RS}_{N]} - \frac{3}{64}\eta^{[R}_{[M}\nabla_{N]}\left(\partial^{S]}\phi/\phi\right) \qquad (1b)$$

where η is flat 10d Minkowski metric.

$$
\begin{aligned}
&-\frac{1}{6}\frac{\kappa^2}{g^2}\phi^{-9/4}\left(\nabla^M H^{NRS}\nabla_R H_{SMN}+\nabla_M H^{MRS}\nabla^N H_{NRS}\right)\\
&+\frac{1}{12}\frac{\kappa^2}{g^2}\phi^{-9/4}\left(5R^{MNRS}H_{MNT}H^T_{RS}-8R^{MN}H_{MNS}H^{RS}_N+R\,(H_{MNR})^2\right)\\
&-\frac{3}{128}\frac{\kappa^2}{g^2}\phi^{-9/4}(H_{MNR})^2\left(\frac{\partial_S\phi}{\phi}\right)^2\\
&+\frac{\kappa^4}{g^2}\phi^{-15/4}\left[\frac{1}{18}(H_{MNR})^2+\frac{1}{12}H^{MNR}H^{ST}_M H^U_{NS}H_{RSU}\right.\\
&\qquad\qquad\left.-\frac{1}{4}H^{MRS}H_{NRS}H^{NTU}H_{MTU}\right] \qquad (1)\\
&-\frac{3}{4}\frac{\kappa^3}{g^2}\phi^{-9/4}H^{MNR}\left(\frac{\partial^S\phi}{\phi}\right)Tr\,(F_{MN}F_{RS})\\
&-\frac{1}{8}\frac{\kappa^4}{g^2}\phi^{-12/4}H^{TM[N}H^{RS]}_T Tr\,(F_{MN}F_{RS})\\
&+\frac{1}{128}g^2\phi^{-9/4}t^{MNRSTUWZ}\left[Tr\,(F_{MN}F_{RS})+\frac{1}{2g^2}\tilde{R}_{MNAB}\tilde{R}^{AB}_{RS}\right]*\\
&\qquad\left[Tr\,(F_{TU}F_{WZ})+\frac{1}{2g^2}\tilde{R}_{TUAB}\tilde{R}^{AB}_{WZ}\right]
\end{aligned}
$$

Doing dimensional reduction one must take into account the overall factor $e^{3\sigma}$ which arise due to the change in the integration measure, namely: $(det(g^{(10)}))^{1/2} \rightarrow (det(g^{(4)}))^{1/2}e^{3\sigma}$. We will include this factor in the following formulae. From now on we will suppress the superscript (4) for the 4d metric.

II. Corrections to the 4d Lagrangian

Now line by line we describe the reduction of the expression (1)

1. $(\nabla H)^2$ terms. Only the first term contribute. In this place one should carefully treat terms like $\nabla_i H_{\mu jk}$ - they do not vanish due to the existence of the nonzero connection $\Gamma^j_{i\mu}$.

$$-\frac{1}{6}\frac{\kappa^2}{g^2}\phi^{-9/4}\left(\nabla^M H^{NRS}\nabla_R H_{SMN} + \nabla_M H^{MRS}\nabla^N H_{NRS}\right) \rightarrow$$
$$4\frac{\kappa^2}{g^2}\phi^{-9/4}e^{-3\sigma}\left[(\partial_\mu\sigma)\partial^\mu|w|^2 - 4(\partial_\mu\sigma)^2|w|^2\right]. \qquad (2a)$$

This we rewrite as

$$\frac{4\xi}{\kappa^2}\left[\frac{9}{4}(\partial_\mu\sigma)(\partial^\mu\phi/\phi) - (\partial_\mu\sigma)^2\right] + \frac{4}{\kappa^2}(\partial_\mu\sigma)\partial^\mu\xi \qquad (2b)$$

where we have introduced notation $\xi \equiv \frac{\kappa^6}{g^2}\phi^{-9/4}e^{3\sigma}|w|^2$.

2. RH^2 terms. There are lots of contributions among which we get the correction to the Einstein gravity. This will force the redefinition of the metric, which will be done after we reduce all the terms.

$$\frac{5}{12}\frac{\kappa^2}{g^2}\phi^{-9/4}5R^{MNRS}H_{MNT}H^T_{RS} \rightarrow 10\frac{\kappa^4}{g^2}\phi^{-9/4}e^{-3\sigma}(\partial^\mu\sigma)^2|w|^2 \qquad (3)$$

$$-\frac{8}{12}\frac{\kappa^2}{g^2}\phi^{-9/4}R^{MN}H_{MNS}H^{RS}_N \rightarrow$$
$$16\frac{\kappa^4}{g^2}\phi^{-9/4}e^{-3\sigma}\left[(\partial_\mu\sigma)\partial^\mu|w|^2 - 3(\partial_\mu\sigma)^2|w|^2 - 9/4(\partial_\mu\sigma)(\partial^\mu\phi/\phi)\right] \qquad (4)$$

In order to obtain this term one has to integrate by parts due to the existence of the covariant derivative of the connection in Ricci tensor (see Appendix 2).

$$\frac{1}{12}\frac{\kappa^2}{g^2}\phi^{-9/4}R(H_{MNR})^2 \rightarrow 4\frac{\kappa^4}{g^2}\phi^{-9/4}e^{-3\sigma}\Big[|w|^2R^{(4)} - 6(\partial_\mu\sigma)\partial^\mu|w|^2$$
$$- 15/2(\partial_\mu\sigma)^2|w|^2 + 27/2(\partial_\mu\sigma)(\partial^\mu\phi/\phi)|w|^2\Big] \qquad (5)$$

Altogether we get the following surprisingly compact contribution

$$\frac{\xi}{\kappa^2}\left[4R^{(4)} - 92(\partial_\mu\sigma)^2\right] - \frac{8}{\kappa^2}(\partial_\mu\sigma)(\partial^\mu\xi) \tag{6a}$$

There are also additional contributions coming from changes of the 4d curvature scalar generated by changes of the metric: $g_{\mu\nu} \to e^{-3\sigma}g_{\mu\nu}$. They are given by

$$R^{(4)} \to R^{(4)} - 9\Delta\sigma + \frac{27}{2}(\partial_\mu\sigma)^2 \tag{7}$$

This modifies the above contribution yielding, after integration by parts the following result:

$$4\frac{\xi}{\kappa^2}\left[R^{(4)} - \frac{19}{2}(\partial_\mu\sigma)^2\right] + \frac{28}{\kappa^2}(\partial_\mu\sigma)\partial^\mu\xi \tag{6b}$$

3. $H^2(\partial\phi)^2$ **term.**

$$-\frac{3}{128}\frac{\kappa^2}{g^2}\phi^{-9/4}(H_{MNR})^2\left(\frac{\partial_S\phi}{\phi}\right)^2 \to -\frac{9}{8}\frac{\xi}{\kappa^2}(\partial^\mu\phi/\phi)^2 \tag{8}$$

4. H^2 **terms.** As before we describe different contributions separately.

$$\frac{1}{18}\frac{\kappa^4}{g^2}\phi^{-15/4}(H_{MNR})^2 \to 16\frac{\kappa^6}{g^2}\phi^{-15/4}|w|^2 * \left[8\kappa^2e^{-9\sigma}|w|^2 + 6e^{-5\sigma}(\partial_\mu a - i\frac{\kappa}{2}(C_x\bar{\mathcal{D}}_\mu\bar{C}_x))^2 + \frac{1}{3}e^{-3\sigma}(H_{\mu\nu\rho})^2\right] \tag{9}$$

$$\frac{1}{12}\frac{\kappa^4}{g^2}\phi^{-15/4}H^{MNR}H_M^{ST}H_{NS}^{U}H_{RSU} \to -24\frac{\kappa^6}{g^2}\phi^{-15/4}|w|^2e^{-5\sigma}(\partial_\mu a - i\frac{\kappa}{2}(C_x\bar{\mathcal{D}}_\mu\bar{C}_x))^2 \tag{10}$$

$$-\frac{1}{4}\frac{\kappa^4}{g^2}\phi^{-15/4}H^{MRS}H_{NRS}H^{NTU}H_{MTU} \to -48\frac{\kappa^6}{g^2}\phi^{-15/4}|w|^2 * \left[2\kappa^2e^{-9\sigma}|w|^2 + e^{-5\sigma}(\partial_\mu a - i\frac{\kappa}{2}(C_x\bar{\mathcal{D}}_\mu\bar{C}_x))^2\right]. \tag{11}$$

All together we get:

$$8\xi\phi^{-3/2}\left[4\kappa^2e^{-6\sigma}|w|^2+3e^{-2\sigma}(\partial_\mu a-i\frac{\kappa}{2}(C_x\bar{\mathcal{D}}_\mu\bar{C}_x))^2+\frac{2}{3}e^{6\sigma}(H_{\mu\nu\rho})^2\right] \qquad (12)$$

5. $H\phi F^2$ term.

$$-\frac{3}{4}\frac{\kappa^3}{g^2}\phi^{-9/4}H^{MNR}\left(\frac{\partial^S\phi}{\phi}\right)Tr\,(F_{MN}F_{RS})\to$$

$$\frac{\xi}{\kappa^2}\left[\frac{27}{2}(\partial^\mu\phi/\phi)^2+18(\partial^\mu\phi/\phi)(\partial_\mu\sigma)\right]+\frac{6}{\kappa^2}(\partial^\mu\phi/\phi)(\partial_\mu\xi) \qquad (13)$$

6. H^2F^2 term.

$$-\frac{1}{8}\frac{\kappa^4}{g^2}\phi^{-12/4}H^{TM[N}H_T^{RS]}Tr\,(F_{MN}F_{RS}) \qquad (14)$$

It is easy to observe, that the contribution which come from the items with all indices on F^2 compact vanish - this follows from the Jacobi identity. The only nonvanishing contribution we get if one of these indices is noncompact. The result reads

$$-3i\frac{\kappa^2}{g^2t_R^4}(\partial_\mu a-i\frac{\kappa}{2}(C_z\bar{\mathcal{D}}_\mu\bar{C}_z))(w\partial^\mu\bar{w}-\bar{w}\partial^\mu w) \qquad (15)$$

7. F^4 term. Finally, we consider the most interesting contribution among discussed in this section, the one involving four 10d stress-energy tensor for gauge fields. In the formula (16) we utilized definition (1b) of the tensor $\tilde{R}$ and we suppressed all the noncontributing R^2 terms. The most feasible term involving H^2 is of the order $(C/\sqrt{T})^{12}$ - then it has been suppressed, too.

$$\frac{\kappa g^2}{128}\phi^{-9/4}t^{MNRSTUWZ}Tr\,(F_{MN}F_{RS})\,Tr\,(F_{TU}F_{WZ}) \qquad (16)$$

So, let us consider case by case contributions coming from the above term. Take the pattern $(F_{\mu n}F_{\nu s})(F_{tu}F_{wz})$ as the first. Reduction of this term results in the correction to chiral matter kinetic energy.

$$2\kappa g^4t_R^{-3}|\mathcal{D}_\mu C^z|^2\left[\frac{9}{2f}(D^\alpha)^2+\frac{8}{3}w_z\bar{w}^z\right] \qquad (17)$$

The pattern $(F_{\mu n}F_{rs})(F_{\nu u}F_{wz})$ gives similary

$$-\frac{20}{3}\kappa g^4 t_R^{-3}(\partial_\mu w)(\partial^\mu \bar{w}) \tag{18}$$

The next terms give upon reduction new contributions to 4d gauge kinetic terms. One of them, $(F_{\mu\nu}F_{\sigma\delta})(F_{tu}F_{wz})$ is diagonal and should be understood as a correction to the S field defined by Witten,

$$-\frac{\kappa g^4}{2S_R t_R^2}\left[\frac{9}{2f}(D^\alpha)^2+\frac{8}{3}w_z\bar{w}^z\right]Tr(F_{\mu\nu})^2 \tag{19}$$

but the second one, $(F_{\mu\nu}F_{rs})(F_{\sigma\delta}F_{wz})$ is nondiagonal and presents itself the highly nontrivial correction to well known expression for the Lagrangian.

$$-\frac{3}{2}\frac{\kappa g^4}{S_R t_R^2}f^{-2}D^\alpha D^\beta F^\alpha_{\mu\nu}F^{\beta\mu\nu} \tag{20}$$

Finally, we list all corrections to the bosonic potential.

$$-\frac{\kappa g^6}{16S_R t_R^4}\left[\frac{1}{4}\left(234+\frac{371}{18f'^2}\right)f^{-4}(D^\alpha)^4+\frac{2909}{81f'^2 f}\left((D^\alpha)^2|w_z|^2+\frac{1}{27f'^2}\right)|w_z|^4\right] \tag{21}$$

In order to obtain the gravitation part in the canonical form one has to redefine once again the 4d metric: $g_{\mu\nu}\rightarrow(1+8\xi)g_{\mu\nu}$. This cancels a term proportional to curvature scalar in (6a) and produces a new correction to the effective Lagrangian.

$$\begin{aligned}8\xi\Big[&-\frac{3}{\kappa^2}(\partial_\mu\sigma)^2+\frac{1}{6}S_R^2(H_{\mu\nu\rho})^2-\frac{9}{16\kappa^2}(\partial^\mu\phi/\phi)^2-3t_R^{-2}(\partial_\mu a-i\frac{\kappa}{2}(C_x\bar{\mathcal{D}}_\mu\bar{C}_x))^2\\&-3t_R^{-1}|\mathcal{D}_\mu C_x|^2-16\kappa^2\phi^{-3/2}e^{-6\sigma}|w|^2-\frac{16}{3}e^{-5\sigma}|w_z|^2-9g^2e^{-5\sigma}(D^\alpha)^2\Big]\end{aligned} \tag{22}$$

Having this information one might try to find corrections to Kähler potential and to the gauge kinetic function of the standard-form 2-derivative 4d SUGRA SY-M theory. We shall comment on this point in the section IV.

For sake of completeness we listed all corrections to the kinetic terms in the Appendix 3.

It should be noted, that some of the terms written above, namely those without derivatives, are a potential source of SUSY breaking through H_{ijk} condensate.

One final comment is that to get dimensions right one should multiply the expressions obtained by the compactification volume which in our conventions is proportional to $\kappa^{3/2}$.

III. General structure of the Lagrangian

The structure of the Witten's Lagrangian (A4),[3] is governed by many symmetries, some of them exact, some of them only approximate. Among the former ones there are two scaling symmetries which are the exact tree-level classical symmetries of the full string theory [4]. They also must be the exact symmetries of the truncated Lagrangian involving higher order terms as well. One may easily check that t-scaling transformation

$$g_{\mu\nu} \rightarrow t g_{\mu\nu},\ \phi \rightarrow t^{-1/3}\phi,\ \sigma \rightarrow \sigma + 1/4\, ln t \tag{23}$$

and r-scaling defined as

$$\begin{aligned} g_{\mu\nu} \rightarrow r^{3/4} g_{\mu\nu}, \quad \kappa \rightarrow r\kappa, \qquad F_{\mu\nu} \rightarrow r^{-3/4} F_{\mu\nu}, \\ \phi \rightarrow r^{2/3}\phi, \quad C \rightarrow r^{-3/4} C,\ H_{\mu\nu\rho} \rightarrow r^{-1/2} H_{\mu\nu\rho}, \end{aligned} \tag{24}$$

under which $L \rightarrow t^{-1} r^{-1/2} L$, are satisfied indeed in our 4d Lagrangian (see Appendix 3 and formula (31).

There exists another (approximate) symmetry of the Witten's Lagrangian, so called Heisenberg symmetry, which is thought to be responsible for vanishing scalar masses at one-loop order in the effective field theory, which in turn is of crucial significance for resulting phenomenology [8]. This symmetry may be traced back to 10d SUGRA symmetry

$$A^{\alpha}_{m} \rightarrow A^{\alpha}_{M} + H^{\alpha}_{M}, \quad B_M N \rightarrow B_{MN} = \frac{1}{2} A^{\alpha}_{[M} H^{\alpha}_{N]} \tag{25}$$

(H^{α}_{M} is a harmonic form) which is exact in 10 dimensions if 10d gauge coupling constant g is set to 0. After reduction to 4 dimensions, the Heisenberg symmetry takes the form

$$\delta C^x = \frac{\epsilon^x}{2}, \quad \delta T = \epsilon^x C^x \tag{26}$$

where ϵ^x are complex parameters. The symmetry holds in the simple Witten's truncation, which should reproduce 1-generation C-Y compactification, but it is generically rather of the orbifold type [5]. This is invalidated if compactification process is affected by setting g to zero, as eg. in the general case of the Calabi-Yau compactification. In the standard case [3] (see Appendix 1) the kinetic terms of the Lagrangian exhibit full invariance under non-compact Heisenberg symmetry, and if W is neglected the multiloop scalar potential depends on C only through the variable $Y = 2ReT - 2|C|^2$, which is the actual variable in the Kähler function. As a consequence the kinetic Lagrangian depends only on the variable Y and terms $(C\mathcal{D}_\mu \bar{C})$ as far as matter fields are concerned.

This is no longer true when higher order corrections are included. One can see from (2)-(14) and (17),(18) that the kinetic energy depends explicitly on the expression denoted by w (proportional to the superpotential in [3]) and on the so called D-terms ($D^\alpha = g\bar{C}\lambda^\alpha C$). So, if the theory presented may be cast into the standard N=1 SUGRA form [2], the modified Kähler function has to depend explicitly on w and D^α, so that the noncompact Heisenberg symmetry is badly broken by higher derivative string corrections. This effect becomes important when the gauge symmetry is no longer unbroken, and fields C are given nonzero vacua. This possibility of breaking of the gauge group with large expectation values for nonsinglet fields is not obvious in the context of the simple Witten's truncation, but it has already been discussed for slightly generalized [16] and quite realistic [9] models, and still another argument in favorer of the effect in question will be pointed out in sect.V.

At this point one should make certain observation. In the modified kinetic energy there is still preserved the structure of the kinetic bosonic terms i.e. $L_{KB}(C,\bar{C}) = L_{KB}(\bar{C}C, \bar{C}\lambda^\alpha C)$ is invariant under overall U(1) rotation $C \to e^{i\beta}C$, which is claimed in [10] to do the same job as the (broken in our model) Heisenberg symmetry (if the theory has the standard form).

To the next group of symmetries to be considered belong those from the coset SU(N+1,1)/SU(N+1)xU(1) nonlinearly realized on the fields T, C. They are in general broken by our corrections as these are all proportional to some power of the 10d gauge coupling g. However special attention should be paid to the SU(1,1) subgroup of SU(N+1,1) which is in fact the symmetry of the full theory presented in [3]. This is the one among coset boosts which commutes with the gauge group and its action reads

$$\delta T = -iaT^2 - bT - i\gamma, \quad \delta C^x = (-iaT + b/2)C^x \tag{27}$$

where a, b, γ are real constants. There is a subgroup of this symmetry, namely noncompact $U_{nc}(1)$, defined by setting $b = \gamma = 0$, which may be extended to the full theory including SUSY breaking terms $W(S) = M_P^3\langle H\rangle + he^{-3S/2b_0}$, see [17],

$$\delta\langle H\rangle = \frac{-3iaT\langle H\rangle}{M_P^3}, \quad \delta h = -3iaTh \tag{28}$$

Under this symmetry $\delta t_R = 2a\, Im(T)t_R$, $\delta W = -3iaTW$ and the standard Kähler function is invariant as well.

$$\delta\left[-3ln(T_R) + ln|W|^2\right] = 0 \tag{29}$$

One can easily check that under this symmetry

$$\delta\left(\frac{|w_x|^2}{t_R^2}\right) = \delta\left(\frac{|C|^2}{t_R}\right) = \delta\left(\frac{(D^\alpha)^2}{t_R^2}\right) = \delta\left(\frac{|w|^2}{t_R^3}\right) = 0 \tag{30}$$

and the improved Lagrangian, including all the corrections, stays invariant under $U_{nc}(1)$. We should emphasize that the proposition for modified Kähler function presented in sect.IV and any function of h (see sect.IV) do respect the exact symmetry $U_{nc}(1)$.

This symmetry in turn is said [11] to be responsible for vanishing of soft gaugino masses at 1-loop order of the field theory limit, and this feature will be retained in our version of the model if it is still locally supersymmetric.

Next comments will consider remaining kinetic part of the Lagrangian (A.1) enriched by (2)-(22). First, one can easily notice that terms (9) and (12) renormalize, in a field dependent way, the kinetic term for the antisymmetric tensor B_{MN}. Second, it is worth of paying attention to expressions (19) and (20). The usual expression for gauge kinetic term is

$$-\frac{1}{4}S_R\delta_{\alpha\beta}F^{\alpha}_{\mu\nu}F^{\beta\mu\nu}$$

The term (19) renormalizes the above diagonally in adjoint gauge indices, but the new terms are now gauge-nonsinglet fields dependent and so is the new effective gauge coupling. Moreover, the piece (20) gives quite a new contribution, which is nondiagonal in adjoint gauge indices, and the diagonal part of which is not proportional to the unit matrix. This means that after the spontaneous breaking of the gauge group through the vacua of C^{*}, there will be additional splitting of the coupling constants for different generators. The required things are the nonzero vacua for the D-terms, which is however quite feasible and takes place e.g. in the SUSY standard model with tgβ different from 1. The nondiagonal part of (20) cannot be understood as a simple redefinition of the superfield S.

Concluding this section we observe, that the corrections we have obtained may be viewed as a series expansion in matter fields C, the actual expansion parameter being $C/\sqrt{t_R}$, $t_R = e^{\sigma}\phi^{3/4}$. Recalling that in general fields like T have a distinguished status as they are so called moduli fields determining the geometry of the compactification manifold, one can see the remarkable feature that the series has a form of the expansion about the manifold of moduli fields. This is the same structure as the one used recently in [7].

IV. The modified form of the Kähler function.

If the theory obtained by the procedure of Witten's truncation is locally supersymmetric, then it should be defined with the help of two functions only. One of them is the gauge kinetic function $f_{\alpha\beta}$, the second one - the Kähler function K. The general form of K which respects the exact classical string symmetries reads [4]

$$K = -ln(S+\bar{S}) - 3ln(T+\bar{T}) + ln|W(C)|^2 + h(C/\sqrt{T}, \bar{C}/\sqrt{T})$$

where function h is inert under (24) and (25). The results presented in sect.II suggest the more specific form of h. As one can see ξ works like an expansion parameter - almost all the terms multiplied by ξ already exist in the original Witten's Lagrangian, so we propose the following form for h:

$$h = a\xi = a\frac{\kappa^6}{g^2}t_R^{-3}|w|^2$$

This simple ansatz reproduces qualitatively all the corrections to the kinetic terms except that given by (17). One should also notice, that in general the nonanalitic redefinitions of the fields T, S, C, (e.g. $t_R \rightarrow (1+\epsilon_T\xi)t_R$), $S_R \rightarrow (1+\epsilon_S\xi)S_R$) similar to that used already in [3] should be allowed and the Lagrangian should take its explicitly supersymmetric form in these new redefined fields. In fact formula (19) suggests that such redefinitions are really needed at least for the S field (we remind that at the lowest order the function $f_{\alpha\beta}$ is proportional to S). The most nontrivial contribution is given by the expression (20). This should give correction to $f_{\alpha\beta}$ but we have not found any simple form ansatz. In spite of this, we assume that our ansatz for K mimics properly the general structure of the 4d effective Lagrangian including higher derivative string contributions.

V. The scalar potential.

At this point it seems to be useful to write down the full scalar potential, including corrections (12),(20),(21), as well as well known lowest order contributions [3]. The corrections are multiplied by the compactification volume $\kappa^{3/2}$, and we take $g = \lambda\kappa^{3/2}$, where λ is assumed to be 0(1) in the light of [12]. Then using variables

u, p, d, y defined as

$$y = \frac{|w_z|^2}{t_R^2},\ u = \frac{|C|^2}{t_R},\ d = \frac{(D^\alpha)^2}{t_R^2},\ u = \frac{|w|^2}{t_R^3}$$

the potential (multiplied by S_R) reads

$$S_R V = \frac{1}{16}p + \frac{1}{48}y + \frac{9}{2}d + 128\kappa^8 p^2 + \frac{128}{3}\kappa^6 py + 72\kappa^6 pd - \frac{\kappa^4}{16}\left[\frac{1}{4f^2 f'^2}\left(234 + \frac{371}{8f'^2}d^2\right) + \frac{2909}{81 f f'^2}dy + \frac{1}{27f'^2}\left(220 + \frac{742}{27f'^2}y^2\right)\right] \tag{31}$$

where different contributions are ordered according to growing powers of κ. The novel feature of (31) is that there are negative contributions to the potential, which make it no longer positive definite. In fact, this is consistent with the observation from the preceding sections, that the classical σ-model SO(N+1,1) symmetries are broken by the terms proportional to g^2. So, the special property of the so-called no-scale structure [13],[19] is lost. It has been argued [20] that in this kind of no-scale models the potential should be positive definite (and as a consequence give a vanishing cosmological constant), but this is not the case here. As a nice illustration take the general supersymmetric case with the Kähler function proposed in section IV. Take h as a function of u only. Then $V = V_1 + V_2 + V_3 + V_D$, where $V_{1,2,D}$ are positive definite (here notation follows that of ref. [16]), but V_3 reads

$$V_3 = \frac{e^h}{16 S_R t_R^3}\frac{\lambda}{\alpha\beta h_u}|uwh_u + 3w|^2 \tag{32}$$

with $\alpha, \beta, h_u > 0$ and $\lambda = h_u - 3h_{uu}$ which may easily be less then zero. Now turn back to eq.(31). One can see that the term containing p, which is a possible vacuum condensate for the superpotential, are all positive. This means that as long as the $\langle W\rangle$ is not supposed to cancel partially the gaugino condensate in the expression for gravitino mass, it is dragged to zero at the vacuum. That is not the case for d and y, which are SUSY breaking D- and F-terms respectively. The terms proportional to κ^4 are all negative, and they clearly destabilize the potential, dragging it to negative infinity for large values of d and y. The values of d and y at which V gets negative are very large, $d, y > \kappa^{-4}$ (or $\langle C\rangle > M_P$, after correcting the normalization of fields from 10d to 4d), in fact one may say unphysical, however the observed phenomenon itself may be significant. Of course, it is possible that still higher order corrections will be positive and will turn the potential up for even larger values of d and y thus producing the true minimum far away from the origin.

One should notice, that what has been said concerns, at the level of Witten's simple model, the gauge non-singlet fields, and large vacua for them mean Planck scale spontaneous gauge symmetry breaking. However, it is possible that in more realistic models the similar higher-order string corrections may touch observable group singlet fields fixing their vacua through nonvanishing $(y)^n$ - F-terms. In our model there still remain undetermined vevs for S_R and t_R fields, and our corrections do not help. So one must consider additional effects to resolve that crucial ambiguity. One possible and very attractive scenario is proposed in [14]. The scalar potential considered there reads

$$\begin{aligned} V =& \frac{1}{16 S_R t_R^3}\left(\left|w - 2S_R\frac{\partial w}{\partial S}\right|^2 + \frac{1}{3}t_R\left|\frac{\partial w}{\partial C} + 2\bar{C}\frac{\partial w}{\partial T}\right|^2 + \right. \\ & \left. \frac{4}{3}t_R^2\left|\frac{\partial w}{\partial T} - \frac{3w}{2t_R}\right|^2 - 3|w|^2\right) + \frac{9}{2S_R t_R^2}(D^\alpha)^2 \end{aligned} \tag{33}$$

The crucial point in this approach lies in the fact that V gets negative at field theory tree-level due to the miscancellation of the last two terms in the parenthesis in (33). Our observation is, that the same role may be played by negative higher order σ-model contribution to the scalar potential described in this work. Of course including non-zero vev for w does not change the qualitative picture.

VI. Conclusions.

In this paper we have made an effort to figure out explicitly higher order (in the number of fields) nonrenormalizable interactions which should be added to the standard no-scale Lagrangian given by Witten in [3] as an approximation to the effective low energy 4d string Lagrangian when 10d heterotic string tree level amplitudes containing up to four derivatives are taken into account. The 10d string loops give corrections of the higher order [18].

It turns out that both kinetic and potential parts of the bosonic Lagrangian are given nontrivial corrections. Assuming, that the resulting expression is still a part of the N=1 4d SUGRA theory, we have proposed an ansatz for the Kähler potential which reproduces qualitative features of the result of truncation and, we believe, may be used as the next reasonable approximation to the actual string Lagrangian.

The corrections to the potential do not retain its positiveness, and destabilize it in the subspace of gauge nonsinglet fields dragging their vacua towards infinity.

We have observed also an interesting phenomenon in the gauge sector, namely the gauge kinetic function taking the full nondiagonal matrix form with nondiagonal terms consisting of gauge invariants built up from matter fields.

The general rule is that the Lagrangian has the form of the series in the field space around the manifold of neutral moduli fields, the expansion parameter being $C/\sqrt{t_R}$.

We have discussed also some phenomenological consequences of our findings, and we hope that this work sheds some more light on the fascinating problem of the truly stringy features of the low energy theory stemming from the heterotic string.

Acknoledgement

We would like to thank Prof H.P.Nilles and Prof.S.Pokorski for useful discussions.

Appendix 1

In this appendix we fix our conventions and write down some useful formulae necessary for (Witten) dimensional reduction. Big latin letters denote 10d vector indices, greek letters - 4d indices, small latin letters are compactified space vector indices - these are also grouped in holomorphic (i,j,k...) and antiholomorfic ($\bar{\mathrm{i}}$, $\bar{\mathrm{j}}$, $\bar{\mathrm{k}}$,...) indices; x,y,z are indices (of the matter fields) of the fundamental representation of the 4d gauge group; a,b,.. are indices of the adjoint representation of the gauge group The reduction for graviton g_{MN} and antisymmetric tensor B_{MN} is defined as follows:

$$g_{MN} \to (g_{\mu\nu},\ g_{mn} = \delta_{mn}e^{\sigma}),$$

$$B_{MN} \to (B_{\mu\nu},\ B_{mn} = \epsilon_{mn}a),$$

where ϵ is antisymmetric tensor and has the following nonzero elements: $\epsilon_{56} = \epsilon_{78} = \epsilon_{910} = 1$. All the fields on the r.h.s. depends only on the points in 4d spacetime. The matter fields C appear by reduction of the gauge fields:

$$F_{MN} \to (F_{\mu\nu},\ F_{\mu i},\ F_{ij},\ F_{i\bar{j}}). \qquad (A.1)$$

where

$$F_{\mu i} = T_{ix}\mathcal{D}_{\mu}C_x,$$

$$F_{ij} = 2g d_{xyz}\epsilon_{ijk}C_x C_y \bar{T}^{kz},$$

$$F_{i\bar{j}} = ig\left(\delta_{i\bar{j}}f^{-1}(C\lambda^{\alpha}\bar{C})\lambda^{\alpha} + f'^{-1}(C\bar{C})\eta^{a}_{i\bar{j}}\eta^{a}\right);$$

λ^{α}, T^{ix}, η^{a} are Lie algebra matrices of the unbroken part, broken part and SU(3) part of the 10d gauge group, respectively; g is 10d gauge coupling constant. In order to derive formula for $F_{i\bar{j}}$ we use the following ansatz

$$[T_{ix}, \bar{T}_{jy}] = i\delta_{i\bar{j}}\lambda^{\alpha}_{x\bar{y}}\lambda^{\alpha} + i\delta_{x\bar{y}}\eta^{a}_{i\bar{j}}\eta^{a}. \qquad (A.2)$$

Using above formulae we can obtain the following rules for dimensional reduction of the stress tensor for the antisymmetric tensor field.

$$H_{MNP} \to \left(H_{\mu np} = (\partial_{\mu}a - i\kappa/2(\bar{C}_x\bar{\mathcal{D}}_{\mu}C_x))\epsilon_{np},\ H_{ijk} = -2\kappa\epsilon_{ijk}w\right), \qquad (A.3)$$

where $w = gd_{xyz}C^xC^yC^z$. In all above one should remember about complex conjugate quantities. We also introduce the following abbreviations

$$w_x \equiv \frac{\partial w}{\partial C_x}, \quad D^\alpha \equiv g(\bar{C}\lambda^\alpha C)$$

The Witten's [3] Lagrangian reads

$$\begin{aligned} e^{-1}L = &-\frac{1}{2\kappa^2}R - \frac{3}{\kappa^2}(\partial_\mu\sigma)^2 - \frac{9}{16\kappa^2}(\partial^\mu\phi/\phi)^2 \\ &- \frac{1}{6}S_R^2(H_{\mu\nu\rho})^2 - \frac{1}{4}S_R Tr\,(F_{\mu\nu})^2 \\ &- 3t_R^{-2}(\partial_\mu a - i\frac{\kappa}{2}(C_x\bar{\mathcal{D}}_\mu\bar{C}_x))^2 - 3t_R^{-1}|\mathcal{D}_\mu C_x|^2 \\ &- 8\kappa^2\phi^{-3/2}e^{-6\sigma}|w|^2 - \frac{8}{3}e^{-5\sigma}|w_x|^2 - \frac{9}{2}g^2e^{-5\sigma}(D^\alpha)^2 \end{aligned} \tag{A.4}$$

We also introduce S, T fields: $S = e^{3\sigma}\phi^{-3/4} - i\theta$, $T = e^\sigma\phi^{3/4} + \bar{C}_xC_x + 3ia$, and notations $S_R = e^{3\sigma}\phi^{-3/4}$, $t_R = e^\sigma\phi^{3/4}$, where $\epsilon_{\mu\nu\rho\sigma}\partial^\rho\theta = \phi^{-3/2}e^{6\sigma}H_{\mu\nu\sigma}$.

Appendix 2

The most difficult part of the calculations is the reduction of the terms which contains curvature tensor. The only nonvanishing components of the connection are

$$\Gamma^\mu_{i\bar{j}} = -\frac{1}{2}g^{\mu\nu}\delta_{i\bar{j}}e^\sigma\partial_\nu\sigma \quad \Gamma^j_{i\mu} = \frac{1}{2}\delta^j_i\partial_\mu\sigma.$$

Curvature tensor contains several nonvanishing components but in the following we need only one

$$R^i_{\bar{j}kl} = -\frac{1}{4}e^\sigma(\partial_\mu\sigma)^2\left(\delta^i_l\delta_{\bar{j}k} - \delta^i_k\delta_{\bar{j}l}\right).$$

The interesting Ricci tensor components are

$$\begin{aligned} R^{(10)}_{\mu\nu} &= R^{(4)}_{\mu\nu} + \left(\nabla_\mu\Gamma^m_{m\nu} + \Gamma^m_{n\mu}\Gamma^n_{m\nu} + (\mu \leftrightarrow \nu)\right), \\ R^{(10)}_{i\bar{j}} &= -\nabla_\mu\Gamma^\mu_{i\bar{j}} + \delta_{i\bar{j}}e^\sigma(\partial_\mu\sigma)^2 \end{aligned}$$

The curvature scalar R has the form

$$R^{(10)} = R^{(4)} + 6\nabla^\mu(\partial_\mu\sigma) - \frac{21}{2}(\partial_\mu\sigma)^2.$$

Appendix 3

Below we list all corrections to the kinetic terms of the 4d Lagrangian.

$$
\begin{aligned}
&\xi\Big[-\frac{64}{\kappa^2}(\partial_\mu\sigma)^2+\frac{27}{4\kappa^2}(\partial_\mu\sigma)(\partial^\mu\phi/\phi)+\frac{63}{8\kappa^2}(\partial^\mu\phi/\phi)^2\\
&\quad+\frac{32}{\kappa^2}(\partial_\mu\sigma)\partial^\mu ln\xi+\frac{6}{\kappa^2}(\partial^\mu\phi/\phi)(\partial_\mu ln\xi)+\frac{20}{3}S_R^2(H_{\mu\nu\rho})^2\Big]\\
&-\left[24t_R^{-1}-2\kappa g^4t_R^{-3}\left(\frac{9}{2f}(D^\alpha)^2+\frac{8}{3}w_z\bar{w}^z\right)\right]|\mathcal{D}_\mu C^z|^2\\
&-3i\frac{\kappa^2}{g^2t_R^4}(\partial_\mu a-i\frac{\kappa}{2}(C_z\bar{\mathcal{D}}_\mu\bar{C}_z))(w\partial^\mu\bar{w}-\bar{w}\partial^\mu w)-\frac{20}{3}\kappa g^4t_R^{-3}(\partial_\mu w)(\partial^\mu\bar{w})\\
&-\frac{\kappa g^4}{4S_Rt_R^2}\left[\left(9f^{-1}(D^\alpha)^2+\frac{16}{3}w_z\bar{w}^z\right)\delta_{\alpha\beta}+6f^{-2}D^\alpha D^\beta\right]F^\alpha_{\mu\nu}F^{\beta\mu\nu}
\end{aligned}
$$

References

1. M.Green, J.Schwarz, E.Witten, "Superstring Theory" (Cambridge U.P., Cambridge, 1987).

2. E.Cremmer, S.Ferrara, L.Girardello, A.van Proyen, Nucl.Phys. B212 (1983) 413.

3. E.Witten, Phys.Lett.B155 (1985) 151.

4. C.Burgess, A.Font, F.Quevedo, Nucl.Phys.B272 (1986) 661.

5. S.Ferrara, C.Kounnas, M.Porrati, Phys.Lett.B181 (1986) 263.

6. D.Gross, J.Sloan, Nucl.Phys.B291 (1987) 41.

7. L.Dixon, V.Kaplunovsky, J.Louis, prep.SLAC-PUB-4959 (1989).

8. P.Binetruy, M.Gaillard, Phys.Lett.B195 (1987) 382; U.Ellwanger, M.Schmidt, prep.HD-THEP-86-21.

9. A.Font, L.Ibañez, P.Nilles, Nucl.Phys.B307 (1988) 109; A.Font, L.Ibañez, P.Nilles, F.Quevedo,prep.CERN-TH5030/88.

10. J.Ellis, A.Lahanas, D.Nanopoulos, prep.CERN-TH.4988/88 (1988).

11. P.Binetruy, M.Gaillard, prep.LAPP-TH-259/89 (1989).

12. V.Kaplunovsky, Phys.Rev.Lett.55 (1985) 1036.

13. A.Lahanas, D.Nanopoulos, Phys.Repts.145 (1987) 1.

14. G.G.Ross, Phys.Lett.B211 (1988) 315.

15. E.Witten, Nucl.Phys.B268 (1986) 79.

16. M.Quiros, Phys.Lett.B196 (1987) 461.

17. M.Dine, R.Rohm, N.Seiberg, E.Witten, Phys.Lett.B155 (1985) 55.

18. K.A.Meissner, J.Pawełczyk, S.Pokorski, Phys.Rev. D38(1988) 1144; J.Pawełczyk, Ph.D.thesis, Warsaw University (1988).

19. N.Dragon, M.G.Schmidt, U.Ellwanger, Phys.Lett.B145 (1984) 192; Nucl. Phys. B255 (1985) 549.

20. P.Binetruy, S.Dowson, M.Gaillard, I.Hi