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ESTIMATION OF PRECIPITABLE WATER FROM SURFACE DEW POINT TEMPERATURE

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ABSTRACT

The Reitan (1963) regression equation which is of the form $\ell n\, w = a + b\, T_d$ has been examined and tested to estimate pricipitable water content from surface dew point temperature at different locations. The study confirms that the slope of this equation (b) remains constant at the value of $.0681\,^{\circ}C$, while the intercept (a) changes rapidly with the latitutide. The use of the variable intercept can improve the estimated result by 2%.

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INTRODUCTION

The mass of water vapour contained in a column of unite cross sectional area extending from surface to height z, may be written as

$$w = \int_0^{z_1} \rho_w dz \qquad gm\ cm^{-2} \tag{1}$$

where ρ_w is the absolute humidity. In more practical uses, w may be expressed as

$$w = \sum_{surface}^{200\,mb} r \Delta p \times \frac{1000}{g} mm$$

where g is the acceleration of gravity and r is the mixing ratio defined as

$$r + \frac{.622\,e}{p-e}$$

The vapour pressure (e) can be calculated using the formula given by Tentens (1930) in terms of dew point temperature (T_d) as

$$e = 6.11 \times 10^{-3} \quad 10^{D(T_d)} m \cdot b \ , \tag{2}$$

where $D(T_d) = \frac{7.5T_d}{237.3+T_d}$.

This expression for $D(T_d)$ was found not to fit well all ranges of dew point temperature. In Fig.(1) different presentations of $D(T_D)$ according to dew point range are given as

$$-10^{\circ}C \leq T_d < 27^{\circ}C \qquad D(T_d) = .02956\,T_d - .00639$$
$$-30^{\circ}C \leq T_d < -10^{\circ}C \qquad D(T_d) = .03956\,T_d + .09653$$

Also another way of presenting $D(T_d)$ below $-10\,^{\circ}C$ are expressed in the equation

$$D(T_d) = .0411T_d + .02806 \qquad T_d \leq -10^{\circ}C\ .$$

Differences in using each one of these equations are clearly shown in Fig.1. Although the difference between these linear approximations is not large at very low temperature below ($-25\,^{\circ}C$), it increases for higher temperatures. To evaluate the mass of the water vapour content (w) in the atmosphere, we need values of T_d at moist levels, and for many purposes there is no possibility of using this procedure unless there is data exist for every single station. Reitan (1963) and Smith (1966) proposed a simple form to calculate (w), using a relation on the form $\ell n\, w = a + bT_d$, where T_d is the surface dew point temperature. Another form for estimating w had been proposed by Adedokun (1986,1988) using a formula

$$w = \alpha q^{\beta}$$

where q is the surface humidity and a, b and α, β remain the coefficients of each model. The objective of this paper is to discuss the different models used for estimating precipitable water and to investigate the physical behaviour of these models.

DATA USED IN THE STUDY

Data for six stations with entirely different location and climate were introduced in this study. These data represent instantaneous radiosonde observations. A list of these stations is given in Table 1:

Station	Latitude N^o	Number of observations
Goose (Canada)	53	60
Buffalo (USA)	45	91
Downsview (Canada)	43	98
Tripoli (Libya)	32	42
Goose (Canada)	53	60
Matrouh (Egypt)	31	324
Cairo (Egypt)	30	320

Table 1
Number of observations for stations used in the study

The aim of using different locations with different climatological conditions is to find out if there is a physical characteristics which may explain the changes of empirical coefficients of these models. These data had been selected to cover equally different seasons, therefore will disregard seasonal changes.

METHODOLOGY

Both models for estimating precipitable water use only surface values of either dew point or humidity. In fact, the surface value will not represent correctly the vertical profile. The vertical distribution of moisture can vary for any given surface humidity and a unique relationship between precipitable water and surface dew point may only be approached for long term means (Smith, 1966).

We consider the vertical change of absolute humidity according to the relation

$$\rho_w(z) = \rho_w(0) e^{-Z/H} \quad (3)$$

where H is the scale height (km) for water vapour and $\rho_w(0)$ is the surface value of the absolute humidity. From Eq.(1), we obtain

$$w = \int_0^{Z_1} \rho_w(Z)\, dZ = \int_0^{Z_1} \rho_w(0)\, \bar{e}^{Z/H} dZ = \left[1 - ^{-Z_1/H}\right] \rho_w(0) H \quad (4)$$

The exponential term in Eq.(4) is therefore simply

$$w = \rho_w(0) H = 10^5 \rho_w(0) H gm/cm^2$$

H will simply represent the vertical extent of the homogeneous atmosphere of water vapour. From equation of state, $\rho_w(0)$ can be expressed in terms of surface vapour pressure e_0 and surface temperature T_0 as

$$w = \frac{10^5 e_0 H}{4.615(273.16 + T_0)} \quad (5)$$

following Tomasi (1981) and taking logarithm of Eq.(5).

$$\ell n\, w = \ell n(10^5/4.615) + \ell n\, H + \ell n\, e_0 - \ell n(273.16 + T_0)\ .$$

Substituting about e_0 by Eq.(2) and taking $D(T_d)$ in the domain from -10 to $27\,^\circ C$, we easily get

$$\ell n\, w = 4.87705 - \ell n(273.16 + T_0) + \ell n H + .068 T_d \quad (6)$$

Comparing Eq.(6) and log linear model equation by Reitan ($\ell n\ w = a + bT_d$) $= a$ assuming the value of $b = .068\,^\circ C^{-1}$ as from Eq.(6), then

$$a = 4.8705 - \ell n(273.16 + T_0) + \ell n\, H \quad (7)$$

Eq.(7) describes well the relation between the intercept (a) and the scale height (H). Many studies had reported values for the coefficients (a, b) for the log linear model. These values were obtained for different time scale of our data and in Table 2, a summary of these values is given:

Time scale	a	b	r
hourly	-.0485	.069	.98
daily	.127	.076	.99
monthly	.115	.066	.97
yearly	.0441	.071	.95

Table 2
Values of (a, b) for different time scale

These obtained values agree with values presented before by Reitan (1963), Bolsenga (1965) and Smith (1966). Simply we can find that a value of a of $(.069)^0 C^{-1}$ is typically reported here and by Reitan (1963) and also deduced from the analysis of Tomasi (1981) [Eq.(6)]. The value of b turns out to be constant even changes on time scale does not exceed (15%), while at the same time a seems to be very sensitive to several factors [(Eq.7)].

By differentiating Eq.(7) with respect to the temperature, it can be easily found that the variation of a with the surface temperature is very small such as

$$\left(\frac{da}{dT}\right)_{T_0=30} = .0033\,°C^{-1} \quad \text{and} \quad \left(\frac{da}{dT}\right)_{T_0=-30} = .0014\,^0C^{-1} .$$

Thus, changes of the intercept with the surface temperature are very small and we will concentrate on changes with the scale height (H). From the chermodynamics of moist air, the scale height defined in Eq.(4) can be given as

$$H = \frac{\epsilon L e}{c_p p(\gamma_a - \gamma_b)} \tag{8}$$

where

$\gamma_a = \frac{g}{c_p}$ dry adiabatic lapse rate
$\epsilon = .622$
L is the latent heat of vaporization
c_p is the specific heat at constant pressure
$\gamma_b = \gamma_a\left(\frac{1+Lr/ART}{1+\frac{\epsilon L}{c_p p}\left(\frac{de_s}{dT}\right)}\right)$ is the actual lapse rate
r is the mixing ratio
p is the atmospheric surface prssure
$\left(\frac{de_s}{dT}\right)$ is the rate of the change of the saturation vapour pressure with the temperature.

Therefore an estimate of H can be easily done using Eq.(8). Changes of H are due to change in vapour pressure (e) and implicitly to the dew point temperature or ($\gamma_a - \gamma_b$) which is the stability index. Fig.2 contains two curves which illustrate the variation of the scale height with stability parameter for both vapour pressure at 2 and 10 $M.B$.

Using now the scale height given by Eq.(8) a variable estimate of the intercept "a" can be achieved, taking the slope a constant of $.068\,°C^{-1}$.

In this way when we do not have a fixed value of ("a"), the variable intercept procedure (VIN) using Eq.(7). The values of (a) obtained from the least square method using measured values will be called as constant intercept procedure (CIN). Values of a (CIN), b and errors of estimate for each station are given in Table 3.

Station	a [2]	b	RMSE% [1]	RMSE % [2]
Goose	-.0594	-.0594	14.5	16.3
Downview	-.0621	.0636	12.6	14.2
Buffalo	-.0648	.0623	13.1	15.3
Tripoli	-.19	.0651	17.4	19.1
Matrouh	-.15	.0681	15.9	16.7
Cairo	-.2	.0665	16.1	17.8

Table 3
Values of a, b and errors for $VIN^{(7)}$ and $CIN^{(6)}$ procedures

As demonstrated in Table 3, the variation of a was quite remarkable, while the slop remains again almost constant. The calculated values from both methods against the measured one was plotted for Downsview, where every point on the scatter diagram represents a number of observations. It is clear that the VIN procedure reduces the scatter along the one–to–one line.

Another trial was done to fit values of (a) given in Table 3 with the latitude of the stations, the best fitting was found to be in the form

$$a = -1.0982 + .0436\,\phi - .000454\,\phi^2 \tag{9}$$

where $52 \geq \phi \geq 30$.

By the aid of the above formula, Fig.4, a rough approximation of (a) can be easily deduced, by using the intercept from this equation and also using the constant intercept error in the order (24 – 27)% in calculating precipitable water is reached. While is not one of the aims of this study to compare models, the formula suggested recently by Adedokun (1986, 1989) had been used for estimating precipitable water for our set of data. Results from this model did not compare well, maybe because of the climate variability in our data set or because this model was verified on monthly data base.

CONCLUSIONS

The log linear model Reitan (1963) had a reasonable performance to estimate precipitable water, and this model may suggest constant slope. The physical discussion of the intercept had been investigated in terms of scale height, and the use of variable intercept (VIN) can reduce the error of estimation. A rough estimating of precipitable water can be achieved through the use of Eq.(9) for intercept and taking b as .0681.

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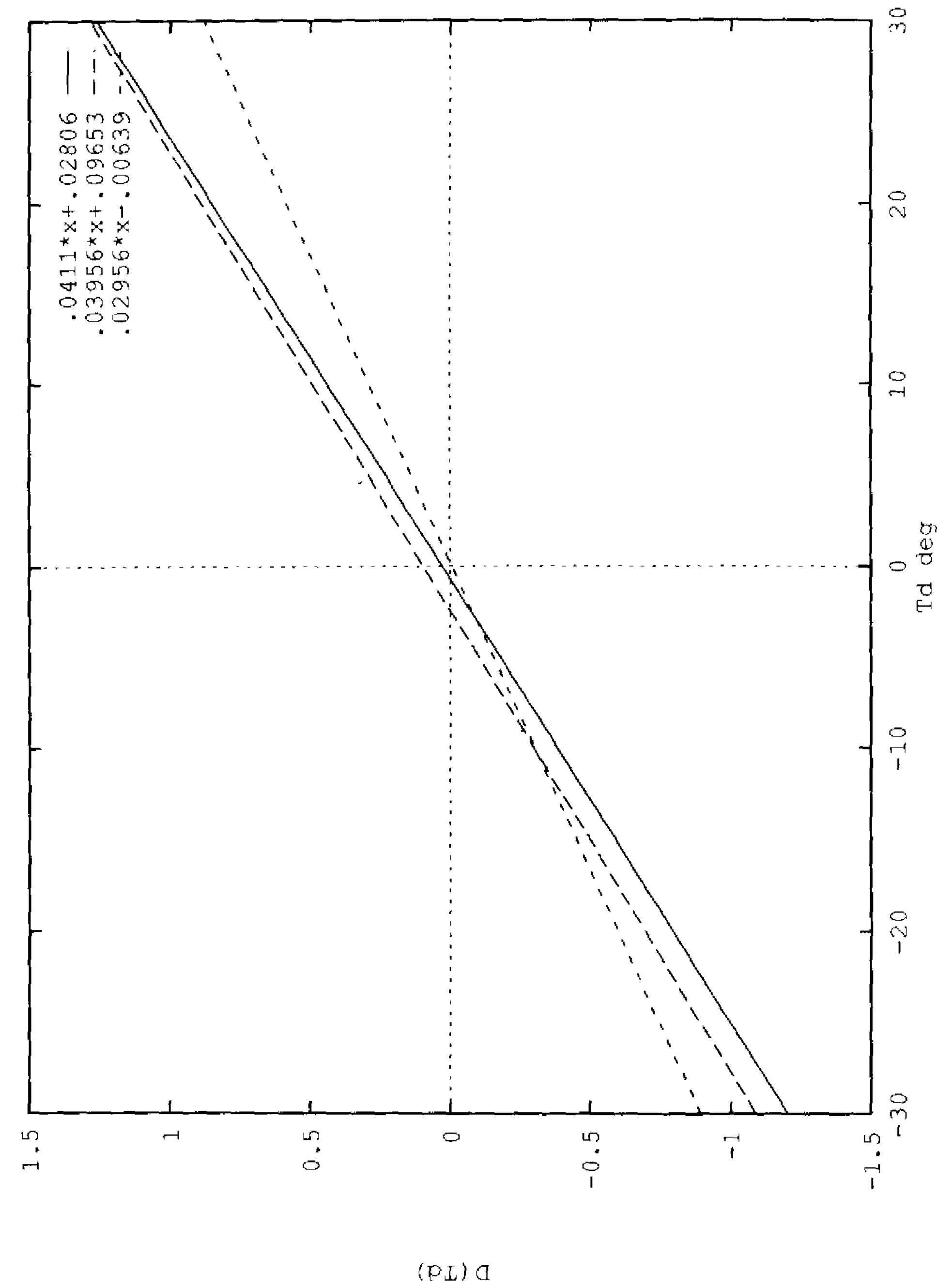

Fig.1
Different presentation of function D(td)

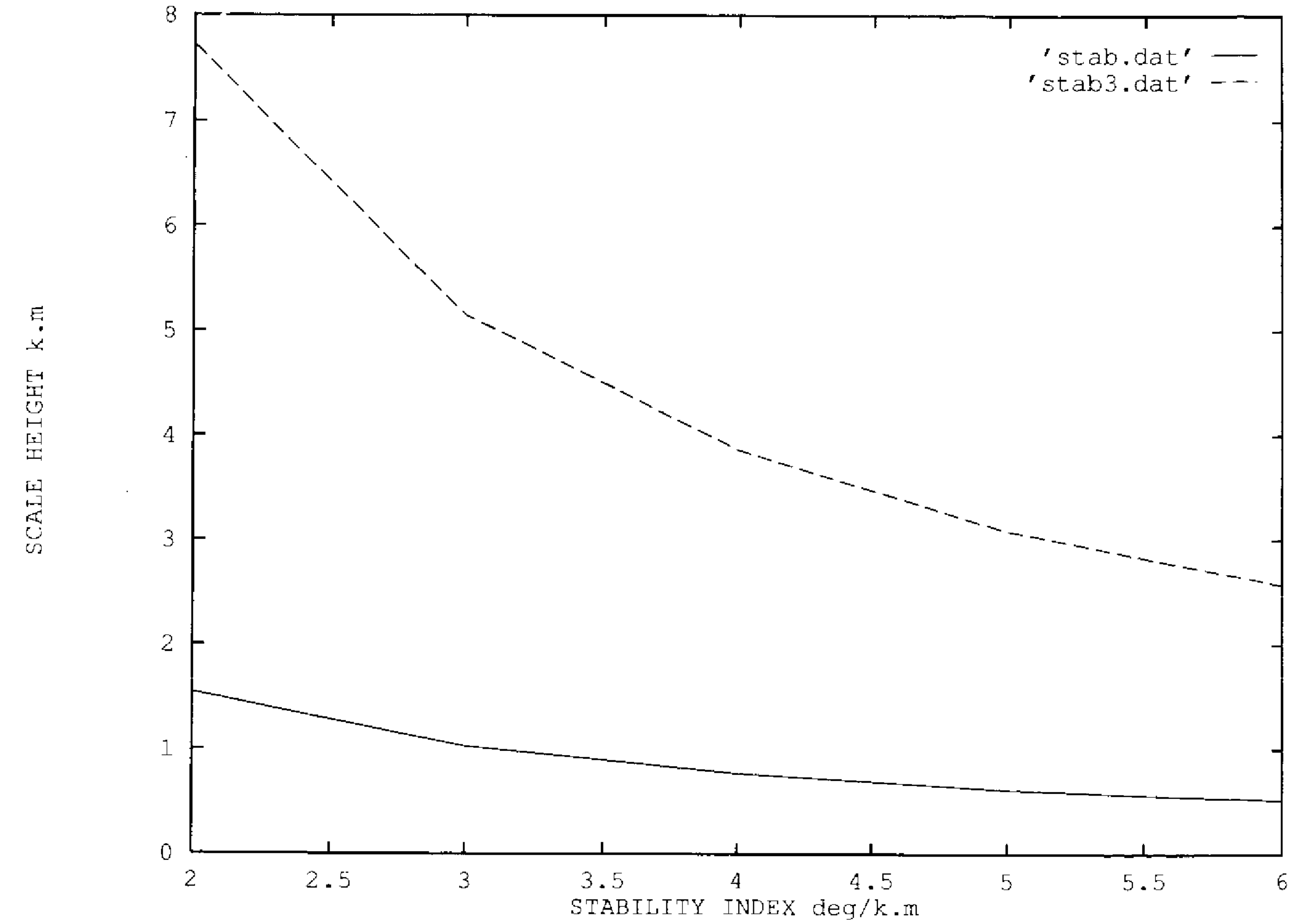

Fig.2

Change of scale height with stability for e = 2,10 m.b.

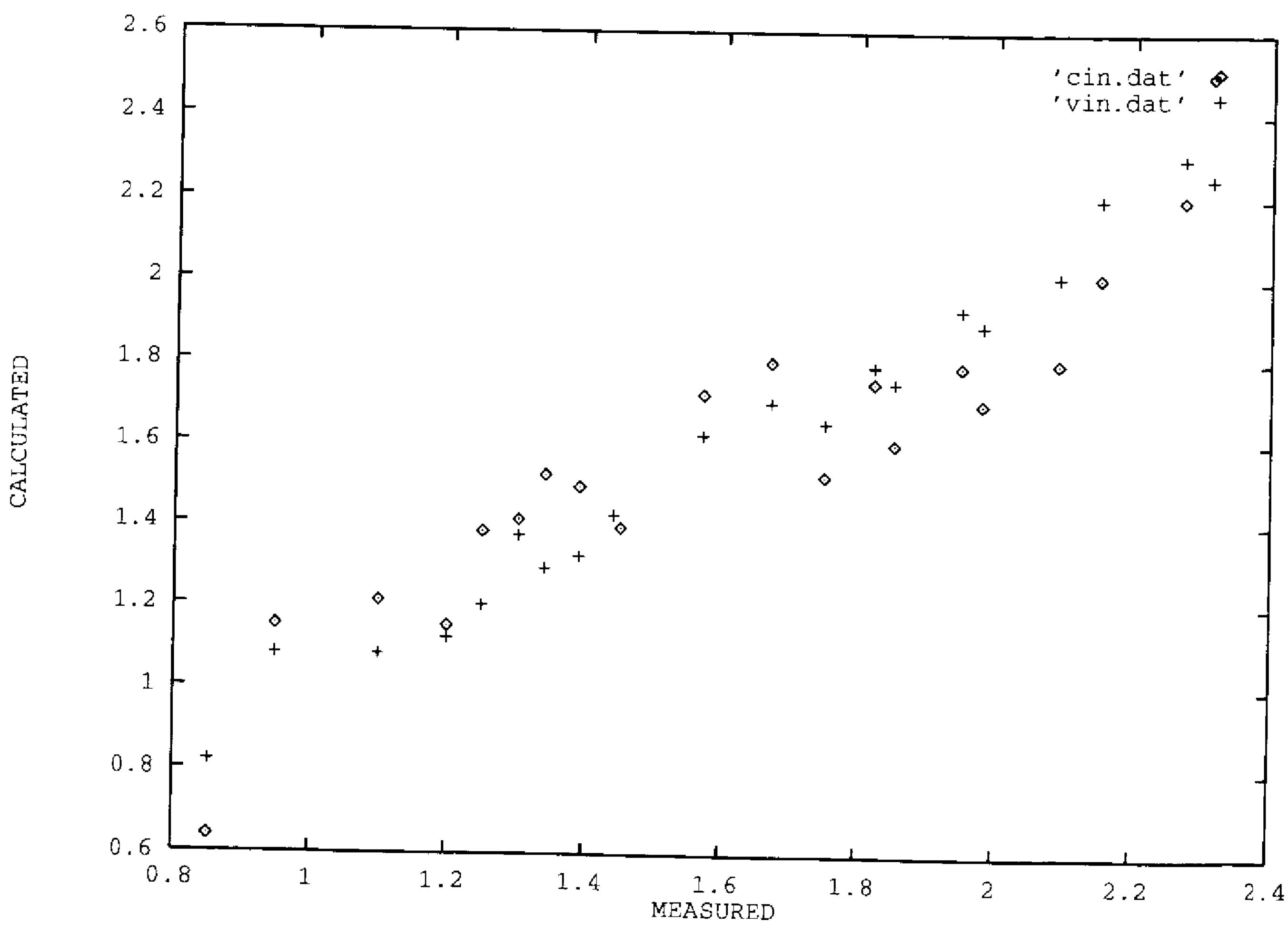

Fig.3

Measured precipitable water against calculated for Downsview (cm)

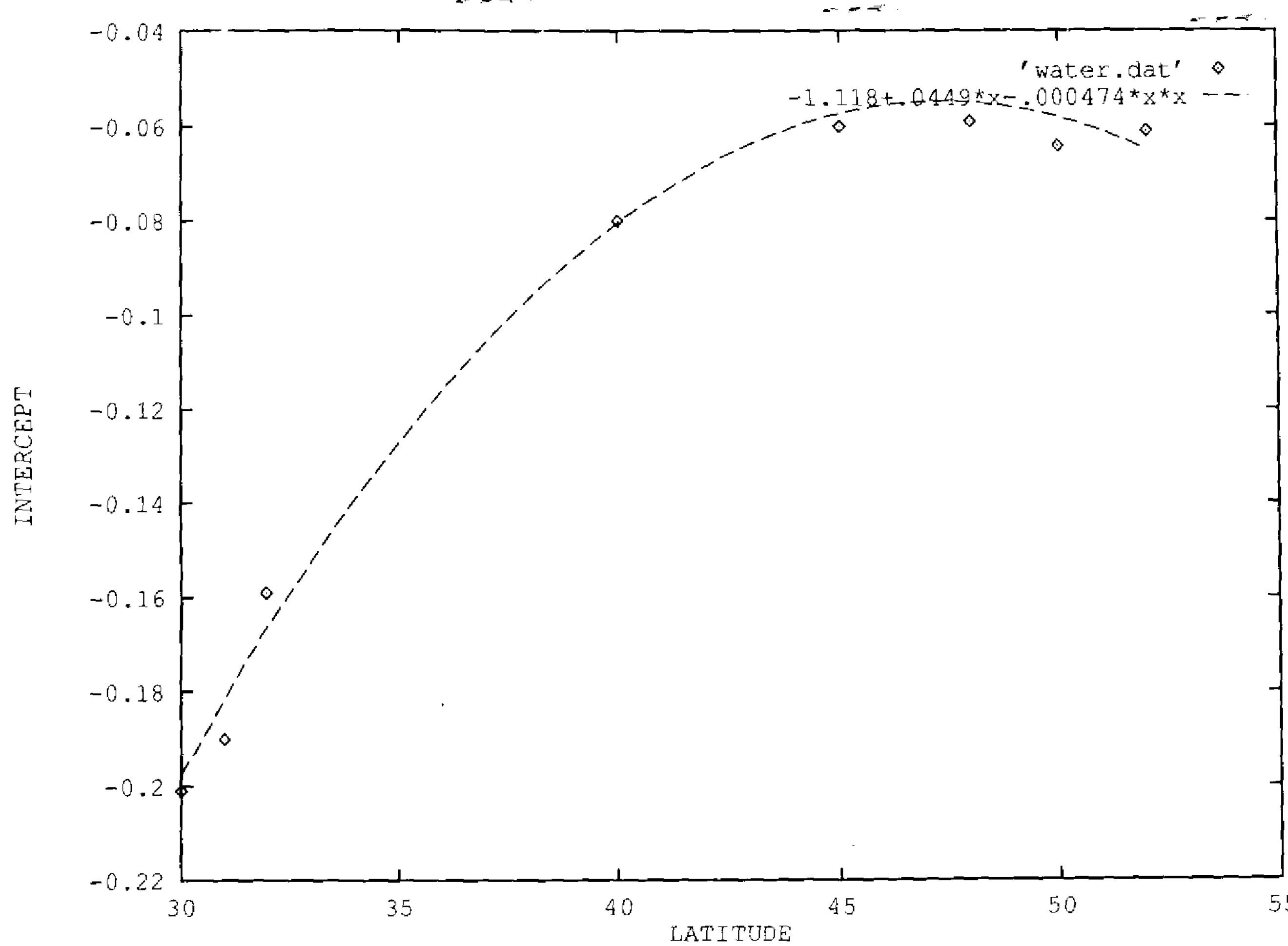

Fig.4

Estimate of intercept with latitude