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EXTENDED VIRASORO ALGEBRA AND ALGEBRA OF AREA PRESERVING DIFFEOMORPHISMS

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ՎԻՐԱՍՈՐՈՅԻ ԸՆԴԼԱՅՆՎԱԾ ՀԱՆՐԱՀԱՇԻՎԸ: ԾԱՎԱԼԸ ՊԱՀՊԱՆՈՂ ԴԻՖԵՈՄՈՐՖԻԶՄՆԵՐԻ ՀԱՆՐԱՀԱՇԻՎԸ:

Ծավալը պահպանող դիֆեոմորֆիզմների հանրահաշիվը կարևոր դեր է խաղում ռելյատիվիստիկ մակերևույթի տեսության մեջ: Մենք ընդգծել ենք այս հանրահաշվի և ընդլայնված Կաց-Մուդիի $G\widehat{(T^2)}$ հանրահաշվին առնչված Վիրասորոյի ընդլայնված հանրահաշվի միջև կապը: Ուսումնասիրված են այս անվերջ չափողականության հանրահաշիվների, ինչպես նաև նրանց ենթահանրահաշիվների ներկայացումները:

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EXTENDED VIRASORO ALGEBRA AND ALGEBRA OF AREA PRESERVING DIFFEOMORPHISMS

The algebra of area preserving diffeomorphisms plays an important role in the theory of relativistic membranes. We pointed out the relation between this algebra and the extended Virasoro algebra associated with the generalized Kac-Moody algebras $\widehat{G}(T^2)$. The highest weight representation of these infinite-dimensional algebras as well as of their subalgebras is studied.

Yerevan Physics Institute

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Т.А.АРАКЕЛЯН

РАСШИРЕННАЯ АЛГЕБРА ВИРАСОРО И АЛГЕБРА ДИФФЕОМОРФИЗМОВ СОХРАНЯЮЩИХ ОБЪЁМ

Алгебра диффеоморфизмов сохраняющих объём играет важную роль в теории релятивистских поверхностей. Мы отмечаем связь этой алгебры с расширенной алгеброй Вирасоро, ассоциированной с обобщенными алгебрами Каца-Муди $\widehat{G(T^2)}$. Исследовано представление старшим весом этих бесконечномерных алгебр,а также их подалгебр.

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1. Introduction

In the light-cone gauge the residual group of symmetry of relativistic surface consists of the area preserving diffeomorphisms of the two-dimensional surface M . The role of this group is close to that of the group of conformal transformations in string theory. For the surfaces of the simplest topology T^2 , S^2 , there were calculated the structure constants and the second group of cohomologies of the algebra of area preserving diffeomorphisms [1] .

In the present paper we are studying the highest weight representation of the extended Virasoro algebra and the algebra of area preserving diffeomorphisms for $M = T^2$. It is in particular shown that the extended Virasoro algebra for $M = T^2$ admits all degenerate representations of the Virasoro algebra as well as contains a subalgebra of the type of $\widehat{SL}^{(n)}(2,R)$ (which differs only in central extension of the corresponding Kac-Moody algebra) whose Verma module structure is close to the Verma module over $\widehat{SL}(2,R)$.

2. sdiff T^2 Algebra and Extended Virasoro Algebra

It is well known that the Virasoro algebra and Kac-Moody algebra are connected with each other, namely the Virasoro algebra is central extension of differentiation algebra of the Kac-Moody algebra $\widehat{G}(S^1)$ [2] . Therefore, it seems natural that the generalization of the Virasoro algebra on the two-dimensional manifold M is central extension of differentiation algebra of the generalized Kac-Moody algebras $\widehat{G}(M)$. By definition the generalized Kac-Moody algebra is central extension of the algebra $G(M)$ of smooth maps from 2-dimensional compact manifold M into the semi-simple Lie algebra G , given by the two-cocycles $\omega_\varphi : G(M) \times G(M) \to C$,

$$\omega_\varphi(X,Y) = \int_M \langle X, dY \rangle \wedge \varphi \qquad (1)$$

where X , $Y \in G(M)$, $\langle \, , \, \rangle$ denote Killing form on G and φ is some closed form on the M [3] . Lie bracket for algebra $\widehat{G}(M)$ looks like

$$[X,Y]_* = [X,Y] + \omega_\varphi(X,Y) \cdot C \, . \qquad (2)$$

Differentiation algebra of $G(M)$ consists of vector fields on M $L = \sum_{i=1}^{2} A^i(\sigma_1, \sigma_2)\partial\sigma_i$ which satisfy a relation:

$$L[X,Y]_* = [LX,Y]_* + [X,LY]_* \, . \qquad (3)$$

With the given 1-form φ condition (3) leads to the following restriction for fields L on the M :

$$\mathcal{L}_L \varphi = 0 \qquad (4)$$

where $\mathcal{L}_L$ is Lie derivation along vector field. From relation $[\mathcal{L}_{L_1}, \mathcal{L}_{L_2}] = \mathcal{L}_{[L_1,L_2]}$ it follows that the vector fields preserving 1 form φ compose differentiation algebra of $\widehat{G(M)}$. Thus, contrary to the S^1 case, here family of nontrivial cocycles define different differentiation algebras of algebra $\widehat{G(M)}$, and therefore Hodge decomposition parametrizes differentiation algebra of algebra $\widehat{G(M)}$. Differentiation algebra of algebra $\widehat{G(M)}$ with cocycles (1) we denote by $\mathcal{D}iff(\widehat{G(M)}, \omega_\varphi)$, whose central extension we call extended Virasoro algebra. Further we discuss algebras $Sdiff\ T^2$ and $\mathcal{D}iff(\widehat{G(T^2)}, \omega_{\varphi_i})$ whose structure constants and cocycles are simplest, where $\varphi_1 = i d\sigma_2$ and $\varphi_2 = i d\sigma_1$ are harmonic 1 forms on the torus $T^2 = S^1 \times S^1$.

Then the basis of algebra $\mathcal{D}iff(\widehat{G(T^2)}, \omega_{\varphi_i})$ has a form:

$$\mathcal{D}iff(\widehat{G(T^2)}, \omega_{\varphi_1}) = \bigoplus_{(n,m)\in \mathbb{Z}^2} c L^{(1)}_{nm} \oplus c L^{(2)}_{00} \tag{5a}$$

$$\mathcal{D}iff(\widehat{G(T^2)}, \omega_{\varphi_2}) = \bigoplus_{(n,m)\in \mathbb{Z}^2} c L^{(2)}_{nm} \oplus c L^{(1)}_{00} \tag{5b}$$

which satisfy the following commutation relations:

$$[L^{(1)}_{nm}, L^{(1)}_{pk}] = (m-k) L^{(1)}_{n+p,m+k}, \tag{6a}$$

for $\mathcal{D}iff(\widehat{G(T^2)}, \omega_{\varphi_1})$

$$[L^{(2)}_{00}, L^{(1)}_{nm}] = -n L^{(1)}_{nm}, \tag{6b}$$

and

$$[L^{(2)}_{nm}, L^{(2)}_{pk}] = (m-k) L^{(2)}_{n+p,m+k}, \tag{7a}$$

$$[L^{(1)}_{00}, L^{(2)}_{nm}] = -n L^{(2)}_{nm}. \tag{7b}$$

for $\mathcal{D}iff(\widehat{G(T^2)}, \omega_{\varphi_2})$

Differentiation algebra $\mathcal{D}iff(G(T^2), \omega\varphi_1)$ admits the following central extension:

$$[L^{(1)}_{nm}, L^{(1)}_{pk}] = (m-k)L^{(1)}_{n+p,m+k} + \frac{c_1}{12}(m^3-m)\delta_{n+p,0}\,\delta_{m+k,0}\ , \tag{8a}$$

$$[L^{(2)}_{00}, L^{(1)}_{nm}] = -nL^{(1)}_{nm}\ . \tag{8b}$$

The same concerns the algebra $\mathcal{D}iff(\widehat{G(T^2)}, \omega\varphi_2)$: due to their isomorphism further we discuss only the algebra $\mathcal{D}iff(G(T^2), \omega\varphi_1)$ with central extension (8a) and (8b), which we call extended Virasoro algebra and denote $EVir_1 = \mathcal{D}iff(\widehat{G(T^2)}, \omega\varphi_1) \oplus CC_1$. Algebras $EVir_1$ and $\mathcal{D}iff(\widehat{G(T^2)}, \omega\varphi_1)$ occupy an important place in the theory of toroidal relativistic membrane. In the light-cone gauge the residual group of symmetry of relativistic membrane is the area preserving diffeomorphism group $Sdiff\,T^2$ whose algebra consists of all divergenceless vector fields on the torus T^2 . Basis of algebra $Sdiff\,T^2$ is expressed through basis of algebras $\mathcal{D}iff(\widehat{G(T^2)}, \omega\varphi_i)$, $i=1,2$ and satisfies the following commutation relations:

$$Sdiff\,T^2 = \bigoplus_{(n,m)\in Z^2} cL_{nm} \oplus cL^{(1)}_{00} \oplus cL^{(2)}_{00} \oplus c\tilde{C}_1 \oplus c\tilde{C}_2\ , \tag{9a}$$

$$L_{nm} = mL^{(1)}_{mn} - nL^{(2)}_{nm},\ [L^{(1)}_{mn}, L^{(2)}_{pk}] = mL^{(1)}_{m+k,n+p} - PL^{(2)}_{n+p,m+k}, \tag{9b}$$

$$[L_{nm}, L_{pk}] = (nk-mp)L_{n+p,m+k} + (\tilde{C}_1 n + \tilde{C}_2 m)\delta_{n+p,0}\,\delta_{m+k,0}\ , \tag{9c}$$

$$[L^{(1)}_{00}, L_{nm}] = -nL_{nm}\ , \quad [L^{(2)}_{00}, L_{nm}] = -mL_{nm}\ . \tag{9d}$$

Note that the central extensions for $EVir_1$ and $EVir_2$ are not connected with the corresponding central extension for

3. Highest Weight Representation

Extended Virasoro algebra $EVir_1$ has the following root space decomposition:

$$EVir_1 = n_- \oplus H \oplus n_+ \tag{10a}$$

$$n_- = \bigoplus_{\substack{n\in Z_+\setminus\{0\} \\ m\in Z}} c L^{(1)}_{-n,m} \bigoplus_{n\in Z_+\setminus\{0\}} c L^{(1)}_{0,-n} , \tag{10b}$$

$$n_+ = \bigoplus_{\substack{n\in Z_+\setminus\{0\} \\ m\in Z}} c L^{(1)}_{+n,m} \bigoplus_{n\in Z_+\setminus\{0\}} c L^{(1)}_{0,+n} , \tag{10c}$$

$$H = cL^{(2)}_{00} \oplus cL^{(1)}_{00} \oplus c C_1 . \tag{10d}$$

in accordance with which we have defined the highest weight representation of algebra $EVir_1$ in the space admitting nonzero vector ν, such that

$$L^{(i)}_{00} \nu = h_i \nu , \tag{11a}$$

$$C_1 \nu = c_1 \nu , \tag{11b}$$

$$L^{(1)}_{+n,m} \nu = L^{(1)}_{0,+n} \nu = 0, \quad n\in Z_+\setminus\{0\}, \; m\in Z , \tag{11c}$$

and the representation space is a linear combination of vectors of the form:

$$L^{(1)}_{-n_s,m_s}\cdots L^{(1)}_{-n_1,m_1}\cdot L^{(1)}_{0,-p_z}\cdots L^{(1)}_{0,-p_1}\nu\,, \tag{11d}$$

$$(n_1,m_1)\leq\cdots\leq(n_s,m_s),\ (0,p_1)\leq\cdots\leq(0,p_z),\ s,\ z\in Z_+\,,$$

$$n_i\in Z_+\setminus\{0\}\,,\quad m_i\in Z\,,\quad i=1,\dots,s\,,$$

$$p_\ell\in Z_+\setminus\{0\}\,,\quad \ell\in 1,\dots,z\,.$$

We assume that all monomials of the form of (11d) are linearly independent and we denote the representation space as $M(h_1,h_2,c_1)$. The corresponding representation is known as Verma representation, and the representation space $M(h_1,h_2,c_1)$ is Verma module over algebra $EViz_1$. An important tool for the investigation of Verma representations is antilinear antiinvolution (Hermitian conjugation) and contravariant form $\langle\cdot|\cdot\rangle$, which give a possibility to study the reducibility (degeneration) and unitarity of Verma representation. Antilinear antiinvolution is defined over basis of algebra $EViz_1$ and by antilinearity continues on the whole algebra:

$$\omega(L^{(1)}_{nm})=L^{(1)}_{-n,-m}\,, \tag{12a}$$

$$\omega(L^{(i)}_{00})=L^{(i)}_{00}\,,\quad i=1;2. \tag{12b}$$

which satisfies

$$\omega([L^{(1)}_{nm},L^{(1)}_{pk}])=[\omega(L^{(1)}_{pk}),\ \omega(L^{(1)}_{nm})]\,; \tag{12c}$$

$$\omega^2=I \tag{12d}$$

Continuing ω from algebra $EViz_1$ over universal enveloping algebra $U(EVir_1)$ we define contravariant form $\langle \cdot | \cdot \rangle$ as

$$\langle R\nu | P\nu \rangle = \langle \nu | \omega(R) P\nu \rangle , \qquad (13)$$

for arbitrary R and $P \in U(EVir_1)$.

Representation space $M(h_1, h_2, c_1)$ decomposes to orthogonal direct sum of proper subspaces $L_{00}^{(1)}$ and $L_{00}^{(2)}$ which can be infinite-dimensional:

$$M(h_1, h_2, c_1) = \bigoplus_{\substack{j_2 \in Z_+ \\ j_1 \in Z}} V(h_1 + j_1 , h_2 + j_2 , c_1), \qquad (14)$$

where $V(h_1+j_1, h_2+j_2, c_1)$ is linear combination of vectors of the form (11) for which $j_1 = -m_1 - \dots - m_s + p_1 + \dots + p_z$ and $j_2 = n_1 + \dots + n_s$.

It is well known that $Ker \langle \cdot | \cdot \rangle$ is maximal proper subrepresentation of Verma representation which also decomposes to orthogonal direct sum of proper subspaces $L_{00}^{(1)}$ and $L_{00}^{(2)}$ whose elements we call null vectors. Subspaces of the form $\bigoplus_{j_1 \in Z_+} V(h_1+j_1, h_2, c_1)$ coincide with Verma modules over Virasoro algebra. Therefore, they reproduce all known degenerate representations of Virasoro algebra, which are well studied [4] . In particular, Kac determinant in the subspace $\bigoplus_{j_1 \in Z_+} V(h_1+j_1, h_2, c_1)$ has a form:

$$\det(h_1+j_1, h_2, c_1) \sim \prod_{\substack{1 \leq rs \leq j_1 \\ r,s \in N}} (h_1 - h_{r,s}(c_1))^{P(j_1 - rs)} . \qquad (15)$$

All other subspaces $V(h_1+j_1, h_2+j_2, c_1)$, for which $j_2 \neq 0$ and $j_2 \in Z_+$, are infinite-dimensional, and we discuss the simplest of them, a subspace of the form $V(h_1, h_2+1, c_1)$ whose structure differs from subspaces of the form

$V(h_1+j_1, h_2, c_1)$:

$$V(h_1,h_2+1,c_1)=\bigoplus_{j\in Z_+\setminus\{0\}} L^{(1)}_{-1,j}\cdot V(h_1+j,h_2,c_1) \qquad (16)$$

We classify null vectors which can arise in the subspace $V(h_1,h_2+1,C_1)$. Although we discuss only this subspace, this approach, apparently, can be applied also for classifications of all other null vectors in the subspaces of the form $V(h_1+j_1,h_2+j_2,C_1)$, $j_1\in Z$ and $j_2\in Z_+$. Null vector in the subspace $V(h_1,h_2+1,C_1)$ we call induced if it is represented in the form $L^{(1)}_{-1,j}\,\vartheta$ for some $j\in Z_+$, and ϑ is null vector in the subspace $V(h_1+j,h_2,C_1)$. In the opposite case null vector is called proper. All proper null vectors in the subspace can be found with the help of orthogonalization process of basis in the $V(h_1,h_2+1,C_1)$, for example, the vector

$$U_2 = L^{(1)}_{-1,1}\,L^{(1)}_{0,-1}\,\nu - \frac{2}{9}(h_1+1)\,L^{(1)}_{-1,2}\,L^{(1)}_{0,-2}\,\nu \qquad (17a)$$

is proper null vector if equation

$$C_1 = 2\left[-4(h_1+1)+\sqrt{97-\frac{81}{h_1+1}}\right]. \qquad (17b)$$

is satisfied.

Algebra LVZ_1 has subalgebra which differs from algebra $SL(2,R)$ only by central extension and this is connected with the fact that the Virasoro algebra has subalgebra $SL^{(n)}(2,R)$ consisting of $\{L_{-n},L_0,L_{+n}\}$. We denote this subalgebra by $\widehat{SL}^{(n)}(2,R)$ which is described in the following way:

$$[L^{(1)}_{m\alpha},L^{(1)}_{p\beta}] = (\alpha-\beta)L^{(1)}_{m+p,\alpha+\beta}+\frac{C_1}{12}(\alpha^3-\alpha)\,\delta_{m+p,0}\,\delta_{\alpha+\beta,0}\,. \qquad (18a)$$

$$[L^{(2)}_{00},L^{(1)}_{m\alpha}] = -m\,L^{(1)}_{m\alpha}\,, \qquad (18b)$$

where α and β take the values $(\pm n, 0)$. Chevalley generators are

$$E_1 = L^{(1)}_{+1,-n}\,, \quad E_2 = L^{(1)}_{0,n}\,, \quad F_1 = L^{(1)}_{-1,n}\,, \quad F_2 = L^{(1)}_{0,-n}\,, \tag{19}$$

Cartan subalgebra coincides with (10d) and satisfies the following defining relations:

$$[E_1, F_1] = -2nL^{(1)}_{00} - \frac{c_1}{12}(n^3 - n) = \gamma_1^{\vee}\,, \tag{20a}$$

$$[E_2, F_2] = 2nL^{(1)}_{00} + \frac{c_1}{12}(n^3 - n) = \gamma_2^{\vee}\,, \tag{20b}$$

$$[E_1, F_2] = [E_2, F_1] = 0\,. \tag{20c}$$

Simple roots are $\gamma_1 = L^{(2)\vee}_{00} - nL^{(1)\vee}_{00}$ and $\gamma_2 = nL^{(2)\vee}_{00}$, where $\langle L^{(i)\vee}_{00}, L^{(j)}_{00} \rangle = \delta^{ij}$, Cartan matrix is

$$A = (\langle \gamma_j, \gamma_i^{\vee} \rangle)_{i;j=1;2}\,, \qquad A = \begin{pmatrix} 2n^2 & -2n^2 \\ -2n^2 & 2n^2 \end{pmatrix} \tag{21}$$

Verma modules are defined (as in the case $EVir_1$) from relations (11a-d) in which only generators $L^{(1)}_{m,\alpha}$, $\alpha \in (\pm n, 0)$ must be written. A simplest subspace in Verma modules is a linear combination of vectors of the form:

$$L^{(1)}_{-1,-n}\nu\,, \quad L^{(1)}_{0,-n} L^{(1)}_{-1,0}\nu\,, \quad L^{(1)}_{0,-n} L^{(1)}_{-1,n}\nu \tag{22}$$

It is easy to check that

$$\Delta^{(1,1)} = 2n^2 L^{(1)}_{-1,-n}\nu - 2nL^{(1)}_{0,-n}\nu + L^{(1)2}_{0,-n} L^{(1)}_{-1,n}\nu \tag{23a}$$

is null vector in the subspace (22) if equation

$$h_1 = -\frac{n^2 - 1}{24} c_1 + n \tag{23b}$$

is satisfied.

For algebra $\mathrm{sdiff}\, T^2$ root space decomposition is realized taking into account the corresponding decompositions for $EViz_1$ and $EViz_2$:

$$\mathrm{sdiff}\, T^2 = n_- \oplus H \oplus n_+ , \tag{24a}$$

$$n_- = \bigoplus_{\substack{n\in Z_+\setminus\{0\} \\ m\in Z}} cL_{m,-n} \quad \bigoplus_{n\in Z_+\setminus\{0\}} cL_{-n,0} , \tag{24b}$$

$$n_+ = \bigoplus_{\substack{n\in Z_+\setminus\{0\} \\ m\in Z}} cL_{m,+n} \quad \bigoplus_{n\in Z_+\setminus\{0\}} cL_{+n,0} , \tag{24c}$$

$$H = cL^{(2)}_{00} \oplus cL^{(1)}_{00} \oplus c\widetilde{C}_1 \oplus c\widetilde{C}_2 , \tag{24d}$$

in accordance with which highest weight representation of algebra $\mathrm{sdiff}\, T^2$ is defined:

$$L^{(i)}_{00} v = h_i v , \quad c_1 v = c_i v \quad (i = 1;2), \tag{25a}$$

$$L_{m,+n} v = 0, \quad L_{+n,0} v = 0 , \quad n \in Z_+\setminus\{0\} , m \in Z , \tag{25b}$$

and the representation space is a linear combination of vectors of the form:

$$L_{m_s,-n_s} \dots L_{m_1,-n_1} L_{-p_z,0} \dots L_{-p_1,0} v \tag{26}$$

where $(m_1, n_1) \leq \dots \leq (m_s, n_s)$, $s \in Z_+ , n_i , p_j \in Z_+\setminus\{0\}$ and $m_j \in Z$, $i = 1, \dots, s$, $j = 1, \dots, z$

Antilinear antiinvolution by antilinearity continues from $L^{(1)}_{nm}$ and $L^{(2)}_{pk}$ over L_{nm} :

$$\omega\, (L_{nm}) = -L_{-n,-m} \tag{27}$$

In this case Verma modules over algebra $\mathrm{sdiff}\, T^2$ admit the following decomposition into an orthogonal direct sum of proper subspaces $L_{00}^{(1)}$ and $L_{00}^{(2)}$:

$$M(h_1,h_2,\tilde{c}_1,\tilde{c}_2) = \bigoplus_{\substack{j_1\in Z\\ j_2\in Z+}} V(h_1+j_1, h_2+j_2, \tilde{c}_1, \tilde{c}_2), \tag{28}$$

where $V(h_1+j_1, h_2+j_2, \tilde{c}_1, \tilde{c}_2)$ is such linear combination of vectors of the form (26) for which $j_1 = -m_s - \dots - m_1 + p_2 + \dots + p_1$ and $j_2 = n_s + \dots + n_1$. Therefore, subspaces of the form $V(h_1+j_1, h_2, \tilde{c}_1, \tilde{c}_2)$ are finite-dimensional and have the following basis:

$$L_{-1,0}^{\mu_1} \dots L_{-j,0}^{\mu_j} \, \nu \qquad \text{, where} \quad 1\mu_1 + \dots j\mu_j = j_1 \tag{29}$$

and matrix of the contravariant form has a diagonal form on the diagonals of which there stand elements:

$$\langle L_{-1,0}^{\mu_1} \dots L_{-j,0}^{\mu_j}\nu \,|\, L_{-1,0}^{\mu_1} \dots L_{-j,0}^{\mu_j}\nu \rangle = \prod_{K=1}^{j} \mu_K!\,(-\tilde{c}_1 K)^{\mu_K} \tag{30}$$

Elements of the form $L_{-j,0}$, $j\in Z$ consist of infinite-dimensional Heisenberg subalgebra in the $sdiff\, T^2$ algebra, which causes such a structure of the subspace $V(h_1+j_1, h_2, \tilde{c}_1, \tilde{c}_2)$ Therefore, when $\tilde{c}_1 \neq 0$ in the subspace of the form $V(h_1+j_1, h_2, \tilde{c}_1, \tilde{c}_2)$ where $j_1 \in Z+$, null vectors are absent.

The next simplest subspace is

$$V(h_1, h_2+1, \tilde{c}_1, \tilde{c}_2) = \bigoplus_{j=0}^{\infty} L_{j,-1} \cdot V(h_1+j, h_2, \tilde{c}_1, \tilde{c}_2) \tag{31}$$

Applying the orthogonalization process for basis of the subspace $V(h_1, h_2+1, \tilde{c}_1, \tilde{c}_2)$ it is easy to show that vector

$$U_2 = L_{0,-1}\nu + \frac{\tilde{c}_2}{\tilde{c}_1} L_{1,-1} L_{-1,0}\nu \tag{32a}$$

is proper null vector if only equation

$$\tilde{C}_2^2 - \tilde{C}_1\tilde{C}_2 + \tilde{C}_1 = 0 \qquad (32b)$$

is satisfied.

Note that due to the absence of null vectors in the subspaces $V(h_1+j_1, h_2, \tilde{C}_1, \tilde{C}_2)$ at $j_1 \in Z_+$, all null vectors in the subspace $V(h_1, h_2+1, \tilde{C}_1, \tilde{C}_2)$ are proper and can be found by orthogonalizing basis in this subspace.

In the process of work I met the article [5] where are discussed Z^N indexed algebras which in the N=2 case are attributed to $EVir_1$ algebra.

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Т.А.АРАКЕЛЯН
РАСШИРЕННАЯ АЛГЕБРА ВИРАСОРО И АЛГЕБРА
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