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P-Shell Hyperon Binding Energies

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A shell model for lambda hypernuclei has been used to determine the binding energy of the hyperon in nuclei throughout the p shell. Conventional (Cohen and Kurath) potential energies for nucleon-nucleon interactions were used with hyperon-nucleon interactions taken from Nijmegen one boson exchange potentials. The hyperon binding energies calculated from these potentials compare well with measured values.

Although many studies have been made of hypernuclear structure[1], most have been concerned with only a small number of hypernuclei. We consider the mass variation of hyperon binding energies in single hyperon hypernuclei throughout the entire p shell. We do so with the assumption that the ground states of p shell hypernuclei are described by $0s$ shell hyperons coupled to complex nuclear cores; cores which have closed $0s$ shells and partially filled $0p$ shells. Hypernuclear wavefunctions can then be determined by the diagonalisation of an appropriate Hamiltonian using the basis formed by such coupled states.

The Hamiltonian to be considered is described in terms of one and two body matrix elements. The one body matrix elements were those of Cohen and Kurath[2] for nucleons but were taken from data for the hyperon. The Cohen and Kurath (8-16)2BME potential energies were used for the two nucleon matrix elements, while the two body hyperon-nucleon matrix elements were calculated from the Nijmegen one boson exchange potentials[3]. These hyperon-nucleon potentials include amplitudes associated with several different meson exchanges, but do not allow for medium effects due to the presence of the other particles, for the transfer of more than one boson, or for basis space truncation. Thus we must solve for a many body Hamiltonian,

$$H = H_0 + \Delta_{12} , \tag{1}$$

and do so using the conventional shell model approach, namely to expand the many body hypernuclear wavefunction in terms of the eigenvectors, $\phi_i(1...A)$, of the single particle Hamiltonian, H_0, i.e.

$$\psi_n(1...A) = \sum_i b_i(n)\, \phi_i(1...A) , \tag{2}$$

where

$$H_0\, \phi_i = \epsilon_i\, \phi_i . \tag{3}$$

Then, upon premultiplication of the Schroedinger equation by each and every one of the basis vectors, viz

$$< \phi_j \,|\, H - E_n \,|\, \psi_n >= 0 , \tag{4}$$

the (infinite) set of homogeneous equations for each state, ψ_n,

$$\sum_{i=1}^{\infty} H(j:i)\, b_i(n) = 0 \tag{5}$$

is obtained where

$$H(j:i) = (\epsilon_i - E_n)\,\delta_{ij} + < \phi_j \,|\, \Delta_{12} \,|\, \varphi_i > \ . \tag{6}$$

The basis space $\{|\varphi_i>\}$ was truncated as defined above. Basis states with the single hyperon in the $0p$ shell can only be part of the ground state wavefunction if the nucleons are also in other orbitals than those considered. That is due to parity constraints. Thus are basis states are

$$|\,\phi_i\,> = \,|\,[\,(p_{1/2})^n_{J_\alpha,T_\alpha}\,(p_{3/2})^m_{J_\beta,T_\beta}\,]_{J_c,T}\,\Lambda_{1/2,0}\,> \ . \tag{7}$$

As the matrix elements are of one or two body type inherently, it is possible to express the A-body wavefunction as a weighted sum of $(A-1)$-body wavefunctions coupled to a single particle (in the one body case), and as a weighted sum of $(A-2)$-body wavefunctions coupled to a pair of particles in the two body case, i.e.

$$|\,\phi_i(1...A)\,> = \sum_{j,k} w_i(j,k)\,\phi_j(1)\,\phi_k(2...A)\ , \tag{8}$$

and

$$|\,\phi_i(1...A)\,> = \sum_{m,n} w_i(m,n)\,\phi_m(1,2)\,\phi_n(3...A)\,, \tag{9}$$

which reduces the problem of calculating many body matrix elements to one of evaluating weighted sums of one and two body matrix elements. Then with the Hamiltonian matrix eqn. (6) defined, standard diagonalisation techniques provide eigenvalues and, therefore, eigenvectors of the hypernuclei. From the lowest energy state the binding energy of the hyperon to the core, B_Λ, can be defined. The value of B_Λ can be obtained in terms of the binding energies of the hypernuclear ground state and nuclear core ground state as

$$B_\Lambda = B.E.({}^A_\Lambda X) - B.E.({}^{A-1}X)\ . \tag{10}$$

The effective interaction between nucleons within a nuclear medium is established reasonably well, given the wealth of spectral and transition data on normal nuclei. Such is not the case as yet for hypernuclei. Hence there are no preferred set of hyperon-nucleon potential energies within the $0p$ shell. Therefore we resorted to evaluation of the hyperon-nucleon potential energies for our shell model calculations from one boson exchange models for the free hyperon-nucleon interaction. Here we consider those of the Nijmegen group, specifically those interactions designated as model D and model F, which give fits to the hyperon-nucleon scattering data. These interactions relate via SU(3) constraints to the nucleon-nucleon potential defined by the same group. The Nijmegen one boson exchange potential is a state dependent hard core potential, the full details of which have been published[3].

With either model interaction, for our purpose, we must evaluate the two particle potential energies as expectations of a g-matrix, viz

$$< V >= \int_0^\infty R_{l_1}(\mathbf{r_1})R_{l_2}(\mathbf{r_2})\ g(|\mathbf{r_1} - \mathbf{r_2}|)\ R_{l_3}(\mathbf{r_1})R_{l_4}(\mathbf{r_2})\ \mathbf{dr_1 dr_2}\ , \qquad (11)$$

wherein $R_l(\mathbf{r})$ are single particle bound state wave functions. Harmonic oscillator wave functions were used, with the nucleon case ($\mathbf{r_1}$) specified with an oscillator energy ($\hbar\omega$) of 41.5 $A^{1/3}$ MeV. We used the value 27 $A^{1/3}$ MeV as suggested by Bouyssy[4] for the oscillator energy of the hyperon state ($\mathbf{r_2}$). In both cases variations of oscillator energy throughout the p shell was ignored and energies associated with $A = 12$ were used for all nuclei. A simple Scott-Moszkowski[5] separation method was then chosen to define the g-matrix from the free hyperon-nucleon model interactions. For simplicity again, we chose the separation radii to be those of the hard cores of the interaction. We note that the precise specification of g-matrices from starting interactions like these can be formed, but as our purpose is solely to specify a reasonable set of pair potential energies for use in shell model estimates of hypernuclear binding energies, such refinement is not essential. To complete the specification of the pair potential energies required in diagonalisation of the hypernuclear Hamiltonian, we chose the (8-16)2BME set of nucleon-nucleon potential energies of Cohen and Kurath[2].

One needs then only the hyperon and nucleon single particle energies to have all data input to the diagonalisation calculations. For nucleons the values given by Cohen and Kurath were assumed. For the hyperon, the $0s$ state energy was taken[6] from the difference in binding energies of ^4He and $^5_\Lambda He$. A value of −3.12 MeV is the result. This method is similar to that used by Cohen and Kurath to calculate the single particle energy of $0p$ shell nucleons.

With these specified sets of one and two body matrix elements hypernuclear shell model calculations were performed for all $0p$ shell, single lambda, hypernuclei, giving binding energies and excitation spectra. The latter we do not consider herein as available data with which to compare is sparse.

The first general result from these calculations was that hypernuclear states of spin $J_c + 1/2$ were more bound than those with $J_c - 1/2$ (for $J_c > 0$), where J_c is the ground state spin of the $A - 1$ particle nuclear system. This is contrary to previous expectations. But the spin-orbit hyperon-nucleon potential energies are weak, and a reversal of sign (of the spin-orbit g-matrix elements) would achieve the expected ordering of states. Therefore we repeated the model F calculations with that change and designate such results hereafter as model F′.

Our results for B_Λ for all $0p$ shell hypernuclei are compared with the measured data in figures 1 and 2. In figure 1 the mass variations of B_Λ are given for model D (open upright triangles) and are compared to calculated values of Bassichis and Gal[7] (open inverted triangles) and with the measured values[6] that are shown by filled circles. Clearly the calculated results are not in good agreement with the data. Bassichis and Gal[7] believe that may be due to strong second order effects, such as those needed to resolve the problem of overbinding in ${}^{5}_{\Lambda}He$. But the changes required to fit the data are so large that it is more reasonable to suggest that the hyperon-nucleon pair potential energies are too strong. The results obtained using model F and model F′ data sets in our shell model calculations are in very good agreement with observation. Those results are depicted by the upright and inverted triangle respectively in figure 2. Clearly the B_Λ values obtained with model F differ from experiment by no more than 1 MeV over the whole mass range and to stress these results, consider the weak coupling limit of no hyperon-nucleon pair potential energy. The hyperon binding energy is then just the hyperon single particle energy (3.18 MeV). This is shown in both figures by the dashed lines.

A shell model study of the binding energies of a lambda hyperon to the nuclear cores in all of the $0p$ shell hypernuclei has been made using nucleon single particle and pair interaction energies of Cohen and Kurath, hyperon single particle energies taken from data and one boson exchange models of hyperon-nucleon interactions to evaluate the necessary hyperon-nucleon pair interaction energies. Diagonalisation of the model Hamiltonians that result gave ground state spin values of $J_c + 1/2$, where J_c is the spin of the ground state of the $A-1$ nuclear system. This was in contradiction to other calculations and a further model was postulated that inverted the spin-orbit coupling. Hyperon binding energies calculated with this model, along with models D and F of the Nijmegen group, were compared to data. It was found that these measured hyperon binding energies were reproduced far more accurately by using the model F one boson exchange interaction for the hyperon-nucleon potential than by using the model D prescription. It was also found that the fit to data of the results using the model F prescription was essentially independent of the sign of the spin-orbit interaction.

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FIG. 1. Mass variation of B_Λ for model D (open triangles) and those of ref. 7 (open squares). The experimental data (filled circles) are taken from ref. 6. The dashed line represents the weak coupling limit, where the hyperon-nucleon interaction is set to zero in all channels.

FIG. 2. Mass variation of B_Λ for model F (open traingles) and model F' (open squares) are compared with the data of ref. 6 (filled circles). Again the dashed line represents the weak coupling limit.

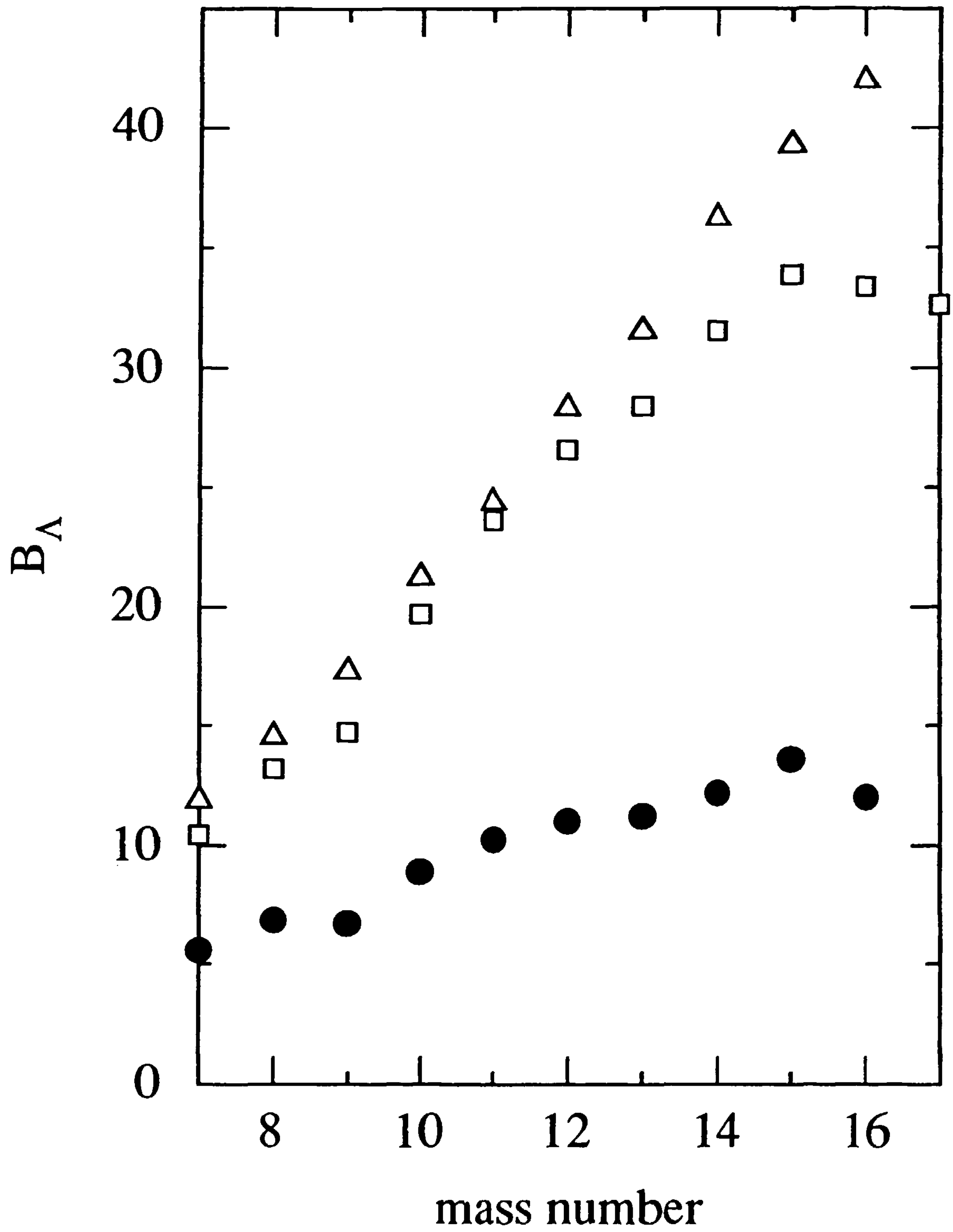

Figure 1

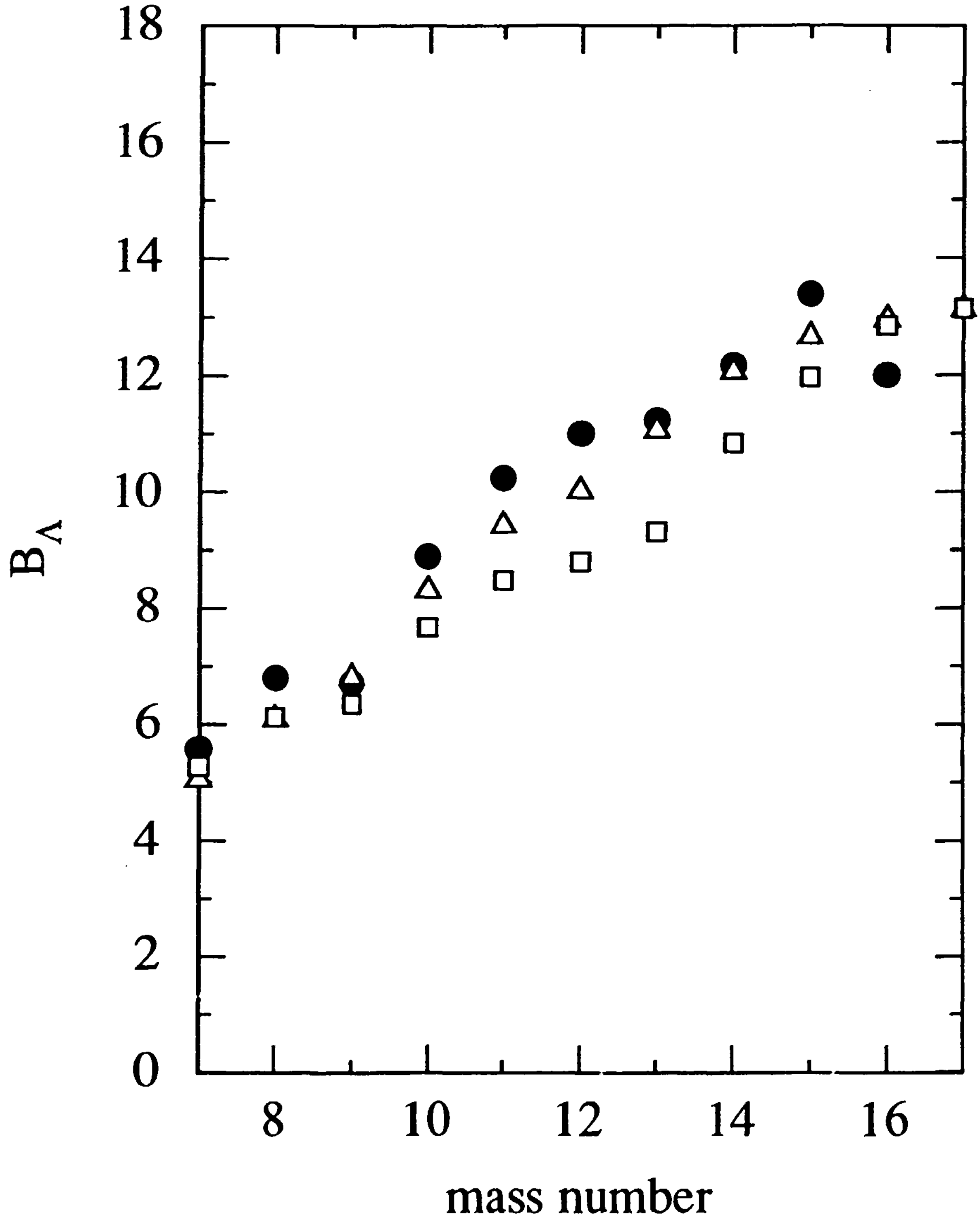

Figure 2