LA-UR -91-3002

LA-UR--91-3002

DE92 000209

Received by OSTI

OCT 04 1991

TITLE: FUZZY RESOURCE OPTIMIZATION FOR SAFEGUARDS

AUTHOR(S): A. Zardecki and J. T. Markin

SUBMITTED TO: Fourth International Topical Conference
on Facility Operations/Safeguards Interface
Albuquerque, New Mexico
September 29-October 4, 1991

Los Alamos

Los Alamos National Laboratory
Los Alamos, New Mexico 87545

FUZZY RESOURCE OPTIMIZATION FOR SAFEGUARDS

Andrew Zardecki and Jack T. Markin
Safeguards Systems Group
Los Alamos National Laboratory
Los Alamos, NM 87545
(505) 665-2812

ABSTRACT

Authorization, enforcement, and verification—three key functions of safeguards systems—form the basis of a hierarchical description of the system risk. When formulated in terms of linguistic rather than numeric attributes, the risk can be computed through an algorithm based on the notion of fuzzy sets. Similarly, this formulation allows one to analyze the optimal resource allocation by maximizing the overall detection probability, regarded as a linguistic variable. After summarizing the necessary elements of the fuzzy sets theory, we outline the basic algorithm. This is followed by a sample computation of the fuzzy optimization.

I. INTRODUCTION

Providing adequate protection for nuclear materials is a complex undertaking that draws on expertise from many technical disciplines. Safeguards systems have steadily improved along with the public's desire to reduce the risk it perceives from safeguards operations. Modern systems studies attempt to describe safeguards in terms of three functions: authorization, enforcement, and verification.[1] The risk analysis of a complex system is an essential element of the integrated system design. A related problem is to assign limited resources to safeguards elements by minimizing the overall risk. Customarily, this has been achieved by maximizing the probability of detection against a range of scenarios—the object of the optimum resource allocation.

As stated by L. A. Zadeh,[2] the pioneer of the fuzzy sets theory, the traditional approaches to risk analysis are based on the premise that probability theory provides the necessary and sufficient tools for dealing with uncertainty and imprecision. The theory of fuzzy sets calls into question the validity of this premise. More specifically, it provides a framework for dealing with linguistic variables, that is, variables whose values are words or sentences in a natural language. Viewed in this perspective, the fuzzy risk analysis allows one to analyze the information in which the principal sources of uncertainty are nonstatistical in nature. Likewise, the resource allocation and optimization, formulated in terms of the fuzzy sets, becomes the fuzzy optimization problem.

The objectives of this paper are twofold. First, we outline a preliminary version of the computer code that performs the fuzzy risk assessment of a hierarchical system. The code consists of two separate modules: the parser module translates the natural language phrases into fuzzy sets, and the fuzzy set module performs the fuzzy arithmetics. Second, we discuss the fuzzy resource optimization algorithm conceived as an extension of the RAOPS model[3] to the situations in which the detection probabilities are replaced by quantitative measures described in terms of fuzzy sets. Initially, we consider a serial configuration of safeguards activities in which an adversary encounters the safeguards elements sequentially in reaching a single target. This will later be generalized to a divergent configuration in which the adversary can follow any one of several paths to multiple targets; the flow of resources branches, then, at specific nodes, as is the case for a hierarchical structure.

II. RESOURCE ALLOCATION IN SAFEGUARDS

In this section, we list some basic results and explain our notation. The basic references are the monographs of Nemhause[4] and of Larson and Casti.[5] In an allocation problem, a conflict of interest arises from the fact that a resource can be used in a number of ways. Each such possible application is called an activity. For a serial arrangement of activities, the multiplicative objective (return) function is

$$J = \prod_{k=1}^{N} L[x(k),u(k),k] \ . \tag{1}$$

Here $x(k)$ and $u(k)$ refer to state variables and decision variables at stage k. In the context of safeguards, the objective function can be thought of as the overall nondetection probability that a diversion at any activity will remain undetected. In other words, $L[x(k),u(k),k]$ is the nondetection probability at stage k, associated with state $x(k)$ and decision $u(k)$. The system equations describe how the state variables at stage $k+1$ are related to the state variables at stage k. These equations are written as

$$x(k+1) = g[x(k),u(k),k] \ , \tag{2}$$

where g is a known function.

For the resource allocation problem, g is simply a difference of x and u:

$$x(k+1) = x(k) - u(k) \quad . \tag{3}$$

The dynamic programming optimization solves the iterative functional equation for the optimum return $I(x,k)$

$$I(x,k) = \min_{u \in U} \left\{ L(x,u,k) \times I[g(x,u,k),k+1] \right\} \tag{4}$$

with $k = 1, \ldots, N-1$, by minimizing the expression in the curly brackets over the set U of decisions. The starting condition is

$$I(x,N) = \min_{u \in U} \{ L(x,u,N) \} \quad . \tag{5}$$

This recursion procedure, called backward recursion, solves the initial state problem in which the optimal N-stage return becomes a function of the input state at stage one. When state inversion is possible, as is the case in the resource allocation problem, one can also use forward recursion to solve a final state problem. In the final state problem, the optimal return is found as a function of the stage output. Finally, the initial-final state optimization consists in finding the optimal return as a function of the input to stage one and the output from stage N.

This basic dynamic programing procedure prevents the combinatorial explosion (curse of dimensionality) from occurring. The generalization to nonserial systems includes both divergent and convergent configurations.[4]

III. FUZZY SETS APPROACH TO RISK AND ALLOCATION

Fuzzy set theory, originally developed by Zadeh, is today a subject of review articles[6] and textbooks.[7] Fuzzy set theory was developed to generalize classical set theory in such a manner as to allow the possibility of partial membership in a set. In everyday life, one can find many examples of sets for which membership is not well defined. Some examples are the set of all tall men, the set of very large trees, or the set of all protective mechanisms that provide security against a certain threat.

We illustrate the notion of a membership function and define the operations of addition and multiplication of fuzzy sets; furthermore, the concept of ordering is introduced. This will be followed by a description of application of these concepts to the risk analysis and resource allocation.

A. Fuzzy Sets

Intuitively, a fuzzy set is a class that admits the possibility of partial membership in it. Let $X = \{x\}$ denote a reference set (universe of discourse). Then a fuzzy set A in X is a set of ordered pairs

$$A = \{x, \chi_A(x)\} \quad , \; x \in X \; , \tag{6}$$

where $\chi_A(x)$ is termed the grade of membership of x in A. We assume for simplicity that $\chi_A(x)$ is a number in the interval [0,1] with grades 1 and 0 representing, respectively, full membership and full nonmembership in a fuzzy set. As an example, consider a linguistic variable *age* with values young, middle-aged, and old viewed as fuzzy sets. With the universe of discourse specified, arbitrarily, as the set of integers 1, ..., 100, the fuzzy sets young, middle-aged, and old are represented graphically in Fig. 1.

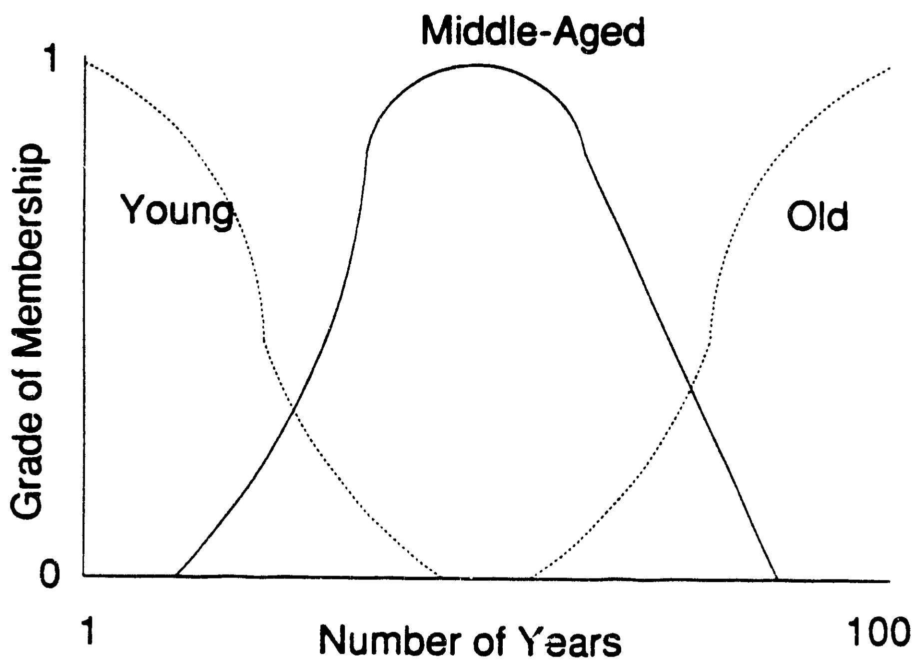

Fig. 1. Membership function for fuzzy sets: young, middle-aged, and old of a linguistic variable age.

A finite fuzzy set A having n elements on X is expressed as

$$A = \sum_{j=1}^{n} \chi_A(x_j)\,/x_j \ . \tag{7}$$

With this notation, the basic algebraic operations are introduced through the so-called extension principle:

$$A + B = \sum_{i,j}^{n} \min\,[\chi_A(x_i),\chi_B(x_j)]\,/\,(x_i + x_j) \ , \tag{8}$$

$$A \times B = \sum_{i,j}^{n} \min\,[\chi_A(x_i),\chi_B(x_j)]\,/\,(x_i \times x_j) \ , \tag{9}$$

$$A\,/\,B = \sum_{i,j}^{n} \min\,[\chi_A(x_i),\chi_B(x_j)]\,/\,(x_i \div x_j) \ . \tag{10}$$

If more than one pair of elements of the reference set is mapped to the same element under these operations, the maximum of the membership grades, defined by Eqs. (8)-(10) is chosen.

The operation of division requires that $x_i \div x_j$ should be reduced to a set of integers. Using the terminology borrowed from object oriented programming, we have overloaded the operators +, *, and /, thus extending their domain of definition to fuzzy sets.

The entities on which the basic operators act constitute either the primaries or hedged primaries of a natural language of linguistic variables. The primaries are *low, medium,* and *high,* as well as the fuzzy numbers between 1 and 9. An example of a hedged primary is *fairly low* or *pretty high.* The complete syntax of the natural language we use follows closely the monograph of Schmucker.[8] This syntax includes more involved constructs, such as relational phrases and composite relations.

To compare two fuzzy numbers, we need a notion of ordering. Kaufmann and Gupta[9] provide a method based on the concept of removal (Fig. 2). This is the mean of the areas bounded by a reference number *k* and the two sides of a fuzzy set. If *k* is identified with the origin, all the fuzzy numbers will have positive removals, which then become a measure of distance from the origin 0. Strictly speaking, removals partition the fuzzy sets into equivalence classes that can be refined by applying other criteria. We ignore these subtleties here.

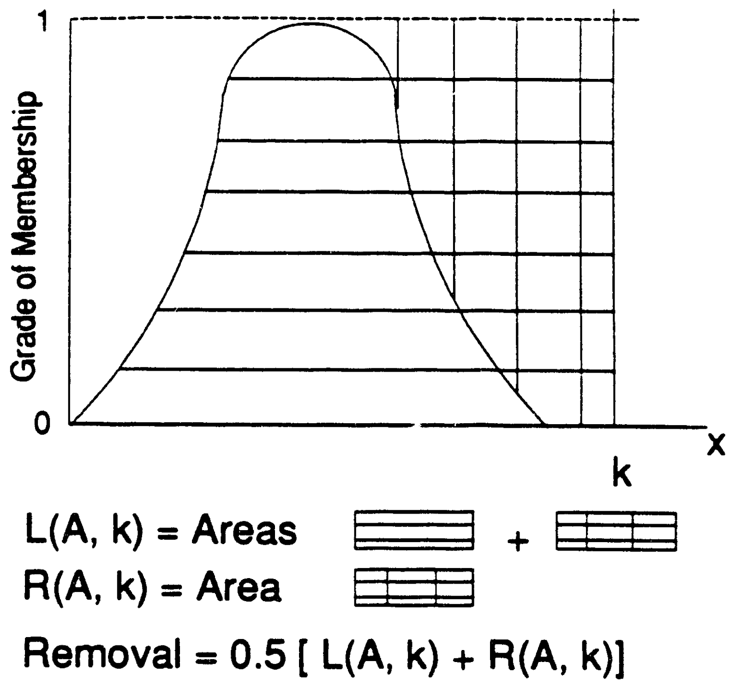

Fig. 2. Computation of removal with respect to *k* of a fuzzy number.

B. Fuzzy Risk Analysis

The fuzzy risk analyzer (FRA) is based on the work of Schmucker,[8] in which a similar algorithm—primarily designed for computer security—has been described.

To conduct a risk analysis, one must identify the components of the system in which risks can be found. A natural way to do this is to decompose the system into its primary components or subsystems and then systematically decompose these into their subcomponents, until further decomposition becomes impractical or offers no significant benefit. Figure 3 illustrates a simplified decomposition of an integrated safeguards system into its component parts.

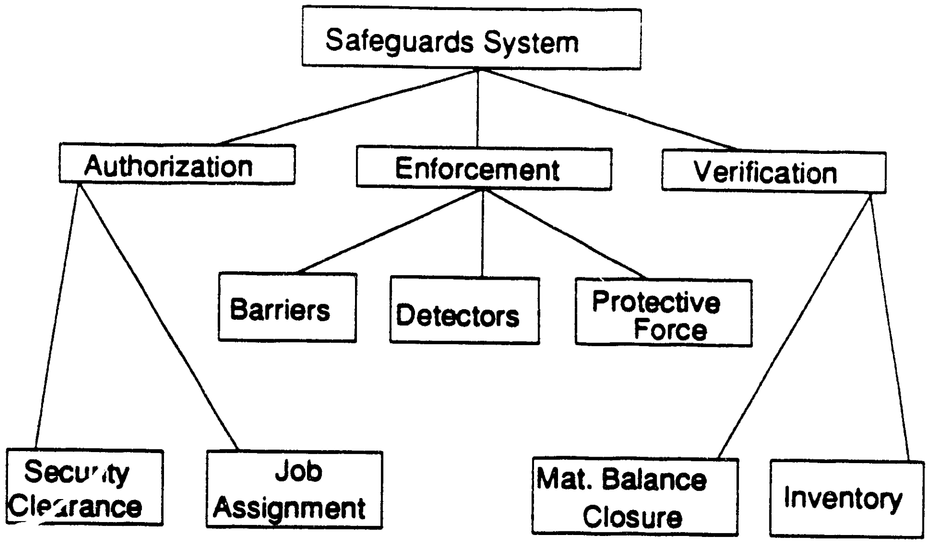

Fig. 3. Decomposition of an integrated safeguards system into subcomponents.

This diagram has the form of a tree data structure. At any level, each node has two text attributes (fields); they are its Component ID and Description. At the lowest level, the terminal nodes (leaves) have three attributes associated with them. They are Likelihood of Loss (Probability of Failure), Severity of Loss, and Confidence Factor. The attribute Confidence Factor is optional and has a unit default value. This means that, when omitted, the Confidence Factor does not modify the risk of a terminal node. The fuzzy product of attributes Likelihood of Loss and Severity of Loss results in a Component Risk Indicator of the node, possibly modified by the Confidence Factor. The Risk Indicators of the components of each parent node are then weighted by the Weight Factors to produce their corresponding parent's node risk indicator. The weight factor, therefore, adds or subtracts weight from the risk indicators as they are merged into higher level indicators. The nonterminal nodes, which rely on their children, do not require the Likelihood of Loss, Severity of Loss, and Confidence Factor fields.

The data structure, shown in Fig. 3, features a tree in which each node has children at most one level in depth. In an unlikely situation, in which the depth of children descending from a given node exceeds one, we apply the same arithmetics as above. The children at different levels are converted into the children at one level.

C. Fuzzy Resource Allocation

In contrast to Sec. 2, in which the detection probabilities are numeric variables, we now describe the probabilities of detection in linguistic terms. Of various feasible objective functions, we consider a simple modification of Eq. (1). This consists in replacing the product of nondetection probabilities by their intersection.

To proceed, we define two more operations on fuzzy sets: complement and intersection. The complement, $\overline{A}$, of a fuzzy set A, given by Eq. (7), is specified by

$$\overline{A} = \{x, \chi_A(x)\} \quad , \; x \in X \; , \tag{11}$$

which replaces the membership grade by its complement with respect to unity. This allows us to convert the linguistic detection probability into a nondetection probability. The fuzzy intersection $A \cap B$ of A and B has the membership grade

$$\chi_{A \cap B}(x) = \min\left[\chi_A(x), \chi_B(x)\right] \; . \tag{12}$$

Equation (12) leads to a new definition of the objective function in which the product of individual objectives is replaced by their intersection. Similarly, the dynamic programming algorithm, Eq. (4), will involve the fuzzy intersection.

It is worth noting that the operation of fuzzy intersection may lead to a fuzzy set that is not one of the sets we have started with. In this case, the best fit procedure selects the fuzzy set that constitutes a solution to the problem.

IV. AN EXAMPLE OF COMPUTATION

An example of the optimization procedure for a nonserial configuration of activities with numeric detection probabilities is contained in Ref. 3. To illustrate our considerations for linguistic data, we consider a simple serial configuration of three activities with the detection probabilities given in Table I.

TABLE I. The dection probabilities for different activities as a function of cost			
	Option Cost		
Activity	1	2	3
A	very low	low to medium	
B	fairly low	low to high	medium
C	lower than medium	medium	

When the amount of available funds is one unit, the overall detection probability is low, with the unit fund assigned to option 1 of activity A. By increasing the amount of funds to five units, the detection probability becomes medium to high; the first two activities receive then two units to defend the second option, the third activity receives one unit for the first option.

The detection probabilities we have just arrived at were obtained by the least-squares fit to an ensemble consisting of eight arbitrarily selected fuzzy sets. By extending this ensemble, we can render the resulting detection probability to be more sensitive to the amount of available funds and to the detection probabilities of the individual options.

V. CONCLUSIONS

We have proposed a novel approach, based on the notion of fuzzy sets, to risk analysis and to the resource allocation problem. As in the investigation of system performance,[10] the key postulate is to replace the numeric attributes by linguistic attributes. This suggests that much of the uncertainty intrinsic in the system analysis is rooted in the fuzziness of the information which is resident in the database and, more particularly, in the fuzziness of underlying probabilities.

We intend to pursue these ideas to model the risk and allocation in a nuclear facility and, specifically, the insider threat, and defence in depth scenarios.

REFERENCES

1. J. W. TAPE, C. A. CCULTER, and J. T. MARKIN, "The Role of System Studies in Safeguards: Designing Integrated Systems," *J. Nucl. Mater. Manage. XV*(4), 79-82 (1987).

2. L. A. ZADEH, Foreword to *Fuzzy Sets, Natural Language Computations, and Risk Analysis*, Computer Science Press, Rockville, Maryland (1984).

3. A. ZARDECKI and J. T. MARKIN, "Defense in Depth and Resource Optimization for Safeguards," *Nucl. Mater. Manage XX* (Proceedings Issue) (to be published).

4. G. L. NEMHAUSER, *Introduction to Dynamic Programming*, John Wiley and Sons, Inc., New York (1966).

5. R. E. LARSON and J. L. CASTI, *Principles of Dynamic Programming*, Parts I and II, Marcel Dekker, New York and Basel (1982).

6. W. S. BRUCE and A. KANDEL, "The Application of Fuzzy Set Theory to a Risk Analysis Model of Computer Security," in *Advances in Fuzzy Sets, Possibility Theory, and Applications*, P. P. Wang, Ed., Plenum Press, New York (1983), pp. 351-376.

7. G. J. KLIR and T. A. FOLGER, *Fuzzy Sets, Uncertainty, and Information,* Prentice Hall, Englewood Cliffs (1988).

8. K. J. SCHMUCKER, *Fuzzy Sets, Natural Language Computations, and Risk Analysis*, Computer Science Press, Rockville, Maryland (1984).

9. A. KAUFMANN and M. M. GUPTA, *Fuzzy Mathematical Models in Engineering and Management Science,* North Holland, Amsterdam (1988).

10. A. ZARDECKI and E. A. HAKKILA, "Fuzzy Methods for System Performance," in *Proc. 13th Annual Symposium on Safeguards and Nuclear Material Management*, edited by L. Stanchi, ESARDA 1991, pp. 667-670.

END

DATE FILMED

12/12/91