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ON COLOR TRANSPARENCY

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Abstract

A quantum mechanical treatment of high momentum transfer nuclear processes is presented. Color transparency, the suppression of initial and final state interaction effects, is shown to arise from using the closure approximation. New conditions for the appearance of color transparency are derived.

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We study a class of high momentum transfer (greater than about 1-2 GeV/c) nuclear processes which proceed by the emission of a single fast nucleon from the nucleus. Examples are the semi-exclusive $(e, e'p)$ and (p, pp) reactions occurring on nuclear targets. In such examples, the momentum transfer is space like.

It has been argued [1,2] that a phenomenon called color transparency occurs for sufficiently large values of the momentum transfer. These arguments are summarized here: To obtain an appreciable amplitude for a high momentum transfer reaction on a nucleon leading to a nucleon, the colored constituents must be close together. If the constituents are close together, their color electric dipole moment is small and the soft interactions with the medium are suppressed. If the particle remains small, it can escape the medium without further interaction. The escape time is proportional to its mass divided its energy times the nuclear diameter. This time is short if the outgoing particle has sufficiently high energy. The feature that the particle suffers no distortion, i.e. that the medium is "transparent", leads to the name "color transparency". Color transparency is expected to hold even when the projectile-nucleus optical potential is not small.

The idea of color transparency has generated much interest from theorists [3–6] and experimentalists [7,8]. This is because of the arguments [1,2] that the existence of color transparency is a testable prediction of quantum chromodynamics (QCD). Moreover, if color transparency holds, one obtains a new way to study the high-momentum tails of nuclear wave functions.

Previous derivations [1,2] of color transparency used a time-dependent approach, classical arguments, and a Fock state decomposition. Here we clarify the arguments using a time-independent, quantum mechanical approach with physical states. A new set of conditions necessary for the appearance and eventual measurement of color transparency effects are determined.

To be definite, consider a high momentum transfer process in which a nucleon leaves the nucleus. The state of the particle immediately after the collision can be written as $J(q)|N\rangle$, in which $J(q)$ is the operator that brings in the high four-momentum, q. The quantity $J(q)|N\rangle$ is the small object that must escape the nucleus. The operator V represents the interaction between the ejected baryon and the nuclear medium. The nuclear interaction can change the momentum of the ejectile and also excite or de-excite the internal degrees of freedom. That is, V acts in the product of the internal N^* space and the ejectile center-of-mass space. Its matrix elements are written as $\langle N|V(\vec{q},\vec{q}')|N^*\rangle$, so that $V(\vec{q},\vec{q}')$ acts on the internal degrees of freedom.

We now treat the effects of the interaction V using first-order perturbation theory. Higher-order terms are discussed below. The amplitude $\mathcal{M}$ is given to first order by:

$$\mathcal{M} = \langle N|J(q)|N\rangle + \langle N|V(\vec{q},\vec{q}')\sum_{N^*}\int d^3q' \, \frac{|N^*\rangle\langle N^*|}{E - E^*(\vec{q}') + i\epsilon} J(q)|N\rangle \ , \qquad (1)$$

where $\vec{q}$ is the (three) momentum of the final (detected) particle, $\vec{q}'$ is the momentum of the intermediate state, and $E^*(\vec{q}') = \sqrt{M_{N^*}^2 + \vec{q}'^2}$ the (virtual) energy of the excited state. The energy of the outgoing nucleon, denoted by $E = \sqrt{M_0^2 + \vec{q}^2}$, is determined by the energy transfer, the nucleon mass and binding energy. The effects of the motion of the bound nucleon (Fermi motion) allow the two momenta $\vec{q}$ and $\vec{q}'$ appearing in the second term of eq. (1) to be different. Indeed,

$$\vec{q}' = \vec{q} + \vec{p}\ , \tag{2}$$

where $\vec{p}$ is the initial momentum of the nucleon. However, our notation suppresses this effect of Fermi motion to simplify the discussion. We stress that the states $|N^*\rangle$ are eigenstates of the internal Hamiltonian; the action of the operator V is the only cause of transitions between different internal states.

Now we use eq. (1) to obtain color transparency. The key is closure. If the $E^*(\vec{q}')$ is approximately the same, say $\overline{E}(\vec{q}')$, for all states contributing significantly to the sum, one may use closure to perform the sum over N^*. Then eq. (1) becomes:

$$\mathcal{M} = \langle N(\vec{q})|J(q)|N\rangle + \langle N|V(\vec{q},\vec{q}')\int d^3q'\,\frac{1}{E - \overline{E}^*(\vec{q}') + i\epsilon}J(q)|N\rangle\ . \tag{3}$$

In the second term, the potential V acts on the small state $J|N\rangle$. If we assume that V acting on such a state is negligible (the statement that under QCD small objects have small interactions) the second term vanishes. The remaining term is just that expected if color transparency occurs, i.e. the impulse approximation with no distortions present.

Let us now examine the closure approximation in more detail. Look at the $\langle N|V$ term of eq. (1). The interaction V is a soft, low-momentum transfer operator. Its action on a nucleon leads mainly to a few low-lying excited states. Suppose the nucleon excitation energies are small compared to the energy of the final nucleon, then $E^*(\vec{q}') \approx E(\vec{q}')$ and the closure of eq. (3) is obtained. To make the condition quantitative it is useful to consider the matrix elements $\langle N|V(\vec{q},\vec{q}')|N^*\rangle$ and $\langle N^*|J(q)|N\rangle$ which depend on the intermediate momentum $\vec{q}'$. The most important value of $\vec{q}'$ is the one for which the pole in the energy denominator occurs, which in turn depends on the mass of the state N^*. (The pole is responsible for the unitarity cut. The principle value term tends to be small at high energies.) For large E, the difference between the locations of the ground state and excited state, N^*, poles is given by $\Delta q_{N^*} = \sqrt{E^2 - M_{N^*}^2} - \sqrt{E^2 - M_0^2} \approx -(M_{N^*}^2 - M_0^2)/(2E)$. Since Δq_{N^*} decreases as E increases, Δq_{N^*} will become zero, closure will hold, and color transparency will be obtained for large enough values of E.

How large is "large enough"? If the matrix elements $\langle N|V(\vec{q},\vec{q}')|N^*\rangle$ and $\langle N^*|J(q)|N\rangle$ vary rapidly with $\vec{q}'_{N^*}$ then the closure approximation can not be valid. To be precise, for the closure approximation to hold, the product of the two matrix elements $\langle N|V(\vec{q},\vec{q}')|N^*\rangle$ and $\langle N|J(\vec{q})|N\rangle$ must not change appreciably when $\vec{q}'$ changes from $\vec{q}'_0$ to $\vec{q}'_{N^*}$. The variation of $\langle N|V(\vec{q},\vec{q}')|N^*\rangle$ is governed by the nuclear rms radius R_A. Nuclear form factors vary as $1 - q^2R_A^2/6$, so one requires:

$$|\Delta q_{N^*}|\ R_A/\sqrt{6} = \frac{R_A(M_{N^*}^2 - M_0^2)}{2\sqrt{6}E} \ll 1\ . \tag{4}$$

For large values of q the variation of the function $\langle N^*|J(q)|N\rangle$ with q is determined by quark counting rules. The low-lying baryon states of relevance here are expected to have similar form factors, with variation as $1/(-q)^{2\nu}$. The dependence on $\vec{q}'$ enters via eq. (2). The amplitude for the nucleon to have an initial momentum $\vec{p}$ is simply the nuclear wave function, governed by the large nuclear size. The nuclear variation with $\vec{q}'$ is more significant than the variation in the baryon form factors. Thus eq. (4) is the only significant condition. Note: color transparency will hold *only* if the sum over intermediate states is restricted to states with excitation energy much less than the energy E of the outgoing particle.

It is interesting to compare our time-independent approach with the time dependent approach of Mueller [1]. The requirements for closure to be valid are essentially the same as those for the escape time to be sufficiently small. This can be understood from the expansion:

$$J(q)|N\rangle = \sum_{N^*} \exp(-iE_N^* t) C_{N^*}(q)|N^*\rangle \ . \tag{5}$$

All the states in the sum have approximately the same energy since, as seen above, only a few low-lying states will contribute. The time dependence is then an overall phase factor, $J|N\rangle$ propagates without change and the "small" system escapes. The replacement of all the energies E_N^* by a common value is simply the closure approximation used to obtain eq. (3). Indeed our result can be rewritten to look like Mueller's. Starting with eq. 4, writing $M_{N^*} = M_0 + \delta M$, and neglecting second order terms in δM we have $M_0 \delta M R_A/(\sqrt{6}E) \ll 1$. The M_0/E is the relativistic time dilation factor, $1/\delta M$ can be identified with the quark transit time defined by Mueller and R_A with the escape time.

The next step is to examine the above ideas using a simple model and to estimate how large E must be for color transparency. For the simple model we take the $|N^*\rangle$'s to be harmonic oscillator states with radius parameter r_0. The small state, $J(q)|N\rangle$, is assumed to be a Gaussian internal state times a center-of-mass wave function $f(\vec{q}')$, recall eq. (2). Explicitly

$$J(q)|N\rangle = |r_s\rangle f(\vec{q}') \text{ with } \langle r|r_s\rangle = N_s/r_s^{3/2} \exp\left(-(r/r_s)^2/2\right) \ , \tag{6}$$

where N_s is the normalization factor. Thus the outgoing particle is a two- fermion system (a meson), with $\vec{r}$ as the separation between fermions. We take V to be product of a center-of-mass potential and a relative potential

$$V = V_{\rm cm} V_{\rm rel} \text{ with } V_{\rm rel} = r^2 \ . \tag{7}$$

The motivation for the r^2 interaction is that the polarization effect of the color electric dipole term $\sim \vec{r}$ leads to disallowed color-non-singlet states. Hence second-order color effects (or old-fashioned meson exchange) effects are required.

The use of the above equations in eq. (1) leads to:

$$\mathcal{M} = \langle 0|r_s\rangle f(\vec{q}) + \int \frac{d^3q' \langle 0|r^2|0\rangle\langle 0|r_s\rangle V_{\rm cm}(\vec{q}-\vec{q}')f(\vec{q}')}{E-(M_0^2+\vec{q}'^2)^{1/2}+i\epsilon} + \int \frac{d^3q' \langle 0|r^2|1\rangle\langle 1|r_s\rangle V_{\rm cm}(\vec{q}-\vec{q}')f(\vec{q}')}{E-([M_0+2\hbar\omega]^2+\vec{q}'^2)^{1/2}+i\epsilon} \tag{8}$$

where the states $\langle 0|$ and $\langle 1|$ are the first two oscillator model states, with energy difference $2\hbar\omega$. That the sum stops at two terms is a consequence of the oscillator model; in general there is an infinite sum. If we assume closure (neglect $2\hbar\omega$) then a cancellation between the last two terms in eq. (8) causes the sum to be of order r_s^2/r_0^2 with respect to the first term.

To see this cancellation explicitly, note that if closure is valid the sum over $|N^*><N^*|$, namely:

$$\langle 0|r^2|0\rangle\langle 0|r_s\rangle + \langle 0|r^2|1\rangle\langle 1|r_s\rangle = \langle 0|r^2|r_s\rangle \ , \tag{9}$$

factors out of the integrals. The equality follows from the completeness of the N^* states or by computation: The term $\langle 0|r^2|r_s\rangle$ is given by $\langle 0|r^2|r_s\rangle = 3C_0(r_0 r_s)^2/(r_s^2+r_0^2)$ where $C_0 = \langle 0|r_s\rangle$. We also have: $\langle 0|r^2|0\rangle = 3r_0^2/2$, $\langle 0|r^2|1\rangle = -(3/2)^{1/2}r_0^2$, and $\langle 1|r_s\rangle = (3/2)^{1/2}(r_0^2 - r_s^2)/(r_0^2 + r_s^2)C_0$. This result is the explicit verification of eq. (9). In the limit of small r_s, which is relevant for color transparency, the two terms on the left hand side individually fall off more slowly than the sum; a cancellation takes place. *Thus if closure holds, we see explicitly that the sum of the last terms in eq. (8) goes as* r_s^2, and hence color transparency is obtained.

In this example, the r^2 nature of V that restricts the sum to low-lying baryon excited states. This keeps the energy low $(2\hbar\omega)$. If one used an excitation energy characterized by $1/r_s$, then color transparency would not hold.

Next we turn to a numerical evaluation of the condition necessary to obtain color transparency. The poles in the integrals in eq. (8) occur at different values of $\vec{q}'$. Thus to get the two terms to cancel we need (consider for example the imaginary part of the integrals) $V_{cm}(\vec{q}-\vec{q}_0') = V_{cm}(\vec{q}-\vec{q}_1')$ and $f(\vec{q}_0') = f(\vec{q}_1')$ where $\vec{q}_0'$ and $\vec{q}_1'$ are the locations of the poles in the two terms. Thus color transparency depends on how quickly V_{cm} and $f(\vec{q}')$ vary as functions of momentum. The potential V_{cm} can be taken as the nuclear optical potential whose shape follows the nuclear form factor, hence the factor of R_A in eq. (4). For an optical potential $R_A = 5.0A^{1/3}$, so that the condition of eq. 4 becomes $E \gg (M_{N^*}^2 - M_0^2)A^{(1/3)}$, with all energies expressed in units of GeV. Now consider the value of the mass M_N^*. For a baryon final state the first excitation, that is not just a spin flip, is the Roper resonance at 1.44 GeV and is thus the lower limit for M_{N^*}. This leads to an estimate of $E \gg 1.2A^{(1/3)}$ or $E \gg 3.6GeV$ for ^{27}Al. This puts an absolute lower limit on the value of E at which color transparency can be expected.

There are variety of reasons for which the limit can be higher then the above estimate. For example the $2\hbar\omega$ baryons states are spread form 1.44 GeV to about 2.0 GeV and thus an average is about 1.7 GeV leading to an estimate of $E \gg 2.0A^{(1/3)}$ or $E \gg 6GeV$ for ^{27}Al. Another feature to note is the oscillator nature of the "baryon" states. If a linear confining potential were used the r^2 interaction acting on the baryon ground state would lead to many excited states, some with a high excitation energy. However, the overlap with truly high energy states is expected to be small. This assumption is crucial for color transparency. Taking M_N^* to be the mass of the lowest $4\hbar\omega$ (G_{17}) to be about 2 GeV [9], and estimating a 40% reduction in the matrix element of the r^2 operator gives $E \gg 1.9A^{1/3}$ GeV, which is again about 6 GeV for ^{27}Al.

Next consider the influence of terms of higher order in the interaction V. A brief examination of a few of the terms, e.g. the second-order term, shows that the relevant values of the arguments of $V(\vec{q},\vec{q}')$ are those for which the $\vec{q}^2 - \vec{q}'^2$ is $2\hbar\omega$ times the average mass of the two N^* states involved. Since this average energy increases with the order in the perturbation we expect that higher orders will require more stringent conditions on E. The rate of convergence of the perturbation expansion is governed by the product of the depth of the optical potential V_0 times R_A. Using the pp 25 millibarn reaction cross-section gives $V_0R_A = 0.6$. So only two or three terms are necessary, and the estimates of the above paragraph will not increase much.

The net result of these considerations is that $E \gg 2A^{1/3}$GeV which for ^{27}Al implies $E \gg 6$ GeV. In the above discussion E is the energy of the outgoing nucleon. For the $(p,2p)$ experiment of ref. 7 the detectors are set for 90° proton-proton elastic scattering in the center-of-mass frame. Thus $E = 1/2(M + E_L) \approx E_L/2$, and the condition on the lab energy becomes

$$E_L \gg 12\text{GeV} \,. \tag{10}$$

For an $(e,e'p)$ experiment $E = E(\vec{q})$ where $\vec{q}$ is the three-momentum transfer to the nucleon. Thus to see color transparency in the $(e,e'p)$ reaction one needs $q \gg 6$GeV.

It is interesting to examine the implications of our conditions for the analysis of the experiment of Carroll *et al.* [7]. The experiment is performed at laboratory beam momenta of 6, 10 and 12 GeV/c, so the condition of eq. (10) is not quite satisfied. To estimate the size of the correction to closure, parametrize the nuclear form factor as a spherical Bessel function (Fourier transform of a uniform spherical density). For a lab momentum $p_L = 6$ GeV/c, $\delta qR = x = 1.28$ and $j_0(x) = .75$ and at $p_l = 12$GeV/c $j_0(x) = .93$. The 18% difference is about the same size as the oscillations observed in the "oscillating color transparency" of ref. 7. Thus the corrections to closure may be as significant as the charm threshold [4] or Lansdhoff [5] effects mentioned elsewhere. For a 3.8 GeV/c momentum transfer $(e,e'p)$ experiment, $x = 1.02$ and $j_0(x) = .84$, so hints of color transparency could also show up at that momentum transfer.

So far we have considered distortions only on the outgoing particle. Distortions on the incoming particle can be calculated by a similar technique and will tend to have the same effect as for the outgoing particle.

QCD predicts [1,2] that the nuclear medium becomes transparent for large enough energies and momentum transfers. We find an absolute lower limit of $E \gg 1.2A^{(1/3)}$. The higher limits presented in this work are influenced by the specific model we use and are therefore rough guidelines rather than rigorous constraints. Nevertheless, our finding is that the effects of color transparency can indeed be measured, provided experiments are designed so that the specific condition of eq. (4) holds.

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