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POTENTIAL FLUCTUATIONS DUE TO THE RANDOMLY DISTRIBUTED CHARGES AT THE SEMICONDUCTOR–INSULATOR INTERFACE IN MIS–STRUCTURES *

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Abstract – A new expression for the Fourier transform of the binary correlation function of the random potential near the semiconductor–insulator interface is derived. The screening due to the image charge with respect to the metal electrode in MIS – structure is taken into account, introducing an effective insulator thickness. An essential advantage of this correlation function is the finite dispersion of the random potential Γ^2 to which it leads in distinction with the so far known correlation functions leading to divergent dispersion. The important characteristic of the random potential distribution Γ^2 determining the amplitude of the potential fluctuations is calculated.

I. INTRODUCTION

MIS - structures represent a proper physical subject for investigation of the disorder at the interface semiconductor-insulator. In this paper we shall be interested in potential fluctuations generated by the randomly distributed charges at or near the interface semiconductor-insulator. As has been pointed out in [1] this charge can be identified by technologically available fixed oxide charge and interface trap charges. We shall investigate the case of a MIS-structure in inversion mode, i.e. the external bias applied to the metal electrode sweeps out the majority carriers in the depth of semiconductor and near the interface,an inversion layer of minority carriers being formed. As has been pointed out by Gergel' and Suris [3] and Brews [1] it is essential that the random potential relief in the surface region is inherently 3-dimensional. Therefore, our considerations will be based on 3-D Poisson equation for the random potential fluctuation.

The aim of this paper is to investigate analytically the statistical properties of the random impurity potential near the interface semiconductor-insulator in order to obtain a finite dispersion of the potential, appropriately accounting for the screening by the gate electrode and the semiconductor. Following Gergel' and Suris [3,4] the essential contribution to the correlation function near the interface (i.e. $\rho, z \ll d$) is given by the short-range charge fluctuations whose potential is purely Coulombic. But the so obtained correlation function gives rise to infinite dispersion as a consequence of a logarithmic divergence [3,7]. This inconsistency is removed in this paper.

II. THEORETICAL BACKGROUND

Consider the electrostatic problem of evaluation of the potential fluctuations of the random potential in MIS-structure in inversion mode. The system is analogous to a parallel plate capacitor with one plate - the metal electrode (gate) and the other one - the edge of the depletion layer. The potential due to a point charge is a superposition of the potential of a charged impurity and its image charge with respect to the metal electrode. Consider the potential of the point charge $G(\boldsymbol{\rho}-\boldsymbol{\rho}';z,z')$ (or the Green's function) generated by a point charge at position $(\boldsymbol{\rho}',z')$ and evaluated at a point $(\boldsymbol{\rho},z)$. The plane $z = 0$ is the semiconductor-insulator interface, the region $-d<z<0$ is the insulator, $z>0$ is the semiconductor and $\boldsymbol{\rho}(x,y)$ is the vector position of points in the interfacial plane or any plane parallel to the interfacial one, z=const. (see Fig.1).

The equation defining the potential of a point charge $G(\boldsymbol{\rho}-\boldsymbol{\rho}';z,z')$ is:

$$\nabla.\left[\varepsilon(z)\ \nabla G\right] = -q\ \delta(z-z')\delta(\boldsymbol{\rho}-\boldsymbol{\rho}') \qquad (2.1)$$

where

$$\varepsilon(z) = \begin{cases} \varepsilon_i, & \text{for } z<0 \\ \varepsilon_s, & \text{for } z>0 \end{cases} \qquad (2.2)$$

The boundary conditions on G are that G vanishes both at the gate ($z=-d$) and at the depletion layer edge ($z=w$). The depletion layer width w is considered as constant and no modulation of the depletion layer width by the point charge is included. This represents the so called parallel plate capacitor approximation [1] and is valid in depletion or in inversion in case when the depletion layer edge is sufficiently remote from the insulator–semiconductor interface.

The above equation is solved by a Fourier transform technique. The following Fourier transform over the two-dimensional co-ordinate $\boldsymbol{\rho}$ of the Green's function is obtained [2,6 p.850]:

$$G(\mathbf{k}_{\parallel},z) = \begin{cases} \dfrac{q}{k_{\parallel}} \quad \dfrac{\mathrm{sh}(k_{\parallel}w)\ \mathrm{sh}[k_{\parallel}(z+d)]}{\Delta_k} & \text{for } -d \leq z \leq 0 \\ \dfrac{q}{k_{\parallel}} \quad \dfrac{\mathrm{sh}(k_{\parallel}d)\ \mathrm{sh}[k_{\parallel}(w-z)]}{\Delta_k} & \text{for } 0 \leq z \leq w \end{cases} \qquad (2.3)$$

where

$$\Delta_k = \varepsilon_i \mathrm{ch}(k_{\parallel}d)\ \mathrm{sh}(k_{\parallel}w) + \varepsilon_s \mathrm{ch}(k_{\parallel}w)\ \mathrm{sh}(k_{\parallel}d) + C_I \frac{\mathrm{sh}(k_{\parallel}d)\mathrm{sh}(k_{\parallel}w)}{k_{\parallel}} \qquad (2.4)$$

C_I is the inversion layer capacitance, considered as a sheet capacitance.

Then the charge fluctuation $\delta\Sigma(\boldsymbol{\rho})$ in the interfacial plane will generate the corresponding potential fluctuation $\delta\phi(\boldsymbol{\rho},z)$ which is a solution of the 3-D Poisson equation [3]:

$$\delta\phi(\boldsymbol{\rho},z) = 4\pi \int d^2\mathbf{k}_{\parallel}\ \delta\Sigma(\mathbf{k}_{\parallel})\ \frac{e^{ik_{\parallel}\cdot\rho}}{k_{\parallel}} \quad \frac{\mathrm{sh}(k_{\parallel}d)\ \mathrm{sh}[k_{\parallel}(w-z)]}{\Delta_k} \qquad (2.5)$$

As shown by Brews and Nicollian [6,p.847], the binary correlation function of the random charge fluctuations in the case of Poisson distribution is:

$$\langle\delta\Sigma(\boldsymbol{\rho})\delta\Sigma(\boldsymbol{\rho}')\rangle = q^2\sigma\ \delta(\boldsymbol{\rho}-\boldsymbol{\rho}') \qquad (2.6)$$

where σ is the sum of the average densities of the positively and negatively charged centers at the interface.

When Fourier transforming:

$$\langle \delta\Sigma(\mathbf{k})\delta\Sigma(\mathbf{k}')\rangle = \left(\frac{q}{2\pi}\right)^2 \sigma\, \delta(\mathbf{k}+\mathbf{k}') \qquad (2.7)$$

Then using this property, the pair correlation function of the potential fluctuation is:

$$W^{(2)}(\rho_1-\rho_2, z_1, z_2) = 4q\,\sigma^2 \int d^2\mathbf{k}_\parallel\, e^{ik_\parallel\cdot(\rho_1-\rho_2)} G(\mathbf{k}_\parallel, z_1)\, G(-\mathbf{k}_\parallel, z_2) \qquad (2.8)$$

In the next section we will examine the case $z,\rho \ll d$ that is the region near the insulator-dielectric interface.

III. RESULTS AND DISCUSSION

The purpose of this work is to evaluate one of the most important statistical characteristics of the random potential - the dispersion (root mean square value) Γ of the potential. The expression for Γ^2 [5] is:

$$\Gamma^2 = \langle \delta\phi(\mathbf{r})\delta\phi(\mathbf{r})\rangle = \int K^{(2)}(\mathbf{k}_\parallel, k_z)\, \frac{d^3\mathbf{k}}{(2\pi)^3} \;. \qquad (3.1)$$

where $K^{(2)}(\mathbf{k}_\parallel, k_z)$ is the Fourier transform of the binary correlator:

$$K^{(2)}(\mathbf{k}_\parallel, k_z) = q^2\sigma \int_{-\infty}^{\infty} d\,(z_1 \pm z_2)\; G(\mathbf{k}_\parallel, z_1)\; G(-\mathbf{k}_\parallel, z_2)\; e^{-ik_z(z_1 \pm z_2)} \qquad (3.2)$$

Using the symmetry property of the Green's function:

$$G(\mathbf{k}_\parallel, z) = G(-\mathbf{k}_\parallel, z) \qquad (3.3)$$

and performing Laplace transform with the corresponding expressions for the Green's function in the two regions we obtain:

$$K^{(2)}(\mathbf{k}_\parallel, k_z) =$$

$$= \frac{q^2\sigma\ sh(k_\| w)sh(k_\| d)\left[sh(k_\| w)ch(k_\| d)\ + sh(k_\| d)ch(k_\| w)\right]}{k_\|\ \ (k_\|^2\ +{}_z^2)\left[\varepsilon_i\, sh(k_\| w)ch(k_\| d) + \varepsilon_s\, sh(k_\| d)ch(k_\| w) + \frac{C_I}{k_\|}\, sh(k_\| d)sh(k_\| w)\right]^2}$$

(3.4

Performing the limiting procedure w >> d, i.e. w → ∞ we obtain in inversion :

$$K^{(2)}(k_\|, k_z) = \frac{q^2\sigma}{2\ \varkappa^2}\ \ \frac{(\ 1\ -\ \exp(-2k_\| d))}{k_\|\ \ (k_\|^2\ +k_z^2)\left[\ 1\ -re^{-2k_\| d}\ +\ \frac{C_I}{2\varkappa k_\|}\ (\ 1\ -\ e^{-2k_\| d})\right]^2} \qquad (3.5)$$

where $\varkappa = \dfrac{\varepsilon_s + \varepsilon_i}{2}$ is the mean dielectric constant,

$r = \dfrac{\varepsilon_i - \varepsilon_s}{\varepsilon_i + \varepsilon_s}$ is the ratio depending on the dielectric constants of insulator and semiconductor.

Following Brews [1] we can approximate the function :

$$f(k_\| d)\ =\ \frac{1\ -\ \exp-2k_\| d)}{\left[\ 1\ -\ re^{-2k_\| d}\ +\ \frac{C_I}{\varkappa k_\|}\ (1\ -\ e^{-2k_\| d})\right]^2} \approx \frac{k_\|\ \tilde{d}}{1\ +k_\|^2\ \tilde{d}^2} \qquad (3.6$$

where

$\tilde{d} = \dfrac{2\varkappa\ d}{\varepsilon_i + qN_I d}$ is an effective thickness of the insulator.

We shall show that the taking into account the screening from the metal electrode leads to a finite value of Γ^2. According to (eq.3.1):

$$\Gamma^2\ =\ \frac{2\pi\ q^2\sigma}{2\varkappa^2\ \ (2\pi)^3}\int_0^\infty \frac{dk_\| . k_\| . k_\| \tilde{d}}{1\ +\ k_\|^2\tilde{d}^2}\int_{-\infty}^{\infty}\frac{dk_z}{k_z{}^2 + \ k_\|{}^2} = \frac{q^2\ \sigma}{16\ \varkappa^2} \qquad (3.7)$$

The potential energy dispersion will be correspondingly

$$\Gamma^2 q^2 = \frac{q^4 \sigma}{16 \varkappa^2} .$$

In a case of a uniform 3-D distribution of charged centers in the bulk of the dielectric the corresponding mean square potential energy should be replaced by:

$$\Gamma^2 = \frac{q^4 n_3 d}{16 \varkappa^2},$$ where n_3 is the bulk density of the charged centers.

If we substitute the constant values for the system MOS (metal $-SiO_2-Si$) , $\varepsilon_i = 3.9$, $\varepsilon_s = 11.5$ and $\sigma = 1.10^{11}$ cm^{-2} typically, we obtain for $\Gamma = 18.6$ meV (when all the charges are at the interfacial plane), and $\Gamma = 5.87\times10^{-7}(n_3 d)^{1/2}$ meV (when the charges are scattered in the bulk of dielectric).

Therefore, the screening from the metal electrode turns out to be substantial in such a Coulomb type problem. The root mean square potential is a measure of the amplitude of the random fluctuations and it is a characteristic quantity determining the statistical distribution of the potential.

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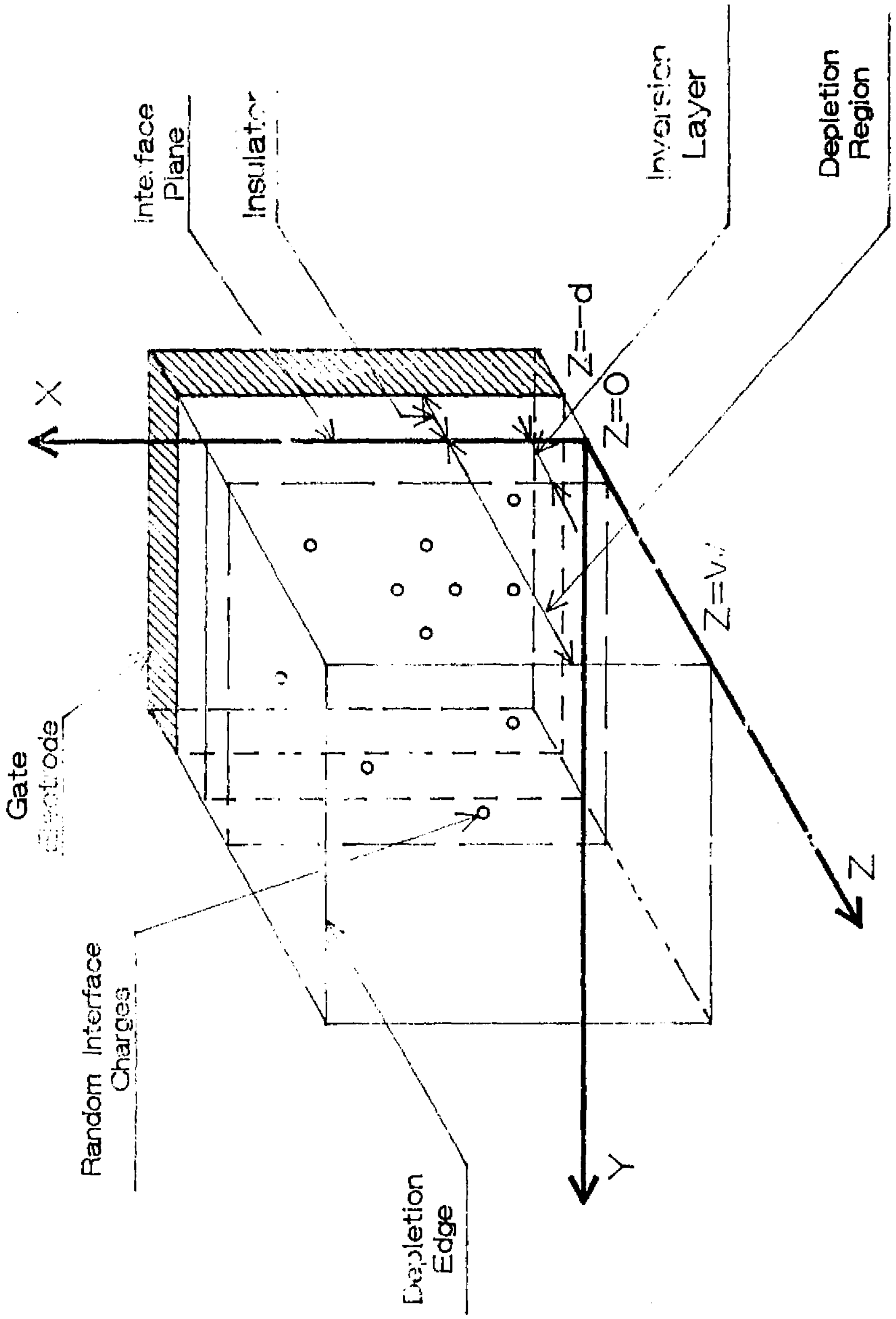

Fig. 1

Fig. 1 - Section of an MIS-structure in inversion mode, indicating the location of the random charge, generating the random potential fluctuations.