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ABSTRACT

We show that the scattering of elementary particles on solitons (monopoles, fluxons, etc.) in supersymmetric gauge theories violates the relations dictated by supersymmetry at tree level. The violation arises because of the discrepancy between the spectra of bosonic and fermionic fluctuations and because of the fermionic nature of the supersymmetry generators.

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In several recent papers [1-5] we discussed the interesting, yet controversial claim that non-perturbative effects break supersymmetry (SUSY) explicitly in supersymmetric gauge theories. The most important results emerge from a new study of the role of instantons in SUSY gauge theories [2,4]. Previous treatments of the problem [6] were always limited to the semi-classical approximation. In this approximation the tunneling amplitude vanishes because of the presence of fermionic zero modes in the classical background field of the instanton. In refs. [2,4] we studied the effect of perturbations. We found that generically the zero modes become low energy resonances. The origin of this effect is the mixing between gaugino and matter zero modes through the scalar fields and the absence of an energy gap separating the zero modes from the continuous spectrum.

As a result of the instability of the fermionic zero modes, the tunneling amplitude is in fact non-zero. Its magnitude is determined by the number of resonances (which is the same as the number of zero modes in the undistorted instanton background). Each pair of resonances reduces the tunneling amplitude by one power of the coupling constant. Since tunneling lowers the vacuum energy density compared to its perturbative value, the vacuum energy density becomes negative. This, in turn, implies an explicit breaking of SUSY as a negative vacuum energy is incompatible with the SUSY algebra.

The instability of the fermionic zero modes is an important effect with applications that run beyond the scope of supersymmetric theories. But given the theoretical as well as phenomenological significance of an explicit non-perturbative breaking of SUSY, we feel that one should try to understand this phenomenon in a broader framework. Also, the instability of the zero modes requires the introduction of special techniques. (The reason is the non-analytic dependence of the fermionic spectrum on the scalar perturbations). We believe that progress towards a better understanding of the explicit breaking of SUSY by non-perturbative effects could be made by proving its existence with a minimal amount of technicalities. By reducing the number of ingredients of the proof to a minimum we hope to identify the key factors responsible for SUSY breaking.

In this letter we calculate the S-matrix for elementary particle – soliton scattering in SUSY gauge theories. We show that the relations between S-matrix elements required by SUSY are violated at *tree level*, and that the breaking of SUSY is an *explicit* one. One needs only the most elementary properties of the soliton sector in the derivation of this result. The violation arises because of (a) the discrepancy between the spectra of bosonic and fermionic fluctuations in the soliton background and (b) the fermionic character of the SUSY generators. The role of each of these factors will be discussed in detail below.

It may be helpful to have some specific models in mind before we begin with the actual derivation. In four dimensions, the soliton is the 'tHooft-Polyakov monopole [7]. The minimal SUSY-Higgs theory which has monopoles in its spectrum (see e.g. ref. [3]) consists of an SU(2) gauge supermultiplet (A^a_μ, λ^a), a matter supermultiplet in the adjoint representation (Φ^a, χ^a) and a neutral supermultiplet (Φ_0, χ_0). The superpotential is $W = y\Phi_0(\Phi^a\Phi^a - v^2)$ where y is a Yukawa coupling. The minimum of the corresponding scalar potential is $\langle\Phi^a\rangle = v\delta_{3a}$ and $\langle\Phi_0\rangle = 0$. At the classical level the gauge symmetry is broken to U(1) while SUSY remains unbroken. The elementary particle spectrum consists of several massive supermultiplets and the massless photon supermultiplet.

In three dimensions a solitonic excitation - the Nielsen-Olesen fluxon [8] - exists in the abelian Higgs model. A discussion of the SUSY version of this model can be found in refs. [1,9]. The structure of this model is very similar to the four dimensional one but there is one important difference: in the SUSY abelian Higgs model all particles are massive.

The discrepancy between the spectra of bosonic and fermionic fluctuations can be easily quantified. Let H_B and H_F be the differential operators which define the bosonic and fermionic fluctuations in the soliton background. In the $A_0 = 0$ gauge, the following identity holds if the domain of H_B is restricted to eigenstates satisfying Gauss' law [1,9]

$$(H_F^2)_{I'J'}\Gamma_{J'J} - \Gamma_{I'I}(H_B)_{IJ} = \gamma_0 V_{I'JK'}\delta\Psi^0_{K'} \,. \tag{1}$$

The ingredients of eq. (1) are as follows. H_F (H_B) is defined from the quadratic part of the fermionic (bosonic) hamiltonian. It is assumed that the left-handed fermion fields together with their charge conjugates are assembled into four component Majorana fields. The primed (unprimed) indices stand for all fermionic (bosonic) indices. $\Gamma_{J'J}$ is a matrix which serves to match the indices of bosonic and fermionic fields belonging to the same supermultiplet. Its non-zero entries are $\Gamma_{\chi\phi} = \frac{1}{2}(1-\gamma_5)$, $\Gamma_{\chi\phi^*} = \frac{1}{2}(1+\gamma_5)$ and $\Gamma_{\lambda A_k} = -(i/\sqrt{2})\gamma_k$. $\delta\Psi^0_{K'}$ is a matrix whose columns are the four (unnormalized) fermionic zero modes. It is obtained by substituting the classical soliton field in the SUSY variation of the fermions. $V_{I'JK'}$ is defined by writing the fermionic interaction hamiltonian as

$$H_F^{int} = \frac{1}{2}\int d^d x\, \overline{\Psi}_{I'} V_{I'JK'}\phi_J \Psi_{K'} \equiv \frac{1}{2}(\Psi|\gamma_0 V(\phi)|\Psi)\,. \tag{2}$$

In eq. (2) $\Psi_{I'}$ (ϕ_J) is a generic name for all Fermi (Bose) fields, and $\overline{\Psi} = \Psi^\dagger\gamma_0 = \Psi^T C$ where C is the charge conjugation matrix. The entries of $V_{I'JK'}$ are therefore products of coupling constants and Dirac matrices. Notice that the r.h.s. of eq. (1) is $O(1)$ since the classical soliton field is $O(1/g)$.

The physical meaning of eq. (1) is simple. The Schrödinger-like operators H_B and H_F^2 define *different* scattering problems. The difference between the scattering potentials of H_B and H_F^2 is given by the r.h.s. of eq. (1). Thus, the corresponding first quantized transition matrices T_B and T_F are different too. Of course, one can loosely say that both bosons and fermions scatter off the classical field of the same soliton. What eq. (1) tells us is that bosons and fermions experience *different* potentials when scattering off the *same* soliton. Since T_B and T_F play the role of the tree level field theoretic transition matrices, one expects SUSY violation to be a direct consequence of the discrepancy between H_B and H_F.

We would like to add a word of caution here. The non-vanishing of the r.h.s. of eq. (1) does not always signal a true discrepancy between the bosonic and fermionic spectra. The exceptional cases are characterized by the presence of a Dirac projection operator in $\delta\Psi^0$. A well known example is the fluctuation spectrum in the background of the BPST instanton [10]. There, $\delta\Psi^0$ involves a projection onto left-handed chirality. As a result, the spectrum of right-handed fermions is identical to the scalar spectrum, and the spectrum of left-handed fermions is related to the scalar spectrum by the action of a local differential operator. Thus, the scattering matrices of all fields are identical. The fluctuation spectrum exhibits a similar behaviour in the background field of solitons in two dimensional scalar theories (see e.g. ref. [11]). However, for solitons in more than two dimensions the non-vanishing of the r.h.s. of eq. (1) represents a genuine discrepancy between the bosonic and fermionic spectra. For example, in both of the above models the neutral scalar field Φ_0 scatters only elastically, whereas its fermionic partner χ_0 can also scatter inelastically into any of the other fermion fields.

Before we study the scattering matrix, let us first mention the requirements that SUSY imposes on the particle spectrum. In both of the above models there are two complex supercharges Q_α , $\alpha = 1, 2$. This means that the monopole (or the fluxon) should belong to a supermultiplet consisting of two bosonic and two fermionic states. At tree level this degeneracy is ensured by the existence of fermionic zero modes in the soliton background. Let us expand the fermionic fields as follows:

$$\Psi(x) = \frac{1}{\sqrt{M}}\delta\Psi^0(x)\mathbf{b} + \text{continuum modes}. \tag{3}$$

The $1/\sqrt{M}$ factor is needed to normalizes the zero modes. $\mathbf{b}$ is the Majorana spinor $(b_1, b_2, -b_2^\dagger, b_1^\dagger)$. b_1 (b_2) is an annihilation operator for the spin up (spin down) zero mode. In the soliton sector one has

$$Q_\alpha = \sqrt{M}\, b_\alpha + O(1) \quad , \quad \alpha = 1, 2\,. \tag{4}$$

Let $|B\rangle$ be the ground state of the soliton sector. This state corresponds to a zero momentum bosonic soliton. At tree level, the other states of the soliton supermultiplet are obtained by acting on $|B\rangle$ with $b_1^\dagger$ and $b_2^\dagger$. There are two spin-$\frac{1}{2}$ states $|F_\alpha\rangle = b_\alpha^\dagger\,|B\rangle$ and a pseudo-scalar state $|B'\rangle = b_1^\dagger b_2^\dagger\,|B\rangle$. Because the soliton mass M is $O(1/g^2)$, soliton states with $\vec{p} \neq 0$ can be constructed perturbatively starting from the $\vec{p} = 0$ states. However, we will not need this generalization since we are interested only in tree order relations.

As a side remark we note that when going beyond tree level, one can show that the monopole sector contains only a single bosonic monopole [3]. Its would-be fermionic partner is in fact unstable, and decays into a monopole and a photino. This effect is a consequence of the instability of the zero modes mentioned earlier, and is related to the presence of a massless fermion – the photino. The situation is different in the SUSY abelian Higgs model. In this model the particle spectrum is completely massive. This guarantees the stability of the four states of the fluxon supermultiplet. It has been shown that these states remain degenerate at one loop [9], although what happens beyond one loop is not clear.

We now turn our attention to the S-matrix. If SUSY is to remain unbroken, certain relations should hold among the S-matrix elements. These relations have been worked out a long time ago by Salam and Strathdee [12]. We will rederive a relation for the scattering of an elementary particle on a soliton which must hold if SUSY is conserved. We will then show that, in fact, this relation is violated at tree level.

More specifically, we will derive a relation for the $2 \to 2$ scattering in the center of mass frame, where both the incoming and the outgoing states contain one soliton and one elementary particle. As explained above, at tree order the soliton can be taken to be at rest. For the elementary particle, however, the three-momentum must be taken into account. Thus, we need the one particle representations of the SUSY algebra for a general momentum. Consider for definiteness a massive supermultiplet in four dimension. At a fixed momentum, the SUSY algebra

$$\{\overline{Q}_\alpha, Q_\beta\} = \sigma^\mu_{\alpha\beta}\, p_\mu\,, \tag{5a}$$

$$\{Q_\alpha, Q_\beta\} = \{\overline{Q}_\alpha, \overline{Q}_\beta\} = 0\,, \tag{5b}$$

reduces to the algebra of two annihilation and two creation anticommuting operators which we denote d_α and $d_\alpha^\dagger$ respectively. To construct the one particle supermultiplet, we start from the state $|b(\vec{p})\rangle$ annihilated by d_α which contains a spin-0 or a spin-1 boson. By acting with $d^\dagger_\alpha$, we obtain two spin-$\frac{1}{2}$ states $|f_\alpha(\vec{p})\rangle = d^\dagger_\alpha\,|b(\vec{p})\rangle$ and a spin-0 state $|b'(\vec{p})\rangle = d^\dagger_1 d^\dagger_2\,|b(\vec{p})\rangle$. On these states, Q_α is represented as

$$Q_\alpha = R(\theta,\varphi)_{\alpha\beta} L(p)_{\beta\gamma} d_\gamma \,. \tag{6}$$

Here

$$L(p) = \begin{pmatrix} \sqrt{E+p} & 0 \\ 0 & \sqrt{E-p} \end{pmatrix} . \tag{7}$$

Up to a multiplicative $\sqrt{m}$ factor, $L(p)$ is the SL(2,C) boost which takes the four vector $(m,0,0,0)$ into $(E,p,0,0)$. $R(\theta,\varphi)$ is the SL(2,C) rotation from the third direction to the (θ,φ) direction. The explicit form of $R(\theta,\varphi)$ will not be needed.

To obtain a relation between S-matrix elements consider the quantity Δ defined as follows

$$\Delta = {}_{out}\Big\langle B, f_\alpha(\vec{p}_1)\Big|\overline{Q}^{in}_\beta - \overline{Q}^{out}_\beta\Big|B, b(\vec{p}_2)\Big\rangle_{in} \,. \tag{8}$$

If SUSY is conserved, $Q^{in} = Q^{out}$ and so Δ should vanish. In order to calculate Δ we act with $\overline{Q}^{in}$ to the right and with $\overline{Q}^{out}$ to the left. Neglecting corrections due to the non-zero soliton momentum which are $O(g^2)$ we obtain

$$\begin{aligned} \Delta &= \sqrt{M}\,{}_{out}\Big\langle B, f_\alpha(\vec{p}_1)\Big|F_\beta, b(\vec{p}_2)\Big\rangle_{in} \\ &\quad + {}_{out}\Big\langle B, f_\alpha(\vec{p}_1)\Big|B, f_\gamma(\vec{p}_2)\Big\rangle_{in} L(p_2)_{\gamma\beta} - {}_{out}\Big\langle B, b(\vec{p}_1)\Big|B, b(\vec{p}_2)\Big\rangle_{in} L(p_1)_{\alpha\beta} \,, \end{aligned} \tag{9}$$

where $p_1 = |\vec{p}_1|$ and $p_2 = |\vec{p}_2|$. Notice that for inelastic scattering the outgoing particle may have a different mass from the incoming one. Thus, p_1 is not necessarily equal to p_2. In eq. (9) we have absorbed the unitary rotation matrix in a redefinition of the *out* states. Taking into account the kinematical factors arising from the normalization of creation and annihilation operators we find

$$\begin{aligned} \Delta &= -2\pi i \delta(E_1 - E_2) \times \\ &\quad \Big\{\sqrt{2EM}\, T_{exc}(\vec{p}_1,\vec{p}_2) + T_F(\vec{p}_1,\vec{p}_2)L(p_2) - T_B(\vec{p}_1,\vec{p}_2)L(p_1)\Big\} \,. \end{aligned} \tag{10}$$

Eq. (10) is valid quite generally. It applies to both four and three dimensional models as well as when the elementary particles involved in the scattering process are massless.

As mentioned earlier, T_B and T_F are the first quantized transition matrices of the operators H_B and H_F. T_{exc} describes the exchange interaction in which an incoming boson plus a fermionic soliton scatter into a fermion and a bosonic soliton. It arises from the fermionic interaction hamiltonian (2) upon substituting a fermionic zero mode for one of the fermionic fields. This interaction is $O(g)$, but must be kept in eq. (10) because $\sqrt{M} = O(1/g)$. Explicitly

$$\sqrt{M}\, T_{exc}(\vec{p}_1, \vec{p}_2) = (\psi_{out}(\vec{p}_1)|\gamma_0 V(\phi_{in}(\vec{p}_2))|\delta\Psi^0)\,. \tag{11}$$

Here ψ_{out} (ϕ_{in}) is an outgoing (incoming) eigenstate of H_F (H_B).

Making use of eqs. (1) and (11) and general formulae for two potentials scattering (see e.g. ref. [13]) the exchange interaction can be related to the bosonic and fermionic T-matrices

$$\sqrt{M}\, T_{exc}(\vec{p}_1, \vec{p}_2) = T_B(\vec{p}_1, \vec{p}_2) - T_F(\vec{p}_1, \vec{p}_2)\,. \tag{12}$$

We can therefore express Δ as

$$\begin{aligned} \Delta &= -2\pi i \delta(E_1 - E_2) \times \\ &\quad \left\{ T_B(\vec{p}_1, \vec{p}_2) \left(\sqrt{2E} - L(p_1)\right) - T_F(\vec{p}_1, \vec{p}_2) \left(\sqrt{2E} - L(p_2)\right) \right\}\,. \end{aligned} \tag{13}$$

It is clear from eq. (13) that Δ does not vanish in general. For example, if the incoming and outgoing particles have the same mass, $p_1 = p_2 = p$ and the expression inside the curly brackets factorizes

$$\left(T_B(\vec{p}_1, \vec{p}_2) - T_F(\vec{p}_1, \vec{p}_2)\right) \left(\sqrt{2E} - L(p)\right)\,. \tag{14}$$

Both factors in the above expression are non-zero in general. This completes the proof that in supersymmetric gauge theories *the scattering of elementary particles on solitons violates supersymmetry at tree level.*

It is clear that the discrepancy between the spectra of bosons and fermions plays a crucial role in the above result. We now want to explain how the fermionic nature of the SUSY generators enters the proof. To this end, it is useful to compare the situation described above to the mechanism which restores translation symmetry in the soliton sector and see where the analogy between the two cases breaks down.

Consider a purely bosonic theory admitting a soliton solution. In the background field of the soliton the bosonic spectrum contains zero modes which correspond to infinitesimal translations of the soliton field. By analogy, in the case of SUSY one has fermionic zero modes corresponding to infinitesimal SUSY transformations of the soliton field. Furthermore, technically the fluctuation spectrum in the soliton background breaks translation symmetry explicitly as the eigenstates are not plane waves.

The crucial step in restoring translation symmetry is the introduction of collective coordinates and, at the same time, the elimination of the bosonic zero modes from the spectrum. This is achieved by the canonical transformation [14]

$$\Phi(x) = \varphi_{cl}(x') + \phi(x')\,, \tag{15}$$

$$x' = x - X\,, \tag{16}$$

where X is the collective coordinate and $\phi(x')$ is orthogonal to the bosonic zero modes. Eq. (15) is supplemented by a transformation of the canonical momenta. In particular, the canonical variable $\hat{p}$ conjugate to X is the total momentum of the system.

The introduction of collective coordinates is not a matter of choice. If one does not perform the canonical transformation (15) the translational zero modes remain in the spectrum and plague the perturbative expansion with infra-red divergences. Once introduced, conservation of momentum follows trivially because the hamiltonian is independent of the collective coordinates. Notice that the new field's argument x' plays the role of a *relative* coordinate. Thus, the fluctuation spectrum does not imply any violation of translation symmetry. Rather, it simply gives the usual description of non-relativistic scattering in terms of the relative coordinate.

Let us now consider SUSY. Since fermions obey a first order equation, fermionic zero mode do not lead to any inconsistencies in the perturbative expansion. This is the crucial difference between bosonic and fermionic zero modes. In fact, as we mentioned earlier, the fermionic zero modes play an important role in the tree level construction of the soliton supermultiplet. The validity of the quantization procedure which retains fermionic zero modes in the spectrum provides the basis for our discussion. It is at this point that the fermionic character of the SUSY generators enters our analysis.

Imagine that, hoping to achieve supersymmetric results, one had tried to build a "supersymmetric collective coordinate". Since we have proved that SUSY is broken explicitly we know that such an attempt is doomed to fail. It may be interesting to figure out what exactly would go wrong. Here we only make a few observations while leaving a more detailed discussion for the future.

In the case of translations, introducing the collective coordinates renders the field's argument x' a *relative* coordinate. As a result, the fact that the spectrum does not consist of momentum eigenstates in the variable x' is harmless. By analogy, the discrepancy between the spectra of bosons and fermions was not harmful if by introducing a supersymmetric collective coordinate the field operators lost their absolute significance as bosons or fermions. In this hypothetical situation, whether a given field describes a boson or a fermion would depend on the value of the supersymmetric collective coordinate. The reader can easily convince himself, however, that this situation could be achieved only at the cost of making the supersymmetric collective coordinate a bosonic variable. Evidently, trying to build a bosonic collective coordinate for SUSY is bound to lead to inconsistencies. A SUSY collective coordinate would have to be fermionic. Thus, the new canonical variables would keep their significance as either bosons or fermions. But this is precisely what gives the discrepancy between the spectra of the bosons and fermions its physical significance!

We conclude with a comment on the vacuum structure of the models discussed in this letter. The question arises whether one could attribute our results to a spontaneous breaking of SUSY by the full vacuum. In the SUSY abelian Higgs model the answer is negative for a trivial reason. This model contains no massless fermion and hence there is no candidate for the goldstino.

In the four dimensional model the answer is negative too. In fact, the vacuum energy of this model is negative [5] and so the vacuum sector too exhibits an explicit breaking of SUSY. Furthermore, this possibility could have been disposed of without any detailed study of the vacuum structure. Since the model is weakly interacting, any non-perturbative contribution to the vacuum is necessarily suppressed by the usual exponential factor. Thus, such contributions cannot explain the behaviour of the S-matrix found above, which violates SUSY at tree level.

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