IHEP-OTF--90-20
(IFVE-OTF--90-20)

USSR STATE COMMITTEE FOR UTILIZATION OF ATOMIC ENERGY
INSTITUTE FOR HIGH ENERGY PHYSICS

И Ф В Э 90-20
ОТФ

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STRESS ENERGY OF ELASTIC GLOBE IN CURVED SPACE AND A SLIP-OUT FORCE

Protvino 1990

УДК 539.1.01 М-24

Abstract

Sokolov S.N. Stress Energy of Elastic Globe in Curved Space and a Slip-Out Force: IHEP Preprint 90-20. - Protvino, 1990.- p. 6, refs. 4.

The energy of stresses in an elastic globe made in the flat space and shifted into curved space is expressed through scalar invariants of the curved space. This energy creates an additional force acting on elastic bodies in a gravitational field.

Аннотация

Соколов С.Н. Энергия натяжений в упругом шаре в искривленном пространстве и сила выскальзования: Препринт ИФВЭ 90-20. - Протвино, 1990. - 6 с., библиогр.: 4.

Энергия натяжений в упругом шаре, изготовленном в плоском пространстве и перемещенном в искривленное пространство, выражена через скалярные инварианты искривленного пространства. Эта энергия порождает дополнительную силу, действующую на упругие тела в гравитационном поле.

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Gravitational field induces stresses in elastic bodies. These stresses are directly measurable, since their energy can be turned into heat. The general relativity (GR)[1], as well as the relativistic theory of gravitation (RTG)[2], predicts two kinds of the induced stresses: those balancing the tidal forces (arising from the difference of accelerations at different points) and those due to non-flatness of the Riemann (in RTG, efficient Riemann) space. They are related to the different parts of metrics and behave differently in a number of aspects.

The tidal forces are related mainly with the time-time component of the metrics. The deformations due to tidal forces and the relevant elastic energy is inversly proportional to the hardness μ of the body. These deformations need certain time for their development (at high frequency of the field they are inversly proportional to the squared frequency). The stresses due to non-flatness of space are related mainly with the spatial part of the metrics. They change instantly with the change of the gravitational field. For a homogeneous body, the deformations due to the non-flatness of space do not depend on the hardness μ and the relevant elastic energy is directly proportional to μ.

The tidal effects are well-known and easily estimated in the Newtonian approximation (GR and RTG add only small corrections). In this paper we concentrate our attention on the less investigated non-flatness effects.

We limit ourselves to the static problem and compute the energy of stresses in an isotropic homogeneous elasic globe prepared without internal stresses in a region where space is (almost) flat and then placed into the region with noticeable non-flatness of space. This energy depends upon the shape of the globe at a new place, so we shall find the minimal energy, corresponding to the least possibly distorted globe not moving with respect to our static gravitational field.

Of course, the globe resting at the given moment, at the next moment will start to move and the tidal accelerations will start stretching the globe, but these effects due to their different behaviour can be , in principle, separated from the effect of non-flatness, if a globe is replaced by a properly designed mechanical device. The replacement of a globe with a device of a comparable size cannot increase the total stress energy, it can only concentrate it in a convenient way. So, the mentioned *minimal* stress energy for the globe is the *maximal useful* energy available for the mechanical device measuring non-flatness effects and is the value that should be compared with the energy of the thermal and other noises, when the sensitivity of a detector is estimated.

Evidently, in our static case and for initially symmetric globe this energy may depend only on the scalar invariants of the 3-dimensional spatial part of the Riemann 4-space. There are 3 independent invariants in a curved 3-space:

$$R=R^i_i, \qquad R_2=R^i_k R^k_i, \qquad R_g= det(R_{ik})/det(g_{ik}),$$

where R, R^i_k are the scalar curvature and the Ricci tensor for the metrics g_{ik}. We shell use also the invariant

$$R_4= R^{ij}_{km} R^{km}_{ij},$$

where R^{ij}_{km} is the Riemann tensor. In the 3-dimensional space

$$R_4=4 R_2- R^2=4 R_3,$$

where $R_3= \tilde{R}^i_k \tilde{R}^k_i$, $\tilde{R}_{ik}=R_{ik} - g_{ik} R/6$. The dependence of the stress energy on these invariants can be found as follows.

Let us use the linear elastisity theory[3] and assume that all displacements within the globe are small, the globe radius ρ compared to the distances, at which the invariants essentially change, is small and only the leading term in the expression for the energy is of interest. Let in the curved 3-space in the place, where the elastic globe is placed, the coordinates be chosen in such a way that the metrics looks as slightly distorted Eucledean one. The general small distortion of the 3-space metrics, which contributes to the leading term, is linear and quadratic in coordinates, and so it is set by 54 arbitrary coefficients.

The position of the globe material in the new coordinates within the accuracy of our approximation can be described as a perturbation of the Cartesean coordinates containing all possible linear, quadratic, and cubic terms. For each coodinate there are 19 such terms, so the perturbation of coordinates contains 57 unknown constants. These constants and coefficients of the curved space metrics completely define the elasic stresses of the globe and the total energy of these stresses. The minimal energy is found by variation of the 57 unknown constants and is a function of the metrics coefficients, of the raduis of the globe, and of elastic properties of its material only. Comparing this expression with the expressions for scalar invariants of the curved 3-space with the same metrics, one may obtain the desired relation.

The actual calculation was done as follows. Since the final formula may contain only a small number of unknown coefficients, it is sufficient to use in the perturbation of the metrics only a few of the 54 free parameters, letting the rest of them to zero. We used 6 parameters and the metrics was chosen to be

$$g_{old}= diag(1,1,1), \quad g_{new}(y)= diag(1+c\ d,1,1),$$

where the perturbation d depends on 6 arbitrary parameters a,b,c,s,t,u:

$$d=a\ x_1^2+b\ x_2^2+c\ x_3^2+s\ x_1x_2+t\ x_2x_3+u\ x_3x_1,$$

and c is a small constant. The terms c^3 and higher were omitted in the calculation (except in the calculation of R_g). The new positions of the points of the globe

$$y_i= x_i+\Sigma_j v_{ij}x_j+\Sigma_{j\leq k} v_{ijk}x_jx_k+\Sigma_{j\leq k\leq l} v_{ijkl}x_jx_kx_l$$

depend on the mentioned 57 coefficients v. The metrics g_{new} in the coordinates x becomes

$$g_{new}(x)_{ij}= g_{new}(y)_{km}\ dy_k/dx_i\ dy_m/dx_j.$$

The difference of the new and old metrics defines local distortions of the material of the globe.

The next step is the calculation of the density of the stress energy through the stress tensor

$$\sigma_{ij} = \lambda\, \delta_{ij}\, (\varepsilon_{11}+\varepsilon_{22}+\varepsilon_{33}) + 2\mu\, \varepsilon_{ij},$$

where λ, μ are the Lame coefficients of the material related to the Young constant E and the Poisson coefficient ν as follows

$$\mu = \frac{E}{2(1+\nu)}, \quad \lambda = \frac{E\,\nu}{(1+\nu)(1-2\nu)} .$$

Calculation of the energy density seems rather difficult, since the finding of stresses at each point of the globe involves, in principle, the transformation of the deformation ellipsoid to the main axis and finding dilatation factors $\lambda_i = 1+2\varepsilon_{ii}$ as the 3 solutions of a *cubic* algebraic equation

$$\det(g_{new} - \lambda\, g_{old}) = 0. \tag{1}$$

Happily, for the isotropic globe and small ε_{ii} the energy density

$$Q = 1/2\, \Sigma\, \sigma_{ii}\, \varepsilon_{ii}$$

is symmetric in ε_{ii}:

$$Q = \mu(\varepsilon_{11}^2 + \varepsilon_{22}^2 + \varepsilon_{33}^2) + \lambda/2\, (\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33})^2 .$$

It gives the possibility to find Q without knowing the directions of the main axis and individual ε_{ii}. Indeed, the roots ε_i of a cubic equation

$$\varepsilon^3 + a_2\, \varepsilon^2 + a_1\, \varepsilon + a_0 = 0$$

satisfy identities

$$\varepsilon_1+\varepsilon_2+\varepsilon_3 = -a_1 , \quad \varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2 = a_1^2 + 2\, a_2,$$

with the help of which one can express Q directly through the coefficients of equation (1). Therefore, Q is just a polynomial in the elements of the metrics g_{new} and (though expression for Q is very lengthy) its integral over the globe can be found in a closed form. The variation of this integral over all v and finding the minimum of energy gives

$$E_{min} = \mu\, \pi\, \rho^7 c^2 [4\, b^2 + 4\, c^2 + 2\, u^2 + (4\, b^2 + 8\, b\, c + 4\, c^2)\, P_1]\, P_2,$$

where $P_1=1/10\ (6\lambda-\mu)/(2\lambda+\mu)$, $P_2=1/105\ (6\lambda+\mu)/(38\lambda+7\mu)$. The scalar invariants R, R_2 for the same metrics g_{new} are

$$R= c\ 2(b+c)+ c^2\ f, \quad R_2= c^2\ (2\ b^2 + 2\ b\ c + 2\ c^2 +1/2\ u^2),$$

where f is some function of x_i. Their combination R_4 is

$$R_4= 2\ c^2\ (2b^2+2c^2+u^2).$$

The invariant R_g is cubic in c:

$$R_g= c^3\ (b +c)\ (b\ c -1/4\ u^2) + O(c^4),$$

and should not enter the expression E_{min} in our approximation. Combining the expressions for E_{min}, R, R_4, we obtain the relation

$$E_{min}= \mu\ \pi\ \rho^7 \left[R_4 + R^2 \frac{1}{10} \frac{6\lambda-\mu}{2\lambda+\mu} \right] \frac{6\lambda+\mu}{105\ (38\lambda+7\mu)} . \qquad (2)$$

(in terms of R, R_2 the expression looks a bit less elegant). In the particular case $\nu=1/4$, $\lambda=\mu$, popular in applications of the elasticity theory, we get

$$E_{min}=\mu\ \pi\ \rho^7\ (\ R_4 + R^2\ 1/6\ )/675.$$

If the curved space is not homogeneous, the invariants R, R_4 and, hence, the stress energy of the globe depend on the point X, where its center is placed. The gradient

$$F_i= -\ \partial\ E_{min}/\ \partial\ X_i$$

is a force pushing the globe in the direction, where the space is less curved. One may say that the globe tends to slip out from the curved places of the space. In a more general case, when the globe is prepared without internal stresses in some place of the curved space, the stress energy evidently grows with the shift from this place and the slip-out force tends to return the globe to its birth-place. If the space is curved due to gravitational interaction, this slip-out force is an additional effect influencing the motion of finite-size test bodies. Two globes of equal size and mass, but having different elasticity, will move differently in the gravitational field due to the force F_i, as well as due to the similar force related with the tidal effects.

There are non-gravitational phenomena able to create stresses in elastic bodies and simulate a non-flatness of space. One of the most important among them is the magnetostriction[4]. Suppose that the material of the elasic globe is isotropic and its logitudinal and transversal dilatations $\Delta l/l$ due to the magnetic field of strength $B(y)$ are $\alpha(B)$, $\beta(B)$. The effect of the magnetic field is mathematically equivalent to the replacement of the flat metrics g_{old} by the metrics

$$g_{eff}(y)_{ij}=(1-\beta(B))\, g_{old\ ij} + (\beta(B)-\alpha(B))n_i n_j$$

of the (3-dimensional) efficient Riemann space, where $n_i=B_i/B$ and where $B_i=B_i(y)$ are the components of the vector of magnetic induction in the material of the globe. Computing the scalar invariants R, R_4 for the metrics g_{eff} and using (2) we obtain the magnetostriction stress energy in the globe for the given magnetic field B_i. Since nonzero magnetostriction is adherent to most construction materials, this effect may mask the curved space effect of gravitation.

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Recieved 29 January 1990

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Энергия натяжений в упругом шаре в искривленном пространстве и сила выскальзывания.

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Подписано к печати 28.02.90. Т-06777. Формат 60x90/16.
Офсетная печать. Печ.л. 0,33. Уч.-изд.л. 0,53. Тираж 150.
Заказ 293. Индекс 3649. Цена 8 коп.

Институт физики высоких энергий, 142284, Протвино Московской обл.

8 коп. Индекс 3649

П Р Е П Р И Н Т 90-20, И Ф В Э, 1990