PAIR CONDENSATES IN REALISTIC SHELL MODEL STATES:
(II) M1 STATES IN \(^{54,56}\text{Cr}\) AND \(^{56,58,60}\text{Fe}\)

P. HALSE

School of Physics, University of Melbourne
Parkville 3052, Australia

ABSTRACT

The possible microscopic realization of IBM collective M1 modes is investigated for \(^{54,56}\text{Cr}\) and \(^{56,58,60}\text{Fe}\), using a shell model space of protons in the \(1f_{7/2}\) orbit and neutrons in the \(2p_{3/2}, 1f_{5/2}\), and \(2p_{1/2}\) orbits. Distributions of the pair-analogues of IBM-2 mixed-symmetry states, and of some g-IBM-2 states, over shell model eigenstates are explicitly determined. For \(J = 1\), the mixed-symmetry \(SD\) strength is generally lower than that of the \(G\)-pair states, and responsible for most of the low-energy M1 strength. Structures incorporating extended sets of levels, including the M1 states, corresponding to the \(sd\)-IBM-2 \(U(6) \supset SO(6) \supset SO(5)\) dynamical symmetry representations \([N-1,1] (N-1,1,0) (\tau)\) are manifest in the shell model spectra for \(^{56}\text{Fe}\) and \(^{54}\text{Cr}\). The correspondence with observed levels is discussed.
I. INTRODUCTION

There is currently much interest in the properties of collective levels at moderate energy excited by M1 mechanisms, and in the various modes in nuclear models by which they may be theoretically described\(^1\). Particular emphasis has been placed on the observation of collective \(J^\pi = 1^+\) levels in rare earth nuclei such as \(^{156}\text{Gd}\) (Ref. 2) and in lighter nuclei such as \(^{46}\text{Ti}\) (Ref. 3), although some \(2^+\) levels in \(^{56}\text{Fe}\) and other nuclei have also been cited\(^4\). Since the “M1” levels are but single members of a presumably extended collective structure (e.g. a rotational band), it is clear that a complete understanding of the mode requires some knowledge of the other members; however, experimental observation has been claimed for only a single level in each nucleus, \(\text{vis.}\) the \(1^+\) in rotational nuclei, the \(2^+\) in less deformed nuclei.

The description of these levels within the Interacting Boson Model\(^5\) (IBM) requires some extension of the simplest version, IBM-1; an equivalent situation exists for the geometrical model\(^6\) (GM). Explicit incorporation of separate proton and neutron degrees of freedom (IBM-2) yields non-totally symmetric, or “mixed-symmetry”, states that can have large M1 matrix elements from the predominantly symmetric ground states\(^7\). The IBM has also been extended to include \(J = 4\) \((g)\) bosons\(^5\), corresponding to the inclusion of hexadecapole deformations in the GM. In that case, states with \(J = 1\) are again produced, both for mixed symmetry and, excepting the two-boson case, for symmetry \([N]\). Strong M1 transitions could then, in principle, also arise from a difference in \(d\)- and \(g\)-boson g-factors\(^8\), although this is not expected to be as important a mechanism as the difference in proton and neutron g-factors.

Thus various possibilities exist for the structure of the M1 levels in the IBM framework, and for the characterization in terms of a most appropriate collective mode, i.e. a dynamical symmetry, including not only the symmetries of the \(sd\) version, but also the relative importance of \(d\)- and \(g\)-bosons. Furthermore, it may even be asked whether such a description, in terms of the same bosons used for the ground states, is in fact consistent with a reasonable realization of those bosons. A phenomenological approach involves the freedom of choosing parameters, and
so cannot be expected to uniquely support a single structure. Moreover, the absence of data on an extended set of levels, noted above, prevents both a comparison with the characteristic ratios of energies and transitions for the dynamical symmetries and the fixing of a quantitative scale (e.g. a moment of inertia in the rotational case).

The invariable, if sometimes only implicit, rationale behind applications of the IBM is that the bosons represent correlated pairs of like shell model nucleons; in this case, details of the collective, *vis. IBM, structure can be extracted directly from the underlying microscopy. In particular, for shell model spaces small enough to allow an exact solution, the probability distribution over realistic eigenstates of the various pair states can be explicitly determined. Such a procedure is clearly not limited to those levels readily seen in any particular experiment.

Here this approach is applied to the nuclei $^{54,56}$Cr and $^{56,58,60}$Fe (Refs. 10-13), a region where collective motion, and in particular the presence of mixed-symmetry states, have been inferred from the data. The validity of the $SD$ pair description, and the existence of an approximate $U(6) \supset O(6) \supset O(5)$ symmetry, for the lowest levels in $^{56,58}$Fe has been discussed previously (Ref. 15, hereinafter referred to as I.). The total $SD$ and $SDG J = 1$ distributions are first compared with the low-energy distribution of M1 strength, allowing the participation of the various pairs in M1 excitation to be determined. Then, for those nuclei where $SD$ components dominate the M1 levels, distributions are presented for the particular $SD$ states corresponding to phenomenologically interesting mixed-symmetry representations of the $U(6) \supset O(6) \supset O(5)$ symmetry appropriate to the ground states, allowing the validity and details of this description to be evaluated. Two sets of mixed-symmetry states in the $O(6)$ limit are of particular phenomenological interest. The most energetically favored representations, $[N-1,1]$ ($N-1,1,0$) ($\tau$), are expected to contain the states excited by M1 transitions from the lowest levels, and have the characteristic spectrum, distinct from that arising for either $U(5)$ or $SU(3)$ symmetry, shown in Table I. The second set of states, with labels $[N-1,1]$ ($N-2,0,0$) ($\tau$), correspond, in the large $N$ limit, to antisymmetric $\beta$ vibrations.
II. DISTRIBUTIONS OF PAIR STATES

A. Shell Model Calculation

The shell model space used for these calculations involves the orbits

\[(\tilde{p}: 1f_{7/2})^{28-Z} \quad (n: 2p_{3/2}, 1f_{5/2}, 2p_{1/2})^{N-28}, \quad (1)\]

with a fitted interaction obtained by Horie and Ogawa\textsuperscript{17,18}, similar\textsuperscript{17} to that derived in a G-matrix approach, as described in I. The calculations are performed using the code OXBASH\textsuperscript{19}, and give spectra in quantitative agreement with experiment\textsuperscript{18,15}.

An effective magnetic dipole operator appropriate for this space is obtained by fitting the usual \(g\)-factors to the measured moments (Refs. 20, 21, 4, 11) for \(^{55}\text{Co}\) (\(g_\text{p}\)) and for \(^{57}\text{Ni},^{58}\text{Ni}, \text{and }^{56}\text{Co}\) (\(g_{n1}\) and \(g_{ns}\)), yielding the values (in units of \(\mu_N^2\))

\[g_\text{p} = 1.378, \quad g_{n1} = 0.420, \quad g_{ns} = -1.741 \quad (2)\]

which may be interpreted as a core contribution to the neutron orbital moment, and a renormalization of the spin strength by 0.65 and 0.45 for the protons and neutrons respectively, approximately equivalent to a renormalization by 0.57 of the Gamow-Teller term.

The pairs used to construct the many-particle \(SD\) and \(SDG\) subspaces

\[A^\dagger(J) = \sum_{j \geq j'} \beta_{j j'}^J / \sqrt{1 + \delta_{jj'}} \quad [a^\dagger(J) a^\dagger(J')]^(J) \quad (3)\]

are also determined as in I; the neutron pairs are taken as the lowest eigenstates for \(J = 0, 2, 4\) in the closed-proton-shell two-neutron system, shown in Table II. (The \(S\) and \(D\) pairs differ slightly from those in I, where the OXBASH interaction contained an incorrect (6% error) \(p_{3/2}\)
single-particle energy, stemming from a misprint in Ref. 18; however, the effect on the results of this is negligible.) The non-$SD$ $SDG$ space, $(S^{N_{5s}} D^{N_{D}} G^{N_{G}})$ with $N_G \geq 1$, denoted $SD (G)$, is here constructed subject to the constraint, effective for $^{56}{\text{Cr}}$ and $^{60}{\text{Fe}}$, that $N_D + N_G \leq 3$.

### B. Results

Since it is unreasonable to assume that the mixing between, and indeed ordering of, levels is well determined in regions of high level density and excitation energy, the probability distributions to be presented are shown collected into bins of 0.5 MeV.

The distributions of the total $SD$ and $SD (G) \ J = 1$ strengths are shown against excitation energy, $E$, for $E \leq 7$ MeV in Figure 1, where comparison should be made with the numbers of states in each sector, tabulated in Table III. Also shown is the distribution of shell model $M1$ strength, $B(M1, 0^+ \rightarrow 1^+_i)$, calculated with the effective operator (Eq. 2), and the experimental values where available, obtained from the $1^+_i \rightarrow 0^+_1$ decays; the calculated $B(M1)$ values arise from approximately equal orbit and spin contributions, this remaining true if the bare $g$-factors are used. It is apparent that the $SD$ strength is generally lower than the $SD (G)$ strength, and is more closely correlated with the $B(M1)$ distribution; although $G$-pair states mix into those dominated by the $SD$ sector, in this system the low-energy $M1$ levels primarily involve the $SD$ degrees of freedom. The only exception is $^{60}{\text{Fe}}$, where no sector can be said to dominate the several $M1$ states, and a consequently more complex description is evident.

The distribution of particular $SD$ states corresponding to the $U(6) \supset SO(6) \supset SO(5)$ dynamical symmetry representations $[N-1,1] \ (N-1,1,0) \ (\tau, (\tau) = (1,0), (1,1), (2,0), \) and $2(1,1)$ (Table I) and $[N-1,1] \ (N-2,0,0) \ (0,0)$ for $^{56,58}{\text{Fe}}$ and $^{54,56}{\text{Cr}} \ (N = 2, 3, 3, \) and $4$ respectively) for $E \leq 6$ MeV are shown in Figure 2, the distribution for $[N] \ (N,0,0) \ (0,0) \ L = 0$ being included for comparison. The approximate realization of the $sd-O(6) \ (N-1,1,0)$ description, with an $SD$ interpretation, for $^{56}{\text{Fe}}$ and $^{54}{\text{Cr}}$ follows from the reasonably sharp localization in energy of the various states, and the appearance of the characteristic degeneracies.
and energy ratios (Table I). In particular, for $^{54}\text{Cr}$ even the $(2,1)$ multiplet, extending from $J = 1 - 5$, is manifest (although significant fragmentation occurs for $J = 4, 5$) and is approximately degenerate apart from a small monotonic dependence of energy on $J$. In contrast, for $^{58}\text{Fe}$ and $^{56}\text{Cr}$ this description is not valid. The "antisymmetric $\beta$ vibration" mode is not realized within the energy range investigated for any of these systems, as might have been expected from the size of the representations involved.

III. DISCUSSION

The extent to which any shell model space allows an adequate description of the realistic states is open to question. Comparison of the observed and calculated M1 properties in this case is satisfactory: although the renormalization of the $g$-factors (Eq. 2) is significant, and the problem of apparent missing strength occurs for the $1^+ \rightarrow 0^+$ transitions (Fig. 1) in $^{56}\text{Fe}$, this is not so for $^{58}\text{Fe}$, and the $2^+ \rightarrow 2^+$ transitions are in agreement; the energy of the M1 states is consistent with experiment, supporting the validity of the Horie-Ogawa interaction for the space used. Moreover, in this paper, the validity of a pair description has been examined for a particular shell model treatment, with the validity of the shell model treatment itself assumed from the work of Horie and Ogawa. However, the stability of the results on expansion of the shell model space, for instance to the full $pf$ shell, or to include neutrons in the $g_{9/2}$ orbit, could be investigated; in such cases more care would presumably be needed in the choice of the optimum collective pairs.

In assigning candidates for the collective states in the observed spectra, the previously noted uncertainty in mixing of shell model states at the relevant excitation suggests that the meaningful quantity is a range of energy (Fig. 2), over which fragmentation may occur; in cases where the $sd-O(6)$ B(M1) values are strong, comparison of the calculated and measured values can be used to provide further information. Thus, although the shell model calculation places
the mixed-symmetry $2^+$ state in $^{56}$Fe predominantly in the second level\textsuperscript{15} (2.748 MeV, observed at 2.658 MeV), comparison of the calculated (0.41 $\mu_N^2$) and measured\textsuperscript{11} B(M1) values suggests that the shell model state, and so also the pair state, is fragmented over the second (0.23 $\mu_N^2$) and third (2.960 MeV, 0.15 $\mu_N^2$) levels, as deduced earlier in Ref. 4 (although in this analysis the admixed state is largely non-SD), or even in addition the fourth level (3.370 MeV, 0.12 $\mu_N^2$). The associated $1^+$ state is expected in the vicinity of 3.3 MeV, with comparison of the B(M1) values favoring the second level (3.449 MeV) over the first (3.120 MeV). Similarly, for $^{54}$Cr (Ref. 10), the major part of the lowest mixed-symmetry O(6) $2^+$ state appears to reside in the second level at 3.075 MeV, with the $1^+$ expected in the vicinity of 3.4 MeV. However, the emphasis of this work is on the identification of a particular mode, in this case that described by the O(6) dynamical symmetry, and on the location of the associated strength where this is not readily done by experiment because M1 transitions to the lowest states vanish or are small, such as for the higher spin states, or those dominated by the higher O(5) multiplets (Fig. 2). For example, the second peak in the $^{54}$Cr SD $1^+$ distribution (5.5 MeV), carrying the O(5) label (2,1), represents another facet of the mode giving rise to the M1 level. It would be of particular interest to observe the rather exotic (2,1) $5^+$ state in $^{54}$Cr, predicted here to be in the vicinity of 4.5 MeV.

A geometrical interpretation of the IBM mode is in this case complicated by the importance of the intrinsic spin. It should be noted that the predominantly orbital, or "scissors", nature\textsuperscript{5} of the IBM-2 M1 mode is a possible geometrical interpretation, not an inherent property; in this study the mode has been realized with a microscopic interpretation, and found to involve both orbital and spin mechanisms, both of which are of interest\textsuperscript{1,6}.

The boson $g$-factors, calculated in first order from the single-pair nuclei, $^{54}$Fe and $^{58}$Ni, are $g_{\pi d}, g_{\pi g}, g_{\nu d}, g_{\nu g} = 1.378, 1.378, 0.361, 0.379$, verifying the expected dominance of the $\pi$-$\nu$ difference over the $d$-$g$ difference, and giving $g_{\pi} - g_{\nu} = 1$ as used in other calculations\textsuperscript{4}. 
The realization of a particular mixed-symmetry collective mode for $^{56}\text{Fe}$ and $^{54}\text{Cr}$, incorporating extended sets of levels including the $J = 1$ low-energy M1 states, described by the $sd$-IBM dynamical symmetry $\text{U}(6) \supset \text{SO}(6) \supset \text{SO}(5)$ representations $[N-1,1] (N-1,1,0) (\tau)$, has been revealed from the underlying microscopy via an interpretation of the bosons as shell model pairs. The characteristic O(6) spectrum can be seen in the (means of the) peaks of the total SD distribution, and the symmetry is confirmed by explicit decomposition. Vestiges of this mode survive in $^{58}\text{Fe}$ and $^{56}\text{Cr}$, but there each state is spread over many eigenstates in a wide energy range, so that the mode does not provide a good description of the nuclear motion. For $^{60}\text{Fe}$, even the lowest levels require in addition G-pairs for a reasonable description. The low-energy M1 $1^+$ level in these calculations is at about 3.5 MeV, approximately the same energy as for the observed levels in the rare earth$^2$ and $f_{7/2}$ nuclei$^3$; as in the latter$^{3,23}$, the M1 strength arises from comparable orbit and spin contributions.

Thus, the M1 states in $^{56}\text{Fe}$ and $^{54}\text{Cr}$ are verified to be largely mixed-symmetry combinations of the $S$ and $D$ pairs involved in the ground states, and to have an approximate O(6) symmetry. Moreover, a more complete realization of the mode has been obtained, in that the states related to the M1 level by a common collective structure, being unified at the O(6) level while differing under reduction to O(5), have been explicitly identified as reasonably sharp superpositions of the numerically determined shell model eigenstates, the lowest such state being a $2^+$ at around 0.5 MeV below the $1^+$. Calculations for much larger spaces are feasible$^{24}$ and are under consideration. For still larger spaces, the mean and variance of the various distributions, which embody much of the important information, could be investigated, perhaps using the methods of statistical spectroscopy. Restricting to a pair subspace, equivalent to finding only the means of the distributions of the boson-eigenstate pair analogues, may underestimate the full extent of fragmentation, giving erroneous conclusions on whether a description is in fact realized.
ACKNOWLEDGEMENT

The author would like to thank Professor B. A. Brown for providing the Horie-Ogawa interaction. This work was supported by an Australian Research Council Grant.
REFERENCES

19. OXBASH-MSU, the Oxford-Buenos Aires-Michigan State University shell model code,
   A. Etchegoyen, W. D. M. Rae, and N. S. Godwin [M.S.U. version by B. A. Brown]
22. Huan Liu and L. Zamick, Rutgers University preprint RU-86-34
Table I

Angular momentum content, eigenvalues $g(\tau)$ of the Casimir operator, and $\Delta = g(\tau) - g(1,0)$, for the lowest few $O(5)$ representations belonging to the $O(6)$ representation ($N-1,1,0$).

<table>
<thead>
<tr>
<th>$\tau$</th>
<th>$J$</th>
<th>$g(\tau)$</th>
<th>$\Delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1,0)</td>
<td>2</td>
<td>4</td>
<td>-</td>
</tr>
<tr>
<td>(1,1)</td>
<td>1,3</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>(2,0)</td>
<td>2,4</td>
<td>10</td>
<td>6</td>
</tr>
<tr>
<td>(2,1)</td>
<td>1,2,3,4,5</td>
<td>12</td>
<td>8</td>
</tr>
</tbody>
</table>
Table II

Structure coefficients $\beta_{jj'}^J$ (Eq. 3) of the $S (J = 0)$, $D (J = 2)$, and $G (J = 4)$ neutron pairs obtained from the proton-closed-shell two-neutron calculation.

<table>
<thead>
<tr>
<th>$J$</th>
<th>$(5/2,5/2)$</th>
<th>$(5/2,3/2)$</th>
<th>$(5/2,1/2)$</th>
<th>$(3/2,3/2)$</th>
<th>$(3/2,1/2)$</th>
<th>$(1/2,1/2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.5227</td>
<td>-</td>
<td>-</td>
<td>0.8019</td>
<td>-</td>
<td>0.2894</td>
</tr>
<tr>
<td>2</td>
<td>0.2041</td>
<td>-0.1545</td>
<td>0.1928</td>
<td>0.7204</td>
<td>0.6151</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>-0.1643</td>
<td>0.9519</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
Number of states with angular momentum $J = 1$ in $^{54,56}$Cr and $^{56,58,60}$Fe for the full shell model, the $SD$ subspace, and the $SD(G)$ ($N_G \geq 1, N_D + N_G \leq 3$) subspace. The numbers of states in the corresponding boson models are also shown, in parentheses, where larger.

<table>
<thead>
<tr>
<th>Space</th>
<th>$^{56}$Fe</th>
<th>$^{58}$Fe</th>
<th>$^{60}$Fe</th>
<th>$^{54}$Cr</th>
<th>$^{56}$Cr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shell model</td>
<td>15</td>
<td>105</td>
<td>188</td>
<td>30</td>
<td>213</td>
</tr>
<tr>
<td>$SD$</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>$SD(G)$</td>
<td>1</td>
<td>9</td>
<td>11</td>
<td>3 (9)</td>
<td>11 (15)</td>
</tr>
</tbody>
</table>
FIGURE CAPTIONS

Fig. 1 Total $J = 1$ probability distributions for the $SD$ subspace and the $SD(G)$ subspace, and distributions of the calculated $B(M1,0^+_1 \rightarrow 1^+_1)$ values, the measured strength being noted where available, for $E \leq 7$ MeV in $^{56,58,60}$Fe and $^{54,56}$Cr. (The observed strength, $> 2 \mu_N^2$, associated with the 4.139 MeV level in $^{58}$Fe has been reported in only one paper, where the lifetimes of several levels differ significantly from other measurements.)

Fig. 2 Probability distributions for $SD$ states classified according to the $sd$-IBM $U(6) \supset O(6) \supset O(5)$ representations $[N-1,1]$ $(N-1,1,0)$ ($\tau$), and $[N-1,1]$ $(N-2,0,0)$ $(0,0)$, with $[N]$ $(N,0,0)$ $(0,0)$ being shown for comparison, for $J = 0 - 5$ and $E \leq 6$ MeV in $^{56,58}$Fe and $^{54,56}$Cr.
<table>
<thead>
<tr>
<th>E(MeV)</th>
<th>SD</th>
<th>SDG</th>
<th>B(M1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For $^{58}$Fe:
- $B(M1)$ values are 0.01, 0.14, and 0.59.

For $^{60}$Fe:
- $B(M1)$ values are 0.01, 0.01, and 0.59.

For $^{54}$Cr:
- $B(M1)$ values are 0.01, 0.01, and 0.59.

For $^{56}$Cr:
- $B(M1)$ values are 0.01, 0.01, and 0.59.