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Pion photoproduction on the nucleon at threshold

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Abstract

Electric dipole amplitudes of pion photoproduction on the nucleon at threshold have been calculated in the framework of the chiral bag model. Our results are in good agreement with the existing experimental data.

(submitted for publication)

I. INTRODUCTION

Recently, considerable interest has been shown in the neutral pion photoproduction at threshold on the nucleon in connection with the low-energy theorem derived by the current algebra and the partially conserved axial-vector current (PCAC) hypotheses.[1-4]

Although charged-pion photoproduction amplitude at threshold can be well predicted up to the first order in $\mu = m_\pi/M$ (the ratio of pion mass to nucleon mass) expansion by the low-energy theorem, the neutral pion case is reported as an example of wrong prediction of the low-energy theorem.[5] As is suggested,[4] it may be important for π^0 photoproduction to take into account the N^* resonance which is apparently dominant at threshold because the Born amplitude vanishes in the limit of $\mu \to 0$.

All previously existing measurements have been carried out by using bremsstrahlung photons with end-point energies exceeding the reaction threshold by more than 15 MeV, and then the slope of the cross section at threshold was extracted by averaging and extrapolation procedures. Therefore, the results contain some uncertainties and are not fully reliable.

In this situation, an absolute measurement of π^0 photoproduction on the proton has been done near threshold using a tagged annihilation photon beam at Saclay.[5] And the electric dipole amplitude $E_{0^+}(p\pi^0) = (-0.5 \pm 0.3) \times 10^{-3}\, m_\pi^{-1}$ has been reported as the experimental value. This value is. however, extracted by subtracting the theoretical values associated with the final-state interaction estimated on the mass shell. Since it is not well justified that the off-mass-shell effects can safely be ignored, for examination of theory it is better to use the electric dipole amplitude before such subtraction.

II. INTERACTION LAGRANGIAN

We report here our theoretical results obtained on the chiral bag model which is quite successful for the charged pion photoproduction in the Δ resonance region.[6] Our calculation is based on the previous one.[6] Our current task is to evaluate the Feynman diagrams shown in Fig. 1. First of all, one needs to derive the interaction Lagrangians.

Starting from the chiral-invariant Lagrangian with σ field,[7] one can derive the effective Lagrangian of KE model[8] by the chiral transformation. In this model, the pion-quark interaction is given in the pseudovector coupling as

$$\mathcal{L}_{\pi qq} = \frac{1}{2f} \sum_{a} \overline{q}_a \gamma^\mu \gamma_5 \vec{\tau} \cdot \partial_\mu \vec{\pi} q_a \theta_v , \qquad (1)$$

where θ_v is the volume step function and f is the pion decay constant.

The photon-quark coupling is generated by introducing the minimal substitution $\partial_\mu \rightarrow \partial_\mu + ieA_\mu$ as

$$\mathcal{L}_{\gamma qq} = -e_q \overline{q}_a \gamma^\mu A_\mu q_a , \qquad (2)$$

$$\mathcal{L}_{\gamma\pi\pi} = ie[\pi^+ \partial^\mu \pi - \partial^\mu \pi^+ \pi] A_\mu , \qquad (3)$$

$$\mathcal{L}_{\gamma\pi qq} = \frac{ie}{2f} \overline{q}_a \gamma^\mu A_\mu \gamma_5 (\tau^- \pi^+ - \tau^+ \pi^-) q_a , \qquad (4)$$

where e_q and e are quark and pion charges, respectively.

The renormalized πNN and γNN vertex functions can be obtained by evaluating the Feynman diagrams given in Figs. 1 and 2. They are functions of the pion and photon energies, respectively. It should be noticed that the pions in the γNN vertex play important roles in describing the pion photoproduction.[6] Particularly, the renormalized pion decay constant $f = 76$ MeV can be derived from the value of the renormalized πNN vertex function at $k_\pi = 0$.

The antiquark contributions are derived with the following $\gamma q\bar{q}$ and $\pi\bar{q}q$ interaction Lagrangians in momentum space:

$$\mathcal{L}_{\gamma qq}(k_\gamma) = -f^{(1)}(k_\gamma)\, b'^{+}_{\alpha} e_q(\vec{\sigma}\cdot\vec{\epsilon}) b_\alpha 2\pi\delta(\omega_{\bar{q}} - \omega_q - \omega_\gamma) \ , \tag{5}$$

$$\mathcal{L}_{\pi qq}(k_\pi) = -i f^{(2)}(k_\pi)\, b^{+}_{\alpha}\, \vec{\tau}\cdot\vec{\pi}\, b'_\alpha\, 2\pi\delta(\omega_q - \omega_{\bar{q}} + \omega_\pi) \quad , \tag{6}$$

where $b'^{+}(b^{+})$ and $b'(b)$ are antiquark (quark) creation and annihilation operators, ω_q and $\omega_{\bar{q}}$ are the quark and antiquark energies and

$$f^{(1)}(k_\gamma) = \omega_\pi \left[K_0\left(k_\gamma;\, j_0^2 - \frac{1}{3} j_1^2\right) + \frac{2}{3} K_2(k_\gamma; j_1^2)\right] \ , \tag{7}$$

$$f^{(2)}(k_\pi) = \frac{1}{2f} K_0(k_\pi;\, j_0^2 + j_1^2) \quad , \tag{8}$$

with

$$K_\ell(k; F) = \omega[2R^3(\omega-1)j_0^2(\omega)]^{-1} \int_0^R d\gamma\gamma^2 j_\ell(k\gamma) F\left(\frac{\omega}{R}\gamma\right) \ . \tag{9}$$

Here $\omega = 2.043$, R is the bag radius and j_ℓ is the spherical Bessel function.

The vector mesons can be introduced by a local gauge transformation in the chiral bag model. The ρ meson is coupled to SU(2) isospin and the ω meson is coupled to U(1) baryon number. The vector meson coupling Lagrangian can be obtained by the minimal coupling

$$\partial_\mu \to \partial_\mu + \frac{1}{2} g_\rho (\vec{\tau}\cdot\vec{\rho}_\mu + \omega_\mu) \tag{10}$$

as

$$\mathcal{L}_{vqq} = \frac{1}{2} g_\rho\, \bar{q}_a(\omega_\mu + i\vec{\tau}\cdot\vec{\rho}_\mu)\gamma^\mu q_a \theta_v \quad , \tag{11}$$

$$\mathcal{L}_{\omega\rho\pi} = -\frac{3e}{8\pi^2 f}\, g_\rho^2\, \epsilon_{\mu\nu\alpha\beta}\, \partial^\mu\omega^\nu \rho^\alpha \cdot \partial^\beta \pi \ . \tag{12}$$

The coupling of vector mesons to the photon is given by the replacements, $\rho_\mu \rightarrow \rho_\mu + (e/g_v)A_\mu$ and $\omega_\mu \rightarrow \omega_\mu + (e/g_v)A_\mu$, as

$$\mathcal{L}_{\gamma\pi\rho} = -\frac{g_v}{8\pi^2 f}\,\epsilon_{\mu\nu\alpha\beta}\, A^\mu k_\gamma^\nu (\partial^\alpha \pi)\cdot \rho^\beta \ , \tag{13}$$

$$\mathcal{L}_{\gamma\pi\omega} = \frac{g_v}{8\pi^2 f}\,\epsilon_{\mu\nu\alpha\beta}\, A^\mu k_\gamma^\nu (\partial^\alpha \pi)\omega^\beta \quad . \tag{14}$$

With these Lagrangians, one can evaluate the scattering amplitude for the (γ, π) reaction on the nucleon at threshold.

III. RESULTS AND DISCUSSION

Our results are listed in Table I. The seagull term, diagram (1a), does not contribute to the neutral pion photoproduction because of a vanishing charge of the pion, while it makes a dominant contribution to the charged pion photoproduction. The second row in Table I are contributions of the diagrams (1b) and (1c) and calculated with the wave functions of $(1s_{1/2})^2\,(1p_{1/2})^1$ states which have a negative parity.[9] The contributions from the diagrams, (1d) and (1e), with antiquarks are given in the third row of Table I. We used the wave function given in Ref. 6 for an antiquark. These contributions are conclusive for the neutral pion photoproduction to which the seagull term makes null contribution. As is seen in Table I, the vector mesons are also found to be important for the π^0 photoproduction.

The sum of the contributions from N^*, Δ^*, antiquarks and vector mesons to the $\gamma p \rightarrow p\pi^0$ amplitude corresponds to the value predicted by the low-energy theorem. Our result, $-2.84 \times 10^{-3}\ \mathrm{m}_\pi^{-1}$, seems to be slightly too large compared with the predictions of the low-energy theorem, $-2.5 \times 10^{-3}\ \mathrm{m}_\pi^{-1}$. This discrepancy might be due to the model wave functions used here for N^* and Δ^*.[9]

The essential point to be stressed in the analysis of pion photoproduction is a treatment of pion rescattering. Particularly, in the case of π^0 photoproduction, it is pointed out that one cannot ignore the second-order process of a charge exchange scattering on the nucleon after the charged pion photoproduction. This is motivated on the fact that the $\pi^{\pm}$ photoproduction amplitudes are one order of magnitude larger than the π^0 photoproduction amplitudes. When the off-mass-shell effects are ignored, the dipole amplitude at threshold is expressed as[10]

$$\mathcal{E}_{0+}^{(p\pi^0)} = E_{0+}^{(p\pi^0)} + iq_{\pi^+} E_{0+}^{(n\pi^+)} a^{(n\pi^+ \to p\pi^0)} , \tag{15}$$

where $a^{(n\pi^+ \to p\pi^0)}$ is the pion charge exchange scattering length. The recent experiment[5] gives the result

$$E_{0+}^{(p\pi^0)} = (-0.5\pm0.3) \times 10^{-3}\, m_\pi^{-1} \quad , \tag{16}$$

which implies[13]

$$\mathcal{E}_{0+}^{(p\pi^0)} = (-1.45\pm0.3) \times 10^{-3}\, m_\pi^{-1} \ . \tag{17}$$

Of course, there is no reason to ignore the off-mass-shell effects. In the conventional calculation of the processes, one should be very careful to avoid double countings based on insufficient separation of the pions photoproduced in the intermediate state from those participating in renormalization of the πNN and γNN coupling constants. Therefore, as mentioned before, it might be better to calculate the quantity $\mathcal{E}_{0+}^{(p\pi^0)}$ instead of $E_{0+}^{(p\pi^0)}$ and to compare it with the experimental value given above.

In our present model, the rescattering processes are clearly described as the pion photoproduced on a quark is rescattered by another quark in the bag. The contributions of these processes are given in the fourth row of Table I. It is obvious that the pion rescatterings are important. The total values are shown in the sixth row. Our results are in good agreement with the data for the charged pion photoproduction, while there is a discrepancy for the π^0 case. In all our calculations, we have used the renormalized pion decay constant $f = 76$ MeV and the bag radius $R = 0.8$ fm. One can easily find that our values given in each row in Table I satisfy the relation

$$f_{0^+}^{(p\pi^0)} - f_{0^+}^{(n\pi^0)} = \frac{1}{\sqrt{2}} \left(f_{0^+}^{(p\pi^-)} + f_{o^+}^{(n\pi^+)} \right) . \tag{18}$$

IV. CONCLUSION

It has been shown that the chiral bag model can well explain the electric dipole amplitudes of pion photoproduction at threshold. We also argue that better agreements can be obtained by adjusting the value of pion decay constant f. For $f =$ 78 MeV, the results are $\mathcal{E}_{0^+}^{(n\pi^+)} = 28.38$, $\mathcal{E}_{0^+}^{(p\pi^-)} = -31.46$, $\mathcal{E}_{0^+}^{(p\pi^0)} = -1.95$ and $\mathcal{E}_{0^+}^{(n\pi^0)} =$ 0.23 in unit of 10^{-3} m_π^{-1}.

The magnetic dipole amplitudes also provide sensitive information on the nucleon structure and interactions. The recent experimental data of the angular distribution for $\gamma p \to p\pi^0$ near threshold[5] are given by the expression $A + B\cos\theta + C\cos^2\theta$. Although they contain large error bars, those data can also be used to examine the theory. Our results will be published elsewhere.

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Table I. Values of the electric dipole amplitudes E_{0+}. The bag radius $R = 0.8$ fm is used. All values are in units of $10^{-3}\ m_\pi^{-1}$.

	(γ,π^+)	(γ,π^-)	$\gamma p \to p\pi^0$	$\gamma n \to n\pi^0$
Seagull term	28.73	−28.73		
N^*, Δ^*	−0.98	−0.24	−1.22	−0.36
antiquark	−0.55	−1.31	−1.84	−0.52
ρ,ω	−0.04	−0.04	0.22	0.28
rescattering	2.07	−2.07	0.88	0.88
Total	29.23	−32.39	−1.96	0.28
Experiment	28.3±0.5[a]	−31.9±5.0[b]	−1.45±0.3[c]	unknown

[a]reference 11.
[b]reference 12.
[c]reference 5.

Figure captions

1. Feynman diagrams for the pion photoproduction. (1a) is the Seagull term. The intermediate states in (1b) and (1c) contain N^* and Δ^* in negative parity. The diagrams (1d) and (1e) show the antiquark propagation. The vector meson contributions are given in the diagram (1f). The last diagram (1g) shows the rescattering process.
2. The πNN coupling.
3. The γNN vertex.

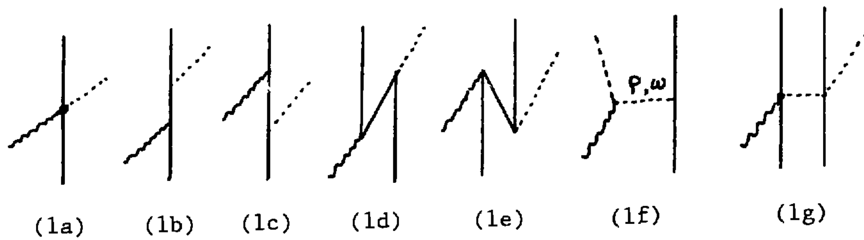

Fig. 1

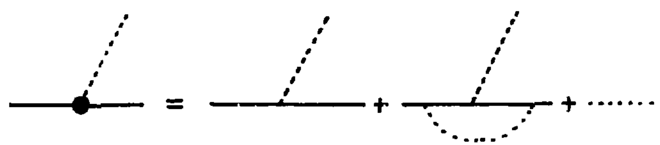

Fig. 2

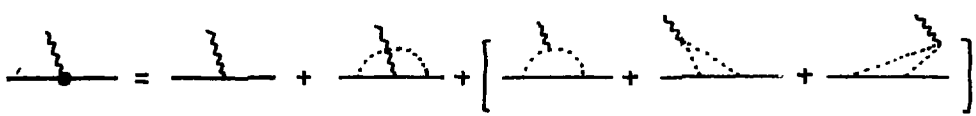

Fig. 3