IC/91/180

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

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ABSTRACT

A model for positron annihilation in the sub-surface region near a fractal surface is proposed. It is found that the power law relationship between the mean positron implantation depth and incident positron energy can be used to measure the fractal dimension of the fractal surface in materials.

MIRAMARE – TRIESTE
July 1991

INTRODUCTION

It is well known that assessing the structure of a rough surface is of importance for many technical processes. Fractal concept has been applied into this field for characterization of the structure of the rough and irregular surface [1],[2],[3]. The development of a variable-energy positron beam technique enables us to study an atomic disorder at the surface and in the sub-surface region [4]. These experiments involved the measurement of the annihilation lineshape parameter S_T (or I_2 of τ_2 in lifetime measurements) as a function of positron implantation energy. In this paper, we are going to show how can we use the power law relationship between the mean positron implantation depth and the incident positron energy E to measure the fractal dimension of the fractal surface.

THE AREA OF A FRACTAL SURFACE

The area of a fractal surface can be calculated by its self-similarity property. Let us consider two mutual perpendicular directions x and y on the surface. We cut the rough surface with planes $x = 0$ and $y = 0$. Their cross-over lines may be considered as Koch curves approximately and statistically. According to Mandelbrot [1], the area of the fractal surface may be given by

$$\frac{A_f(\varepsilon)}{A_0} \sim \frac{\ell_x \ell_y}{\ell_0^2} = \varepsilon^{2-(D_x+D_y)} = \varepsilon^{2-D_s} \quad (1)$$

where, ℓ_0 is the length of the initiation of a Koch curve, and $\varepsilon = \eta/\ell_0$, is the yardstick length, η, normalized with respect to ℓ_0. D_x and D_y are the fractal dimensions of Koch curves on x and y directions, $D_s = D_x + D_y$, is the fractal dimension of the fractal surface S.

THE POSITRON IMPLANTATION DEPTH AS A YARDSTICK LENGTH

A positron as a light particle will undergo appreciable angular deflections during the inelastic collisions due to positron-electron and positron-phonon interactions and the elastic scattering by nuclei. The angular deflection by collision becomes so large in the final stages of thermalization that the correlation between the positron path in the solid and its initial direction is completely lost [5]. We think that this process may smear the finer structure of the fractal surface shape as the frontier surface of thermalized positrons. The deeper the positron penetrates, the flatter the frontier surface would be. The one dimensional schematic figure is in Fig.1.

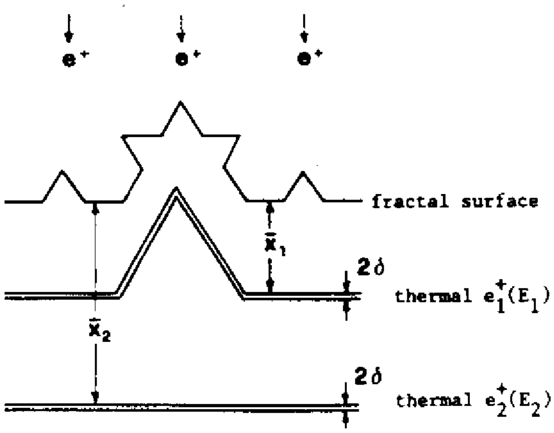

Fig.1 Schematic positron of $e_1^+(E_1)$ and $e_2^+(E_2)(E_2 > E_1, \bar{x}_2 > \bar{x}_1$ and $2\delta \sim 1000$ Å).

In the similar sense as Tsallis suggested on the measure of fractal dimensionalities through the traditional skin effect [6], we assume that the mean positron penetration depth $\bar{x}$ can play a role of the yardstick length for measuring the length of a fractal line (one dimensional structure of the fractal surface). Then

$$\eta = c\bar{x}, \quad or\ \varepsilon = c\bar{x}/\ell_0 \propto E^{1.6} \tag{2}$$

where c is a constant.

THE FRACTAL DIMENSION OF THE SURFACE

The r.m.s. deviation δ from the mean penetration depth $\bar{x}$ appears to be energy independent [5]. We may consider it as a constant. It seems that two different cases should be distinguished. If $\bar{x} > \delta$, thermalized positrons would annihilate in a constant volume, $A_f \cdot 2\delta$. However, if $\bar{x} < \delta$, thermalized positrons would annihilate in a variable volume, $A_f \cdot 2\delta(\bar{x})$.

1. Case 1: $\bar{x} > \delta$
From equations 2 and 3

$$S_f/S_T = (A_f(\varepsilon)2\delta)/(A_0 \cdot 2\delta) = A_f(\varepsilon)/A_0 \quad \propto \varepsilon^{2-D_s} \propto E^{1.6(2-D_s)}\ . \tag{3}$$

Then,

$$\ell n S_f = \text{const.} + 1.6(2 - D_s)\ell n E \tag{4}$$

and

$$D_s = 2 - \frac{1}{1.6}\left(\frac{d\ell n S_f}{d\ell n E}\right) \tag{5}$$

Instead of S_f, the same relationship can be obtained for the I_2 by similar argument. Due to $\frac{dS_f}{d\bar{x}}$ or $\frac{dS_f}{d\varepsilon} < 0$. Therefore, a value of D_s between two and three can be expected.

2. Case 2: $\bar{x} < \delta$ The positron diffusion constant and the effective annihilation rate in the lattice are dependent on the implantation depth beneath the surface. The diffusion length $\delta(E)$ is a variable which is much reduced due to the small $\bar{x}$ and then due to the large defect density near the surface [10]. Roughly, the relation may be written as follows

$$\frac{S_f(\varepsilon)}{S_T} \propto \frac{A_f(\varepsilon)(2\bar{x})}{A_0(2\delta)} \simeq (\frac{\ell_0}{\delta c})\varepsilon^{3-D_s} \tag{6}$$

where, we have assumed $\delta(E) = \delta(\bar{x}) = b\bar{x}(b = 1)$. This expression satisfies the conditions: (1) $S_f = 0$ at $\bar{x} = 0$ ($E = 0$) (2) It reduces to equation (4) at $\bar{x} = \delta$. Then

$$D_s = 3 - \frac{1}{1.6}\left(\frac{d\ell n S_f}{d\ell n E}\right) \tag{7}$$

We notice that $\frac{d\ell n S_f}{d\ell n E} > 0$ due to $\frac{dS_f}{d\bar{x}} > 0$ and when $\bar{x} < \delta$. A value of D_s between two and three can also be expected.

DISCUSSION

Up to now we did not find any experimental work performed along this line. Fig.2 is the log-log plots of two typical experimental data obtained by Brusa et al. [7] and Schödelbauer et al. [8]. It seems that the data are consistent with the double logarithm relationship in a certain range of E. The slopes of the two straight lines correspond to the values of fractal dimensions being 2.03 and 2.28. This shows that this effect is large enough to be detected by positron annihilation experiments if the fractal dimension of the fractal surface is larger than 2.03 which usually appear in many polished material surfaces [9].

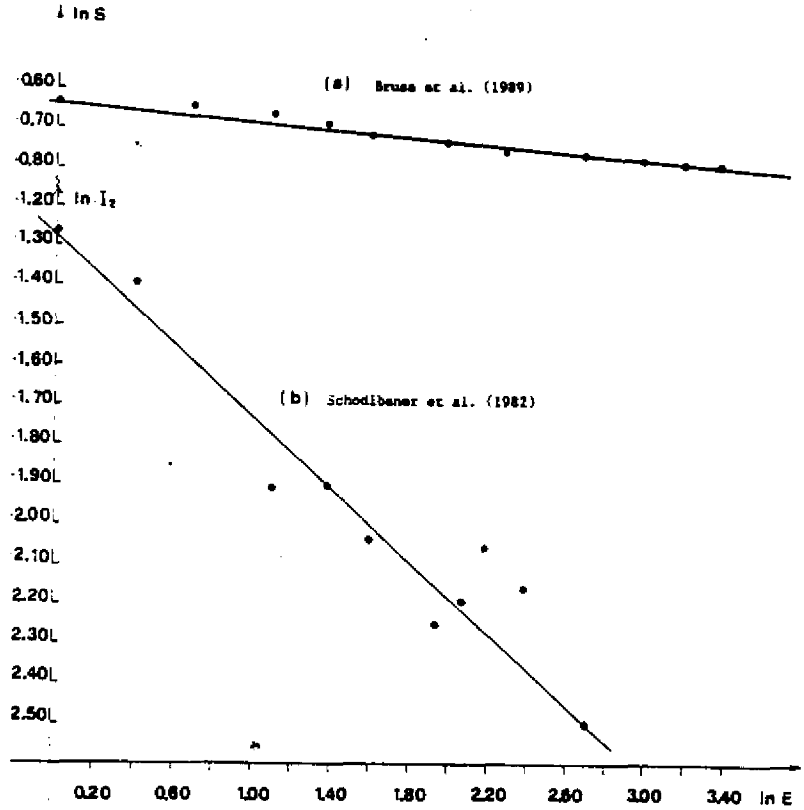

Fig.2 Positron annihilation parameters as a function of the positron beam energy: (a) S in annelead Cu [7], (b) I_2 in the in He implanted Ni [8].

See Fig.3 in the paper written by Vehanen [10]. The shape of the curve below 1.5 kev seems very like case 2 though it perhaps may be predicted as the effect of other causes by previous authors. Anyhow, the monotonic increasing character of the function $S_f(E)$ when $\bar{x} < \delta$ is reasonable because $S_f(0)$ should be zero at $E = 0$. There would be no positron penetrating into the material and then no positron annihilation effect would be expected.

The model being consistent with some experimental data shows that it cannot be excluded from predictions of experimental results in monoenergetic positron studies. It may superpose with other physical processes. Experiments along this line are needed.

ACKNOWLEDGMENTS

One of the authors (C.W.L.) would like to thank Professor Abdus Salam, the International Atomic Energy Agency and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste. He would also like to thank Professor Stig Lundqvist for his encouragement on fractals in materials.

REFERENCES

1) Mandelbrot, B.B.: The Fractal Geometry of Nature, 1983, Freeman San Francisco.

2) Lung, C.W.: in Fractal in Physics, Eds. Pietronero L. and Tosatti E., 1986, p. 189 Elsevier, New York.

3) Lung C.W. and Mu Z.Q.: Phys. Rev. B, 1988, 38, 11781.

4) Triftshauser, W. and Köger, G.: Phys. Rev. Lett., 1982, 48, 1741.

5) Paulin, R.: Positron Solid-State Physics, Proceedings of the International School of Physics, "Enrico Fermi", Course LXXXIII, 14-24 July 1981, Eds. Brandt W. and Dupasquier A., (North Holland Publishing Company, 1986) p. 565.

6) Tsallis, C.: Fractal in Physics, Eds. Pietronero L. and Tosatti E., 1986, p.65, Elsevier Science Publishers B.V.

7) Brusa, R.S., Dupasquier, A., Grisenti, R., Liu, S., Oss, S. and Zecca, A.: Positron Annihilation, Proc. ICPA-8, Eds. Dorikens-Vanprast L., Doriklens M. and Segers D. 1989, p. 410, World Scientific, Singapore.

8) Schödelbauer, D., Kögel, G., Sperr, P. and Triftshäuer, W.: Phys.Stat. Sol. 1987, A102, 549.

9) Liu, S.H.: ICTP Lecture Notes in "Working Party on Electrochemistry", SMR 524-22, 1990; or Proceeding of the Working Party on Electrochemistry (to be published).

10) Vehanen, A.: Positron Annihilation, Proc. ICPA-8, Eds. Dorikens-Vanpraet L. Dorikens M. and Segers D., 1989, p. 42 World Scientific, Singapore.