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ABSTRACT

A massive Yang–Mills field theory with conformal invariance [1] and gauge invariance is proposed. It involves gravitational and various gauge interactions, in which all the mass terms appear as a uniform form of interaction $m(x) = K\Phi(x)$. When the conformal symmetry is broken spontaneously and gravitation is ignored, the Higgs field emerges naturally, where the imaginary mass μ can be described as a background curvature.

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1. Introduction

The mass origin of the bosons is one of the fundamental problems in gauge field theories. In the Weinberg-Salam model[2] the masses of $W^{\pm}$ and Z^0 are generated via the Higgs mechanism[3]. However, the Higgs mechanism is a prioritheoretically, the Lagrangian

$$L_\phi = -\frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \frac{\mu^2}{2}\phi^2 - \frac{\lambda}{4}\phi^4. \tag{1}$$

requires the Higgs field ϕ to have an imaginary mass μ and a quartic self-interaction term with $\lambda > 0$, meanwhile the existence of Higgs mesons is so far unobserved experimentally. Therefore, studying its origin or finding an alternative reasonable theory is meaningful.

Massive Yang-Mills field theory with gauge invariance[4] was an alternative, its Lagrangian for $SU(n)$ case is that

$$L = \frac{1}{2}Tr(F^{\mu\nu}F_{\mu\nu}) + \frac{1}{2}m^2Tr(A_\mu - \partial_\mu UU^{-1})^2 \tag{2}$$

where $U(x)$ is the element field of $SU(n)$ group and $\partial_\mu UU^{-1}$ is associate pure gauge. The theory is equivalent with the nonlinear σ-model. Unfortunately, it is not a renormalizable theory[5].

At present, the conformal symmetry brings wide attention, this is mainly because of: (1). the scaling invariance of particles in high energy, (2). Hung Cheng's [6] introduction of Weyl's vector meson to absorb remaining degree of freedom. the magnitude of the Higgs field. (3). The coupling constant in Einstein's gravitation being dimensional.

In this note we propose a massive Yang-Mills field theory with conformal invariance, in which gravitational and various gauge interactions have dimensionless coupling constants and all the mass terms become uniform form of interaction $m(x) = K\Phi(x)$. When the conformal symmetry is broken spontaneously, the Higgs field theory emerges. It is renormalizable and unitary with gravitation ignored.

2. Massive gauge field theory with conformal invariance

The conformal transformation means a local scale transformation satisfied by the metric $g_{\mu\nu}(x)[\eta_{\mu\nu} = (-+++)]$ and the general field function $F(x)$,

$$g'_{\mu\nu}(x) = \Omega^2(x)g_{\mu\nu}(x), \qquad F'(x) = \Omega^w(x)F(x). \tag{3}$$

where $\Omega(x)$ is a continuous, non-vanishing real function and w is called as Weyl weight of $F(x)$. The convariant derivative can be defined as

$$d_\mu = \partial_\mu + wb_\mu(x), \qquad (d_\mu F)' = \Omega^w d_\mu F. \tag{4}$$

where $b_\mu(x)$ is Weyl gauge field and its field express is $H_{\mu\nu} = \partial_\mu b_\nu - \partial_\nu b_\mu$. There exists pure gauge for the conformal group C,

$$\hat{b}_\mu = \phi^{-1}\partial_\mu\phi, \qquad if \;\; \hat{H}_{\mu\nu} = 0. \tag{5}$$

where $\phi(x)$ is the Weyl scalar field with $w = -1$.

By use of $\phi(x)$ and $g_{\mu\nu}(x)$ we can construct the intergrable (metricable) Weyl geometry[7], in which metric, affine connection and scalar curvature are

$$\hat{g}_{\mu\nu} = \phi^2 g_{\mu\nu}, \tag{6}$$

$$\hat{\Gamma}^\lambda_{\mu\nu} = \Gamma^\lambda_{\mu\nu} - \phi^{-1}(\delta^\lambda_\mu\partial_\nu\phi + \delta^\lambda_\nu\partial_\mu\phi - g_{\mu\nu}\partial^\lambda\phi), \tag{7}$$

$$\hat{R} = \phi^{-2}(R - 6\phi^{-1}\Box\phi). \tag{8}$$

respectively. Then the Lagrangian describing spacetime should be

$$L_G = \sqrt{-\hat{g}}(\frac{\alpha}{4}\hat{R} + \frac{\lambda}{4}) = \sqrt{-g}\{\frac{\alpha}{4}\phi^2(R - 6\phi^{-1}\Box\phi) + \frac{\lambda}{4}\phi^4\}. \tag{9}$$

and the Lagrangian for the gauge field $A_\mu = iI_a A^a_\mu$ is

$$L_A = \sqrt{-g}\{\frac{1}{2g_c^2}Tr(F_{\mu\nu}F^{\mu\nu}) + \frac{1}{2n}K^2\phi^2 Tr(A_\mu - \partial_\mu UU^{-1})^2\} \tag{10}$$

which is conformal invariant and K is a dimensionless constant. Generally speaking, the theory involving Eqs.(9) and (10) is not renormalizable with gravitation ignored.

Let us put $K^2 = 3\alpha$, The total Lagrangian becomes

$$L = \sqrt{-g}\{\frac{\alpha}{4n}Tr(\Phi^\dagger\Phi)R - \frac{\lambda}{4n^2}(Tr\Phi^\dagger\Phi)^2 + \frac{1}{2g_c^2}Tr(F_{\mu\nu}F^{\mu\nu}) - \frac{1}{2n}K^2 Tr[D_\mu\Phi(D^\mu\Phi)^\dagger]\}. \tag{11}$$

where $\Phi = \phi U$ is the element field of $C \times SU(n)$ direct product group, $\phi(x)$ is its magnitude and $U(x)$ the phase, as well as $D_\mu\Phi = \partial_\mu\Phi - A_\mu\Phi$. This is the massive gauge field theory with conformal and gauge invariance and has following character:

(1). The $g_{\mu\nu}(x)$ and $\phi(x)$ describe gravitation, being of the geometry field, A_μ is the matter field, and all the coupling constants are dimensionless.

(2). In the above equation the last term is just a linear σ-model in the integrable Weyl space, i.e.,

$$-\frac{1}{2n}K^2 Tr[D_\mu\Phi(D^\mu)^\dagger] = \frac{1}{2n}K^2(\Phi\Phi^\dagger)Tr(A_\mu - \partial_\mu\Phi\Phi^{-1})^2 \tag{12}$$

where $\Phi(x)$ is a linear field,

$$\Phi(x) = \frac{1}{2}\phi_0 I_0 + i\phi_a I_a \tag{13}$$

This means that introducing the conformal symmetry may improve renormalizability of the massive Yang-Mills field theory.

(3). The Lagrangian for the spinor field ($w = \frac{3}{2}$) with $SU(n)_L \times SU(n)_R$ chiral and conformal symmetries is

$$L_\psi = \sqrt{-g}\{\bar{\psi}\Gamma^\mu D_\mu \psi - K_\psi(\bar{\psi}_L \Phi \psi_R + \bar{\psi}_R \Phi^\dagger \psi_L)\}. \tag{14}$$

Therefore, in the theory all the mass terms appear as a uniform form of interaction $m(x) = K\Phi(x)$.

(4). For the case of $SU(2) \times U(1)$ group, this theory corresponds to the Weinberg-Salam model with local scale invariance. It is different from reference[6] because we do not introduce the Weyl gauge field $b_\mu(x)$.

3. Spontaneous breaking symmetries and Higgs mechanism

At the limit of low energy, the quantum effect should result in the spontaneous breaking of the conformal and gauge symmetries. From the Lagrangian (11) we can read off the field equations of $g_{\mu\nu}$, Φ and A_μ as

$$\begin{aligned} &\tfrac{\alpha}{4n} Tr(\Phi^\dagger\Phi)(R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R) + \tfrac{1}{4n}Tr[g_{\mu\nu}\Box(\Phi^\dagger\Phi) - \nabla_\mu\partial_\nu(\Phi^\dagger\Phi)] + \tfrac{\lambda}{8n^2}K^2(Tr\Phi^\dagger\Phi)^2 \\ &= \tfrac{1}{g_c^2}Tr(F_{\mu\alpha}F^\alpha_\nu - \tfrac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}) - \tfrac{1}{4n}K^2 Tr[g_{\mu\nu}D_\alpha\Phi(D^\alpha\Phi)^\dagger - 2D_\mu\Phi(D_\nu\Phi)^\dagger]. \end{aligned} \tag{15}$$

$$(\Box + \frac{\alpha}{2K^2}R)\Phi = \frac{\lambda}{nK^2}[Tr(\Phi^\dagger\Phi)]\Phi, \tag{16}$$

$$\tilde{D}_\nu F^{\mu\nu} + \frac{1}{4n}K^2 g_c^2[\Phi(D^\mu\Phi)^\dagger - (D^\mu\Phi)\Phi^\dagger] = 0. \tag{17}$$

where $\tilde{D}_\mu = \nabla_\mu - A_\mu$ and $\Box = \tilde{D}_\mu\tilde{D}^\mu$.

Taking the vacuum expectation values, we have

$$\langle A_\mu\rangle_o = 0, \quad \langle\psi\rangle_o = 0, \quad \langle\phi\rangle_o = v, \quad \langle U\rangle_o = I. \tag{18}$$

where v is a non-vanishing constant and I unit matrix. Then the solution of $\phi(x)$ and $U(x)$ will cause the spontaneous breaking of conformal and gauge symmetries. As well as the solution of the metric is that

$$\langle R\rangle_o = \frac{2\lambda v^2}{\alpha,} \quad \langle g_{\mu\nu}\rangle_o = \eta_{\mu\nu} + \frac{kx_\mu x_\nu}{1 - kx^\alpha x_\alpha} \tag{19}$$

where $k = -\frac{1}{12}\langle R\rangle_o$. This is of the constant curvature solution, i.e., the maximally symmetric space that describes a stable vacuum state.

Ignoring the perturbation of quantum gravitation in a small range, we can use $\eta_{\mu\nu}$ and $\langle R\rangle_o$ instead of $g_{\mu\nu}$ and R, then the Lagrangian (11) becomes

$$L = \frac{1}{2g_c'^2} Tr(F_{\mu\nu}F^{\mu\nu}) - \frac{1}{2} Tr[D_\mu\Phi(D^\mu\Phi)^\dagger] - \frac{1}{2}\mu^2 Tr(\Phi^\dagger\Phi) - \frac{\lambda'}{4}[Tr(\Phi^\dagger\Phi)]^2 \qquad (20)$$

where

$$\mu^2 = -\frac{1}{6}\langle R\rangle_o = -\frac{1}{3\alpha}\lambda v^2. \qquad (21)$$

When $\lambda > 0$, one obtains the Higgs mechanism naturally, in which the imaginary mass μ is associated with the background curvature $\langle R\rangle_o > 0$, and the quartic self-interaction term is required by the conformal symmetry. Now the theory is renormalizable and unitary.

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