Proton Induced Nucleon Knockout from $^{40}$Ca in the Dirac Impulse Approximation

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Abstract

The (p,2p) reaction on $^{40}$Ca at incident proton energies of 200 and 300 MeV is examined within a Dirac distorted wave impulse approximation. The formalism is similar to that developed in ref. [1] except that the relativistic Love-Franey t-matrix is evaluated at the nucleon-nucleon laboratory energy (as defined within the plane wave approximation), rather than the nucleon-nucleus laboratory energy, as in ref. [1]. Particular attention is paid to the sensitivity of the calculated cross sections and analyzing powers to the properties of the bound states employed. It is found that the analyzing powers depend very little on the bound state properties, while the cross sections depend significantly only on the rms radii of the bound state wave functions. A major success of the model is its ability to fit the cross section data over a particular range of momentum transfers at both 200 and 300 MeV with the same bound state potential. Outside this momentum transfer range, the predicted cross sections are too low. The calculated analyzing powers agree well with the data at 200 MeV, but disagree with the data at 300 MeV. Values for the rms radii of the $1D_{3/2}$ and $1D_{5/2}$ states in $^{40}$Ca are derived from the requirement that the peak positions of the calculated cross sections at 300 MeV agree with the empirical peak positions. Some preliminary results are also reported for neutron knockout from $^{40}$Ca at 150 MeV.
1 Introduction

Proton knockout reactions have long been of interest as a means of studying the momentum distributions of single nucleon overlap functions in nuclei (see ref. [2] for a comprehensive review). Most of the existing studies of proton knockout are non-relativistic studies based on the zero-range approximation and are only able to achieve limited agreement with the available data. Recently, a relativistic, fully finite-range DWIA formalism was developed for these reactions [1] with the idea of testing the single nucleon knockout mechanism with as few approximations as possible. Initial results were presented for proton knockout from $^{40}$Ca at an incident proton energy of 300 MeV in the lab. These results confirmed earlier zero range calculations [3] which obtained analyzing powers significantly different from those obtained in non-relativistic calculations. The better agreement with the experimental analyzing powers achieved in the relativistic calculations, as compared with the non-relativistic ones, suggests that the problem is more appropriately treated using the Dirac equation than the Schrodinger equation.

Despite this improvement in the analyzing powers, the results of the relativistic, fully finite range calculations of ref. [1] did not agree well with the data. In ref. [1] an attempt was made to study the dependence of the results on the details of the relativistic distortion potentials and the bound state wave functions (single nucleon overlap functions) employed in the calculations. It was found that neither the cross sections nor the analyzing powers are sensitive to the distortion potentials, provided these potentials yield a good description of the elastic nucleon-nucleus data. On the other hand, modification of the bound state wave functions did yield substantial changes in the results obtained. In particular, it was found that an increase in the rms radius of the bound state wave function increases the normalization of the cross section and shifts the peak in the cross section to higher values of the energy difference $E_1 - E_2$ between the outgoing protons.

In this paper, we continue the analysis initiated in ref. [1] with particular emphasis on the sensitivity of the results to the bound state wave functions used. Results are presented not only for 300 MeV protons but also 200 MeV protons incident on $^{40}$Ca, for which data is available at a variety of angle pairs. We have also examined neutron knockout from $^{40}$Ca and present here some preliminary results for the (p,np) cross sections and analyzing powers with a $^{40}$Ca target.

In section 2 we describe the relativistic model that is employed and summarize the various inputs to the calculations. Since the model has been discussed extensively in ref. [1], only a brief review, which emphasizes differences with the previous approach, is presented here. For a more detailed description, the reader should consult the earlier reference.

Section 3 contains our results for the (p,2p) reaction on $^{40}$Ca at an incident lab energy of 300 MeV. In agreement with our earlier results [1], we find that the cross
sections at this energy are sensitive primarily to the rms radius of the bound state wave function used. By adjusting the rms radius accordingly, the peak in the calculated cross sections as a function of \( E_1 - E_2 \) can be brought into accordance with the peak in the data. The analyzing powers are, by contrast, much less sensitive to the bound state rms radius; the major effect of variations in the bound state is just to scale the analyzing powers up and down.

Most of the general trends manifested in the 300 MeV results characterize the 200 MeV results, as well, which are presented in section 4. In particular, the cross sections are strongly sensitive to the rms radius of the bound state. A pleasing feature of our results is that with a single bound state, we are able to reproduce much of the data at both 200 and 300 MeV. Other data, particularly the 200 MeV data at large angles (i.e. the \((47^\circ, -47^\circ)\) and \((54^\circ, -54^\circ)\) data sets) and the 300 MeV analyzing powers, remain difficult to reproduce. Our predictions for the \((p, np)\) reaction on \(^{40}\text{Ca}\) are presented in section 5, while section 6 contains a summary and conclusions.

2 (\(p, 2p\)) in the Dirac Impulse Approximation

Within the Dirac impulse approximation, the \((p, 2p)\) reaction is viewed as a single nucleon knockout process, the matrix element of which consists of a nucleon-nucleon (NN) \(t\)-matrix sandwiched between a relativistic bound state and distorted wave in the initial state and two distorted waves in the final state. In position space, the direct part of this matrix element has the form

\[
<F | t_{NN} | I>_d = \int \int d^4x d^4x' \bar{\Psi}^{\mu_1}(k_1, x) \Psi^{\mu_2}(k_2, x') t_{NN}(s, x - x') \Psi^{\mu_1}(k_I, x) \Psi^M_B(E_B, x')
\]

where \(\mu_1, \mu_2\) and \(k_1, k_2\) are the spin projections and momenta of the outgoing protons; \(\mu_I\) and \(k_I\) are the spin projection and momentum of the incoming proton; and \(M_B\) and \(E_B\) are the angular momentum projection and separation energy of the single nucleon overlap function (bound state wave function). The complete matrix element consists of this matrix element minus an exchange term arising from the requirement that the wave function for the two outgoing protons be antisymmetric.

As a model for the NN \(t\)-matrix, we employ a relativistic Love-Franey form from ref. [4], in which the Lorentz invariant coefficients are parameterized in momentum space as series of phenomenological meson exchanges modified by form factors. The meson masses are assumed to be independent of energy and are chosen in agreement with dynamical one boson exchange models of the NN interaction. The coupling strengths and the form factor cutoffs, on the other hand, are energy dependent and fit to the
elastic NN data. After Fourier transforming to position space, one obtains

$$t_{NN}(s, |\vec{x} - \vec{x}'|) = \sum_{\alpha} t_{\alpha}(s, |\vec{x} - \vec{x}'|) \lambda_\alpha^1 \cdot \lambda_\alpha^2,$$  

(2.2)

where

$$Re[t_{\alpha}(s, r)] = \sum_j \frac{g_{\alpha j}^2}{4\pi} \left[ \left( \frac{\Lambda_{\alpha j}^2}{\Lambda_{\alpha j}^2 - m_{\alpha j}^2} \right)^2 \left( \frac{\exp(-\mu_{\alpha j} r)}{r} - \frac{\exp(-\Omega_{\alpha j} r)}{r} \right) - \frac{1}{2} \left( \frac{\Lambda_{\alpha j}^2}{\Lambda_{\alpha j}^2 - m_{\alpha j}^2} \Omega_{\alpha j} \right) \exp(-\Omega_{\alpha j} r) \right]$$  

(2.3)

with

$$\mu_{\alpha j} = (m_{\alpha j}^2 - q_0^2)^{1/2}$$

$$\Omega_{\alpha j} = (\Lambda_{\alpha j}^2 - q_0^2)^{1/2}. \quad (2.4)$$

The imaginary part of $t_{\alpha}(s, r)$ is given by an expression similar to that of eqn. (2.3). The index $\alpha$ labels a particular Lorentz invariant with corresponding Dirac operators $\lambda_{\alpha}$ at the two vertices.

Although the NN t-matrix has been written as a function of the squared energy in the NN centre of momentum (cm) frame, it is actually parameterized in ref. [4] as a function of the NN equivalent laboratory energy. In ref. [1], this energy was simply set equal to the laboratory energy in the nucleon - nucleus system, a choice which is correct only if the struck nucleon is at rest. To arrive at a better choice, we note that in the plane wave limit, the momentum of the struck nucleon is just the negative of the recoil momentum. In that case, the energy at which the t-matrix should be evaluated is given by

$$T_{lab} = -2M + s_{12}/2M \quad (2.5)$$

where $M$ is the proton mass and $s_{12}$ is the squared cm energy in the nucleon-nucleon system. We believe that this energy is the most appropriate energy at which to evaluate the t-matrix, even when distortions are included, and have used it throughout the present work. Since a Love-Franey fit of the t-matrix is only available at particular input energies, it is necessary to interpolate to the energy at which the t-matrix is required. We employ a linear interpolation procedure.

The importance of this change in the t-matrix prescription is indicated in figure 1, which presents results obtained for knockout from the $1D_{3/2}$ state in $^{40}$Ca. Both curves here correspond to 300 MeV incident protons in the nucleon-nucleus laboratory system and to a geometry where the outgoing protons emerge at angles $30^\circ$ and $-55^\circ$ relative to the incident beam direction. The dashed curve was obtained using the prescription of ref. [1], i.e., using the nucleon-nucleus laboratory energy, whereas the solid curve
was obtained using the prescription described above. As can be seen, the two curves differ somewhat, but the differences are not large. Comparisons at other energies and geometries yield similar results, which indicates that the Love-Franey t-matrix is not strongly energy dependent and gives us some confidence that the prescription adopted here is a reasonable one.

The distorted waves in both the incoming and outgoing states have the form,

$$\Psi^\mu(\vec{z}, \vec{k}) = \sum_{JMLm} \frac{1}{r} \left( \frac{f_{LJ}(r)}{-i\vec{\sigma} \cdot \vec{g}_{LJ}(r)} \right) (Lm \frac{1}{2} \mu | JM)Y_{Lm}(\vec{k})Y_{LJ}^M(\vec{r}),$$

(2.6)

where

$$Y_{LJ}^M(\vec{r}) = \sum_{mLMs} (LmL \frac{1}{2}m_s | JM)Y_{Lm}(\vec{r})\chi_{ms}$$

(2.7)

The radial functions here are obtained through solution of the Dirac equation with vector and scalar global optical potentials (GOPs). These optical potentials are symmetric Woods-Saxon forms,

$$V(E, r) = V(E)f(r, R_1, a_1) + iW(E)f(r, R_2, a_2) + iX(E)f'(r, R_3, a_3),$$

(2.8)

with

$$f(r, R, a) = \frac{1}{[1 + \exp((r - R)/a)][1 + \exp(-(r + R)/a)]},$$

(2.9)

where the energy dependent strengths and the geometric parameters, assumed to be energy independent, are fit to the elastic nucleon-nucleus data over a wide range of energies.

Most of the results reported here have been obtained with the potential GOP1 of ref. [1]. This potential is fit to the $^{40}$Ca elastic data over the energy range 21 to 1040 MeV and incorporates surface absorption. For the sake of comparison, we have also obtained some results with the potential GOP2, which is fit over the somewhat reduced range 80 to 300 MeV and contains no surface absorption. The parameters of both potentials are given in table 1 of ref. [1]. Note in that table that there is a small copying error which we would like to correct here. In the first column, line 6, the range parameter $R$ of the real vector potential should be 3.55957, not 3.538693, as reported in the table. This error is a copying error only; the correct radius was used in all of the $(p,2p)$ calculations reported in ref. [1].

The results of ref. [1] indicate that the dependence of both the cross sections and the analyzing powers on the details of the distorting potentials is quite weak. We have confirmed that this is also true at other incident energies and for a variety of outgoing
angle pairs. Consequently, in subsequent sections, we display only those results that have been obtained with GOP1.

The single nucleon overlap function, approximated by a simple bound state wave function, is also obtained through solution of the Dirac equation with vector and scalar distorting potentials. Most of the results obtained in ref. [1] employed a potential with the same geometric parameters as in GOP1 but with potential strengths (at zero energy) adjusted to fit the separation energies of the bound state and its spin-orbit partner. In the present study, we have allowed the geometric parameters to vary as well in an attempt to find a potential which simultaneously fits the separation energies of the 1D_{3/2}, 1D_{5/2}, and 2S_{1/2} states and which also gives a reasonably good description of the (p,2p) data. This task proved to be difficult using the ordinary symmetric Woods-Saxon form given by eqn. (2.9); however, using the modified parabolic form,

\[ f(r, R, a, w) = \frac{1 + w(r/R)^2}{[1 + \exp((r - R)/a)][1 + \exp(-(r + R)/a)]}, \]  

with \( w = -0.161 \), in accordance with the charge distribution as determined in electron scattering measurements, we were able to fit the two 1D state separation energies and to obtain a 2S_{1/2} separation energy of 9.946 MeV, which is reasonably close to the observed separation energy of 11.0 MeV. Unfortunately, the extra freedom provided by the parameter \( w \) in eqn. (2.10) yields only a marginal improvement in the (p,2p) results obtained; in fact, as indicated in the next section, the (p,2p) cross sections are sensitive primarily to the rms radii of the bound states employed and are relatively insensitive to any other parameter.

3 The 300 MeV results and the rms radii of single nucleon overlap functions

In an attempt to elucidate the sensitivity of the model to various input parameters, we have carried out a series of calculations of the (p,2p) reaction on \(^{40}\text{Ca}\) at 300 MeV. The results, reported in this section, all correspond to a coplanar reaction geometry where the two outgoing protons emerge at angles of 30° and 55° on opposite sides of the beam in the laboratory frame. Since the normalization of the cross section data at this energy is arbitrary [5], our discussion of the calculated cross sections will emphasize the peak positions and widths, rather than the cross section magnitudes.

Results for the cross sections and analyzing powers obtained with the parabolic Woods-Saxon bound state potential, eqn. (2.10), are displayed in figures 2a, 2b, and 2c for the 1D_{3/2}, 1D_{5/2}, and 2S_{1/2} bound states, respectively. The geometric parameters for this potential appear on the first line of table 1; the corresponding potential strengths have been fit to the 1D_{3/2} and 1D_{5/2} separation energies which we have taken to be 8.3 MeV and 15.5 MeV respectively. With the empirical cross sections renormalized to
fit approximately the peak strengths of the calculated cross sections, the model results, on the whole, are not too bad. In particular, for the two D-states, the peak positions are correct, although the peak widths are too small. These results can be interpreted in terms of the momentum transferred to the nucleus since in a fixed geometry, particular values of the outgoing proton energy difference correspond to particular values of this momentum transfer. For outgoing angles 30° and -55°, the range in $E_1 - E_2$ over which the calculated and empirical cross sections agree corresponds to momentum transfers in the range $90 \text{ MeV/c} \leq q \leq 140 \text{ MeV/c}$ with $q = 100 \text{ MeV/c}$ at the peak. Outside this range, the model seems to have difficulty, perhaps in the bound state. As will become evident in the next section, this difficulty manifests itself at 200 MeV as well. In contrast with the cross sections, the model predictions for the analyzing powers are rather poor, especially for the two D states.

Table 1

Bound state parameters

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<tr>
<th>State</th>
<th>$a_{vec}$</th>
<th>$a_{sca}$</th>
<th>$r_{vec}$</th>
<th>$r_{sca}$</th>
<th>$w_{vec}$</th>
<th>$w_{sca}$</th>
<th>rms-D$_{3/2}$</th>
<th>rms-D$_{5/2}$</th>
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<tr>
<td>1</td>
<td>0.80</td>
<td>0.83</td>
<td>5.35</td>
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<td>-1.161</td>
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<td>2</td>
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<td>0.65</td>
<td>4.67</td>
<td>4.79</td>
<td>0</td>
<td>0</td>
<td>5.265</td>
<td>4.970</td>
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<td>3</td>
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<td>4.45</td>
<td>0</td>
<td>0</td>
<td>4.897</td>
<td>4.623</td>
</tr>
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<td>0.65</td>
<td>4.00</td>
<td>4.12</td>
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<td>0</td>
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<td>4.29</td>
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<td>0</td>
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<td>4.567</td>
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<td>0</td>
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<tr>
<td>7</td>
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<td>4.33</td>
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<td>4.897</td>
<td>4.623</td>
</tr>
<tr>
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<td>0.59</td>
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<td>4.52</td>
<td>0</td>
<td>0</td>
<td>4.891</td>
<td>4.650</td>
</tr>
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</table>

The sensitivity of the model results to the rms radius of the bound state employed is illustrated in figure 3, where the cross section and analyzing power for the $1D_{3/2}$ state are displayed for several bound state wave functions. The solid curves are identical to those in fig. 2a. The dashed, dotted, and dash-dotted curves were generated using the geometric parameters listed respectively in the second, third, and fourth lines of table 1 and again adjusting the potential strengths to fit the separation energies of the $1D_{3/2}$ state and its spin-orbit partner. Comparison of the four cross section curves shows a pronounced and systematic sensitivity to the bound state rms radius with the peak generally increasing in magnitude and shifting to higher values of the outgoing proton energy difference as the rms radius is increased. Note that the bound states yielding the solid and dotted curves have approximately the same rms radius but different values of the parameter $w$, so that differences in these two curves reflect the influence of the parabolic term in eqn. (2.10). Aside from this minor dependence on $w$, the rms radius is
the only radial parameter associated with the bound states (which are normalized) that significantly affects the calculated cross sections; i.e., different choices of the potential parameters, $r_{MC}$ and $r_{loc}$, which yield the same bound state rms radius also yield nearly identical cross section curves. This is true for the $1D_{5/2}$ state, as well as the $1D_{3/2}$ state. The $1D_{5/2}$ cross sections also vary with the rms radius in the same way as the $1D_{3/2}$ cross sections displayed in fig. 3.

In contrast with the cross sections, the analyzing powers do not exhibit a strong dependence upon the bound state rms radius, merely scaling up and down without changing shape as the rms radius is varied. Nor do the analyzing powers depend significantly on any other parameter associated with the bound states. In one sense, this is a pleasing result in that it provides a fairly unambiguous test of the model. It is rather disconcerting, however, that the calculated results do not lie closer to the data. At present, we have no explanation for the discrepancy between the theoretical and empirical analyzing powers at this energy; however, the difficulty seems to be energy dependent. The model yields considerably better analyzing powers at 200 MeV, as indicated in the next section.

Having established that the positions of the calculated cross section peaks depend sensitively on the rms radii of the bound states used, it is of interest to determine whether the cross sections are sensitive to any other properties of the bound states. In particular, one might expect that the widths of the peaks will depend on the diffuseness of the bound state potential employed. That this is not the case is demonstrated in figure 4, which displays results for the $1D_{3/2}$ state using four bound state wave functions with nearly identical rms radii but generated from potentials with different diffuseness parameters. The particular parameters employed are listed in the last four lines of table 1. Only the cross sections are presented in the figure, as the corresponding analyzing powers are virtually indistinguishable from one another. Comparison of the various curves reveals that, provided that the radial parameters are adjusted to give the same rms radius, variation of the diffuseness parameters has virtually no effect on either the peak position or width and alters the magnitude of the cross section by at most ten percent over the whole parameter range considered. The $1D_{5/2}$ results are similarly insensitive to these diffuseness parameters.

The sensitivity of the cross sections to the bound state rms radii and the lack of sensitivity to any other parameters associated with the bound states, suggests that $(p,2p)$ cross sections are providing an unambiguous means for determining rms radii. More specifically, one can determine the rms radius of a particular bound state, or more accurately a single nucleon overlap function, by requiring that the peak position of the calculated cross section coincide with the the empirical peak position. Using the 300 MeV $^{40}$Ca data, this procedure yields rms radii of about 4.9 fm and 4.6 fm for the $1D_{3/2}$ and $1D_{5/2}$ states respectively.

A significant success of the model at 300 MeV is its ability to fit the peak positions of both the $1D_{3/2}$ and $1D_{5/2}$ cross sections with the same bound state potential.
However, for neither state does the model predict the width of the peak correctly. As demonstrated by the curves in fig. 4, this difficulty does not appear to be associated with the diffuseness parameters of the potential; different parameter choices do not alter the cross sections significantly. Since all other aspects of the bound state wave functions are fixed by requiring that the wave functions be normalized, fit the $1D_{3/2}$ and $1D_{5/2}$ separation energies, and yield the correct positions of the cross section peaks, it is apparent that modification of the bound state wave functions will not improve the widths. We therefore sought to modify the widths by modifying the distorted waves in the region of the nuclear potentials; in particular, by enhancing or suppressing the distorted waves by factors of the order of 5%, as might result from some sort of relativistic Perey mechanism. Such experiments were to no avail, merely altering the cross section normalizations ever so slightly, and without substantially affecting the peak widths.

The discrepancy between the theoretical and empirical cross section widths may not have its origin in the model itself. The detectors which count the outgoing protons have finite angular resolution, so that at each angle all protons that leave the interaction region within a certain angular range are counted, rather than just those emerging at a particular angle. Since the momentum transfer in the reaction is angle dependent, this angle smearing in the detectors could conceivably broaden the empirical cross sections as compared with the theoretical ones. In the 300 MeV TRIUMF experiment where the low energy proton is detected with an angular resolution of $\pm 4^\circ$ and the high energy proton with a resolution of $\pm 1.5^\circ$, the effect could be significant. To investigate this possibility, we recomputed the cross section for the $1D_{5/2}$ state at the center and edges of each angular window (nine curves in all) and then integrated over the windows by means of a two dimensional Simpson's rule, assuming uniform angular acceptances for both detectors. The results are presented in figure 5. Comparing the heavy solid curve, which is the window-integrated cross section, with the middle thin solid curve, which is the cross section at the center of both windows, reveals that smearing the theoretical result over a range of angles does indeed broaden the cross section peak, but not nearly enough to bring it in accord with the data.

4 The 200 MeV results

The results described above have all been carried out for an incident laboratory kinetic energy of 300 MeV. It is of considerable interest to determine how successful the model is at other energies and to what extent characteristics of the model evident at 300 MeV persist at these other energies. With this in mind, we have extended our analysis of the $(p,2p)$ process on $^{40}$Ca to reactions occurring at an incident laboratory kinetic energy of 200 MeV. The existing data at this energy is of poorer quality than the 300 MeV data, but is considerably more extensive, spanning a range of outgoing proton angle pairs, all with coplanar geometry. Because different angle pairs correspond to different momentum transfer ranges, the existence of data for several angle pairs
permits the (p,2p) process to be studied within several different momentum transfer ranges. The 200 MeV data, unlike the 300 MeV data, is also normalized, thereby providing additional constraints on the model.

The calculated cross sections and analyzing powers at 200 MeV exhibit much the same parameter dependence as the 300 MeV results. In particular, for both the 1D_{3/2} and 1D_{5/2} states, the analyzing powers at 200 MeV are remarkably stable with respect to variations in the bound state wave functions, even more stable than those at 300 MeV. The cross sections at 200 MeV, like those at 300 MeV, are relatively insensitive to variations in the diffuseness parameters of the bound state potentials and depend on the radial parameters only through the bound state rms radii.

In figures 6ab, 7ab, and 8ab, we present model results obtained at 200 MeV for the three angle pairs (30°, -30°), (29°, -47°), and (54°, -54°) respectively. The figures labelled a correspond to the 1D_{3/2} state, while those labelled b correspond to the 1D_{5/2} state. In each figure the three cross section curves were generated from three bound state wave functions with different rms radii. These three wave functions are, in fact, the same three used to generate the dashed, dotted, and dash-dotted curves in figure 3. For the two symmetric angle pairs, represented by figs. 6 and 8, the cross sections must be symmetric in the outgoing proton energy difference so that the peak positions cannot depend on any parameters of the model. This is not true for the asymmetric angle pair, however; as seen in figs. 7, the cross section peaks shift to larger values of $E_1 - E_2$ as the rms radii increase, just as the 300 MeV peaks do. In all three cases, the magnitudes of the cross sections increase with increasing rms radii, again mimicking the results at 300 MeV.

Comparing the model results with the data reveals that the bound states which fit the peak positions in the 300 MeV cross sections yield excellent fits of both the cross sections and analyzing powers for the (30°, -30°) angle pair at 200 MeV. This result strongly supports the viability of the model because it demonstrates that the model can successfully reproduce data in different kinematical regions with the same set of parameters. For the other two angle pairs the model results are not quite as good. In both cases, the calculated analyzing powers are reasonable, certainly much better than at 300 MeV. For the (29°, -47°) angle pair, the cross sections generated with the best bound states peak in the right places and have about the right magnitudes but, like the 300 MeV cross sections, are too narrow. For the (54°, -54°) angle pair, on the other hand, the widths seem to be about right, but the cross sections are too small.

The cross section results presented here can be interpreted in the same manner as the 300 MeV results, namely, in terms of the momentum transfer to the nucleus. For the (30°, -30°) angle pair, this momentum transfer lies in the range $90 \text{ MeV/c} \leq q \leq 140 \text{ MeV/c}$, which is precisely the range where we obtained good fits to the cross sections at 300 MeV. As seen in figs. 6, we obtain good fits to the 200 MeV cross sections as well in this range of momentum transfer. For the (54°, -54°) pair, the momentum transfer lies above 140 MeV/c, and we underestimate the cross section over the whole
range of energy transfer. For the $(29^\circ, -47^\circ)$ pair, the range of momentum transfers extends below 90 MeV/c and above 140 MeV/c. In the range between 90 MeV/c and 140 MeV/c, the calculated cross sections agree with the data, while outside this range, the cross sections are again too low.

In an attempt to discover why the cross sections are too low outside a certain band of momentum transfers, we have investigated the relationship between the cross sections and the bound state wave functions. By comparing the momentum transfer dependence of the cross sections with the Fourier transforms of the wave functions, we discovered that the $(p,2p)$ process directly measures the Fourier content of the bound state wave functions. Thus, it is possible to increase the cross sections over any range of momentum transfers by just enhancing the Fourier components of the wave functions over that range. This does not solve the problem, however, because of the constraint that the wave functions be normalized; if we seek to improve the cross sections within the high momentum transfer band, for example, by enhancing the high momentum components of the wave functions, the normalization constraint forces us to reduce the intermediate momentum components of the wave functions, so that the calculations will then be too low in the momentum transfer region where they were previously successful. In other words, the $(p,2p)$ cross section data contains extra strength over certain momentum transfer ranges relative to the bound state wave functions, which we cannot account for at present. It would be very useful if more detailed and more precise experiments could be performed to explore this effect.

We conclude that for both the $1D_{3/2}$ state and the $1D_{5/2}$ state, the success of the model in predicting cross sections depends only on the momentum transferred to the nucleus and is independent of any other kinematical considerations. The model does well around the quasi-elastic peak; away from the quasi-elastic peak, it underestimates the cross sections. By contrast the quality of the calculated analyzing powers seems to be energy dependent. On the whole, the relativistic $(p,2p)$ results at 200 MeV are superior to the non-relativistic results reported in ref. [6].

5 Results for the $(p,np)$ process

Since the NN t-matrix employed here is fit to both pn data and pp data, it can be used to study neutron knockout from $^{40}$Ca, as well as proton knockout. A detailed study of the $(p,np)$ process can provide information concerning the isospin dependence of single nucleon overlap functions which cannot be obtained from a study of the $(p,2p)$ process alone. Recently, some preliminary data has become available for the $(p,np)$ process on $^{40}$Ca at 150 MeV [7]. This is compared in figure 9 with some $(p,np)$ results obtained with the Dirac model using the bound state potential that provides the best description of the $(p,2p)$ data. This bound state is obtained using the potential parameters from the first line in table 1, turning off the Coulomb interaction, and then readjusting the two strengths to get the correct binding energy and spin-orbit splitting.
for the $1D_{3/2}$ wave function, which then has an rms radius of 4.66 fm. The same potentials are used to generate the $2S_{1/2}$ wave function, except that the scalar potential is rescaled slightly to attain agreement with the empirical neutron separation energy (18.5 MeV).

As can be seen, the calculated (p,np) cross sections, although not as good as the (p,2p) cross sections, are not too bad, especially since no attempt has been made here to fit the (p,np) data by readjusting the bound state rms radius. It is particularly interesting that the (p,2p) bound state potential parameters yield the correct peak positions of the (p,np) cross sections. The fact that the calculated cross sections are too large near the peaks may have its origin in several factors. The calculated cross sections assume spectroscopic factors of unity; whereas the actual spectroscopic factors are probably significantly less than unity. Another possibility is that an incident energy of 150 MeV is simply too low for the impulse approximation to be reliable. It is rather remarkable, in fact, that the impulse approximation for (p,2p) is so successful at 200 MeV. In this regard, it would be of considerable interest to obtain (p,np) data at higher energies, where the theoretic foundation for the model is more secure.

6 Conclusions

To recapitulate, we have re-examined the (p,2p) process on $^{40}$Ca at 300 MeV, concentrating particularly on the role of the bound state wave function. Our results indicate that the only property of the bound states which significantly affects the calculated cross sections is the rms radius and that both the magnitudes and the peak positions of the cross sections depend quite sensitively on the rms radii of the states employed. This enabled us to estimate the rms radii of the $1D_{3/2}$ and $1D_{5/2}$ states in $^{40}$Ca by requiring that the positions of the peaks in the calculated and empirical cross sections occur at the same value of the outgoing proton energy difference. We then examined the 200 MeV data and discovered that the bound states which yield the best fits to the cross section data at 300 MeV also yield the best fits to the 200 MeV data. Moreover, the optimal choices for both the $1D_{3/2}$ and $1D_{5/2}$ wave functions are obtained with the same bound state potential parameters. This result strongly supports the viability of the model; it also gives us confidence that the extracted rms radii are sensible ones.

Unfortunately, the model is not without some difficulties. In particular, the calculated cross sections agree with the data only over a limited range of momentum transfers. We have tried various schemes for improving the fit outside this range, without success. The cross sections do not depend significantly on the diffuseness of the bound state potentials and seem to be insensitive to any properties of the distorted wave potentials. Including a quadratic term in the bound state potential increases the peak widths somewhat, but not nearly enough to bring the calculated cross sections in agreement with the data. The calculated analyzing powers are also not entirely satisfactory. While the model reproduces the empirical analyzing powers quite well
at 200 MeV, the theoretical analyzing powers at 300 MeV are not even qualitatively correct. We have no explanation at present for this energy dependence in the quality of the calculated analyzing powers. Unlike the cross section discrepancies, the analyzing power discrepancies do not appear to be correlated with momentum transfer.

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References

Figure Captions

Figure 1: Proton knockout from the $1D_{3/2}$ state at 300 MeV using different NN t-matrix prescriptions: present prescription described in section 2 (solid curve); prescription of ref. ([1]) (dashed curve).

Figure 2a: Proton knockout from the $1D_{3/2}$ state at 300 MeV using bound state parameters from the first line of table 1. Data from ref. ([8]).

Figure 2b: Proton knockout from the $1D_{5/2}$ state at 300 MeV using bound state parameters from the first line of table 1. Data from ref. ([8]).

Figure 2c: Proton knockout from the $2S_{1/2}$ state at 300 MeV using bound state parameters from the first line of table 1. Data from ref. ([8]).

Figure 3: Proton knockout from the $1D_{3/2}$ state at 300 MeV using bound states with different rms radii. The solid, dashed, dotted, and dash-dotted curves were obtained with bound state parameters from the first, second, third, and fourth lines of table 1 respectively. Data from ref. ([8]).

Figure 4: Proton knockout from the $1D_{3/2}$ state at 300 MeV using bound state potentials with different diffuseness parameters. The solid, dashed, dotted, and dash-dotted curves were obtained with bound state parameters from the fifth, sixth, seventh, and eighth lines of table 1 respectively. Data from ref. ([8]).

Figure 5: Proton knockout from the $1D_{5/2}$ state at 300 MeV for different angle pairs spanning the angular widths of the detectors. The thin solid curves, dotted curves, and dash-dotted curves correspond to the choices $\theta_2 = -55^\circ$, $-59^\circ$, and $-51^\circ$ respectively. Within each trio, the three curves correspond to $\theta_1 = 28.5^\circ$, $30^\circ$, and $31.5^\circ$ from right to left. The heavy solid curve is the integrated result, as described in the text. Data from ref. ([8]).

Figure 6a: Proton knockout from the $1D_{3/2}$ state at 200 MeV with the outgoing protons emerging at angles $30^\circ$ and $-30^\circ$. The bound state parameters employed are from the second line (dashed curves), third line (dotted curves), and fourth line (dash-dotted curves) of table 1. Data from ref. ([9]).

Figure 6b: Proton knockout from the $1D_{5/2}$ state at 200 MeV for outgoing angles $30^\circ$ and $-30^\circ$ using the same bound state parameters as in fig. 6a. Data from ref. ([9]).

Figure 7a: Same as fig. 6a for outgoing protons at angles $29^\circ$ and $-47^\circ$.

Figure 7b: Same as fig. 6b for outgoing protons at angles $29^\circ$ and $-47^\circ$.

Figure 8a: Same as fig. 6a for outgoing protons at angles $54^\circ$ and $-54^\circ$.

Figure 8b: Same as fig. 6b for outgoing protons at angles $54^\circ$ and $-54^\circ$.

Figure 9: Neutron knockout from the $1D_{3/2}$ state (left) and $2S_{1/2}$ state (right) at 149.5 MeV using the bound state potential parameters from the first line of table 1 (note that the rms radii for neutron bound states are different from those appearing in the table). Data from ref. ([7]).