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A treatment of the Final-state Interaction for Photonuclear Reactions

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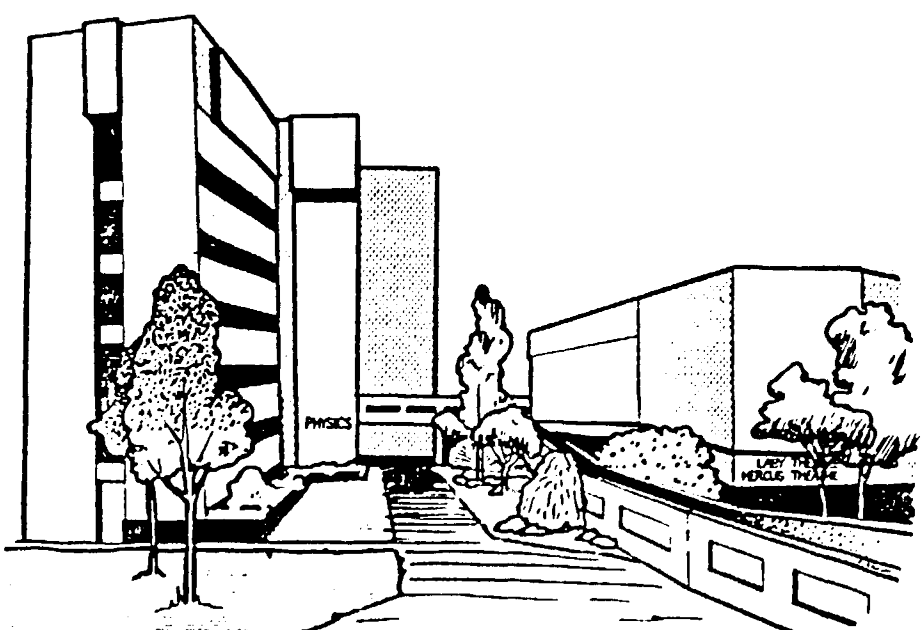

A treatment of the Final-state Interaction for Photonuclear Reactions

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I. INTRODUCTION

The final-state interactions (FSI) for photonuclear reactions are the processes whereby energetic photonucleons suffer interactions as they escape the residual nucleus. The 'surface refraction' and 'absorption' processes are considered separately as distinct contributions to the final-state interaction. The surface refraction process, allowing photonucleons to refract as they emerge from the nuclear potential well, cannot change the magnitude of the total cross section, but may redistribute the strength in the angular dependence of the differential cross section. The absorption process accounts for the loss of photonucleons as a result of inelastic collisions while escaping the nucleus.

In principle, the FSI could be calculated in a Born approximation, with the real and imaginary parts of the optical potential representing the surface refraction and absorption processes respectively. However the absorption process is relatively strong, and cannot be reliably calculated in a Born approximation. Following Gottfried's approach, the two corrections are treated as separate problems. The surface refraction correction is calculated in a plane-wave Born approximation (PWBA) and the absorption correction is presented as a simple development of earlier phenomenological treatments.

II. SURFACE REFRACTION CORRECTION

A. Gottfried's Approach

Gottfried[1] derived the transition amplitude for the (γ, pn) reaction as

$$\mathcal{T}^{\lambda f}_{nln'l'LM} = \mathcal{T}_0 + \mathcal{T}_n + \mathcal{T}_p \tag{1}$$

where the direct contribution is,

$$\mathcal{T}_0 = \mathcal{M}_{\lambda f} f_{nln'l'L}(p_q) Y_{LM}(\hat{p}_q) \tag{2}$$

and the surface refraction terms for each of the photonucleons takes the form,

$$\mathcal{T}_N = \sum_\alpha \mathcal{M}_{\lambda\alpha} \frac{\langle \Psi_f^{(-)} | \mathcal{V} | \Phi_\alpha \rangle}{(E_f - E_\alpha + i\eta)} f_{nln'l'L}(p_q') Y_{LM}(\hat{p}_q') \tag{3}$$

where,

$$\vec{p_q}\,' = \vec{p_q} + \vec{p_N}\,' - \vec{P_N} \tag{4}$$

The terms appearing in equations (1-4) are defined as follows,

$f_{nln'l'L}(p_q) Y_{LM}(\hat{p}_q)$ is the amplitude for finding a pn-pair with bound-state momentum $\vec{p_q}$. The explicit form of $f_{nln'l'L}(p_q)$ is given in ref 2.

$\vec{p_n}$, $\vec{p_p}$ are the bound-state momenta of the neutron and proton before they absorb the photon.

$\vec{p_q}\,'$, $\vec{p_N}\,'$ are the momenta of the pn-pair and nucleon in the intermediate state Φ_α, before the nucleon is refracted into the final state $\Psi_f^{(-)}$ with momentum $\vec{P}_N$.

$\mathcal{M}_{\lambda f}$ is the photo-absorption matrix element giving the amplitude for a photon of polarisation λ exciting the pn-pair into the observed final state, $\Psi_f^{(-)}$,

$\mathcal{M}_{\lambda\alpha}$ is the off-energy-shell photo-absorption matrix element giving the amplitude for a photon of polarisation λ exciting the pn-pair into the intermediate state Φ_α,

$\langle \Psi_f^{(-)} | \mathcal{V} | \Phi_\alpha \rangle$ is the exact scattering amplitude for the optical-model potential, and,

$E_f - E_\alpha + i\eta$ is the propagator for the intermediate state.

An exact and explicit form for equation (3) is not available since the magnitude and phase of the off-energy shell matrix elements are not known. To make progress, Gottfried assumed that $\mathcal{M}_{\lambda\alpha} \sim \mathcal{M}_{\lambda f}$, i.e. all the off-shell transition matrix elements are assumed to be the same. (In the Quasi-Deuteron Model (QDM), $|\mathcal{M}_{\lambda f}|^2$ is approximated by the deuteron photo-absorption cross section, and the FSI processes which introduce $\mathcal{M}_{\lambda\alpha}$ are completely ignored). If small angle scattering dominates the surface refraction process, the kinematics will be similar to the direct process, so the approximation $\mathcal{M}_{\lambda\alpha} \sim \mathcal{M}_{\lambda f}$ may be acceptable. In any event, the variation of $\mathcal{T}_N$ with Φ_α is expected[2] to be dominated by $f_{nln'l'L}$. Using only the real part of the optical-model potential, Gottfried writes the total transition amplitude as,

$$\mathcal{T}_N = \mathcal{M}_{\lambda f}\frac{2Mv}{(2\pi^3)}\int \frac{\langle \vec{P}_N |\mathcal{V}| \vec{p}_N\,'\rangle}{P_N^2 - p_N'^{\,2} + i\eta} f_{nln'lL}(p_q')Y_{LM}(\hat{p}_q')d\vec{p}_N\,' \tag{5}$$

where $\langle \vec{P}_N |\mathcal{V}| \vec{p}_n\,'\rangle$ is the calculated scattering amplitude for the real optical-model potential, M is the nucleon mass and v is the normalization volume.

Equation (1) gives the transition amplitude for a well defined final state, and includes three terms corresponding to direct photonucleon emission, neutron refraction and proton refraction. For the direct term, no surface refraction is acknowledged, so that

$$\vec{p}_q = \vec{p}_n + \vec{p}_p - \vec{p}_\gamma \tag{6}$$

The pictorial representation of the direct term is given in figure 1. The treatment of the kinematics describing the photo-absorption process is similar to other calculations[3,4], but is subtly different from that of Gottfried's, since the present calculation acknowledges the difference between the internal and external photonucleon energies. The distinction becomes important only when the internal and external photonucleon energies are quite different, *i.e.*, when nucleons are ejected with small kinetic energies or from deeply bound orbitals.

The neutron and proton surface refraction processes are shown in figures 2(i) and 2(ii) respectively. Refraction of both nucleons is ignored to first order in the Born approximation.

The calculation of the neutron and proton surface refraction terms is the same, but for definiteness, consider the neutron surface refraction calcultion, where direct photodisintegration of the pn-pair leads to an intermediate nuclear state Φ_α with photoneutron momentum $\vec{p}_n\,'$ and photoproton momentum is $\vec{p}_p$. This implies (see equation (4)) that the bound state momentum of the pn-pair was,

$$\begin{aligned}\vec{p}_q\,' &= \vec{p}_n\,' + \vec{p}_p - \vec{p}_\gamma \\ &= \vec{p}_q + \vec{p}_n\,' - \vec{p}_n\end{aligned} \tag{7}$$

Since figure 2 represents *all* intermediate states in the neutron refraction process, it is necessary to intergrate over all intermediate-state photoneutron phase space, and determine the amplitudes for

1. finding a pn-pair with momentum $\vec{p}_q\,'$,

2. the photon coupling to the pn-pair and causing a transition to an intermediate state where the nucleons have not yet escaped the nuclear potential well, and have momenta $\vec{p}_n\,'$ and $\vec{p}_p$,

3. the neutron refraction process $\vec{p}_n{}' \rightarrow \vec{P}_n$, where the internal photoneutron transfers momentum to the $(A-2)$-system, resulting in the required (external) neutron momentum.

The assumption $\mathcal{M}_{\lambda\alpha} \sim \mathcal{M}_{\lambda f}$, means that step 2 is ignored, and the off-shell photoabsorption matrix element is taken to be the same as it was for the direct term.

Gottfried used plane waves for the intermediate and final states, so equation (5) becomes,

$$\mathcal{T}_N = \mathcal{M}_{\lambda f}\frac{2M}{(2\pi^3)}\int\int \frac{e^{i(\vec{p}_N-\vec{P}_N).\vec{r}}}{P_N^2 - p_N'{}^2 + i\eta}\,\mathcal{V}(r)\; f_{nln'lL}(p_q')Y_{LM}(\hat{p}_q')d\vec{p}_N{}' \tag{8}$$

Defining $h_{nln'l'LM}$ such that,

$$f_{nln'l'L}(p_q)Y_{LM}(\hat{p}_q) = (2\pi)^{-3/2}\int e^{-i\vec{p}_q.\vec{R}}\,h_{nln'l'LM}(\vec{R})\,d\vec{R} \tag{9}$$

Gottfried obtained,

$$\mathcal{T}_N = -\mathcal{M}_{\lambda f}\frac{M}{(2\pi)^{5/2}}\int \frac{e^{iP_N|\vec{r}-\vec{R}|}}{|\vec{r}-\vec{R}|}\,e^{-i\vec{P}_N.\vec{r}}\,e^{-i\vec{q}_N.\vec{R}}\,\mathcal{V}(r)\,h_{nln'l'LM}(\vec{R})\,d\vec{r}d\vec{R} \tag{10}$$

where

$$\vec{q}_N = \vec{p}_q - \vec{P}_N \tag{11}$$

Gottfried then adopted several simplifying assumptions, one of which was to consider only the $1s$ harmonic oscillator shell. In the following section, Gottfried's treatment is extended to include any set of single-particle orbitals for the bound-state pn-pair.

B. Extension to Higher Orbitals

Direct numerical evaluation of equation (10) is not convenient, so the exponential terms are expanded as partial waves:

$$\frac{e^{iP_N|\vec{r}-\vec{R}|}}{|\vec{r}-\vec{R}|} = 4\pi P_N \sum_{l_1=0}^{\infty}\sum_{m_1=-l_1}^{l_1} j_{l_1}(P_N r_<)h_{l_1}^{\dagger}(P_N r_>)Y_{l_1m_1}^*(\hat{r})Y_{l_1m_1}(\hat{R}) \tag{12}$$

$$e^{-i\vec{P}_N.\vec{r}} = 4\pi \sum_{l_2=0}^{\infty}\sum_{m_2=-l_2}^{l_2} i^{-l_2}j_{l_2}(P_N r)Y_{l_2m_2}^*(\hat{P}_N)Y_{l_2m_2}(\hat{r}) \tag{13}$$

$$e^{-i\vec{q}_N.\vec{R}} = 4\pi \sum_{l_3=0}^{\infty}\sum_{m_3=-l_3}^{l_3} i^{-l_3}j_{l_3}(|q_N|R)Y_{l_3m_3}^*(\hat{q}_N)Y_{l_3m_3}(\hat{R}) \tag{14}$$

An explicit form for $h_{nln'l'LM}$ is also required. Taking the Fourier transform of equation (9) gives,

$$h_{nln'l'LM}(\vec{R}) = (2\pi)^{-3/2} \int f_{nln'l'L}(p)\, Y_{LM}(\hat{p})\, e^{i\vec{p}.\vec{R}}\, d\vec{p} \tag{15}$$

This is recast into a more convenient form by substituting,

$$e^{i\vec{p}.\vec{R}} = 4\pi \sum_{l_4=0}^{\infty} \sum_{m_4=-l_4}^{l_4} i^{l_4} j_{l_4}(pR) Y^*_{l_4 m_4}(\hat{p}) Y_{l_4 m_4}(\hat{R}) \tag{16}$$

into equation (15), and since,

$$\int_{\Omega_p} Y^*_{LM}(\hat{p}) Y_{l_4 m_4}(\hat{p}) d\Omega_p = \delta_{Ll_4}\delta_{Mm_4} \tag{17}$$

equation (15) becomes,

$$h_{nln'l'LM}(\vec{R}) = \left(\frac{2}{\pi}\right)^{1/2} Y_{LM}(\hat{R}) i^L g_{nln'l'L}(R) \tag{18}$$

where

$$g_{nln'l'L}(R) = \int_0^\infty f_{nln'l'L}(p)\, j_L(pR)\, p^2 dp \tag{19}$$

Equations (12), (13), (14) and (18) are then substituted into equation (10). After separating radial and angular parts, and noting that,

$$\int_{\Omega_r} Y^*_{l_1 m_1}(\hat{r}) Y_{l_2 m_2}(\hat{r}) d\Omega_r = \delta_{l_1 l_2}\delta_{m_1 m_2} \tag{20}$$

and

$$\int_{\Omega_R} Y_{l_1 m_1}(\hat{R}) Y_{l_3 m_3}(\hat{R}) Y_{LM}(\hat{R}) d\Omega_R = \frac{\hat{l}_1 \hat{l}_3 \hat{L}}{\sqrt{4\pi}} \begin{pmatrix} l_1 & l_3 & L \\ m_1 & m_3 & M \end{pmatrix} \begin{pmatrix} l_1 & l_3 & L \\ 0 & 0 & 0 \end{pmatrix} \tag{21}$$

a manageable expression for $\mathcal{T}_N$ is obtained,

$$\mathcal{T}_N = -\mathcal{M}_{\lambda f} \frac{8 M_N P_N}{\sqrt{\pi}} \sum_{l_1 l_3} \mathcal{R}_{nln'l'Ll_1l_3}(|\vec{q}_N|, \vec{P}_N)\, \mathcal{A}_{Ll_1l_3}(|\hat{q}_N|, \hat{P}_N) \tag{22}$$

where

$$\mathcal{R}_{nln'l'Ll_1l_3}(|\vec{q}_N|, \vec{P}_N) = \int_R j_{l_3}(|\vec{q}_N|R)\, g_{nln'l'L}(R)\, R^2$$
$$\times \int_r j_{l_1}(P_N r) j_{l_1}(P_N r_<) h^\dagger_{l_1}(P_N r_>) \mathcal{V}(r) r^2 dr dR \tag{23}$$

and

$$\mathcal{A}_{Ll_1l_3}(\hat{q}_N, \hat{P}_N) = (i)^{L-l_1-l_3} \hat{l}_1 \hat{l}_3 \hat{L} \begin{pmatrix} l_1 & l_3 & L \\ 0 & 0 & 0 \end{pmatrix}$$
$$\times \sum_{m_1 m_3} (-)^{m_1+m_3} \begin{pmatrix} l_1 & l_3 & L \\ m_1 & m_3 & M \end{pmatrix} Y_{l_1 -m_1}(\hat{P}_N) Y_{l_3 -m_3}(\hat{q}_N) \tag{24}$$

and $\hat{L} = \sqrt{2L+1}$.

The real part of the optical-model potential is taken as,

$$\mathcal{V}(r) = V_c - V f(x_0) + \left(\frac{h}{m_\pi c}\right)^2 V_{so} \left(\vec{\sigma}.\vec{l}\right) \frac{1}{r}\frac{d}{dr} f(x_{so}) \tag{25}$$

where

$$V_c = \begin{cases} ZZ'e^2/r & \text{if } r \geq R_c \\ (ZZ'e^2/2R_c)(3 - r^2/R_c^2) & \text{if } r \leq R_c \end{cases} \tag{26}$$

$$f(x) = \frac{1}{(1+e^x)}, \quad x = \frac{(r - r_0 A^{1/3})}{a_0} \tag{27}$$

$$\vec{\sigma}.\vec{l} = \begin{cases} l & \text{if } j = l + \frac{1}{2} \\ -(l+1) & \text{if } j = l - \frac{1}{2} \end{cases} \tag{28}$$

and

$$\left(\frac{h}{m_\pi c}\right)^2 \cong 2.000 fm^2 \tag{29}$$

C. Surface Refraction and the Gottfried Factorized Cross Section

The transition amplitude for the (γ, pn) reaction which acknowledges the surface refraction process can be obtained from equations (1), (2) and (22). The approximation $\mathcal{M}_{\lambda\alpha} \sim \mathcal{M}_{\lambda f}$, ensures that the transition amplitude can still be factorized into two terms, representing the photo-absorption matrix element and the momentum density for the pn-pair. Hence it is possible to write the momentum density as,

$$\mathcal{F}_{nln'l'}(p_q) = (\mathcal{T}_0 + \mathcal{T}_n + \mathcal{T}_p) / \mathcal{M}_{\lambda f} \tag{30}$$

The momentum densities, for direct $F(p_q)$, or direct+refraction $\mathcal{F}(p_q)$, may be inserted to any Gottfried factorized cross section. For example, the QDM expression for the coplanar $(\phi_n = \phi_p - \pi)$ (γ, pn) cross section is given by[2]

$$\left.\frac{d^4\sigma_{qdm}}{dT_n d\Omega_n dT_p d\Omega_p}\right|_{nlj;n'l'j'} = c_1 L \frac{N_{nlj} Z_{n'l'j'}}{A} F_{nln'l'}(p_q) \left\{\frac{d\sigma_d(E'_\gamma, \vartheta''_p)}{d\Omega''_n}\right\} J_{pn} p_q^2 \delta_{\vec{p}_q} \tag{31}$$

so that by replacing $F(p_q)$ in equation (31) with $\mathcal{F}(p_q)$, the QDM is modified to include the surface refraction process.

D. Results of the Surface Refraction Correction

The present calculation of the surface refraction process for the (γ,pn) reaction is more general than the QDM, however, so the importance of the surface refraction process is examined by a comparison of numerical results for $F(p_q)$ and $\mathcal{F}(p_q)$ for the ^{40}Ca(γ,pn) reaction. Comparisons are presented in figures 3-6 for a variety of initial and final states. The initial states are specified by the photon energy and the single-particle shells $(nlj;n'l'j')$ from which the pn-pair is ejected. The references to j and j' are needed to fix the relationship between the internal and external kinematics, and to determine the spin-orbit contribution to the optical-model potential. The final states are limited to co-planar photonucleon momenta. This restriction has *not* been used in deriving the transition amplitude of equation (22), but it is adopted now to limit the choices of interesting final states to be examined.

In the deeply bound orbitals the number of quasi-deuteron pairs is relatively small, and the probability that the photonucleons escape is expected to be significantly lower than for the outer shells. Hence the cross section arising from the photodisintegration of pn-pairs whose constituent nucleons occupy the $1s$ and $1p$ orbitals is fairly small, and is restricted to large missing energies ($E_m=E_\gamma$-T_n-T_p-T_{A-2}-$Q_{(\gamma,pn)}$). In addition, the results for the $1p_{1/2}$ and $1p_{3/2}$ are quite similar because the single-particle energies[2] do not distinguish the spin-orbit splitting of the $1p$ harmonic oscillator level, and the spin-orbit contribution to the optical-model potential[6] is less important than the Coulomb and volume absorption terms. Hence, the momentum densities shown in figures 3-6 are restricted to $1p$ and $2s-1d$ shells.

An inspection of figures 3-6 shows that,

1. For the lower photon energies the refraction process enhances the momentum density in all shells, indicating that the Born approximation is failing.

2. For the higher photon energies the fine details in the shape of the momentum density alter when the refraction process is acknowledged.

3. The gross details in the shape of the momentum density are not changed except at small opening angles where the enhancement due to the refraction process is very large.

4. The sharp minima, occurring in the momentum densities for pn-pairs with a unique L, are smoothed out.

The gross shape of the momentum density is almost unaltered, so it may be concluded that small angle scattering amplitudes dominate the refraction terms in the transition amplitude, particularly at large photon energies. This is an important point. It means that the on- and off-shell kinematics are usually fairly similar, and this lends some support to the approximation, $\mathcal{M}_{\lambda\alpha} \sim \mathcal{M}_{\lambda f}$.

The quality of the results depends on the accuracy with which the momentum density and scattering amplitudes are calculated, on the reliability of the assumption $\mathcal{M}_{\lambda\alpha} \sim \mathcal{M}_{\lambda f}$, and on the validity of the Born approximation.

Gottfried estimates that a momentum density calculated from realistic wavefunctions would be experimentally indistinguishable from the harmonic oscillator case, although more recent studies by Zabolitzky[12] indicate that short-range correlations substantially enhance the momentum density at very large bound-state pn-pair momenta. Zabolitzky's findings suggest that the simple harmonic oscillator wavefunctions used in the present treatment will lead to an underestimation of the momentum density at small opening angles, and therefore to an overestimation of the enhancement due to the refraction process. It is important to note that although the (initial-state) NN-correlations and the surface refraction process are two distinct considerations in any photonuclear reaction calculation, their influence on the momentum density (for a Gottfried factorized cross section) is very similar.

The scattering amplitudes might be adequately represented provided the nucleon energy is not too small, but it must be admitted, for the lower photon energies of interest here, the photonucleons are not well represented by plane waves. Further problems arise because the kinematics describing the photo-absorption process for the direct and refraction terms are very different when the pair momenta is (assumed by the direct process to be) very large. This means the kinematics that should be used to calculate the on- and off-shell photo-absorption matrix elements are quite different, and hence the approximation $\mathcal{M}_{\lambda\alpha} \sim \mathcal{M}_{\lambda f}$, may become unreliable.

At small opening angles, where the direct reaction requires the pn-pair momentum to be very large, the momentum density for the direct term becomes very small. Here the refraction terms completely dominate the transition amplitude. The very large enhancement of the momentum density arises principally because the refraction terms are integrated over the internal phase space, and thus include contributions from the more readily available small pn-pair momenta. However, the Born approximation is not reliable when the refraction terms dominate the transition amplitude.

The refraction process should not alter the photo-absorption strength for transitions from the initial state to any definite intermediate state; it should serve only to redistribute the strength into different final states. Hence, an accurate Born approximation calculation should satisfy the normalization requirement for the probability density function,

$$4\pi \int p_q^2 \mathcal{F}_{nln'l'}(p_q) dp_q = 1 \qquad (32)$$

The present calculation is expected to be accurate only when both photonucleons have large kinetic energies and the pn-pair momentum is relatively small, i.e. at large photon energies, and at nucleon emission angles where the cross section is close to its maxima. At lower nucleon energies, the results are interesting mainly because they establish that the refraction corrections are significant, and should not be ignored. Since a better (distorted wave) calculation would probably retain the present approach as a leading term, it may be that the present results give the correct general trends even at lower nucleon energies, although this remains to be seen.

E. The Inclusive Photonucleon Reaction

The discussion of the surface refraction effect has so far assumed a (γ, pn) reaction and has therefore required specification of both the neutron and proton momenta in the final state. Since the inclusive (γ, n) cross section at high missing energy (E_m=E_γ-T_n-T_{A-1}-$Q_{(\gamma,n)}$) is dominated by the (γ, pn) reaction[5], the inclusive (γ, n) cross section can be calculated by integrating the calculated (γ, pn) cross section over the phase space of the (undetected) photoproton. (The same argument applies to the calculation of the (γ, p) cross section).

Under these circumstances the transition amplitude is given by,

$$\mathcal{T}^{\lambda f}_{nln'l'LM} = \mathcal{T}_0 + \mathcal{T}_n \tag{33}$$

i.e., if the transition amplitude is to be integrated over all photoproton phase space, there is no need to calculate the proton scattering amplitudes. A numerical evaluation of this form is a straightforward extension of the procedures outlined earlier, but is quite cumbersome, and has not been attempted.

The importance of the nucleon refraction process in the inclusive (γ, n) reaction is expected to be considerably less important than for the (γ, pn) reaction. For the inclusive (γ, n) reaction, where the proton is free to emerge at any angle, experiments cannot select out final states where a direct reaction requires extreme pn-pair momenta. Hence the data is dominated by events with small pn-pair momentum. In the (γ, pn) reaction, these kinematical conditions are found to produce only minor changes to the momentum density.

III. ABSORPTION CORRECTION

A. Introduction

The nucleon transparency η, is the probability that a single photonucleon will escape the nucleus without interaction. As noted earlier, the absorption correction is too strong to be reliably treated by the Born approximation. However, earlier calculations[9,7] have estimated the nucleon transperancy as,

$$\eta = \frac{1}{V}\int e^{-X(\vec{r})/\lambda} d\vec{r} \tag{34}$$

where the integration is restricted to the nuclear volume, V, and,

λ is the mean free path of the nucleon (or pion) inside the nucleus,

X is the thickness of the nucleus through which the nucleon must travel in order to escape, and this depends on the starting position $\vec{r}$ inside the nuclear volume.

There are a few implicit assumptions that are inappropriate for the present purposes:

1. The density distribution of photo-absorption sites is taken as uniform throughout the nucleus,
2. The nuclear volume is taken as a sphere of definite radius R,
3. The nuclear density is assumed to be constant,
4. The mean free path is assumed to be independent of neutron kinetic energy.

B. Development of Earlier Treatments

The estimation of η can easily be improved by replacing equation (34) with,

$$\eta_{nl}(Tn) = \frac{\int \rho_{nl}(r) e^{-\sigma_{NN'}(T_n)\int_{t=0}^{\infty}\rho_A(r'(t))dt} d\vec{r}}{\int \rho_{nl}(r) d\vec{r}} \tag{35}$$

where the volume integrations extend over all space, and

t is the position on the escape trajectory, defined to start at the photo-absorption site, (r, θ, ϕ), and be directed towards the detector,

$\rho_A(r'(t))$ is the nuclear density at position t on the escape trajectory,

$\rho_{nl}(r)$ is the density of nucleons in the harmonic oscillator shell (nl) at radius r.

$\sigma_{NN'}(T_n)$ is the cross section for a neutron of kinetic energy T_n, interacting with the residual (A-2) nucleus.

Ideally, the cross section, $\sigma_{NN'}$ governing the photonuclcon absorption process should taken as the sum of cross sections for reactions induced by the nucleon for the residual (A-2) system, except those for which the final state consists exclusively of the continuum nucleon and the (A-2) nucleus. Unfortunately, these cross sections are presently unavailable, so $\sigma_{NN'}$ is approximated with the ^{40}Ca neutron reaction cross section[8].

Inclusion of the nucleon absorption process requires the (γ, pn) cross section to be scaled by η^2, and the (γ, N) cross section to be scaled by η. This treatment of the nucleon transparency implies the absorption processes for the neutron and proton are taken to be completely independent. Although such an approximation is commonly used[4,10,11], it is not strictly true. Clearly if one of the nucleons does not escape, the reaction Q must change, and the phase space in the final state will depend on the energies of the excited states in a different residual nucleus. However, within the present simplistic treatment, a more rigorous analysis of the available phase space for the absorption process is not warranted.

C. Results of the Absorption Correction

The probability that a photoneutron will escape the nucleus without being absorbed, or lost to a different reaction channel, is given by equation (35), and the neutron energy dependence is shown for various harmonic oscillator shells in figure 7. The results seem intuitively reasonable, with the probability of escape being isotropic, and greater for high nucleon energies and for valence orbitals. For the present calculation, the dependence on single-particle orbitals is more important than nucleon energy.

IV. CONCLUSIONS

For final states where the direct reaction requires a very large pn-pair momentum, the direct term in the transition amplitude becomes very small. However, the surface refraction process offers a mechanism where pn-pairs with smaller momentum can contribute. In such cases the surface refraction process completely dominates the cross section, and the direct assumption is expected to be quite wrong. However, these same conditions of very large pair momenta render the approximation $\mathcal{M}_{\lambda\alpha} \sim \mathcal{M}_{\lambda f}$ rather doubtful, and the Born approximation becomes unreliable. At low nucleon energies, additional problems are expected from the use of plane waves to represent the continuum photonucleons. Future calculations should seek primarily to avoid the use of plane waves. The calculation presented here is most likely to be accurate at large nucleon energies where the opening angle is near that at which the momentum density is maximum. At smaller photonucleon energies, the present results are numerically unreliable, but this stems principally from the intuitive result that the refraction process makes large alterations to the internal (intermediate state) phase space available to any definite final state. There is a clear need for a better (distorted wave) treatment of the surface refraction process.

The importance of the nucleon refraction process in the inclusive (γ, n) reaction is expected to be considerably less important. This follows directly from phase space considerations.

The neutron absorption correction is presented as a simple development of earlier treatments, and is shown to cause a depletion of the cross section by a factor dependent on both the neutron energy and, more importantly, on the bound-state orbital from which the nucleon is ejected.

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[1]K. Gottfried, Nucl. Phys. **5**, 557, (1958).

[2]J. A. Eden, Ph.D thesis, The University of Melbourne, (1990), unpublished.

[3]J. L. Matthews, M.I.T. Laboratory for Nuclear Science, Internal Report 112 (1967), (unpublished).

[4]H. Schier and B. Schoch, Nucl. Phys. **A229**, 93 (1974).

[5]J. A. Eden, G. J. O'Keefe, R. P. Rassool, D. J. McLean, T. Suda, I. Nomura, J. Yokokawa, O. Konno, T. Terasawa and Y. Torizuka, Paper submitted for publication in Phys. Rev. C., (1991).

[6]C. M. Perey and F. G. Perey, Atomic Data and Nuclear Data Tables, **17**, 1, (1976).

[7] K. A. Brueckner, R. Serber, and K. M. Watson, Phys. Rev. **84**, 258, (1951).

[8] H. S. Camarda, T. W. Phillips and R. M. White, Phys. Rev. **C34**, 810, (1986).

[9] S. Fernbach, R. Serber, and T. B. Taylor, Phys. Rev. **75**, 1352, (1949).

[10] P. D. Harty, Ph.D thesis, The University of Melbourne, (1986).

[11] T. Suda, Ph.D. thesis, Tohoku University (1987), unpublished, and privite communication.

[12] J. G. Zabolitzky and W. Ey, Phys. Lett. **76B**, 527, (1978).

Figure Captions

FIG. 1. Pictorial representation of direct photo-absorption by a pn-pair.

FIG. 2. Pictorial representation of surface refraction contributions to the transition amplitude for (i) neutron refraction, and (ii) proton refraction.

FIG. 3. 'Direct' (dotted line) and 'Direct + Refraction' (solid line) quasi-deuteron momentum densities for E_γ = 100 MeV, $T_n/T_p = 1$, and $\theta_n = 45°$

FIG. 4. 'Direct' (dotted line) and 'Direct + Refraction' (solid line) quasi-deuteron momentum densities for E_γ = 100 MeV, $T_n/T_p = 1$, and $\theta_n = 45°$

FIG. 5. 'Direct' (dotted line) and 'Direct + Refraction' (solid line) quasi-deuteron momentum densities for E_γ = 300 MeV, $T_n/T_p = 1$, and $\theta_n = 45°$

FIG. 6. 'Direct' (dotted line) and 'Direct + Refraction' (solid line) quasi-deuteron momentum densities for E_γ = 300 MeV, $T_n/T_p = 1$, and $\theta_n = 45°$

FIG. 7. Neutron transparency for specified harmonic oscillator shells.

Figure 1 of "A treatment...."

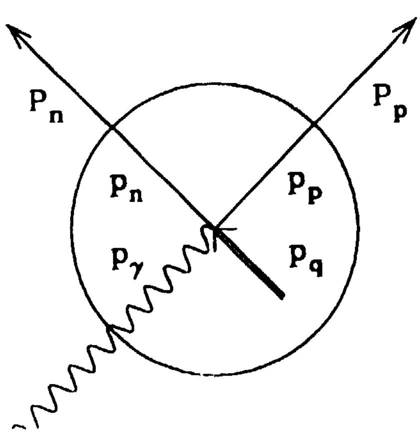


Figure 2 of "A treatment .... "

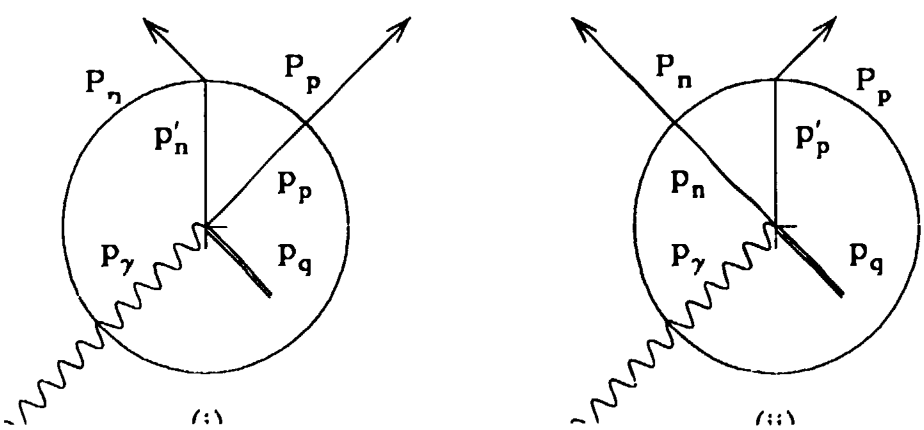


Figure 3 of "A treatment ..."

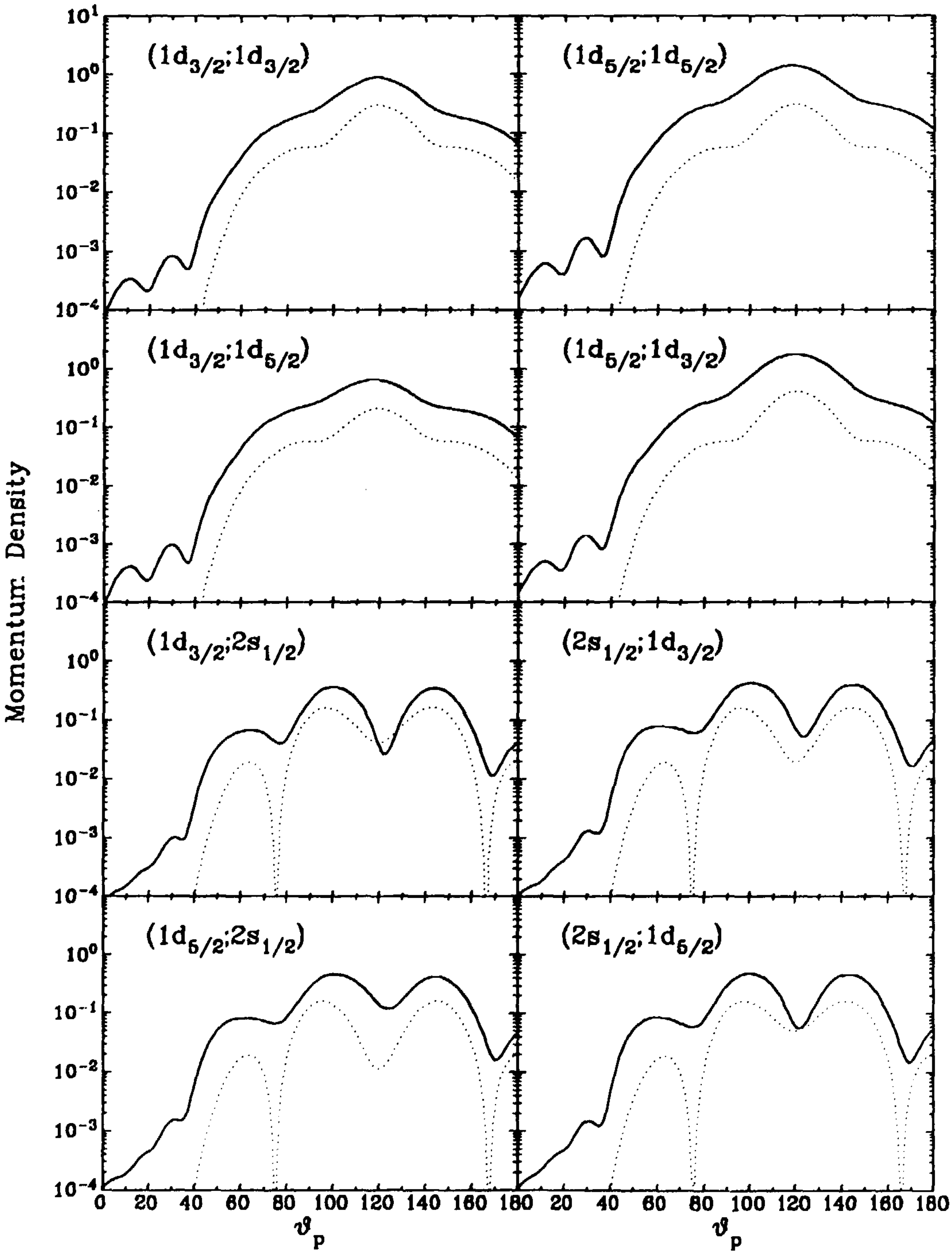


Fig 2. of "A treatment ...

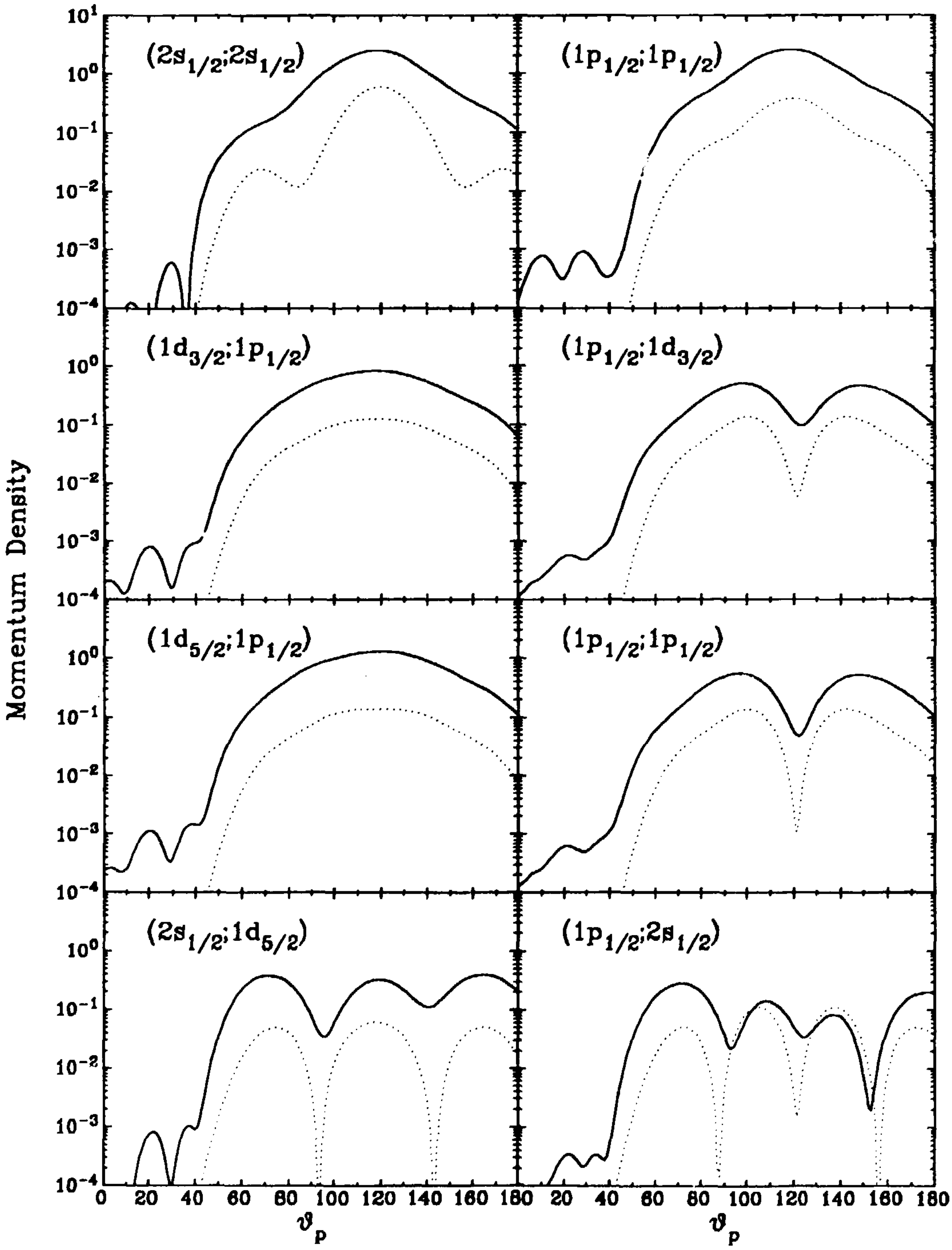


Figure 5 of "A treatment ...."

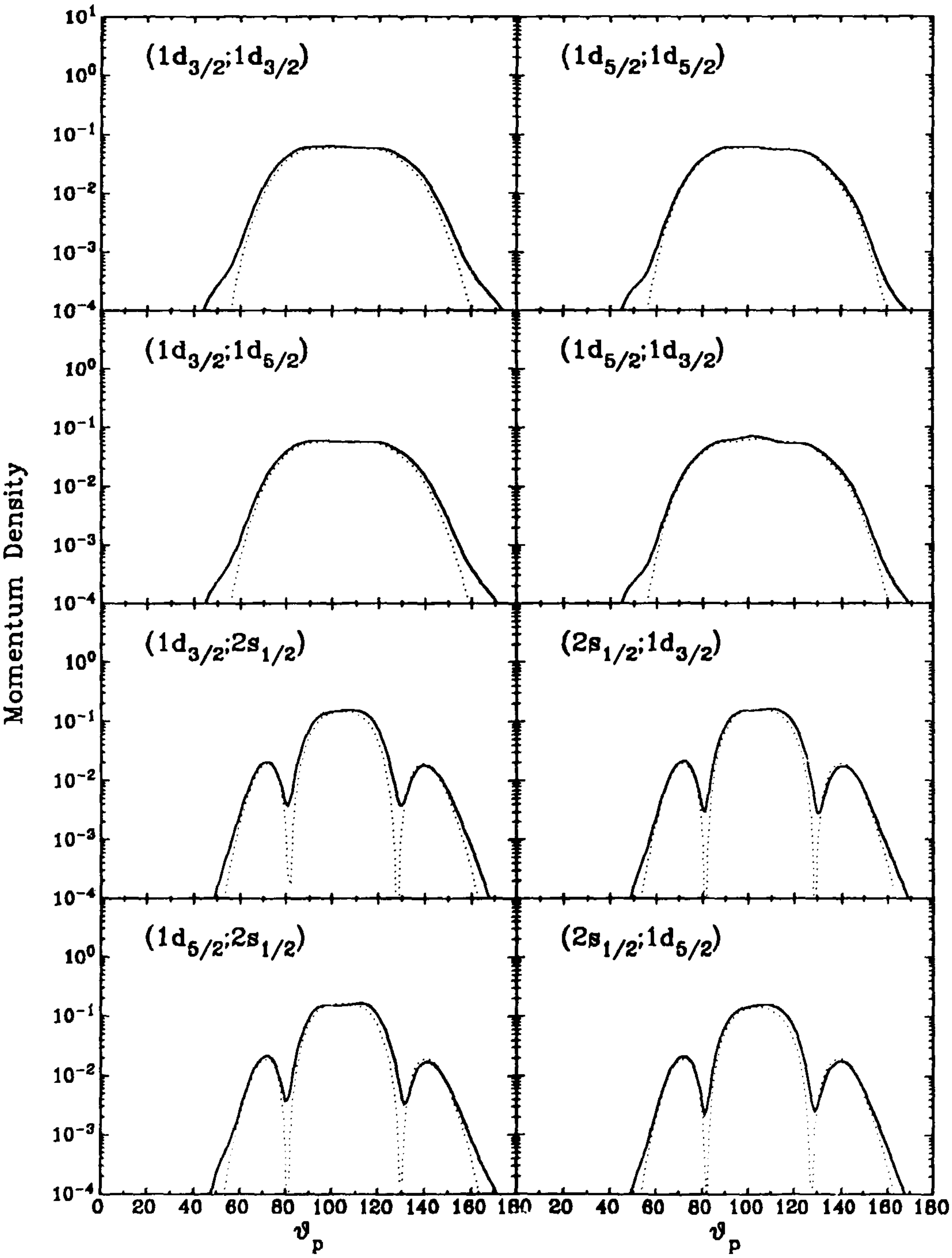
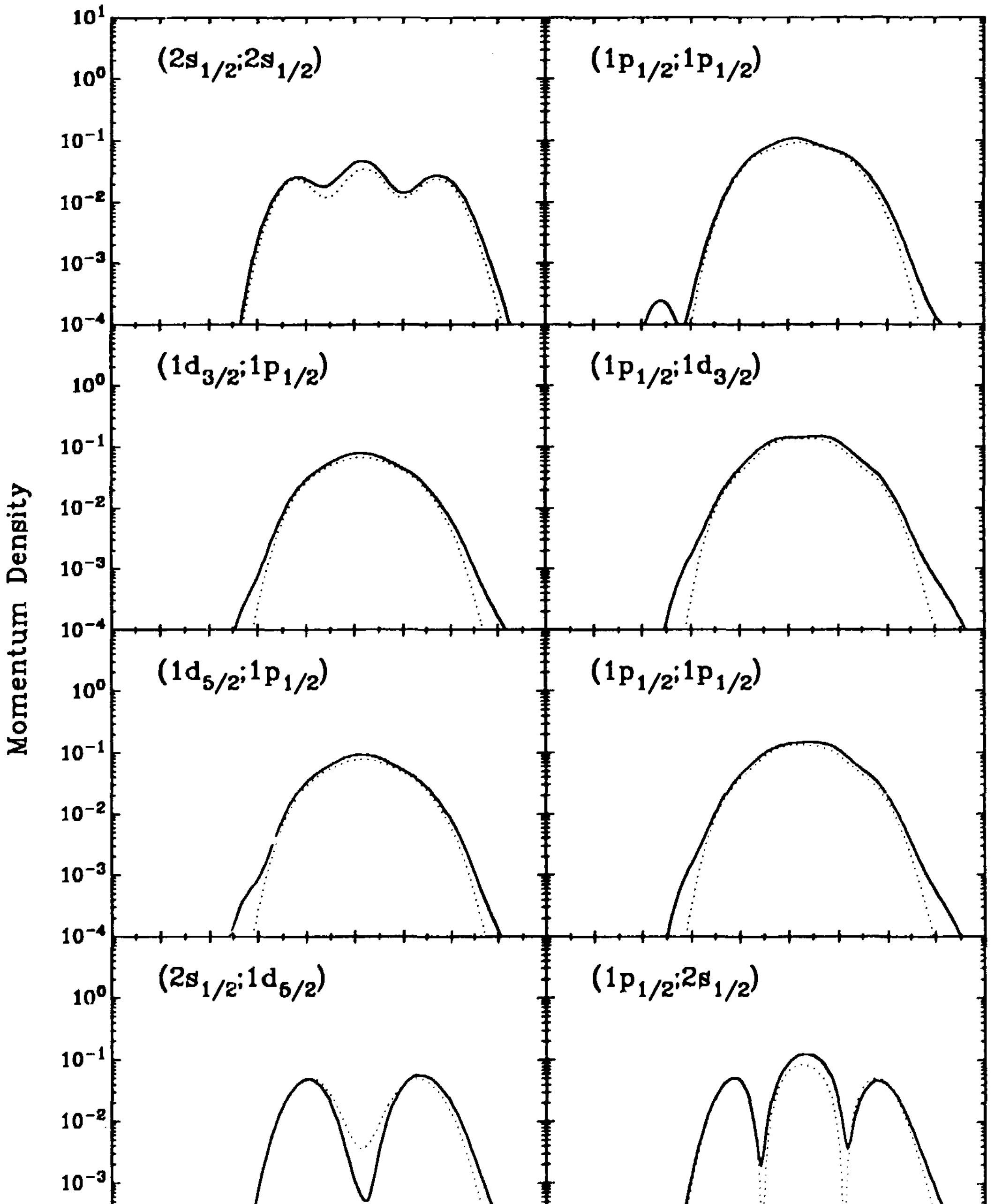
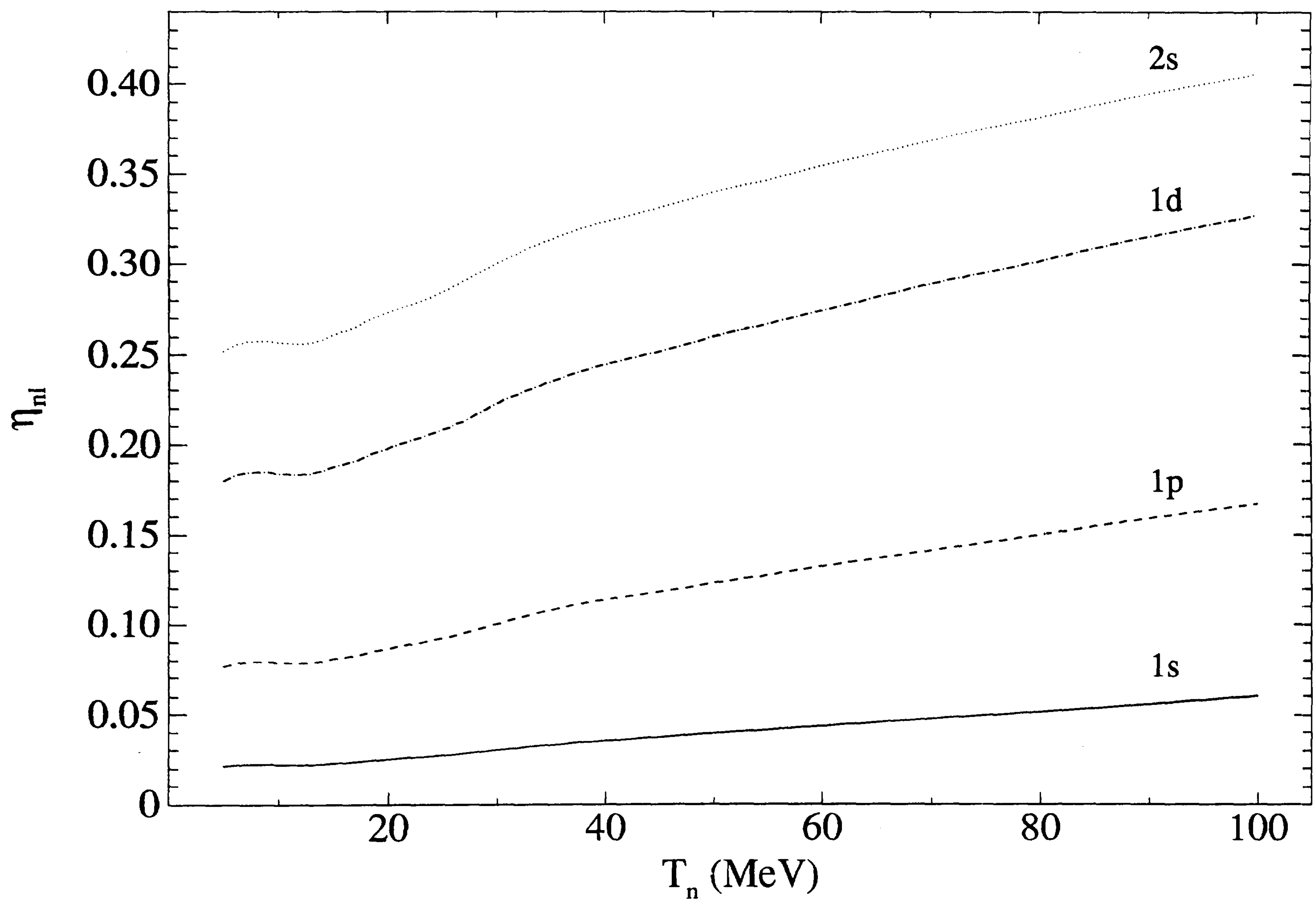


Figure. 6 of "A treatment ... "





2s
1d
1p
1s
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0
20
40
60
80
100
η_{nl}
T_n (MeV)