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ABSTRACT

Specific properties of the longitudinal current fluctuations in a coasting ion beam, moving in a storage ring with the beam cooling, are reviewed in the paper. As the cooling changes both the debunching mechanism and the dielectric constant of the beam, it modifies the longitudinal Schottky noise spectra of both low-intensity and high-intensity beams. At the statistical equilibrium the longitudinal beam spectra are described by the universal formula, where contributes the beam dielectric constant only. This enables one to create the fitting code for the diagnostic of the beam parameters as well as its interaction with surrounding electrodes in storage rings with the beam cooling.

I. INTRODUCTION

The measurement of Schottky noise spectra provides the powerful diagnostic tool for (especially coasting) beams in storage rings [1]. Since it is based on the analysis of the noise signals, which the beam induces in pickup electrodes, this is non-distructive method and results of the measurements generally reflects the mutual processes in the beam. Nevertheless the interpretation of such measurements is simple only for low-intensity uncooled beams, when the noise spectrum coincides with the revolution frequency distribution function in the beam and the power with the beam number of particles N.

For high-intensity or cooled beams the noise spectra can be distorted by either collective effects or by the cooling and accompanying diffusion. Thus the interpretation of experimental data becomes more complicated and generally needs some fitting code though can yield much more information concerning beam parameters and its interaction with surroundings. Referring to ion beams in storage rings with electron cooling this was first pointed out in Refs [2, 3].

In this paper we shall review some phenomena, which can be observed in the longitudinal Schottky spectra in storage ring with electron cooling as well as some technical details, which can be useful for the models of fitting.

2. THE LONGITUDINAL SCHOTTKY NOISE OF THE NORMAL BEAM

Let us discuss first the basic properties of the current noise in the uncooled, coasting beam. Below we shall call such a beam as a normal one. The linear microscopic density in the coasting beam has the form

$$\rho(\vartheta, t) = \sum_{a=1}^{N} \delta(\vartheta - \vartheta_a(t)) , \tag{2.1}$$

where ϑ is the azimuth and $\vartheta_a(t)$ is its instantaneous value for a-th particle, N is number of particles in the beam, $\delta(\vartheta)$ is the periodic δ-function. Using:

$$\delta(\vartheta) = \sum_{n=-\infty}^{\infty} \frac{\exp(i n \vartheta)}{2\pi} , \tag{2.2}$$

one can get for Fourier harmonic of ρ:

$$\rho(\vartheta, t) = \sum_{n=-\infty}^{\infty} \frac{\rho_n(t) \exp(i n \vartheta)}{2\pi} , \tag{2.3}$$

$$\rho_n(t) = \sum_{a=1}^{N} \exp(-i n \vartheta_a(t)) . \tag{2.4}$$

Due to the thermal motion of particles both ρ and $\rho_n(t)$ are random functions of the time with average values:

$$\langle \rho \rangle = N , \quad \langle \rho_n(t) \rangle = 0 , \quad n \neq 0 . \tag{2.5}$$

The r.m.s. value of $|\rho_n|^2$, which can be measured from pickup electrodes is connected with two-time correlation function:

$$K_n(t_1, t_2) = \langle \rho_n(t_1) \, \rho_n^*(t_2) \rangle . \tag{2.6}$$

Generally, the calculation of $K_n(t_1, t_2)$ can be very complicated problem. Therefore, some simplifying assumptions become necessary to find out this function in particular cases. For instance, for the beam of low intensity one can neglect the interaction of particles and, hence, write:

$$\vartheta_a(t) = \omega(\Delta p_a)\, t + \varphi_a = \omega_s\, t + \omega_0'\, \Delta p_a\, t + \varphi_a\,, \tag{2.7}$$

$$\omega_0' = \frac{\omega_s \eta}{p_s}, \qquad \eta = \gamma^{-2} - \alpha\,, \qquad \Delta p_a = p_a - p_s\,,$$

where α is the momentum compaction factor of the ring. The substitution of these formulae into eq. (2.6) after the averaging over initial phases φ_a yields:

$$K_n(\tau) = \Big\langle \sum_{a=1}^{N} \exp(-in\omega_a \tau) \Big\rangle, \qquad \tau = t_1 - t_2\,. \tag{2.8}$$

Introducting the momentum distribution function:

$$f_0(\Delta p) = \Big\langle \frac{1}{N} \sum_{a=1}^{N} \delta(\Delta p - \Delta p_a) \Big\rangle\,, \tag{2.9}$$

one can rewrite eq. (2.8) in the form:

$$K_n(\tau) = N \int_{-\infty}^{\infty} d\Delta p\; f_0(\Delta p)\; \exp(-i\, n\, \omega(\Delta p)\, \tau)\,. \tag{2.10}$$

The spectrum analyser measures the Fourier amplitudes of $K_n(\tau)$:

$$K_n(\omega) = \int_{-\infty}^{\infty} d\tau\; K_n(\tau)\; \exp(i\omega\tau)\,. \tag{2.11}$$

For the normal beam of the low intensity this coincides with the beam frequency distribution function:

$$K_n(\Delta\omega_n) = 2\pi N \int_{-\infty}^{\infty} d\Delta p\, f_0(\Delta p)\, \delta(\Delta\omega_n - n\omega_0'\,\Delta p)\,, \tag{2.12}$$

$$\Delta\omega_n = \omega - n\omega_s\,.$$

Generally, $K_n(\Delta\omega_n)$ is centered around $n\omega_s$ and has the width proportional to the revolution frequency spread in the beam $n\Delta\omega$. The power of the noise spectrum (2.12) is

$$W_n = K_n(0) = \int_{-\infty}^{\infty} \frac{d\Delta\omega_n}{2\pi} K_n(\Delta\omega_n) = N . \tag{2.13}$$

As can be seen, for the low-intensity beam the correlator K_n depends only on the difference of times $\tau = t_1 - t_2$, which means that K_n describes the stationary noise. Thus, during the measurements K_n can be stored.

For beams with the interaction between particles the Schottky noise becomes stationary in the statistical equilibrium, but interaction generally distortes the shape of the noise spectra from that described by eq. (2.12). For typical measurements, when harmonic numbers n are not too high, one may expect that such interaction can be associated with fields, which the beam induces in surrounding electrodes. Then for the Fourier amplitudes:

$$\rho_n(\omega) = \int_0^{\infty} dt\, \rho_n(t)\, e^{i\omega t}$$

the calculations based on the linearized Vlasov's equations yield (see Refs [2, 4] for details):

$$\rho_n(\omega) = \frac{\rho_n^{(0)}(\omega)}{\varepsilon_n(\omega)} . \tag{2.14}$$

Here $\rho_n^{(0)}(\omega)$ is the Fourier harmonic of $\rho_n(t)$ calculated without the interaction between particles,

$$\varepsilon_n(\Delta\omega_n) = 1 + \frac{\Omega_n^2}{n\omega_0'} \int_{-\infty}^{\infty} d\Delta p \, \frac{\partial f_0/\partial\Delta p}{\Delta\omega_n - n\omega_0' \Delta p}, \qquad \mathrm{Im}\,\omega > 0 \tag{2.15}$$

is the dielectric constant of the beam, which by means of the dispersion equation

$$\varepsilon_n(\Delta\omega_n) = 0 \tag{2.16}$$

yields eigen-frequencies of longitudinal coherent oscillations in the beam, and

$$\Omega_n^2 = n^2 \frac{N(ze)^2 \omega_0 \omega_0'}{\Pi} (-i Z_n/n) , \tag{2.17}$$

is the squared longitudinal coherent tune shift, ze is the charge of ions, Π is the perimeter of the orbit. These equations imply that the interaction of the beam with surrounding electrodes is described by the longitudinal broad band impedance:

$$E_n(\omega) = -\frac{Nze\omega_0}{\Pi} Z_n \rho_n(\omega) . \tag{2.18}$$

where $E_n(\omega)$ are harmonics of the longitudinal electric field induced by the beam. Using eq. (2.14) we can rewrite eq. (2.18) in the form

$$E_n(\omega) = \frac{E_n^{(0)}(\Delta\omega_n)}{\varepsilon_n(\Delta\omega_n)}, \quad E_n^{(0)} = -\frac{Nze\omega_0 Z_n}{\Pi} \rho_n^{(0)}(\Delta\omega_n) , \tag{2.19}$$

which demonstrates that collective reaction of the beam reduces external fields in $\varepsilon_n(\Delta\omega_n)$ times.

From eq. (2.14) one can find out for $K_n(\omega)$ (see in Ref. [4]):

$$K_n(\Delta\omega_n) = \frac{K^{(0)}(\Delta\omega_n)}{|\varepsilon_n(\Delta\omega_n)|^2}, \tag{2.20}$$

$$K_n^{(0)}(\Delta\omega_n) = 2\pi N \int_{-\infty}^{\infty} d\Delta p\, f_0(\Delta p)\; \delta(\Delta\omega_n - n\omega_0' \Delta p) .$$

Equation (2.20) describes so-called Schottky noise suppression in high-intensity beams and was calculated in many papers (see, for instance, in Refs [2, 4, 5, 6]). It is caused by the propagation of coherent oscillations along the beam and the effect is as stronger as longer is the life time of such oscillations. The usage of eq. (2.20) to fit Schottky spectra measured for normal beams enables one to get the number of particles in the beam, its frequency spread $\Delta\omega$, both parts of $Z_n = Z_n' + Z_n''$ and, in principle, the momentum distribution function.

Some general properties of the noise spectra for intense beams can be seen directly from eq. (2.20). It may have sharp peaks in the close vicinity of roots of eq. (2.16) provided decrements of coherent oscillations are small enough. In fact, this can be realized in two cases:

1) if coherent oscillations are stable and the beam is cold enough;
2) if coherent oscillations can be unstable and the beam is tuned close enough to threshold of the instability.

Since the impedance Z_n generally has both real and imaginary parts, even for symmetrical distribution functions, the spectrum

$K_n(\Delta\omega_n)$ can be asymmetrical in $\Delta\omega_n$ and such asymmetry increases, if the beam intensity tends to any thresholds of coherent instabilities.

For the low and medium energy ion storage ring one may expect the dominant contribution into the impedance Z_n from Coulomb forces:

$$(-iZ_n/n) = \frac{1}{v\gamma^2}\begin{cases} \ln(b/a)+1/2, & n \ll n_0 = R_0/a, \\ (n_0/n^2), & n \gg n_0, \end{cases} \quad (2.21)$$

where b and a are respectively vacuum chamber and beam radii, $R_0 = \Pi/2\pi$. With this impedance and for Gaussian momentum distribution in the beam

$$f_0(\Delta p) = \frac{1}{\sqrt{2\pi}\,\sigma}\exp(-\Delta p^2/2\sigma^2) \quad (2.22)$$

eq. (2.20) can be written in the form ($\varepsilon = \varepsilon' + i\varepsilon''$)

$$K_n(\Delta\omega_n) = \frac{2N}{\zeta}\,\frac{\varepsilon_n''(\Delta\omega_n)}{\Delta\omega_n\,|\varepsilon_n(\Delta\omega_n)|^2}, \quad (2.23)$$

$$\zeta = \Omega_n^2/n^2\Delta\omega^2, \qquad \Delta\omega = \omega_0'\sigma.$$

For cold beams ($\zeta \gg 1$) one has in eq. (2.23) $|\varepsilon''| \ll |\varepsilon'|$ for all $\Delta\omega_n$ except those, which satisfy the dispersion equation

$$\varepsilon_n'(\Delta\omega_n) \simeq 1 - \Omega_n^2/\Delta\omega_n^2 = 0.$$

Then, using the substitution

$$\lim_{\Delta\to 0}\frac{\Delta}{x^2+\Delta^2} = \pi\delta(x),$$

one can replace eq. (2.23) by

$$K_n(\Delta\omega_n) = (N_c/2)\,[\,\delta(\Delta\omega_n - \Omega_n) + \delta(\Delta\omega_n + \Omega_n)\,], \quad (2.24)$$

$$N_c = \eta\,(\sigma/p_s)^2\,\frac{2\pi p v}{z^2e^2\omega_s}\,(n/-iZ_n),$$

describing the double-peak spectrum. The power of this spectrum obviously is N_c and thus is proportional to the longitudinal temperature of the beam [2]. Let us also note that the power of the spectrum (2.23) can be calculated exactly using the dispersion relation for the beam response function:

$$\chi(\Delta\omega_n) = 1 - 1/\varepsilon_n(\Delta\omega_n) = \chi' + i\chi'',$$

$$\chi'(\Delta\omega_n) = \mathrm{PV}\int_{-\infty}^{\infty} \frac{d\omega'}{\pi}\, \frac{\chi''(\omega')}{\omega' - \Delta\omega_n}, \qquad (2.25)$$

where PV$\int$ denotes the calculation of the principal value of the integral. The result is [4]

$$W_n = N/(1+\zeta) = N/(1+N/N_c). \qquad (2.26)$$

Let us discuss now some properties of Schottky noise for beams near the thresholds of coherent instabilities [4]. In this region $\varepsilon_n(\Delta\omega_n)$ can be expanded in powers of $(\Delta N/N)$, $\Delta N = N_{th} - N \ll N$ (N_{th} is the number of particles at the threshold). For the broad band impedance in the first approximation one has

$$\varepsilon_n(\Delta\omega_n) = (\Delta N/N) + i\,\varepsilon_n''(\Delta\omega_n). \qquad (2.27)$$

The substitution of this expression into eq. (2.20) yields

$$K_n(\Delta\omega_n) = \frac{2\pi^2 N}{1 - N/N_{th}}\, \delta[\varepsilon_n''(\Delta\omega_n)] \int_{-\infty}^{\infty} d\Delta p\; f_0(\Delta p)\; \delta(\Delta\omega_n - n\omega_0'\Delta p). \quad (2.28)$$

As the position of peaks now is determined by the roots of virtually unstable solutions of eq. (2.16)

$$\varepsilon_n''(\Delta\omega_n) = 0, \qquad (2.29)$$

in contrast with eq. (2.24), the shape of the spectra (2.28) can be very asymmetric in respect to the point $\Delta\omega_n = 0$, provided Re $(Z_n) \neq 0$. In particular, it can consist even of one narrow peak. The power of the spectrum (2.28) is

$$W_n = \frac{\pi N}{1 - N/N_{th}} \sum_{\beta} |\partial\varepsilon''/\partial\omega_\beta|^{-1} \int_{-\infty}^{\infty} d\Delta p\; f_0(\Delta p)\; \delta(\Delta\omega_n^\beta - n\omega_0'\Delta p), \quad (2.30)$$

where β marks solutions of eq. (2.29). For instance, for the negative mass instability ($d\omega_0/dp < 0$) in a Gaussian beam one has

$$N_{th} = N_c, \quad \Delta\omega_n = 0, \quad |\partial\varepsilon''/\partial\omega|_{th} = \frac{\sqrt{\pi/2}}{n\Delta\omega}$$

and, hence

$$W_n = N/(1 - N/N_c) \tag{2.31}$$

the noise power increases, when N tends to the threshold value. We have to note that the same increase in the noise power can take place below transition energy of the ring, if the impedance has large inductive component.

Since for high harmonic numbers Z_n decreases, asymptotically one has $\Omega_n^2 \to \text{const}$, $\zeta_n \to 0$, and therefore there will be no signal suppression ($W_n \to N$) for harmonic numbers above

$$n = n_D = \sqrt{\Omega_n^2 / \Delta\omega^2}\,, \tag{2.32}$$

which corresponds to Debye shilding radius $r_D = R_0 / n_D$.

3. THE LONGITUDINAL SCHOTTKY NOISE OF COOLED BEAM

If the beam is cooled by any mechanism, one more parameter — the cooling decrement λ can affect the shape of its noise spectra. As from λ and $n\Delta\omega$ one can construct the dimensionless parameter

$$q = (n\Delta\omega/\lambda)^2 \tag{3.1}$$

the different behaviour for spectra might be expected, depending on whether q is small or large. The physical reasons are the following.

In the coasting beam any distortion of phase density can dilute only due to the relative motion of particles along the beam caused by their energy deviations. If the cooling is weak ($q \gg 1$), particles leave and enter the sample of azimuthal size $1/n$ faster then cooling starts to work. Hence, in this region of parameters one may expect small distortion of the noise spectra by cooling.

If the cooling is strong ($q \ll 1$) after the cooling time $1/\lambda$ particles change their azimuthal position by $\Delta\vartheta_a = \Delta\omega_a/\lambda$ and then stop. In this region the dilution becomes possible only due to associated with cooling random kicks (or collisions), which provide the necessary deviations in the particle energies. Once these kicks are random, one has

$$\overline{\Delta\vartheta_a} = \overline{\Delta\omega_a}/\lambda = 0\,, \quad \overline{\Delta\vartheta_a^2} = \overline{\Delta\omega_a^2}/\lambda\,,$$

where the solid line means the averaging over collisions. Taking into account that the average frequency of kicks is λ, one can get that the distortion in phases will dilute due to weak diffusion of particles

$$\overline{\frac{d}{dt}\langle\Delta\vartheta^2\rangle} = \frac{\Delta\omega^2}{\lambda}. \tag{3.2}$$

Generally, such dilution mechanism is specific for that in dense media (see in Ref. [8]), when collision frequency is high. It obviously can be realized, if collisions occur more frequently then the thermal motion produces new fluctuations. For the Schottky band near the frequency $n\omega_s$ this requires $\lambda \gg n\Delta\omega$.

The influence of the described phenomenon on the equilibrium Schottky spectra can be calculated using the Fokker—Planck equation [7]:

$$\frac{\partial f}{\partial t} + \omega_0(p)\frac{\partial f}{\partial\vartheta} + zeE(\vartheta, t)\frac{\partial f_0}{\partial\Delta p} = \frac{\partial}{\partial\Delta p}\left[\lambda\Delta p f + \frac{d}{2}\frac{\partial f}{\partial\Delta p}\right]. \tag{3.3}$$

To calculate values referring to the beam equilibrium state, we can adopt that both λ and the diffusion coefficient d are constants, while $f_0(\Delta p)$ is Gaussian distribution (2.22) with $\sigma^2 = d/2\lambda$. Before making the particular calculations we have to remind that actually eq. (3.3) is not usual kinetic equation because entering there function $f(\Delta p, \vartheta, t)$ is not the conventional distribution function, but is only its fluctuational part averaged over collisions. Thus, its average value over the beam fluctuations is $\langle f(\Delta p, \vartheta, t)\rangle = 0$, while the second momenta are related to the beam noise characteristics [4]. In fact, $f(\Delta p, \vartheta, t)$ must be calculated from the microscopic equation

$$\frac{\partial f}{\partial t} + \omega_0(p)\frac{\partial f}{\partial\vartheta} + zeE(\vartheta, t)\frac{\partial f_0}{\partial\Delta p} = -\frac{\partial}{\partial\Delta p}\{(-\lambda\Delta p + d(t))f\},$$

where $d(t)$ is the random function describing collisions. Therefore, the use of eq. (3.3) is based on the assumption that the correlation time of external noise is much shorter then that of beam fluctuations.

The simple calculations with eq. (3.3) yield for harmonics of phase density $\rho_n(\omega)$ the same equation as eq. (2.14) (see in Refs [4, 7] for details)

$$\rho_n(\omega) = \rho_n^{(0)}(\Delta\omega_n)/\varepsilon_n(\Delta\omega_n),$$

but with

$$\varepsilon_n(\Delta\omega_n)=1+\zeta e^q \sum_{l=0}^{\infty} \frac{(-q)^l}{l!} \frac{q+l}{\frac{-i\Delta\omega_n}{\lambda}+q+l}, \tag{3.4}$$

which is quite different from that of eq. (2.15). Therefore, we can use for spectra eq. (2.20) after the calculation of $K_n^{(0)}(\Delta\omega_n)$ for cooled, low-intensity beam. The last can be done directly using eqs (2.4), (2.6):

$$K_n^{(0)}=\left\langle \sum_{a=1}^{N} \overline{\exp(-in[\vartheta_a(t+\tau)-\vartheta_a(t)])} \right\rangle \tag{3.5}$$

and equations of the particle motion:

$$\dot{\omega}_a=-\lambda\omega_a+D(t), \quad \omega_a=\omega_0'\Delta p_a, \quad \dot{\vartheta}_a=\omega_a, \tag{3.6}$$

where collisions are described by the stationary white noise $D(t)$:

$$\overline{D(t)\,D(t')}=\overline{D^2}\,\delta(t-t'), \quad \overline{D^2}=2\lambda\,\Delta\omega^2,$$

$$\overline{D(t_1)\dots D(t_{2k+1})}=0, \tag{3.7}$$

$$\overline{D(t_1)\dots D(t_{2k})}=\sum_P \overline{(D(t_1)\,D(t_2))}\dots\overline{(D(t_{2k-1})\,D(t_{2k}))}$$

and P means the summation over all combinations of couples. The simple calculations yield [4]

$$K_n^{(0)}(|\tau|)=\exp\{-q\Psi(\lambda|\tau|)\}\left\langle \sum_{a=1}^{N} \exp\left\{\frac{in\omega_a}{\lambda}(1-e^{-\lambda|\tau|})\right\}\right\rangle,$$

$$\Psi(x)=x-2(1-e^{-x})+(1-e^{-2x})/2,$$

or, using eq. (2.9),

$$K_n^{(0)}(|\tau|)=\exp\{-q\Psi(\lambda|\tau|)\}\int_{-\infty}^{\infty} d\Delta p\, f_0 \exp\left\{\frac{in\omega_0' p}{\lambda}(1-e^{-\lambda|\tau|})\right\}. \tag{3.8}$$

In the region $\lambda\tau\gg 1$ and, hence, $\Psi(\lambda\tau)\simeq\lambda\tau$, this function decays with the time constant

$$\tau_n = \frac{\lambda}{(n\Delta\omega)^2} \gg (n\Delta\omega)^{-1}$$

in agreement with eq. (3.2).

For the Gaussian distribution function eq. (3.8) can be rewritten in the form

$$K_n^{(0)}(\Delta\omega_n) = \frac{2N}{\lambda} e^q \sum_{l=0}^{\infty} \frac{(-q)^l}{l!} \frac{q+l}{(\Delta\omega_n/\lambda)^2+(q+l)^2}. \quad (3.9)$$

The power of this spectrum is the same ($W_n = N$) as that for the normal, low-intensity beam, but widths of Schottky bands are much smaller provided the cooling is strong:

$$K_n^{(0)}(\Delta\omega_n) = 2N \frac{\lambda q}{\Delta\omega_n^2 + (q\lambda)^2}, \quad q \ll 1. \quad (3.10)$$

The Schottky noise spectrum for the cooled beam can be obtained by the substitution of eqs (3.4) and (3.9) into eq. (2.20). As the comparison of these equations gives

$$K_n^{(0)}(\Delta\omega_n) = \frac{2N}{\Delta\omega_n} \operatorname{Im}\left\{\frac{\varepsilon_n - 1}{\zeta}\right\}, \quad (3.11)$$

one can write $K_n(\Delta\omega_n)$ using only one function — $\varepsilon_n(\Delta\omega_n)$:

$$K_n(\Delta\omega_n) = \frac{2N}{\Delta\omega_n |\varepsilon_n(\Delta\omega_n)|^2} \operatorname{Im}\left\{\frac{\varepsilon_n - 1}{\zeta}\right\}. \quad (3.12)$$

It is easy to verify that for normal beams with Gaussian momentum distribution eq. (2.20) coincides with eq. (3.12), which thus gives the universal representation for Schottky spectra of both normal and cooled beams. This fact also can be traced by direct calculation of the asymptotic for $\varepsilon_n(\Delta\omega_n)$ when $q \to \infty$ (Refs [4, 7]):

$$\varepsilon_n = 1 + \zeta + iy\zeta e^q \sum_{l=0}^{\infty} \frac{(-q)^l}{l!} \frac{1}{-iy+q+l} =$$

$$= 1 + \zeta + iy\zeta \int_0^{\infty} ds \exp\{isy - qs + q(1-e^{-s})\} =$$

$$= 1 + \zeta + iy\zeta \int_0^\infty ds \exp\{isy - qs + q(1 - (1 - s + s^2/2 + \ldots))\} =$$

$$= 1 + \zeta + i\Delta\omega_n \zeta \int_0^\infty dt \exp\left\{ i\Delta\omega_n t - \frac{(n\Delta\omega_n)^2 t^2}{2} \right\}, \qquad (3.13)$$

$$y = \Delta\omega_n/\lambda, \qquad q \gg 1 .$$

In some particular cases eq. (3.12) can be simplified. If, for instance, $\mathrm{Im}\,\zeta = 0$ (i. e. $\mathrm{Re}\,Z_n = 0$), eq. (3.12) gives for the spectrum eq. (2.23) as it was calculated in Ref. [2]. Some more usuful representations for ε_n can be found in Ref. [4].

The behaviour of Schottky spectra described by eq. (3.12) for some particular parameters is shown in Figs 1—3. It indicates well pronounced asymmetry in spectra, if $\zeta'' \neq 0$. Such effect was recently detected in measurements at TSR (see in Ref. 9).

4. THE LONGITUDINAL COHERENT OSCILLATIONS OF COASTING BEAM WITH STRONG COOLING

As for $q \gg 1$ the dielectric constant for cooled beam coincide with that for normal one, we shall concentrate here on the brief discussion of properties of longitudinal coherent oscillations for strongly cooled ($q \ll 1$) coasting beam. Before making calculations, let us mention also that with the given $\Delta\omega$ and λ the comparison of q with 1 separates coherent oscillations (and, hence, Schottky bands) by the harmonic number. From this point of view, below we shall discuss the properties of longitudinal coherent oscillations with relatively small n:

$$|n| \ll \lambda/\Delta\omega . \qquad (4.1)$$

For $q \ll 1$ only two first items in r.h.s of eq. (3.4) are important. Therefore the dispersion equation takes the form

$$\varepsilon_n(\Delta\omega_n) = 1 - \frac{\zeta q}{(y + iq)(y + iq + i)} = 0 . \qquad (4.2)$$

It has the roots

$$y = -iq - \frac{i}{2} \pm \left\{ \frac{\Omega_n^2}{\lambda^2} - \frac{1}{4} \right\}^{1/2},$$

or

$$\Delta\omega_n = \pm\Omega - i\delta_{\pm}, \qquad (4.3)$$

$$\Omega = \left\{\frac{X+U-\lambda^2/4}{2}\right\}^{1/2}, \qquad (4.4)$$

$$\delta_{\pm} = \frac{\lambda}{2} + q\lambda \pm \left\{\frac{X-U+\lambda^2/4}{2}\right\}^{1/2}, \qquad (4.5)$$

$$X^2 = (U-\lambda^2/4)^2 + V^2, \quad \Omega_n^2 = U + iV.$$

Using eq. (4.5) one can find out that oscillations will be stable ($\delta > 0$), if

$$U + (n\Delta\omega)^2 > V^2/\lambda^2. \qquad (4.6)$$

In particular, this means that the beam cooling does not stabilize the negative mass instability [7]. Nevertheless, it increases the size of the stability diagram in the plane (U, V) along the axis V especially in the region $U > 0$. It is interesting to note that for fixed values of q (not necessarily small) the stability diagram asymptotically $(U \to \infty)$ follows the parabolic law (4.6). This can be seen directly from eq. (3.4) by the calculation of ε_n for the asymptotic region $(\mathrm{Im}\,\Delta\omega_n = 0,\ \Delta\omega_n \to \infty)$:

$$\varepsilon_n(\Delta\omega_n) = 1 - \frac{\zeta q}{y^2}(1 + i/y) = 0, \quad y \to \infty,$$

or

$$(U/\lambda^2) = \frac{y^4}{1+y^2}, \quad (V/\lambda^2) = -\frac{y^3}{1+y^2};$$

$$(V/\lambda^2) = \pm U^{1/2}/\lambda, \quad U \gg \lambda^2.$$

In the region $q \gg 1$ there can be the situation, when before reaching the parabolic asymptotic the stability diagram shrinks in V like it does that for the normal Gaussian beam. This time the stability diagram will have the waist as it is shown on Fig. 4. Respectively, during the storaging it can occur initial deterioration and subsequent enhancement of beam collective stability, while beam current increases. Anologous variations in the stability of coherent oscillations have been observed in experiments at TSR (see in Ref. 9). The same behaviour of stability diagram was recently obtained in Ref. [6] using quite different collisional integral.

The threshold current of the beam can be found from the stability condition (4.6):

$$N_{th}/N_c = \xi/2 + \sqrt{\xi(1+\xi/4)}\,, \quad \xi = Z''_n/qZ'_n\,. \tag{4.7}$$

In the region $d\omega_0/dp > 0$ and $\xi > 0$ it can significantly exceed the threshold of negative mass instability N_c

$$N_{th} = N_c\,\xi \propto \lambda^2 (Z''_n/Z'_n) \tag{4.8}$$

if $\xi \gg 1$.

From eq. (4.5) one can see that with strong cooling ($q \ll 1$) Schottky bands of intense beam are broad enough $\Delta\omega \approx \lambda/2$, if the beam is not close to threshold of a coherent instability. Though, if the beam intensity is not high ($U < \lambda^2$), or cooling is very strong, the spectrum will have only one well pronounced peak, which is centered around $\Delta\omega_n = 0$ and has the width (see in Ref. [2] and also in Fig. 1):

$$\delta_- = \frac{(n\Delta\omega)^2 + U}{\lambda}. \tag{4.9}$$

The splitting of the spectrum 2Ω at high beam intensities ($U \gg \lambda^2$) is proportional to $N^{1/2}$, while at $U \simeq \lambda^2$ this scaling changes for (see eq. 4.4))

$$\Omega \sim \sqrt{U - \lambda^2/4}\ .$$

Close behaviour of the splitting recently has been observed at TSR [9].

The behaviour of the spectra close to thresholds of coherent instabilities has been discussed before.

5. DISCUSSION

The presented results shows that both the spectra and the power of the Schottky noise of the coasting beam are very sensitive to collective behaviour of the beam. This can be used for fitting of Schottky noise measurements and recalculation of beam parameters, parameters of cooling device and beam surroundings. The use of the universal representation for spectra by eqs (3.12), (3.4) can simplify the fitting.

The sensitivity of Schottky spectra to thresholds of coherent instabilities can be used to study that, making measurements below the thresholds. In storage rings with beam cooling the distance to the threshold can be controlled by manipulation with the cooling rates.

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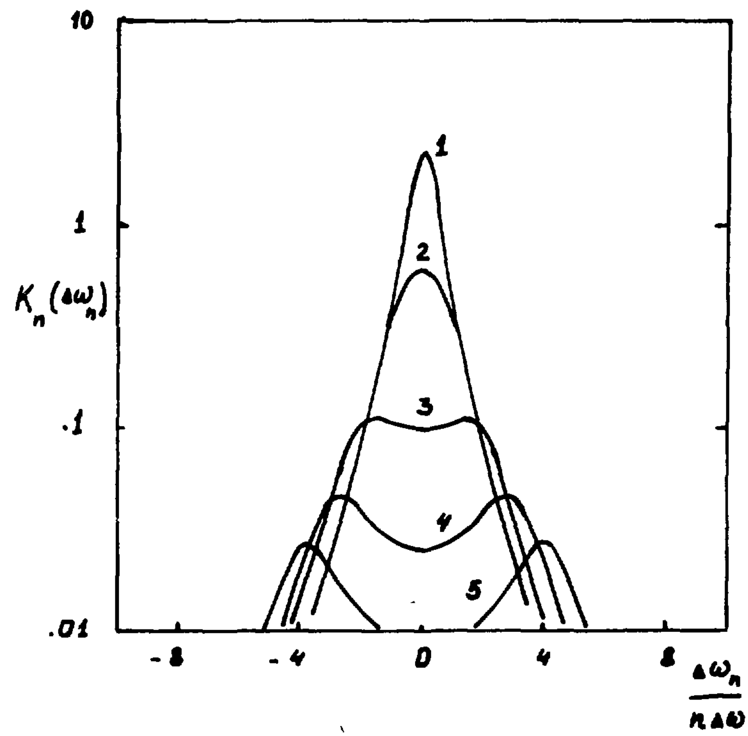

Fig. 1. The Schottky spectra of cooled beam calculated using eq. (3.12) with $q=0.25$, $(\zeta=\zeta'+i\zeta'')\,\zeta''=0$ and *1*: $\zeta'=0$, *2*: $\zeta'=1$, *3*: $\zeta'=4$, *4*: $\zeta'=9$, *5*: $\zeta'=16$.

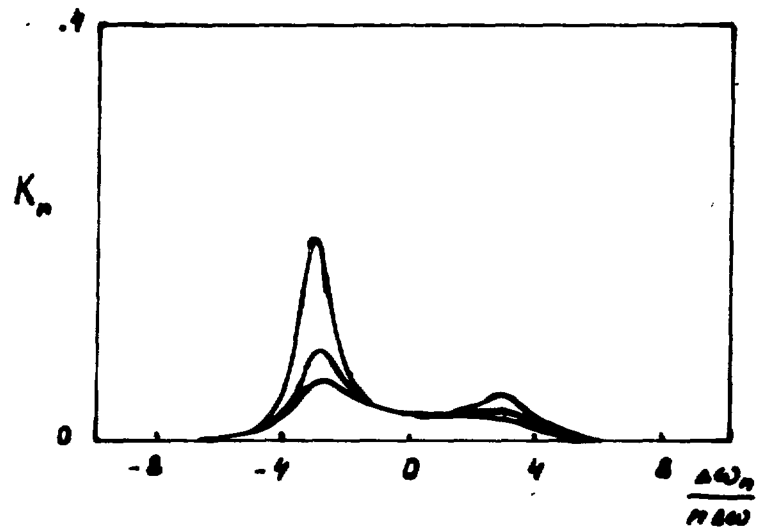

Fig. 2. The dependence of the shape of the Schottky spectra for cooled beam on ζ''; $q=0.25$, $\zeta'=9$, from the top to the bottom $\zeta''=-4, -2, -0.5$.

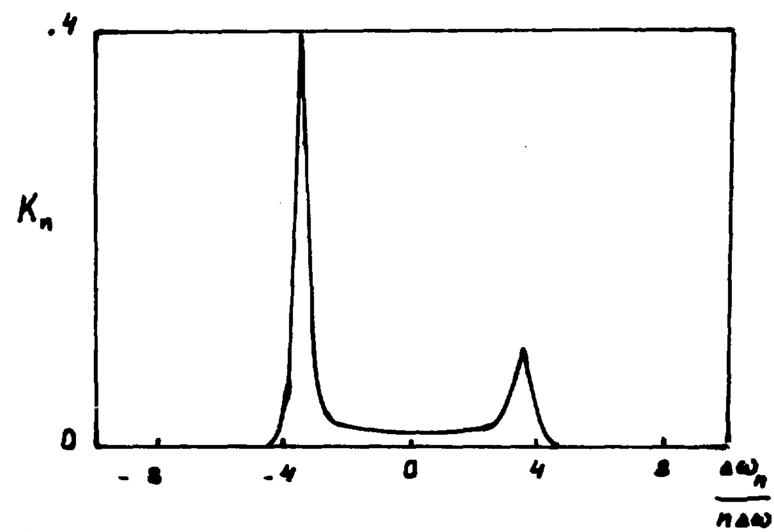

Fig. 3. The same as for Fig. 2 but with $q=25$, $\zeta'=9$, $\zeta''=-0.5$.

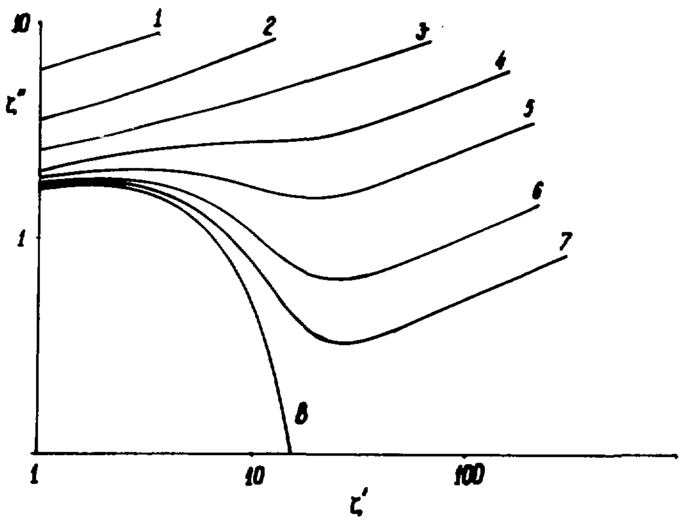

Fig. 4. The upper right quater of the stability diagram for the cooled beam; *1*: $q=1/16$, *2*: $q=1/4$, *3*: $q=1$, *4*: $q=4$, *5*: $q=16$, *6*: $q=100$, *7*: $q=400$, *8*: $q\rightarrow\infty$.

D.V. Pestrikov

The Longitudinal Schottky Noise of Intense Beam

Д.В. Пестриков

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