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INEQUIVALENCE OF INTERIOR AND EXTERIOR DYNAMICAL PROBLEMS

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ABSTRACT

We begin a series of notes with the review of the historical distinction by Lagrange, Hamilton, Jacobi and other Founding Fathers of analytic dynamics, between the *exteriordynamical problem*, consisting of motion in vacuum under action–at–a–distance interactions, and the *interior dynamical problem*, consisting of motion within a resistive medium with the additional presence of contact, nonlinear, nonlocal and nonhamiltonian internal forces. After recalling some of the historical reasons that led to the contemporary, virtually complete restriction of research to the exterior problem, we show that the interior dynamical problem cannot be reduced to the exterior one. This establishes the open character of the central objective of these notes: the identification of the space–time symmetries and relativities that are applicable to interior, nonlinear, nonlocal and nonhamiltonian systems.

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The earlier studies of analytic dynamics, such as those by Lagrange [1], Hamilton [2], Jacobi [3] and others, were based on the distinction between:

1) The *interior dynamical problem*, which is essentially the study of dynamics in the interior of the minimal surface containing all matter of the celestial body considered, including any possible atmosphere; and

2) The *exterior dynamical problem*, which is essentially the study of dynamics in the empty space (vacuum) outside the above minimal surface, assumed as homogeneous and isotropic.

In fact, the original Lagrange's [1] and Hamilton's [2] equations were formulated with external terms precisely to represent the forces of the interior dynamics which were known to be outside the representational capabilities of the Lagrangian or Hamiltonian functions by their very originators. Similarly, Jacobi [3] formulated his celebrated theorem, not for the contemporary equations (those without the external terms) but for the original analytic equations with external terms.

The distinction between the exterior and the interior problem gradually disappeared from the scientific scene as a result of an evolutory process that does not appear to have been sufficiently studied by historians in the field until now. Without any claim of completeness, we can mention here:

a) The birth of *Lie's theory* [4] which identified the algebraic structure of Hamilton's equations *without* external terms, and its impact in mathematical and physical studies;

b) The discovery of the Special Relativity by Lorentz [5], Poincare [6] Einstein [7,8] and others with its strictly Lie-Hamiltonian character, and the profound influence in the scientific thought which resulted from its experimental verification;

c) The successes of the quantum mechanical description of the atomic structure with its strictly Lie-Hamiltonian character in operator form (see, e.g., ref. [8] and quoted historical contributions); and numerous other factors.

Birkhoff [9] identified a generalization of Hamilton's equations also derivable from a first-order variational principle, but its algebraic structure remained unknown. Also, he applied his equations to typical exterior problems, such as the stability of planetary trajectories.

Despite the general lack of recent interest, the historical motivations that led Lagrange, Hamilton, Jacobi and other

Founders of analytic dynamics to formulate interior problems with external terms, were sound indeed. In fact, as we shall review shortly, the contact interactions of interior trajectories generally possess *nonlinear* as well as *nonlocal forces* (see, e.g., Refs. [10,11] and quoted papers). Moreover, quantitative studies on the irreducibility of the interior to the exterior problem were initiated by H. Helmholtz [12] who established the *nonlagrangian and nonhamiltonian character* of the external terms in the analytic equations, via the so-called conditions of *variational selfadjointness*.

Some of the most comprehensive studies of nonconservative interior conditions of this century are those conducted by I. Prigogine and his collaborators (see Refs. [13-15] and quoted papers), primarily along statistical lines but with rather deep analytic and operator counterparts. Additional studies in interior dynamics have also been conducted by several other researchers (see, e.g., Refs. [16-17] and quoted literature.

Somewhat stimulated by Prigogine's pioneering work, this author initiated his research [18,19] with a reinspection of the original Lagrange's and Hamilton's equations, and the indication that the brackets of the time evolution, when properly written, violate the Lie algebras axioms and verify instead those of a covering of Lie algebras known in mathematics as *Lie-admissible algebras* [20]. These studies were then continued later on in Refs. [21-24].

As a complement to these Lie-admissible studies, this author submitted an alternative formulation of the interior dynamics based on the so-called *Lie-isotopic theory* [21] which, as we shall see, is based on the most general possible realization of the conventional axioms of Lie's theory. It was then shown (loc. cit.) that Birkhoff's equations [9] possess precisely a Lie-isotopic structure and, consequently, a conventional symplectic character. Finally, Birkhoff's equations were proved to be *directly universal*, i.e., capable of representing all possible interior trajectories in local and analytic formulation (universality), directly in the frame of the experimenter (direct universality) (loc. cit.). These Newtonian results were then expanded in Refs. [25,26].

The objective of the alternative approach [21,25,26] is to show that, in the transition from the exterior to the interior dynamics, there is no need to abandon the conventional analytic, algebraic and geometric structures of contemporary physics, but only the need to pass from their simplest possible (Hamiltonian) realizations to their most general possible (Birkhoffian) realizations.

This series of notes is primarily devoted to the Lie-isotopic formulation of interior dynamics. The reader should however be aware that all our results can be reformulated in terms of the broader Lie-admissible approach. In fact, the central topics of these notes, the Lie-isotopic generalization of the Galilei and Poincaré symmetries, are particular cases of expected, still more general, Lie-admissible generalizations of the same symmetries [24].

A majestic classical example of the dicothomy exterior vs interior problem is provided by Jupiter, whose exterior center-of-mass trajectory is manifestly stable and verifies all conventional symmetries and physical laws; nevertheless, the interior dynamics is manifestly unstable, nonconservative, and nonlagrangian-nonhamiltonian.

A conceptual guidance of the classical studies of these notes is therefore provided by Jupiter's structure. In fact, these notes can be essentially considered as attempting the identification of the generalizations of conventional Galilean, relativistic and gravitational treatments which can provide a classical, direct, representation of Jupiter, as it appears to our experimental observations, and without hypothetical, seemingly inconsistent (see below), conservative reductions.

In subsequent papers we hope to indicate the possibility of achieving a consistent operator formulation of our results, and apply them to the problem of the hadronic structure, with particular reference to the possibile identification of the hadronic constituents with (suitably modified forms of) massive physical particles freely produced in the spontaneous decays. In different terms, the hope of these studies is that of investigating whether the historical identification of the atomic and nuclear constituents freely produced in the spontaneous decays, can also be extended, in due time, to the hadronic structure, of course, under suitably generalized formulations. For this purpose, it may be of some assistance to identify a conceivable operator counterpart of the above classical guidelines.

Hadrons are currently conceived as being strictly Lagrangian or Hamiltonian, with a strictly local-differential and potential structure. This essentially implies the treatment of the hadronic structure as an exterior dynamical problem, i.e., the tacit assumption that *the hadronic structure is equivalent to that of the* *atomic structure*, i.e., the quark constituents freely orbit inside hadrons along an atomic-type structure. Still equivalently, we can say that a necessary condition for the consistency of current theories is the conception of hadrons as ideal empty spheres with massive points moving in stationary interior orbits.

In ref. [22] we submitted the conjecture that the hadronic structure is an operator version of the classical structure of Jupiter, much along the historical open legacy of the ultimate nonlocality of strong intreractions. In fact, in their exterior center-of-mass dynamics (e.g., in a particle accelerator) hadrons obey all conventional symmetries and relativities, as is the case for the motion of Jupiter in the Solar system. Nevertheless, the interior dynamics of hadrons could well be generalized, along Jupiter's interior structure.

In fact, hadrons are the densest objects measured in laboratory until now. Moreover, clear experimental evidence establishes that all massive particles (such as the electrons) have a wavepacket of the order of 1F (= 10^{-13} cm), which is precisely the order of magnitude of the size of all hadrons, as well as of the range of the strong interactions themselves. Needless to say, the hadronic constituents can indeed have a point-like charge (such as the electron), but "point-like wavepackets" do not exist in the physical reality. In order for the constituents of hadrons to be physical massive particles, their wavepacket is therefore expected to be finite and of the order of the size of all hadrons.

It then follows that, while in the atomic structure we have very large mutual distances as compared to the size of the wavepackets of the constituents, in the hadronic structure we have instead mutual distances of the same order of magnitude of the wavepacket of the constituents. This results in a quark motion which is typical of all interior problems.

As a result, *the hadronic structure, could therefore be analytically equivalent to that of Jupiter in the sense that motion of a hadronic constituent inside a hadron could be analytically equivalent to the motion, say, of a space-ship in Jupiter's atmosphere, resulting in both cases in the presence of internal forces of contact, nonlocal and nonhamiltonian type* [22,23].

In conclusion, a conceptual guidance for our studies can also be given by the hadronic interior problem conceived precisely along the historical teaching by Lagrange, Hamilton and Jacobi [1,2,3].

We now begin by identifying the physical differences between the exterior and the interior problem.

Consider a test particle in the gravitational field of a celestial object, say, Jupiter. When considering the exterior trajectory, motion occurs in empty space which is known to be homogeneous and isotropic to the best of our approximations. Under these conditions, the actual size and shape of the test particle do not affect its dynamical evolution. The particle can then be effectively assumed as being dimensionless, resulting in Galilei's concept of *"massive point"*.

In turn, this implies the exact validity of a local-differential geometry. Since points cannot collide, the only admissible forces are the conventional action-at-a-distance, potential (selfadjoint [12,21,25]) forces. The orbits are therefore necessarily stable, with consequential validity of conventional symmetries, such as the rotational symmetry O(3) or the Galilei's symmetry G(3.1).

We can therefore say that:

> ***The exterior dynamical problem** can be characterized by a local-differential geometry; it consists of motion of point-like particles in vacuum under potential* (selfadjoint), and *therefore Lagrangian-Hamiltonian forces; and it can be effectively represented via the conventional (in today's standards) Lagrange's and Hamilton's equations, those without external terms.*

When the same test body penetrates Jupiter's atmosphere, thus passing to the interior dynamical problem, the physical framework is profoundly different. To begin, we now have motion within a physical medium which is evidently inhomogeneous (e.g., because the density of Jupiter tends to zero with the increase of the distance from its center), and anisotropic (e.g., because Jupiter's intrinsic angular momentum creates a preferred direction in the medium considered). Also, the actual size and shape of the test particle can no longer be ignored, because they directly affect its dynamical evolution. As a result, we can no longer claim motion of point-like particles in vacuum, but we have instead motion of an extended object within a generally inhomogeneous and anisotropic physical medium.

Moreover, the acting forces are given by the conventional potential (say, gravitational) forces which remain unaffected by the interior dynamics, plus the contact forces, that is, forces caused by the actual contact of the extended object with the physical medium.

In particular, besides being nonlinear, these latter forces are known to be :

1) of *nonlocal type*, in the sense of requiring a surface or volume integrals for their representation [10,11];

2) of *nonlagrangian-nonhamiltonian type*, first, because the notion of potential has no physical meaning for contact interactions and, more deeply, because they are *nonselfadjoint* [12,21,25], i.e., they violate the necessary and sufficient conditions for the existence of a Lagrangian or a Hamiltonian representation; and, last but not least,

3) of *zero range* in the sense that they occur at the instant of mutual "contact" , thus being *instantaneous* by conception.

Finally, as a result of the latter interactions, the orbits are manifestly unstable, e.g., because of the decay of the angular momentum, thus resulting in an apparent (see subsequent notes) breaking of the rotational symmetry O(3). The local breaking of Galilei's symmetry G(3.1) is then consequential, as requested, e.g., by the inhomogeneous and anisotropic character of the interior media, the lack of canonical formulations, etc..

In fact, the insistence in the exact validity for the interior trajectories of the same symmetries of the exterior problem would directly imply excessive approximations, such as the acceptance of the perpetual motion in a physical environment, trivially, because of the necessary conservation of the angular momentum.

In conclusion:

The interior dynamical problem generally requires a nonlocal integral *geometry; it consists of the motion of extended (and therefore deformable) test particles within generally inhomogeneous and anisotropic material media (with the understanding that the underlying space remains homogeneous and isotropic); and it requires for an effective treatment the original Lagrange's and Hamilton's equations, those with external terms representing the contact forces precisely along the original conception of the Founders of analytic dynamics* [1,2,3].

A primary task of these studies is therefore to identify

suitable formulations for the quantitative treatment of the above interior conditions, as a preparatory step for possible operator treatments.

We now pass to the study of the irreducibility of the above two problems.

Numerous (seemingly unpublished) objections are informally voiced against the differentiations between the exterior and the interior problems. It is important for this analysis to review the most important objections and show their inconsistencies or lack of technical feasibility.

In particular, we shall point out that the conventional concepts of the exterior problem, when applied to the interior problem as a result of protracted use without a technical scrutiny, generally lead to insidious inconsistencies as well as fundamental physical misrepresentations.

The first objection refers to the (classical and) local treatment of the problem and consists of the belief that the conventional Lagrange's and Hamilton's equations are sufficient to represent interior trajectories. The fact that this is indeed the case for numerous physical systems is undeniable (see, e.g., the examples of Refs. [25]). However, the lack of general applicability of the conventional equations is rigorously established by the violation of the conditions of variational selfadjointness. Besides, the insistence in the general use of the conventional analytic equations would lead to evident, excessive approximations of physical reality.

In fact, nonlocal forces can be well approximated via power series in the velocities truncated (via suitable coefficients) at given powers. The point is that, to avoid excessive approximations, such powers must remain arbitrary, thus precluding the general existence of a direct Lagrangian or Hamiltonian representation.

As an example, computerized guidance systems in contemporary rocketry require the use of up to the tenth power in the velocity or more. It is evident that, under these conditions, no Lagrangian or Hamiltonian can be effectively computed. On the contrary, the existence of a direct Birkhoffian representation is ensured [26], with numerous methodological possibilities, e.g., the consequential direct applicability of the optimal control theory.

A second objection, also of classical and local nature, is that the above nonlagrangian and nonhamiltonian systems can be transformed into suitable frames in which a conventional Lagrangian or Hamiltonian exists. Under sufficient topological conditions (regularity, locality and analyticity) the *Lie-Koening* *theorem* ensures that there always exist a transformation under which a nonlagrangian or a nonhamiltonian Newtonian system admits a conventional Lagrangian or Hamiltonian representation (see the geometric and analytic proofs of ref. [26], Sect. 6.2). However, the transformation is necessarily *noncanonical* (evidently because the original system is nonhamiltonian by assumption), and generally nonlinear in all variables. Therefore, the admitted frames are strictly *noninertial* and, as such, *incompatible with the applicable relativity.* Besides, *the transformed frames are not realizable in experiments,* thus having a purely mathematical meaning. As a result, the objection here considered is not physically sound.

This is the reason for the insistence in Refs. [21-26] that *the methodological formulations of the interior problem must be directly applicable in the frame of the experimenter,* and this insistence will be kept throughout our analysis. Only after the achievement of this basic physical objective, the use of the transformation theory may have a physical value.

We can therefore state that *a quantitative, classical treatment of interior trajectories requires:*

i) the treatment directly in the frame of the observer, to avoid mathematical, noninertial frames nonrealizable in actual experiments and other inconsistencies;

ii) a generally nonlocal and nonhamiltonian theory, to represent the interior forces as they occur in the physical reality; and

iii) if locality is admitted in first approximation, the theory must remain nonhamiltonian to avoid excessive approximations of the type of perpetual motion in a physical environment.

A final objection is that the distinction between the exterior and the interior problem is "illusory" because, when the test body and the surrounding atmosphere are all reduced to their elementary constituents, one regains motion of point-like particles in vacuum, in which case all distinctions between the exterior and the interior problems cease to exist.

This latter objection itself has been proven to be "illusory" because intrinsically inconsistent and not technical realizable [27]. Consider, again, the space-ship in Jupiter's atmosphere. Its trajectory is manifestly noncanonical and nonhamiltonian, as established by clear experimental evidence. On the contrary, the elementary constituents of the space-ship are evidently *assumed*

to have unitary and Hamiltonian time evolutions. The following property can then be readily proved.

THEOREM 1 [27]: Under sufficient topological conditions, a classical noncanonical and nonhamiltonian interior system cannot be reduced to a finite collection of unitary and Hamiltonian particles; and, viceversa, a finite collection of unitary and Hamiltonian particles cannot produce a classical noncanonical and nonhamiltonian ensemble under the correspondence or other limits.

The proof is trivial. In fact, a macroscopic unstable orbit simply cannot be decomposed into a finite number of stable elementary trajectories; and, viceversa, a collection of stable elementary trajectories simply cannot result in a classical unstable ensemble. On the contrary, a classical, stable system can indeed be decomposed into a collection of unstable trajectories [22], as we shall review in the next note.

The symmetry counterpart of the above property is then predictably given by the following

THEOREM 2 [27]: Under sufficient topological conditions, a classical Galilei- (or Lorentz-) noninvariant system cannot be reduced to a finite collection of Galilei- (or Lorentz-) invariant particles; and, viceversa, a finite collection of Galilei- (or Lorentz-) invariant particles cannot produce a Galilei- (or Lorentz-) noninvariant ensemble under the correspondence or other limits.

In fact, the local validity of the Galilei (or Lorentz) symmetry necessarily implies the stability of the constituents' orbits which, as such, cannot result in a nonconservative and, therefore, Galilei- (or Lorentz-) noninvariant ensemble.

The above theorems essentially establish that classical systems such as a satellite during re-entry in Earth's atmosphere with its continuously decaying angular momentum, is an experimental reality outside the arena of applicability of conventional symmetries and relativities, and its conceptual reduction to the elementary constituents in the hope of regaining conventional knowledge, cannot be technically realized in a consistent way.

Stated in different terms, the above theorems indicate that the contact, nonlocal and honhamiltonian forces of the satellite during re-entry, by no means, are "illusory" and can be made to "disappear" in the reduction of the satellite to its elementary constituents.

In fact, we find, first, exactly the same forces in the region of contact of the satellite with the atmosphere and, more particularly, in the overlapping of the wavepackets of the peripheral atomic electrons at the region of contact under extremely high local pressures.

Furthermore, along the open historical legacy of the ultimate nonlocal nature of the strong interactions, we expect to find again the nonlocal forces in the mutual overlapping of the wavepackets of the constituents in the hadronic as well as, to a lesser extent, in the nuclear structure [22]. The fact that current particle theories cannot accomodate this historical legacy, is not sufficient ground to conclude that strong interactions are necessarily local.

An objective of these studies is therefore that of abandoning conceptual abstractions of dubious scientific value, and confronting the problem of the mathematical representation of interior trajectories as they appear in our physical reality, that is, with internal nonconservations, of course, in a way compatible with conventional settings in the exterior problem.

Equivalently, we shall attempt a reconciliation between the exact character of local relativities in the center-of-mass behaviour (exterior problem), and the open historical legacy of the ultimate nonlocality of the hadronic structure (interior problem).

The necessity of a joint study of the problem at the Newtonian, as well as relativistic and gravitational levels can now be identified. In fact, the excellent results of current quark theories (see, e.g., reprints [28]) indicate the possibility that, after all, the nonlocal and nonhamiltonian internal effects under consideration here could be small in the hadronic structure, and therefore ignorable at a first nonrelativistic and nongravitational treatment. This could be the case if the wavepackets of quarks could experimentally be proved to be very small as compared to the size of the hadrons, in order to truly have a hadronic structure of the atomic type.

However, when passing to the gravitational treatment, the physical evidence of the ultimate nonlocal structure of gravitation cannot be ignored on any true scientific ground. As an example, in

a star undergoing gravitational collapse, we have not only the *total mutual penetration of the wavepackets of the constituents* (whatever their size), but also their *compression* in large numbers within the same very small region of space. The emerging, essentially nonlocal, and therefore nonselfadjoint structure of the interior gravitational problem is then incontrovertible, as we shall see in more details in subsequent studies.

At the same time, nonlocal and nonselfadjoint interior problems cannot be solely studied at the gravitational level, but require for consistency their study also at the preceding Newtonian and relativistic levels, as done in these notes.

In conclusion, the historical distinction between the exterior and interior problems was established by the Founding Fathers of analytic mechanics [1,2,3] on rather sound physical grounds and direct experimental evidence; it is confirmed by theoretical studies on their inequivalence, and all known (and seemingly unpublished) objections are not technically consistent with this writing.

This establishes the open character of the central objective of these papers: the construction of the space-time symmetries and relativities of interior nonlinear, nonlocal and nonhamiltonian problems. In the subsequent notes of this series, we shall summarize our main results. A more detailed and comprehensive presentation is contained in ref. [29].

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