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SATURATED POROUS CAVITY**

***Maria Laura Martins Costa¹
Rogério M. Saldanha da Gama
Rubens Sampaio²***

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Maria Laura Martins Costa¹
Rogério M. Saldanha da Gama
Rubens Sampaio²

¹ Part of the material presented here was developed while this author was at Mechanical Engineering Department, PUC/RJ.

² Department of Mechanical Engineering, PUC/RJ.

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ABSTRACT

The natural convection flow in a two-dimensional fluid-saturated porous cavity is modelled by means of a Theory of Mixtures viewpoint in which fluid and porous medium are regarded as continuous constituents of a binary mixture, coexisting superposed. A local description, that allows distinct temperature profiles for both fluid and solid constituents is obtained. The model, simplified by the Boussinesq approximation, is simulated with the help of the Control Volumes Method.

The effect of some usual parameters like Rayleigh, Darcy and Prandtl numbers and of a new dimensionless number, relating coefficients associated to the heat exchange between fluid and solid constituents (due to its temperature difference) and coefficients of heat conduction for each constituent, is considered. Stream lines for the fluid constituent and isotherms for both fluid and solid constituents are presented for some cases.

Qualitative agreement with results using the classical approach (Darcy's law and additional terms to account for boundary and inertia effects, used as momentum equation) was obtained.

INTRODUCTION

Natural convection in porous media is present in a broad range of engineering applications, such as geothermal energy extraction, fiber and granular insulation, heat losses from underground storage systems, solidification of castings, post-accident heat removal from nuclear reactor rubble beds or the identification of geological repositories (deep ocean seabeds or deep earth rock layers) for the storage of nuclear waste material.

The main purpose of the present work is the study of natural convection in a fluid-saturated porous cavity, a subject that attracts a great deal of research attention, not only for the fundamental nature of the problem, but also for the large number of applications, as previously mentioned.

The phenomenon is mathematically described with the help of the Continuum Theory of Mixtures, which considers fluid and porous medium as continuous constituents of a binary mixture, coexisting superposed in a material region (the mixture). Since both constituents are continuous, there exists, simultaneously, at each spatial point, two temperatures and two velocities, giving rise to new parameters (not present in the classical single continuum approach) in order to take into account the thermomechanical interaction between both constituents. The existence of two temperatures at a single point gives rise to an Energy Generation Function, while the existence of two velocities gives rise to an Interaction Force (a momentum source), providing, respectively, thermal and dynamical interactions.

Although natural convection in fluid-saturated porous cavities has already been extensively studied, it is not usual in the literature to allow a distinct temperature field for each constituent of the mixture (fluid and porous medium), as considered in the present work. Thus, the extensively used thermal equilibrium hypothesis is not employed in the problem formulation, which, besides allowing two temperature fields, is supported

by a thermodynamically consistent theory.

MATHEMATICAL MODEL

Since the porous medium is assumed rigid and at rest, the mass and linear momentum balance equations need only to be considered for the fluid constituent, while the energy balance must be satisfied by both constituents.

The continuity equation for the fluid constituent, assumed chemically inert, for steady-state flow regime, can be written as [1]:

$$\text{div}(\rho_F \underline{v}_F) = 0 \quad (1)$$

in which ρ_F is the fluid constituent mass density (locally, the ratio of mass of the fluid to the corresponding volume of mixture) and \underline{v}_F is the fluid constituent velocity. Since the porous medium is saturated by the fluid constituent, the field ρ_F is given as follows:

$$\rho_F = \varphi \rho \quad (2)$$

where ρ is the actual fluid density and φ the porosity.

The balance of linear momentum is given by [1]:

$$\rho_F (\text{grad} \underline{v}_F) \underline{v}_F = \text{div} \underline{\sigma}_F + \underline{m}_F + \rho_F \underline{g} \quad (3)$$

where $\underline{\sigma}_F$ is the partial stress tensor [1], analogous to the stress tensor in Continuum Mechanics, \underline{m}_F an interaction force, which acts as a momentum source and provides dynamical interaction between solid and fluid constituents [1] and \underline{g} is the specific body force.

The energy balance [1] must be satisfied by each constituent of the mixture. For steady-state flow, low velocities, no heat generation, it can be stated as:

$$\rho_i c_i (\text{grad } T_i) \cdot \underline{v}_i = -\text{div } \underline{q}_i + \psi_i \quad (4)$$

in which $i \equiv S$ and $i \equiv F$ stand for the solid and the fluid constituents, respectively, ρ_i represents the i -constituent density, T_i its temperature, \underline{q}_i its partial heat flux, ψ_i its energy generation function and, finally, c_i the specific heat of the i -constituent, regarded as a continuum.

The following constitutive equations for the partial stress tensor and the interaction force were proposed, considering the hypothesis stated by Williams [2] and Martins Costa [3]:

$$\underline{\sigma}_F = -p\underline{1} + \gamma \lambda \varphi^2 \mu (\text{div } \underline{v}_F) \underline{1} + 2\lambda \varphi^2 \mu \underline{D}_F \quad (5)$$

in which μ represents the actual fluid viscosity, λ a scalar parameter, always positive, which depends on the porous matrix microstructure [2], γ a scalar that, to satisfy the second law of Thermodynamics, must satisfy the relation: $\gamma \geq -2/3$ [3], \underline{D}_F the symmetrical part of the velocity gradient and p the pressure. Since the porous medium is at rest and saturated by the fluid, the interaction force assumes the form:

$$\underline{m}_F = -\frac{\varphi^2 \mu}{K} \underline{v}_F \quad (6)$$

where K is the specific permeability, a scalar, since the porous matrix is considered isotropic.

Equation (4) also requires some constitutive hypotheses. The partial heat fluxes for the solid (\underline{q}_S) and the fluid (\underline{q}_F) constituents, assuming low velocities, are stated, according to the model proposed by Saldanha da Gama [4]:

$$\underline{q}_S = -\Lambda k_S(1 - \varphi)\text{grad } T_S \quad (7)$$

$$\underline{q}_F = -\Lambda k_F \varphi \text{grad } T_F \quad (8)$$

where Λ represents an always positive parameter which may depend on both the internal structure and the kinematics of the mixture and k_S and k_F are the solid and the fluid thermal conductivities.

The total heat flux (per unit of time and area) for the mixture is given by:

$$\underline{q} = \underline{q}_F + \underline{q}_S \quad (9)$$

The energy generation function, ψ , which is an internal contribution, represents the energy supply to a given constituent, arising from its (thermal) interaction with the other constituents of the mixture. The ψ function is zero at a given point only if all the constituents are at the same temperature at this point. According to Saldanha da Gama [4] and Martins Costa [3], the following linear constitutive relations can be considered for both constituents energy generation function:

$$\psi_S = R_{SF}(1 - \delta|\underline{v}_F|)(T_F - T_S) \quad (10)$$

$$\psi_F = R_{FS}(1 - \delta|\underline{v}_F|)(T_S - T_F)$$

as the solid constituent is supposed at rest.

The coefficients R_{SF} , R_{FS} and δ are positive valued parameters which depend on both constituents' thermal properties and on the mixture internal structure. In order to satisfy the energy balance for the mixture [3],

which states that the sum $\psi_F + \psi_S$ must be zero, it is necessary to ensure that $R_{SF} \equiv R_{FS} \equiv R$.

The influence of the fluid constituent's velocity on the energy generation function (accounting for convective heat transfer), for simplicity, will be neglected in the present work. In other words, δ will be considered zero. Under this assumption, the energy generation function can be represented by the following equation:

$$-\psi_S = \psi_F = R(T_S - T_F) \quad (11)$$

The classical Boussinesq approximation can be stated as:

$$\rho_F \approx \rho_0(1 - \beta(T_F - T_0)) \quad (12)$$

where $\rho_0 = \rho\varphi$, β is a coefficient of thermal expansion and T_0 is a reference temperature, which, in this work, is the lowest temperature.

AN APPLICATION

In this section the natural convection phenomenon in a cavity, as presented in figure 1, is taken into consideration. Assuming constant thermal properties, low velocities, isocoric flow and the mentioned Boussinesq approximation, the problem, in its two-dimensional form ($0 < x < L$, $0 < y < H$), is reduced to:

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0$$

$$\rho(U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y}) = -\frac{\partial P}{\partial x} + \lambda\varphi\mu \left[\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} \right] - \frac{\varphi\mu}{K} U$$

$$\rho \left(U \frac{\partial V}{\partial x} + V \frac{\partial U}{\partial y} \right) = - \frac{\partial P}{\partial y} + \rho \beta g T_F + \lambda \varphi \mu \left[\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} \right] - \frac{\varphi \mu}{K} V \quad (13)$$

$$U \frac{\partial T_F}{\partial x} + V \frac{\partial T_F}{\partial y} = \frac{\Lambda k_F}{\rho c_F} \left[\frac{\partial^2 T_F}{\partial x^2} + \frac{\partial^2 T_F}{\partial y^2} \right] + \frac{R}{\rho c_F} (T_S - T_F)$$

$$0 = \frac{\partial^2 T_S}{\partial x^2} + \frac{\partial^2 T_S}{\partial y^2} + \frac{R}{\Lambda k_S (1 - \varphi)} (T_F - T_S)$$

where U and V are \underline{v}_F components on x and y directions, respectively and $P = p + \rho(1 - \beta T_0)gy$ is a modified pressure. The following boundary conditions are to be satisfied:

$$T_F = T_S = T_C, \quad U = V = 0 \quad \text{at } x = 0, \quad 0 < y < H$$

$$T_F = T_S = T_H, \quad U = V = 0 \quad \text{at } x = L, \quad 0 < y < H$$

$$\frac{\partial T_F}{\partial y} = \frac{\partial T_S}{\partial y} = 0, \quad U = V = 0 \quad \text{at } y = 0, \quad 0 < x < L \quad (14)$$

$$\frac{\partial T_F}{\partial y} = \frac{\partial T_S}{\partial y} = 0, \quad U = V = 0 \quad \text{at } y = H, \quad 0 < x < L$$

NUMERICAL PROCEDURE

Since no analytical solution to the system of equations describing the natural convection flow is known, numerical approximations to its solution

are searched with the help of a Control Volumes Finite Difference approach, proposed by Patankar [5]. In this methodology, the balance laws are applied over finite size control volumes, surrounding the grid nodes. An integration over these control volumes furnishes the discretization equations. To present equation (13) in the form stated in [5], the Energy Generation Function (a linear function on both T_F and T_S , present in the fluid and the solid constituents' energy equations) and the interaction force between both constituents (a linear function on \underline{v}_F , acting as a momentum sink in the fluid constituent's momentum equation), which are not present in a classical Continuum Mechanics viewpoint, are considered as part of the source terms. (The first one, a temperature dependent source and the second one, a velocity dependent source.)

When the natural convection in a cavity is studied, both momentum and energy equations of the fluid inside the cavity are coupled. However, if the cavity is a porous one and the problem is modelled through a Theory of Mixtures viewpoint, not only fluid constituent's momentum and energy equations are coupled to one another, but also to the solid constituent's energy equation. So, in the present work, a Control Volumes method is applied to find numerical approximations to the solution of the system stated in equations (13) and (14).

The discretization equations are solved using the TDMA line by line algorithm [5], a combination of the TDMA (tridiagonal matrix algorithm) direct method to the Gauss-Seidel method, which is applied first to the fluid constituent's momentum and continuity equations, then to its energy equation and, at last, to the solid constituent's energy equation.

Since no appreciable differences are observed as mesh size varies from 21x21 to 28x28 uniformly distributed nodal points, it can be considered that reasonable accuracy was obtained.

In fact two different (but coupled) problems are being solved: the fluid constituent's problem (composed by mass, momentum and energy equa-

tions) and the solid constituent's problem (reduced to the energy equation, since the solid constituent is supposed rigid and at rest), and each problem's solution acts as input for the other one. The fluid constituent's problem is solved with the help of an initial estimate for T_S , which is present in the fluid constituent's energy equation. Then the values obtained for both components of \underline{v}_F (U and V) are kept in auxiliary vectors (the components U and V for the solid constituent are zero) and the ones for T_F are used in the solid constituent's energy equation, whose solution gives new T_S values. The previously obtained values for U and V are then recovered for a new iteration of the fluid constituent's problem. This process is repeated until convergence for U , V , T_F and T_S is obtained.

After their solving, the discretization equations are put in a dimensionless form (for $0 < x < L$, $0 < y < H$):

$$\frac{\partial u}{\partial \xi} + \frac{\partial v}{\partial \eta} = 0$$

$$u \frac{\partial u}{\partial \xi} + v \frac{\partial u}{\partial \eta} = -\frac{\partial p'}{\partial \xi} - \frac{Pr_F}{Da} u + \lambda Pr_F \left[\frac{\partial^2 u}{\partial \xi^2} + \frac{\partial^2 u}{\partial \eta^2} \right]$$

$$u \frac{\partial v}{\partial \xi} + v \frac{\partial v}{\partial \eta} = -\frac{\partial p'}{\partial \eta} - \frac{Pr_F}{Da} v + Ra_F Pr_F \theta_F + \lambda Pr_F \left[\frac{\partial^2 v}{\partial \xi^2} + \frac{\partial^2 v}{\partial \eta^2} \right] \quad (15)$$

$$u \frac{\partial \theta_F}{\partial \xi} + v \frac{\partial \theta_F}{\partial \eta} = \Lambda \left[\frac{\partial^2 \theta_F}{\partial \xi^2} + \frac{\partial^2 \theta_F}{\partial \eta^2} \right] + \Lambda R_F^* (\theta_S - \theta_F)$$

$$0 = \frac{\partial^2 \theta_S}{\partial \xi^2} + \frac{\partial^2 \theta_S}{\partial \eta^2} + R_S^* (\theta_F - \theta_S)$$

with the following associated boundary conditions :

$$\theta_F = \theta_S = 0, \quad u = v = 0 \quad \text{at } \xi = 0, \quad 0 < \eta < H/L$$

$$\theta_F = \theta_S = 1, \quad u = v = 0 \quad \text{at } \xi = 1, \quad 0 < \eta < H/L$$

$$\frac{\partial \theta_F}{\partial \eta} = \frac{\partial \theta_S}{\partial \eta} = 0, \quad u = v = 0 \quad \text{at } \eta = 0, \quad 0 < \xi < 1 \quad (16)$$

$$\frac{\partial \theta_F}{\partial \eta} = \frac{\partial \theta_S}{\partial \eta} = 0, \quad u = v = 0 \quad \text{at } \eta = H/L, \quad 0 < \xi < 1$$

The dimensionless equations were obtained with the help of the following dimensionless variables:

$$\begin{aligned} \xi &= \frac{x}{L} & \eta &= \frac{y}{L} \\ u &= \frac{UL}{\alpha_F} & v &= \frac{VL}{\alpha_F} & p' &= \frac{PL^2}{\rho\alpha_F} \\ Pr_F &= \frac{\varphi\eta}{\rho\alpha_F} & Da &= \frac{K}{L^2} \\ Ra_F &= \frac{\rho g \beta L^3 (T_H - T_C)}{\varphi\eta\alpha_F} \\ \theta_F &= \frac{T_F - T_C}{T_H - T_C} & \theta_S &= \frac{T_S - T_C}{T_H - T_C} \\ R_F^* &= \frac{RL^2}{\Lambda k_F \varphi} & R_S^* &= \frac{RL^2}{\Lambda k_S (1 - \varphi)} \end{aligned} \quad (17)$$

where $\alpha_F = k_F / \rho c_F$, L is the cavity's length and T_H and T_C are, respectively, the highest and the smallest cavity's temperatures. The new dimensionless parameters R_F^* and R_S^* relate the coefficients associated to the heat

exchange between both constituents (due to its temperature difference) to coefficients of heat conduction, for the fluid and the solid constituent, respectively.

RESULTS

In this section some results showing stream lines and isotherms for both fluid and solid constituents are presented. Figures 2,3 and 4 show the influence of the parameter R on these three isovalues curves, when R varies from 10^3 to 10^5 . (Consequently the dimensionless parameters R_F^* and R_S^* vary from 8.33×10^2 to 8.33×10^4 and from 8.33 to 8.33×10^2 , respectively.) A comparison between the stream lines presented in these three figures shows that they are not significantly affected by the variation of R : stream lines shapes are essentially the same in these three cases, while stream function values tend to increase as R increases. A comparison among both constituents isotherms, presented in figures 2,3 and 4, shows that they get closer as the parameter R grows (for $R = 10^5$ they are almost superposed). This trend could be anticipated by the examination of both constituents' energy equations, as the parameter R is present in both equations coupling term (corresponding to the thermal interaction between fluid and solid constituents), meaning that the more R grows, more significant becomes the thermal coupling. For small values of R ($R_F^* \leq 8.33 \times 10^1$, $R_S^* \leq 8.33 \times 10^{-1}$) solid constituent's isotherms are almost vertical straight lines, showing a large difference if compared to fluid constituent's. It can be concluded that the usual thermal equilibrium hypothesis can be considered a good one for large values of R_F^* and R_S^* only.

Alterations in Rayleigh number cause alterations in the buoyancy term, which governs natural convection flow. Figure 5 shows isovalues when Rayleigh number is ten times greater ($Ra_F = 2.92 \times 10^{12}$) than the de-

fault value (2.92×10^{11}) considered in figures 2 to 4. A comparison between figures 5 and 3, valid since the only varying parameter is the Rayleigh number, shows that as it increases a significant increase in stream function values occurs, but no meaningful alteration on the stream lines form is observed. A decrease of the same order in Rayleigh number, on the other hand, causes not only a decrease in stream function values, but also makes stream lines tend to circumferences.

Solid and fluid constituents' isotherms are also affected by Rayleigh number. Comparing figures 5 and 3 it can be noticed that isotherms tend to become more vertical as Rayleigh number decreases. (For a Rayleigh number ten times smaller than that used in figure 3 ($Ra = 2.92 \times 10^{10}$), the isotherms tend to vertical straight lines.) Examining the isotherms presented in figures 3 and 5 it can also be noticed that the difference between solid and fluid constituents' isotherms, for each figure, increases as Rayleigh number increases.

Darcy number is of great importance in porous media problems as, for a fixed dimensions porous matrix, it is a measure of the porous medium permeability. Figures 6 and 7 present $Da = 10^{-4}$ and $Da = 10^{-6}$, respectively ten times greater and ten times smaller than the default value (10^{-5}) used in figure 3. A comparison among the stream lines presented in these three figures shows that they tend to circumferences, while stream function values decrease, as Darcy number decreases.

A decrease in Darcy number also makes both constituents' isotherms tend to vertical lines. Besides, the difference between solid and fluid constituents' isotherms, for a given figure, increases as Darcy number increases.

A comparison between figures 6 (when $Da = 10^{-4}$) and 5 ($Ra_F = 2.92 \times 10^{12}$) shows a strong similarity among the three isovalues curves. An analogous behaviour can be noticed for $Ra = 2.92 \times 10^{10}$, whose isovalues curves are almost the same of those shown in figure 7, when $Da = 10^{-6}$.

Prandtl number variation was also considered in the present work but no meaningful influence on the three isovalues curves is observed as Prandtl is made ten times greater or ten times smaller than the default value ($Pr = 3.39$, obtained for water).

FINAL REMARKS

The model presented in this work allows a local simulation of the natural convection flow in a fluid saturated porous cavity, in a context of thermal nonequilibrium, based on a systematic theory. This model, which considers fluid and solid (porous matrix) as continuous constituents of a binary mixture, was stated in such a way that the classical balance equations are recovered when the number of constituents is reduced to one.

The effects of a new dimensionless parameter (R_F^* or R_S^*) as well as those caused by Rayleigh, Prandtl and Darcy numbers variation on fluid constituent's velocity and on both fluid and solid constituents' temperature were studied.

In order to allow comparisons among results obtained with this two-temperatures model and with the usual thermal equilibrium hypothesis (solid and fluid with the same temperature distribution) it is sufficient to consider a very high value for R . Since $R(T_S - T_F)$ is bounded, $R \rightarrow \infty$ implies that $T_S \rightarrow T_F$ everywhere.

A comparison with the study of Beckermann et al [6], which uses a single continuum approach, supposes thermal equilibrium between fluid and porous medium, neglects inertial terms and considers a darcian term (analogous to the interaction force), a Brinkman term (analogous to the diffusive term) and a Forchheimer term (without analogy in this work, as the interaction force is supposed linear in \underline{v}_F [2]) to account for the porous medium, has shown qualitative agreement.

Some previous works (e. g. Vafai and Sozen [7] and Lage and Bejan [8]) have already considered a two-temperatures model, using a Continuum Mechanics viewpoint. A term, analogous to the energy generation function, takes into account fluid and solid temperatures difference. Both temperatures are calculated as intrinsic volume averages, each one in a volume associated to the respective phase. On the other hand, in a Mixture Theory approach both temperatures are evaluated in the same region, since fluid and solid constituents coexist superposed, occupying the whole volume of the mixture.

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NOMENCLATURE

c_i	i -th specific heat
\underline{D}_F	symmetrical part of the fluid constituent velocity gradient
Da	Darcy number
e_i	i -th constituent internal energy
\underline{g}	gravitational acceleration
H	height of enclosure
K	specific permeability
k_i	i -th thermal conductivity
L	length of enclosure
\underline{m}_F	fluid constituent interaction force
p	pressure
p'	dimensionless modified pressure
P	modified pressure
Pr_F	fluid constituent Prandtl number
\underline{q}	heat flux
\underline{q}_i	i -th constituent partial heat flux
R	heat transfer coefficient between both constituents (also denoted by R_{FS} or R_{SF})
R_F^*	dimensionless R for fluid constituent
R_S^*	dimensionless R for solid constituent
Ra_F	fluid constituent Rayleigh number
T_i	i -th constituent temperature
u	dimensionless x-component of fluid constituent's velocity
U	x-component of fluid constituent's velocity
v	dimensionless y-component of fluid constituent's velocity
V	y-component of fluid constituent's velocity
\underline{v}_F	fluid constituent velocity (vector field)
α_F	fluid constituent thermal diffusivity

β	coefficient of thermal expansion
δ	coefficient depending on thermal properties and on mixture structure
η	dimensionless vertical coordinate
θ_i	dimensionless i-th constituent temperature
λ	parameter depending on porous matrix
Λ	parameter related to mixture structure
μ	fluid viscosity
ξ	dimensionless horizontal coordinate
ρ	fluid density (as a continuum)
ρ_i	i-th constituent density
$\underline{\sigma}_F$	fluid constituent partial stress tensor
φ	porosity
ψ_i	i-th constituent energy generation function

Subscripts

F	fluid constituent
S	solid constituent

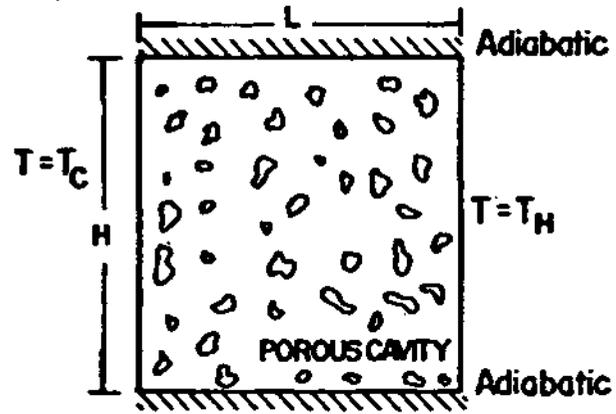
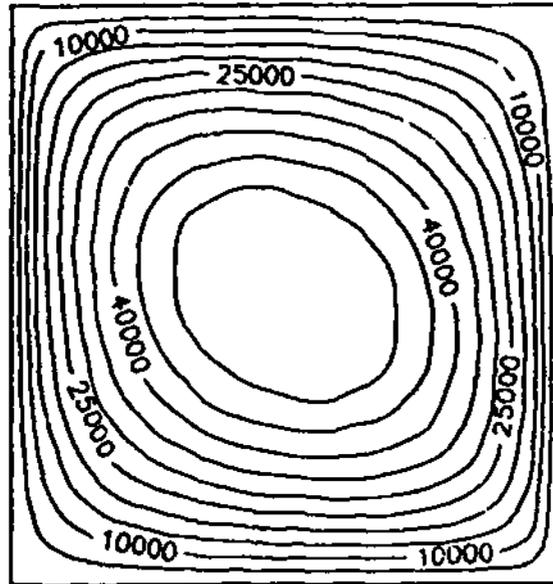
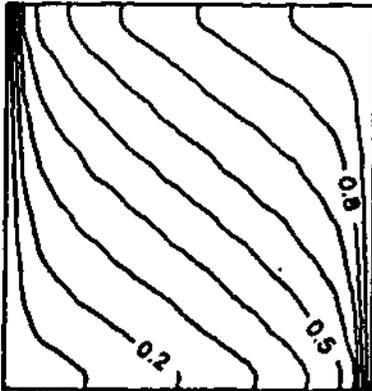


Figure 1 - Problem's Scheme

STREAM LINES



ISOTHERMS - FLUID



ISOTHERMS - SOLID

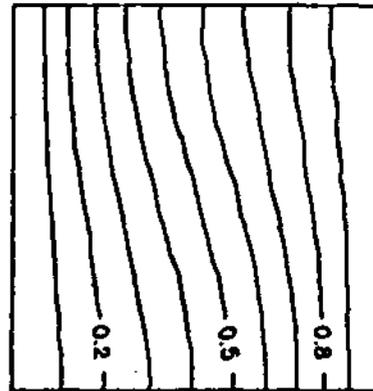
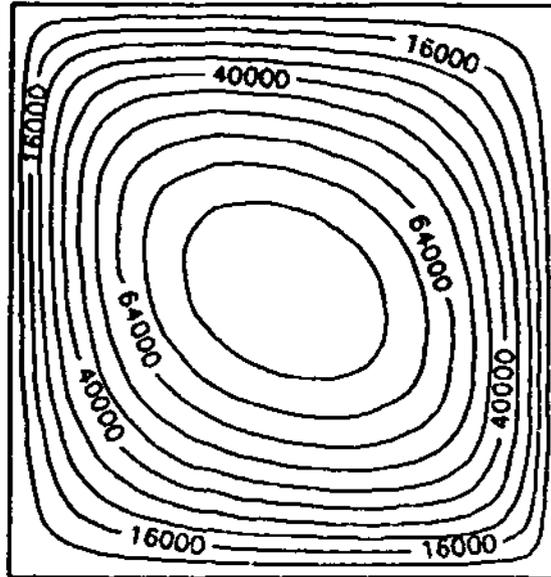
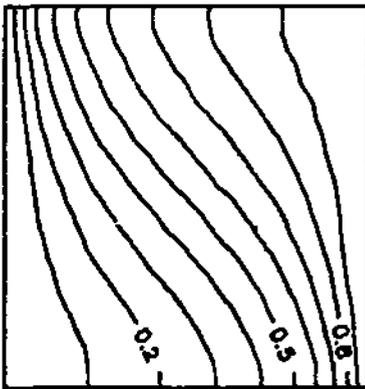


Figure 2 - Stream Lines and Isotherms
(for $R = 10^9$, $Pr_F = 3.39$, $Ra_F = 2.92 \times 10^{11}$, $Da = 10^{-6}$)

STREAM LINES



ISOTHERMS - FLUID



ISOTHERMS - SOLID

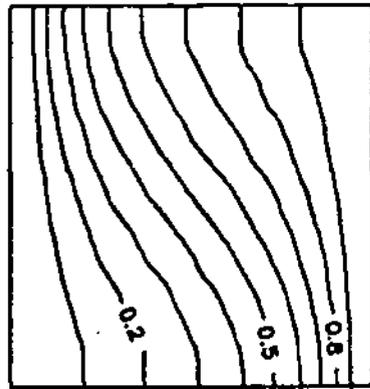
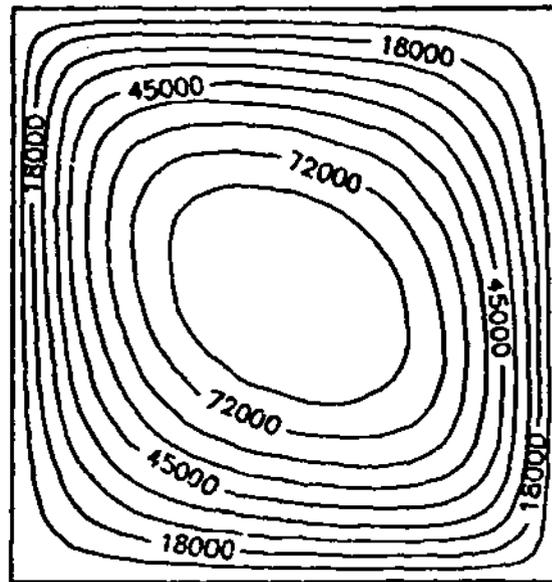
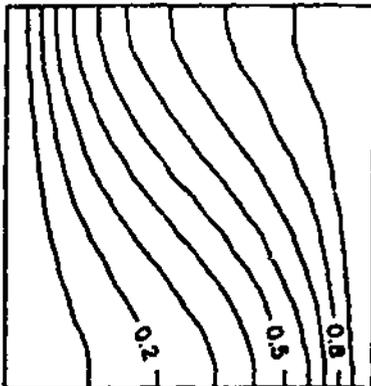


Figure 3 - Stream Lines and Isotherms
(for $R = 10^4$, $Pr_F = 3.39$, $Ra_F = 2.92 \times 10^{11}$, $Da = 10^{-5}$)

STREAM LINES



ISOTHERMS - SOLID



ISOTHERMS - FLUID

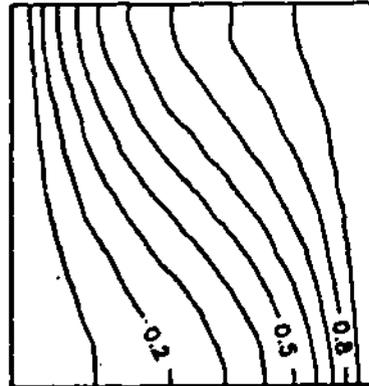
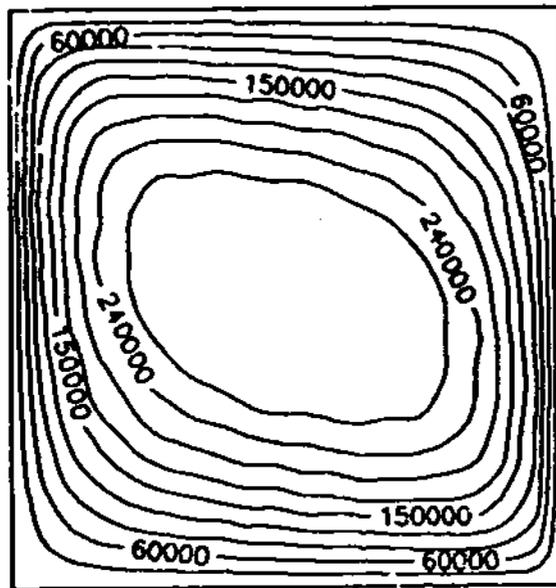
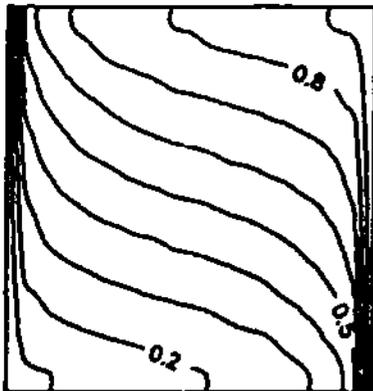


Figure 4 - Stream Lines and Isotherms
(for $R = 10^6$, $Pr_F = 3.39$, $Ra_F = 2.92 \times 10^{11}$, $Da = 10^{-6}$)

STREAM LINES



ISOTHERMS - FLUID



ISOTHERMS - SOLID

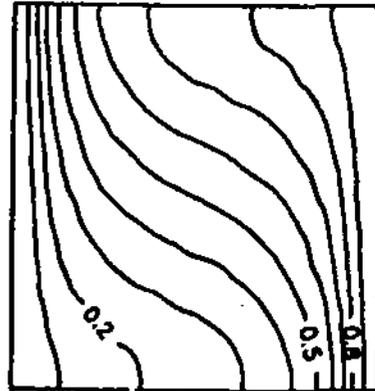
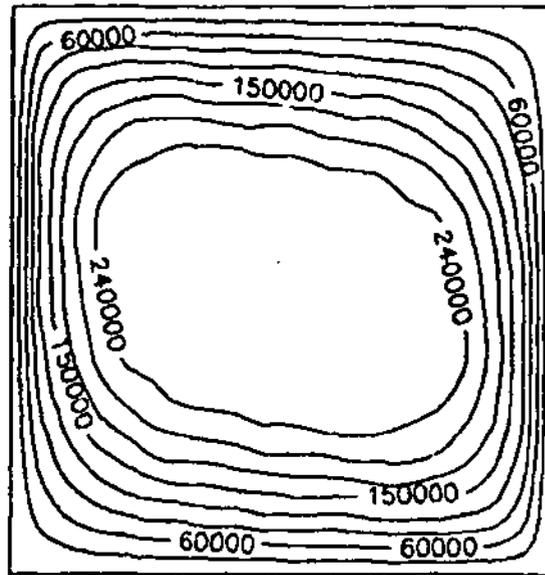
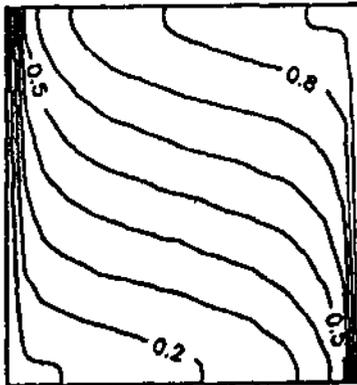


Figure 5 - Stream Lines and Isotherms
(for $R = 10^4$, $Pr_F = 3.39$, $Ra_F = 2.92 \times 10^{12}$, $Da = 10^{-6}$)

STREAM LINES



ISOTHERMS - FLUID



ISOTHERMS - SOLID

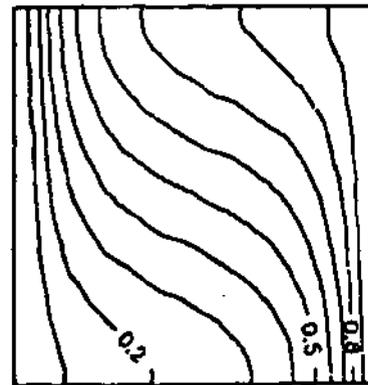


Figure 6 - Stream Lines and Isotherms
(for $R = 10^4$, $Pr_F = 3.39$, $Ra_F = 2.92 \times 10^{11}$, $Da = 10^{-4}$)

STREAM LINES

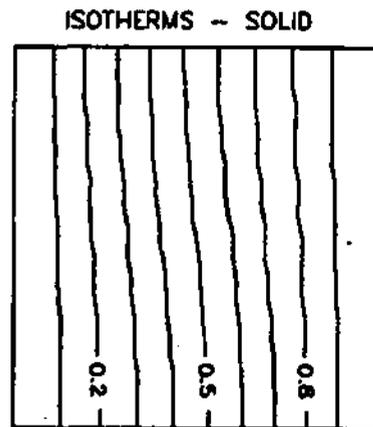
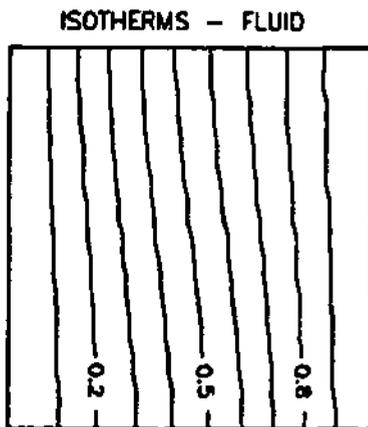
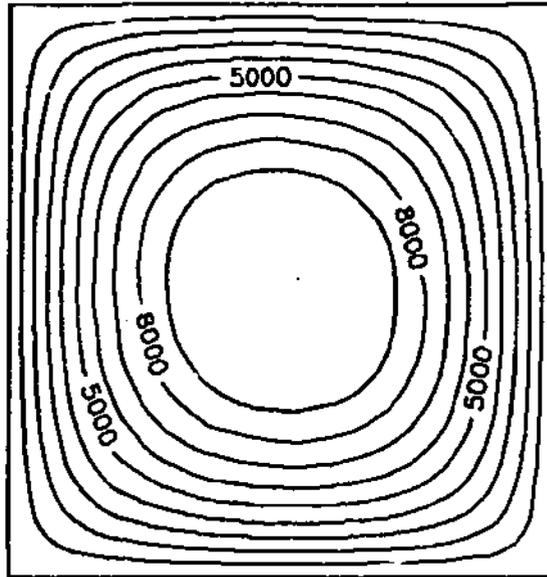


Figure 7 - Stream Lines and Isotherms
(for $R = 10^4$, $Pr_F = 3.39$, $Ra_F = 2.92 \times 10^{11}$, $Da = 10^{-6}$)

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Secretaria da Ciência e Tecnologia
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Laboratório Nacional de Computação Científica
Rua Lauro Müller, 455 – Caixa Postal 56018 – 22290 – Rio de Janeiro – RJ – Brasil
Telex: (21) 22653 LCCN BR – FAX: (021) 295-8499