A.Ya. Kraftmakher

PARITY NONCONSERVATION IN ZEEMAN ATOMIC TRANSITIONS

PREPRINT 90-54
Institute of Nuclear Physics

A.Ya. Kraftmakher

PARITY NONCONSERVATION IN ZEEMAN ATOMIC TRANSITIONS

PREPRINT 90-54

NOVOSIBIRSK
1990
Parity Nonconservation in Zeeman Atomic Transitions

A.Ya. Kraftmakher

Institute of Nuclear Physics
630090, Novosibirsk, USSR

ABSTRACT

We consider the abilities to observe the parity violation at the radiofrequency transitions between the hyperfine and Zeeman terms of the atomic levels. The ratio of the P-odd E1-amplitudes to the M1-amplitudes for the Zeeman transitions appeared to be significantly higher, than for the hyperfine ones. Moreover, the E1-amplitudes for the Zeeman transitions of heavy atoms in weak magnetic fields are larger, than for the light atoms hyperfine transitions at the same wavelength.
It is known, that the P-odd interaction in the atoms contains the part, which depends of the spin of the nucleus. In heavy atoms this part is determined by the interaction of the electrons with the anapole moment of the nucleus, caused by the P-odd nuclear forces [1, 2]. This interaction causes the dependence of P-odd atomic properties of the total angular momentum [3, 4]. Recently such dependence was observed in the precise measurement of P-odd E1-amplitudes of 6s, \(F-7s, F'\) transitions in caesium [5]. The uncertainty of measured anapole moment of the \(^{133}\text{Cs}\) nucleus was more than 50%. The spin-dependent effect was observed on the background of the not spin-dependent E1-amplitude, caused by the weak interaction of electron axial and nucleon vector currents.

Another way is to measure the E1-amplitude of the radio-frequency transitions between the hyperfine terms of the same atomic level [6]. The «background» not spin-dependent E1-amplitude is absent there. In Ref. [7] such experiment was considered in details for the hydrogen, potassium and caesium atoms. For caesium the E1-amplitude of hyperfine transitions 6s, \(F-6s, F'\) is about 50 times higher, than for potassium, but the last has the much less transition frequency. This is an important advantage because no resonators are necessary for the experiment. The work [7] also considered the abilities of the experiment in a strong constant magnetic field, when the background M1-transitions are suppressed, and E1—not.

In 1987 V.V. Flambaum supposed to put the atoms in the weak constant magnetic field and to measure the P-odd E1-amplitude of
the transitions between the Zeeman terms: $F$, $F_z - F$, $F_z\pm 1$. Here the transition frequency is proportional to the external magnetic field and can be chosen arbitrary small. Then the background alternating magnetic field, which is caused by the alternating electric field and is proportional to its frequency will be suppressed. Moreover, we can choose the external magnetic field to make the experiment with the different atoms, including the heavy ones, at the same transition frequency. The heavy atoms are important, because they have the P-odd E1-amplitudes much larger, than the light ones. We must note, that the P-odd E1-amplitudes of Zeeman transitions in weak magnetic fields are suppressed by the factor of $\hbar \omega/\Delta E_{hf}$, where $\omega$ is the transition frequency and $\Delta E_{hf}$—the hyperfine splitting. However, the E1-amplitude increases with the nuclear charge as $\langle \tilde{d} \rangle \propto Z^2 A^{2/3} R_a$, where $A$—the atomic height, $R_a$—the atomic relativistic factor (look, for example, [8]), while the hyperfine splitting $\Delta E_{hf} \propto Z$. So the E1-amplitudes of Zeeman transitions increase faster, than $Z A^{2/3}$ when we use the heavy atoms in weak magnetic fields at fixed transition frequency. For example, the Zeeman E1-amplitude in caesium at the frequency of hyperfine transition of potassium is an order of magnitude higher, than the potassium hyperfine E1-amplitude.

Here we consider the abilities of different experimental schemes (without the external magnetic field, in weak field, in strong field) to found the most favourable atomic levels and experimental conditions. We calculate the P-odd E1-amplitudes $\langle \gamma | \tilde{d} | \gamma' \rangle$ and the ratios $R = \langle \tilde{d} \rangle / \omega \langle \tilde{\mu} \rangle$, where $\tilde{\mu}$ is the electron magnetic moment. This ratios characterize the ratio of P-odd E1-transitions to the background M1-transitions (the background alternating magnetic field $H_a \propto \omega E_a$, $\langle E1 \rangle / \langle M1 \rangle \propto E_a \langle \tilde{d} \rangle / H_a \langle \tilde{\mu} \rangle \propto R$).

We consider the $^{39}$K, $^{41}$K, $^{133}$Cs, $^{205}$Tl atoms. For all of them the electron angular momentum $J = \frac{1}{2}$ and only two values of total atomic angular momentum $F = I = \frac{1}{2}$ ($I$ is the nucleus spin) are possible. In presence of external constant magnetic field the states with fixed $F$ are splitted into the states with fixed projections $M$ onto the external field direction. The total angular momentum is not conserved there and the new states appears:

$$| 1, M \rangle = \alpha_{1, M} | I - 1/2, M \rangle + \beta_{1, M} | I + 1/2, M \rangle,$$

$$| 2, M \rangle = \alpha_{2, M} | I - 1/2, M \rangle + \beta_{2, M} | I + 1/2, M \rangle,$$

$$| 1, M \rangle \xrightarrow{H \to 0} | I - 1/2, M \rangle, \quad | 1, M \rangle \xrightarrow{H \to \infty} | I_z = M - 1/2, J_z = 1/2 \rangle,$$
The new state energies and mixing coefficients $\alpha, \beta$ are calculated for the arbitrary value of external field from the secular equation with the hyperfine interaction and interaction with external field taken together. This equation decomposes itself to the independent second-order equations for each value of $M$. Then

\[ \beta_{2, M} = \alpha_{1, M}, \quad \alpha_{2, M} = -\beta_{1, M}, \quad \alpha_{1, M}^2 + \beta_{1, M}^2 = 1 \quad (2) \]

for each $M$ except $M = \pm (I + 1/2)$, when $|1, M\rangle = 0$, $|2, M\rangle = \pm |I + 1/2, M\rangle$.

The dependence of external field for the $^{133}$Cs 6s$_{1/2}$ hyperfine and Zeeman terms is presented at Fig. 1. In the weak fields the energy difference of hyperfine terms $|1\rangle$ and $|2\rangle$ is constant and equal to $\Delta E_{hf}$, of Zeeman terms $|1, M\rangle$ and $|1, M \pm 1\rangle$, $|2, M\rangle$ and $|2, M \pm 1\rangle$ is proportional to the field. In the strong fields the division of the level structure onto the hyperfine and Zeeman terms has no sense because the states have no fixed total angular moments but the fixed projections of nucleus spin $I_z$ and electron angular momentum $J_z = \pm 1/2$. However we shall call the $|1\rangle - |2\rangle$ transitions the hyperfine ones and $|1, M\rangle - |1, M \pm 1\rangle$, $|2, M\rangle - |2, M \pm 1\rangle$—the Zeeman ones like for the weak field. In the strong fields the hyperfine transition energy increases proportionally to the field and the Zeeman one is constant equal to $\Delta E_{hf}/(2I + 1)$.

The $P$-odd E1-amplitudes in thallium and potassium were calculated semiempirically, as in [4, 6—8]. The E1-amplitude for potassium coincides with the result [7]. For the caesium atom the E1-amplitude was calculated by the relativistic Hartree—Fock method with Brueckner orbitals. Such a calculation for 6s, $F - 7s$, $F'$ transitions in caesium is described in [9]. The result $\langle 6s, F || \mathbf{d} || 6s, F' \rangle = 4.3 \times 10^{-12} \text{ixea}_B$ ($F = 3$, $F' = 4$) appeared to be slightly less, than the semiempirical result [6] $(5.0 \times 10^{-12} \text{ixea}_B)$, where constant $\kappa$ characterizes the value of the anapole moment [1].

For the hyperfine transitions without the external field

\[ R_0 = \frac{\langle 1 || \mathbf{d} || 2 \rangle}{\omega \langle 1 || \mathbf{\tilde{d}} || 2 \rangle} = \left( \frac{I + 1/2}{I(I + 1)} \right)^{1/2} \frac{\langle I - 1/2 || \mathbf{d} || I + 1/2 \rangle}{g\mu_0 \Delta E_{hf}}. \quad (3) \]

Here $g$ is a $g$-factor of the external electron (for caesium and potassium it is $s_{1/2}, g = 2$, for thallium $- p_{1/2}, g = 2/3$). In the weak
fields the E1-amplitudes behave itself just like the transition frequencies (they are proportional to the field for Zeeman transitions and constant for the hyperfine ones). So the \( R \)-parameters for the hyperfine transitions do not depend of the external field while it is weak. For Zeeman transitions

\[
R_{1,M-1,M\pm 1} = (2I+2) \: R_0, \quad R_{2,M-2,M\pm 1} = 2I \: R_0.
\]  

(4)

In the strong fields the hyperfine transitions \( |1, M\rangle - |2, M\rangle \) do not present an interest: \( |1, M\rangle - |2, M\pm 1\rangle \) corresponds to the change of nucleus spin projection \( \Delta I_z = 2 \), the P-odd E1-amplitude is forbidden (the selection rule for it is \( \Delta I_z = 0, \pm 1 \)), \( |1, M\rangle - |2, M\pm 1\rangle \) corresponds to \( \Delta I_z = 0 \), the M1-amplitude is allowed and \( R \propto 1/\omega \) decreases inversely to the field. For \( |1, M\rangle - |2, M\rangle \) transitions the E1-amplitude appears to be independent of the external field at all. It is connected with the anti-symmetry of \( \alpha, \beta \) mixing coefficients for the same \( M \) (relations (2)) and with the imaginarity of the P-odd E1-amplitude:

\[
\langle I + \frac{1}{2}, M | \bar{d} | I - \frac{1}{2}, M \rangle = \langle I - \frac{1}{2}, M | \bar{d} | I + \frac{1}{2}, M \rangle^* =
\]

\[
= - \langle I - \frac{1}{2}, M | \bar{d} | I + \frac{1}{2}, M \rangle.
\]

\[
\langle 1, M | \bar{d} | 2, M \rangle = \alpha_{1,M} \beta_{2,M} \langle I - \frac{1}{2}, M | \bar{d} | I + \frac{1}{2}, M \rangle +
\]

\[
+ \beta_{1,M} \alpha_{2,M} \langle I + \frac{1}{2}, M | \bar{d} | I - \frac{1}{2}, M \rangle =
\]

\[
= \alpha_{1,M} \alpha_{1,M} \langle I - \frac{1}{2}, M | \bar{d} | I + \frac{1}{2}, M \rangle -
\]

\[
- \beta_{1,M} \beta_{1,M} \langle I + \frac{1}{2}, M | \bar{d} | I - \frac{1}{2}, M \rangle =
\]

\[
= (\alpha_{1,M}^2 + \beta_{1,M}^2) \langle I - \frac{1}{2}, M | \bar{d} | I + \frac{1}{2}, M \rangle = \langle I - \frac{1}{2}, M | \bar{d} | I + \frac{1}{2}, M \rangle.
\]

The M1-transitions with \( \Delta I_z = 1 \) are \( \mu_{nuc}/\mu_0 \) times suppressed. In not very strong external fields \( (\Delta E_{hf}/\mu_0) \ll H \ll (\Delta E_{hf}/\mu_{nuc}) \) the M1-amplitudes of \( |1, M\rangle - |2, M\rangle, |1, M\rangle - |1, M\pm 1\rangle, \) and \( |2, M\rangle - |2, M\pm 1\rangle \) transitions are determined by the small mixing
of the states $|1, M\rangle = |I_z = M - 1/2, I_z = +1/2\rangle$ and $|2, M\rangle =
|I_z = M + 1/2, I_z = -1/2\rangle$. This mixing coefficients are inversely proportional to the field. Then the $R_{1,M-2,M}$ parameter appears to be independent of the magnetic field and always equal to $R_0$. For Zeeman transitions in strong field the E1-amplitudes, like the frequencies, rich the constant values comparable with that for the hyperfine transitions, and the $R$ parameters increases proportionally to the field due to the suppression of M1-transitions:

$$R_{1,M-1,M\pm1} = R_{2,M-2,M\pm1} = (I + 1/2)xR_0,$$

where $x = H/H_c$, $H_c = \Delta E_M/2g\mu_0$ is the scale of external magnetic field.

In Table 1 the main data for the considered atoms is presented: the nucleus spin and hyperfine splitting, the scale of external field $H_c$, the constants of the nucleus anapole moment $\alpha$ and the P-odd E1-amplitudes, the $R$ parameters for weak and strong field limits.

Such experimentally important values as P-odd E1-amplitudes $\langle \vec{d} \rangle$, parameters $R = \langle \vec{d} \rangle / \omega \langle \vec{\mu} \rangle$ and the transition frequencies $\omega$ depend of the external magnetic field $H$, but it is useful to present the $\langle \vec{d} \rangle$, $R$ and $H$ values as the functions of the frequency. Such dependencies for $\langle \vec{d} \rangle$ and $R$ are presented at Figs 2, 3. The way of the curves from left to right and from down to up corresponds to the increasing of external field from zero (for hyperfine transitions) or 10 G (for Zeeman ones) to 20 kG. The points on the curves correspond to the values of external field 0 or 10 G, 100 G, 1 kG, 10 kG. For $^{41}$K the growth of frequency and E1-amplitude of Zeeman transitions finishes at the field about 200 G, for $^{39}$K — 500 G. For hyperfine transitions in $^{133}$Cs the growth begins at about 1 kG, $^{205}$Tl — 5 kG. For every atom the transitions with the largest E1-amplitude and $R$ are showed: the $|1, 0\rangle - |2, 0\rangle$ and $|1, 0\rangle - |1, 1\rangle$ for caesium and potassium, $|1, 0\rangle - |2, 0\rangle$ and $|2, 0\rangle - |2, -1\rangle$ for thallium.

At the fixed wavelengths of 260 and 470 cm the Zeeman transitions in $^{39}$K and $^{41}$K in strong field gives the largest ratio of the P-odd E1 and background M1 transitions but it is obtained by the suppression of the M1-amplitudes and gives no gain in the P-odd signal.
<table>
<thead>
<tr>
<th>Atom, level</th>
<th>( I )</th>
<th>( \Delta E_M ) (cm(^{-1}))</th>
<th>( H_c ) (G)</th>
<th>( \alpha )</th>
<th>( \langle I - \frac{1}{2} | \bar{d} | I + \frac{1}{2} \rangle ) ((10^{-12} , \text{m}^2))</th>
<th>( R ) ((10^{-2} \alpha/2 , \text{Ry}))</th>
<th>( x = 0 )</th>
<th>( x &lt; 1 )</th>
<th>( x \gg 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{39}\text{K}) 4s(_{1/2})</td>
<td>3/2</td>
<td>0.0154</td>
<td>320</td>
<td>0.12</td>
<td>0.018</td>
<td>0.35 (65)*</td>
<td>1.75 (520/x)</td>
<td>0.7 \cdot x (260)</td>
<td></td>
</tr>
<tr>
<td>(^{41}\text{K}) 4s(_{1/2})</td>
<td>3/2</td>
<td>0.0085</td>
<td>180</td>
<td>0.12</td>
<td>0.018</td>
<td>0.63 (118)</td>
<td>3.15 (944/x)</td>
<td>1.25 \cdot x (470)</td>
<td></td>
</tr>
<tr>
<td>(^{133}\text{Cs}) 6s(_{1/2})</td>
<td>7/2</td>
<td>0.307</td>
<td>1640</td>
<td>0.25</td>
<td>1.08</td>
<td>0.73 (3.26)</td>
<td>6.6 (52.2/x)</td>
<td>2.9 \cdot x (26.1)</td>
<td></td>
</tr>
<tr>
<td>(^{205}\text{Tl}) 6p(_{1/2})</td>
<td>1/2</td>
<td>0.710</td>
<td>11400</td>
<td>0.38</td>
<td>3.75</td>
<td>7.5 (1.41)</td>
<td>7.5 (5.64/x)</td>
<td>7.5 \cdot x (2.82)</td>
<td></td>
</tr>
</tbody>
</table>

*) In brackets—the transition wavelength.
Conclusion

The considered transitions at the available external magnetic fields covers the range of wavelength from less then 1 cm to 10 m. In the reasonable range 10 cm—1 m the Zeeman transitions in caesium and thallium in the external fields of 1—10 kG seems more favourable: in comparison with the hyperfine potassium transitions at the same wavelength their E1-amplitudes and R-parameters are an order of magnitude higher.

So, the observation of the parity violation at the Zeeman transitions of the heavy atoms has the significant advantages in comparison with the hyperfine transitions considered earlier.

The author is grateful to V.V. Flambaum for the proposed idea of the work and for useful discussions, and to M.G. Kozlov and V.F. Ezhov for the interest to the work and useful critical comments.

REFERENCES

Fig. 1. Zeeman splitting of the hyperfine terms of $^{133}\text{Cs}$ 6s level as a function of external field. $H_e=1640$ G — the scale of the field.
Fig. 2. P-odd E1-transition amplitudes between the Zeeman and hyperfine terms of different atoms as a functions of the external field and transition frequency. The points on the curves correspond to the values of magnetic field (from left to right and from down to up) 0, (for hyperfine transitions) or 10 G (for Zeeman ones), 100 G, 1 kG, 10 kG.
Fig. 3. The parameters of the ratio of the useful signal to the background as a functions of the external magnetic field and transition frequency. Notations like at Fig. 2.
A.Ya. Kraftmakher

Parity Nonconservation in
Zeeman Atomic Transitions

A.Я. Крафтмахер

Нарушение четности в переходах
между зеемановскими компонентами
атомных уровней

Ответственный за выпуск С.Г.Попов

Работа поступила 4 января 1990 г.
Подписано в печать 20.04 1990 г. МН 08560
Формат бумаги 60×90 1/16 Объем 0,9 печ.л., 0,8 уч.-изд.л.
Тираж 210 экз. Бесплатно. Заказ № 54

Набрано в автоматизированной системе на базе фотонаборного автомата ФА1000 и ЭВМ «Электроника» и отпечатано на ротапринте Института ядерной физики СО АН СССР,
Новосибирск, 630090, пр. академика Лаврентьева, 11.