KFKI-1991-23/A

V.Sh. GOGOHIA
Gy. KLUGE

DYNAMICAL CHIRAL SYMMETRY BREAKING AND PION DECAY CONSTANT

Hungarian Academy of Sciences
CENTRAL
RESEARCH
INSTITUTE FOR
PHYSICS

B U D A P E S T

KFKI-1991-23/A
PREPRINT

DYNAMICAL CHIRAL SYMMETRY BREAKING AND PION DECAY CONSTANT

V.Sh. GOGOHIA, Gy. KLUGE
Central Research Institute for Physics
H-1525 Budapest 114, P.O.B. 49, Hungary

HU ISSN 0368 5330

V.Sh. Gogohia, Gy. Kluge: Dynamical chiral symmetry breaking and pion decay constant. KFKI-1991-23/A

ABSTRACT

Flavour non-singlet, chiral axial-vector Ward-Takahashi identity is investigated in the framework of dynamical chiral symmetry breaking. We propose to use the condition of stationarity for the bound state amplitude in order to fully determine this quantity and the regular piece of the corresponding axial vertex. This makes it possible to express the pion decay constant in terms of the quark propagator variables only. We find an exact expression for the pion decay constant in current algebra and in Jackiw-Johnson representation as well. We also find a new expression for the pion decay constant in the Pagels-Stokar-Cornwall variables within the framework of Jackiw-Johnson representation.

В.Ш. Гогохия и Д. Клуге: Динамическое нарушение киральной симметрии и пионная константа распада. KFKI-1991-23/A

АННОТАЦИЯ

В рамках динамического нарушения киральной симметрии исследовано соответствующее несинглетное аксиал-векторное тождество Уорда-Такахаши. Для того, чтобы определить волновую функцию пиона, а также регулярную часть аксиал-векторной вершины, предложено использовать условия стационарности для волновой функции связанного состояния. Это позволило выразить пионную константу распада только в терминах соответствующего кваркового пропагатора. Мы нашли точные выражения для пионной константы распада, а также новое выражение для нее в так называемых переменных ПСК.

Gogohia V.Sh., Kluge Gy.: A dinamikus chiral szimmetria sértés és a pion bomlási állandó. KFKI-1991-23/A

KIVONAT

A nem-szinglet axial-vektor Ward-Takahashi azonosságot vizsgáljuk a dinamikus chiral szimmetria sértés keretében. A pion hullámfüggvény és az axial-vertex reguláris részének meghatározására a kötöttállapoti hullámfüggvény stacionaritásának feltételét használtuk. Így a pion bomlási állandó kifejezhető csupán a megfelelő kvark propagátor változókkal. Meghatároztuk a pion bomlási állandó pontos kifejezését és új alakját a Pagels-Stokar-Cornwall (PGS) változókkal.

1. Introduction

Dynamical chiral symmetry breaking (DCSB) [1-4] is one of the most important and most difficult nonperturbative problems in quantum chromodynamics (QCD). On quark (microscopic) level, DCSB means that there exist nonperturbative solutions to the quark Schwinger- Dyson (SD) equations in the chiral limit $m_0 = 0$, where m_0 denotes the current ("bare") mass of a single quark (dynamical quark mass generation). On the hadronic (macroscopic) level, DCSB means that, at least, the lightest hadrons (for example, pions) are the Goldstone particles, according to the 't Hooft anomaly conditions [3,5]. Indeed, the standard QCD Lagrangian is actually symmetric under the chiral group $SU_R(N) \times SU_L(N) \times U_B(1) \times U_A(1)$ in massless quark limit. Here N is the number of different flavours and $U_B(1)$ and $U_A(1)$ correspond to baryon (observed) and axial baryon (unobserved) numbers' conservation, respectively. If the chiral $SU(N) \times SU(N)$ symmetry is spontaneously broken to $SU(N)$ in the vacuum, then the Goldstone theorem implies the existence of an $(N^2 - 1)$ multiplet of massless pseudoscalar bosons (Goldstone states). For three flavours (N=3) this multiplet must be identified with the pseudoscalar octet (π^0, $\pi^\pm$, K^0, $\bar{K}^0$, $K^\pm$, η). Obviously, in this case, the Goldstone states associated with the dynamically broken chiral symmetry should be considered as the quark - antiquark bound - states. (see, for example, Refs. 4).

The wave functions of such bound-states can be obtained from the axial-vector, chiral Ward-Tahakashi (WT) identity, as the residue of the corresponding axial-vector vertex at pole ($q^2 = 0$), according to DCSB. Moreover, the correct treatment of this identity provides complete information on its regular piece at zero momentum transfer ($q = 0$) (see section 2). The main problem in the analytical calculations of bound-states amplitudes and axial vertices through the corresponding axial-vector WT identities is the dependenceot the arbitrary form factor on these quantities.

Usually these form factors are neglected by assuming that they are of order g^2 in coupling constant. In this paper we shall show that these perturbative arguments are not valid for treating such nonperturbative effect as DCSB in the framework of the axial WT identity. To resolve this essential problem, we propose to use the condition of stationarity for the bound-state amplitude (the residue of the axial vertex at pole). This condition of the extreme of the residue at zero momentum transfer enables one to remove all arbitrary form factor dependence from the bound-states amplitudes and corresponding axial vertices. In other words, these quantities become dependent only on the quark propagator variables. These very quantities are required for analytical and numerical calculations of such important physical parameters as the pion decay constant, meson charge radii, and so on.

Pion decay constant F_π is an important constant in nature. On the one hand, it determines the rate of the pion semileptonic decays $\pi^\pm \to e^\pm \nu_e, \mu^\pm \nu_\mu$. On the other hand, it is one of the two (together with the quark condensate) important chiral QCD parameters, determining a scale of chiral dynamics. All other parameters in chiral QCD are expressed through these two independent basic parameters. In QCD, there is an exact representation of the pion decay constant F_π, derived by Jackiw and Johnson (JJ)[6]. In comparison with the current algebra (CA) representation (see section 3), their representation is free from overlapping divergences, but it requires the explicit investigation of the corresponding Bethe-Salpeter (BS) scattering kernel. We have derived an exact expression for the pion decay constant in both representations. Moreover, using our condition of stationarity for the bound-state amplitude at zero momentum transfer, we were able to determine these expressions completely (i.e., we were able to express them in terms of the quark propagator variables only). We also derived a new correct expression for the pion decay constant within the JJ representation in the so-called Pagels-Stokar-Cornwall (PSC) variables (see section 4).

The paper is organized as follows. In section 2 we investigate flavour non-singlet, chiral axial-vector WT identity in the framework of DCSB. Using the proposed condition of stationarity for the bound-state amplitude, we obtained exact expressions for this quantity itself and for the regular piece of the corresponding axial-vector vertex. In sections 3 and 4, we obtain an exact expression for the pion decay constant in the CA and JJ representations, respectively. In section 4, we also obtain a new correct expression for the pion decay constant within the JJ representation in the PSC variables. Section 5 is devoted to the renormalization program. In section 6, we summarize our results and draw conclusions.

2. Chiral, axial-vector WT identity

Let us consider flavour non-singlet, axial-vector WT identity in the chiral limit $m_0 = 0$, where m_0 is the current "bare" quark mass

$$iq_\mu \Gamma^i_{5\mu}(p+q,p) = \left(\frac{\lambda^i}{2}\right)\{\gamma_5 S^{-1}(p) + S^{-1}(p+q)\gamma_5\}, \tag{2.1}$$

where $q = p' - p$ and the quark propagator is given by

$$-iS(p) = \hat{p}A(-p^2) + B(-p^2). \tag{2.2}$$

In connection with (2.1) one has to point out that in general λ^i is a $SU(N)$ flavour matrix and in the massless case the quark propagator is proportional to the unit matrix in the flavour space. The inverse quark propagator in our parametrization Eq. (2.2) is expressed as

$$\{-iS(p)\}^{-1} = \hat{p}\overline{A}(-p^2) - \overline{B}(-p^2), \tag{2.3}$$

where

$$\begin{aligned} \overline{A}(-p^2) &= A(-p^2)D^{-1}(-p^2), \\ \overline{B}(-p^2) &= B(-p^2)D^{-1}(-p^2), \\ D(-p^2) &= p^2A^2(-p^2) - B^2(-p^2). \end{aligned} \tag{2.4}$$

Substituting the inverse quark propagator from eq. (2.3) into the WT identity eq. (2.1), one obtains

$$\begin{aligned} iq_\mu \Gamma^i_{5\mu}(p+q,p) = i\left(\frac{\lambda^i}{2}\right)\gamma_5\{&[\overline{B}(-p^2) + \overline{B}(-(p+q)^2)] + \hat{q}\overline{A}(-(p+q)^2) \\ &+ \hat{p}[\overline{A}(-(p+q)^2) - \overline{A}(-p^2)]\}, \end{aligned} \tag{2.5}$$

where $\overline{A}(-(p+q)^2)$ and $\overline{B}(-(p+q)^2)$ are defined by (2.4) with the substitution $p \to p+q$.

Using now the standard decomposition of the inverse quark propagator Eq. (2.3), DCSB can be implemented as satisfying the following condition

$$\{S^{-1}(p), \gamma_5\}_+ = 2i\gamma_5\overline{B}(-p^2) \neq 0, \tag{2.6}$$

so that the γ_5 invariance of the quark propagator is broken. This condition leads to the zero mass boson(Goldstone state) in the flavour, axial-vector WT identity (2.1). On the other hand, the quark must have a nonzero mass (dynamical) even if the current "bare" mass of a single quark is equal to zero (dynamical quark mass generation). Indeed, it follows from (2.5) that one gets the nonzero dynamical quark mass, defined by (2.6), if and only if $\Gamma^i_{5\mu}(p+q,p)$ has a pseudoscalar pole at $q^2 = 0$ (Goldstone state) and vice versa. If the right hand side of Eq. (2.5) is finite at zero momentum transfer ($q = 0$) then axial-vector vertex $\Gamma^i_{5\mu}(p+q,p)$ must have a sigularity (pole) at $q^2 = 0$. Having in mind these mathematical grounds of the DCSB, let us solve the WT identity explicitly.

The Lorentz invariance and parity allow one to write the axial-vector vertex in terms of twelve independent form factors [7, 8]

$$\begin{aligned}\Gamma^i_{5\mu}(p+q,p) = (\frac{\lambda^i}{2})\gamma_5\{&\gamma_\mu G_1 + p_\mu G_2 + p_\mu\hat{p}G_3 + \hat{p}\gamma_\mu G_4 \\ &+ q_\mu G_5 + \hat{q}q_\mu G_6 + \hat{q}p_\mu G_7 + \hat{q}\gamma_\mu G_8 \\ &+ \hat{p}q_\mu G_9 + \hat{p}\hat{q}p_\mu G_{10} + \hat{p}\hat{q}q_\mu G_{11} + \gamma_5\epsilon_{\mu\nu\lambda\sigma}\gamma_\nu p_\lambda q_\sigma G_{12}\},\end{aligned} \tag{2.7}$$

where $G_j = G_j(-p^2, -p'^2, -q^2)$ $(j = 1,2,3...12)$. Obviously only form factors G_5, G_6, G_9, and G_{11} can contribute to the pole-like structure at $q^2 = 0$ of the axial-vector vertex (2.7) (because only they are multiplied by q_μ) whereas all other form factors can be considered as the regular functions of their arguments. Substituting this decomposition into Eq. (2.5) and comparing its two sides, one obtains four important relations between the form factors G_j , namely

$$G_1 + q^2G_6 + (pq)G_7 = \overline{A}(-(p+q)^2) \tag{2.8}$$

$$(pq)G_2 + q^2G_5 + q^2G_8 = \overline{B}(-(p+q)^2) + \overline{B}(-p^2) \tag{2.9}$$

$$(pq)G_3 + q^2G_9 = \overline{A}(-(p+q)^2) - \overline{A}(-p^2) \tag{2.10}$$

$$G_4 + q^2G_{11} + (pq)G_{10} = 0. \tag{2.11}$$

Solving these relations with respect to G_5, G_6, G_9 and G_{11}, substituting them into the expression (2.7) and after some algebraic manipulations, the axial-vector vertex can be represented in the following way

$$\Gamma^i_{5\mu}(p+q,p) = \left(\frac{\lambda^i}{2}\right)\gamma_5 \frac{q_\mu}{q^2}[\overline{G}_1 + \hat{q}\overline{G}_2 + \hat{p}\overline{G}_3 + \hat{p}\hat{q}\overline{G}_4] + \Gamma^{iR}_{5\mu}(p+q,p), \qquad (2.12)$$

where

$$\begin{aligned}
\overline{G}_1 &\equiv \overline{G}_1(p+q,p) = [\overline{B}(-p^2) + \overline{B}(-(p+q)^2] - (pq)G_2,\\
\overline{G}_2 &\equiv \overline{G}_2(p+q,p) = [\overline{A}(-(p+q)^2] - G_1 - (pq)G_7,\\
\overline{G}_3 &\equiv \overline{G}_3(p+q,p) = [\overline{A}(-(p+q)^2] - \overline{A}(-p^2)] - (pq)G_3,\\
\overline{G}_4 &\equiv \overline{G}_4(p+q,p) = -G_4 - (qp)G_{10}
\end{aligned} \qquad (2.13)$$

and

$$\Gamma^{iR}_{5\mu}(p+q,p) = \left(\frac{\lambda^i}{2}\right)\gamma_5\{\gamma_\mu G_1 + p_\mu G_2 + p_\mu \hat{p} G_3 + \hat{p}\gamma_\mu G_4 + O_\mu(q)\} \qquad (2.14)$$

is a regular piece of the axial-vector vertex and $O_\mu(q)$ denotes the terms of order q, which play no role in future analysis and therefore we will not write them down explicitly. In the most interesting cases (see the following sections), the terms proportional to the form factors G_7 and G_{10} in Eqs. (2.12-2.13) can be neglected, because they are of order q^2 terms. For this reason, the determination of the form factors $G_j = G_j(-p^2)(j = 1,2,3,4)$ at zero momentum transfer ($q = 0$) makes it possible to find exactly the bound-state amplitudes up to terms of order q as well as the regular piece of the axial vertex at $q = 0$ (see below, Eqs. (2.15) and (2.17)). Thus, the axial-vector vertex $\Gamma^i_{5\mu}(p+q,p)$ has indeed a singularity (pole) at $q^2 = 0$ (Goldstone pole corresponding to the massless pion) with residue proportional to the pion decay constant F_π

$$G^i_5(p+q,p) = -\frac{1}{F_\pi}\left(\frac{\lambda^i}{2}\right)\gamma_5\{\overline{G}_1 + \hat{q}\overline{G}_2 + \hat{p}\overline{G}_3 + \hat{p}\hat{q}\overline{G}_4\}. \qquad (2.15)$$

Obviously $G^i_5(p+q,p)$ represents the proper pseudoscalar meson-quark-antiquark vertex function and the right hand side of this relation is nothing other than the general decomposition of the BS pion wave function into independent matrix strutures.

In order to calculate certain important physical parameters, e.g. the pion decay constant (see below), meson form factors and their charge radii, one needs to know the bound-state wave function Eq. (2.15) and the regular piece of the axial-vector vertex $\Gamma^{iR}_{5\mu}(p+q,p)$ at zero momentum transfer ($q = 0$). The former can be obtained directly from Eqs. (2.15) and (2.13) in the limit $q = 0$

$$G^i_5(p,p) = -\frac{1}{F_\pi}\left(\frac{\lambda^i}{2}\right)\gamma_5\{2\overline{B}(-p^2)\} \tag{2.16}$$

and the latter is

$$\Gamma^{iR}_{5\mu}(p,p) = \left(\frac{\lambda^i}{2}\right)\gamma_5\{\gamma_\mu G_1 + p_\mu G_2 + p_\mu \hat{p} G_3 + \hat{p}\gamma_\mu G_4\} \tag{2.17}$$

where $G_j = G_j(-p^2)(j = 1,2,3,4)$. Relations (2.8-2.11) require more careful consideration.

As was mentioned above, only the form factors G_5, G_6, G_9 and G_{11} need have pole singularities at $q^2 = 0$. For this reason, let us express these form factors as follows (j=5,6,9,11)

$$G_j(p,q) = \frac{1}{q^2}R_j(p,p) + G^R_j(p,q), \tag{2.18}$$

where $R_j(p,p)$ and $G^R_j(p,q)$ are the residues and the regular parts of the corresponding form factors, respectively. In relations (2.8) and (2.11) it is sufficient to pass to the limit $q = 0$ directly, taking into account Eq. (2.18), while relations (2.9) and (2.10) should be differentiated with respect to q_ν and then setting $q_\nu = 0$ *

Finally, one obtains the following system of equations

$$\begin{aligned} G_1(-p^2) &= \overline{A}(-p^2) - R_6(-p^2) \\ G_2(-p^2) &= -2\overline{B}'(-p^2) \\ G_3(-p^2) &= -2\overline{A}'(-p^2), \\ G_4(-p^2) &= -R_{11}(-p^2), \end{aligned} \tag{2.19}$$

* If one takes the limit $q = 0$ directly in relation (2.9), taking into accont (2.18), then one obtains that only the residue $R_5(p,p) = 2\overline{B}(-p^2)$ of the form factor $G_5(p,q)$ completely determines the residue at pole $q^2 = 0$ of the corresponding axial-vector vertex $\Gamma^i_{5\mu}(p,q)$, i.e. (2.16) as it is expected.

where the primes denote differentiation with respect to the argument $(-p^2)$. Going now over the Euclidean space $(p^2 \to -p^2)$ and dimensionless variables

$$A(p^2) = \mu^{-2}A(t), \qquad B(p^2) = \mu^{-1}B(t), \qquad t = p^2/\mu^2, \tag{2.20}$$

where μ is the appropriate mass parameter, the system (2.19) for dimensionless form factors should read

$$\begin{aligned} G_1(t) &= -\overline{A}(t) - R_6(t) \\ G_2(t) &= 2\overline{B}'(t) \\ G_3(t) &= 2\overline{A}'(t) \\ G_4(t) &= -R_{11}(t), \end{aligned} \tag{2.21}$$

From now on, primes denote differentiation with respect to the dimensionless Euclidean momentum variable t, and

$$\begin{aligned} \overline{A}(t) &= A(t)E^{-1}(t), \\ \overline{B}(t) &= B(t)E^{-1}(t), \end{aligned} \tag{2.22}$$

$$E(t) = p^2A^2(t) + B^2(t). \tag{2.23}$$

Thus, system (2.21) is the general solution to the regular piece of the axial-vector vertex at zero momentum transfer (2.17) in the chiral limit. This solution depends on only two arbitrary functions $R_6(t)$ and $R_{11}(t)$, which are the residues of the corresponding form factors. In order to find these functions, let us recall that the residue at pole $q^2 = 0$ of the axial-vector vertex $\Gamma^i_{5\mu}(p+q,p)$ (2.12) is determined completely by the behaviour of the meson-quark-antiquark wave function $G^i_5(p+q,p)$ (2.15) at zero momentum transfer $(q = 0)$. For this reason, it is worth applying the condition of stationarity for the wave function at this point, i.e.,

$$\{\frac{\partial}{\partial q_\nu} G^i_5(p+q,p)\}_{q_\nu=0} = 0, \tag{2.24}$$

which makes it possible to establish completely all form factors $G_j(t)$ $(j = 1,2,3,4)$. It is obvious that in the general case it is impossible to establish the sign of the second derivative of the wave function $G_5^i(p+q,p)$ at $q_\nu = 0$. It depends only on the concrete forms of the solutions to the quark SD equations. Therefor we are not able to establish whether the residue at pole $q^2 = 0$ gets its maximum or minimum at this point. But in any case, it is only the condition of stationarity that is sufficient to find all form factors completely. Indeed, differentiating wave function (2.15) (taking into account (2.13)) according to the condition of stationarity (2.24) and adopting the dimensionless Euclidean variables (2.20), one finally obtains the following system of equations

$$\begin{aligned} G_1(t) &= -\overline{A}(t) \\ G_2(t) &= 2\overline{B}'(t) \\ G_3(t) &= 2\overline{A}'(t) \\ G_4(t) &= 0. \end{aligned} \tag{2.25}$$

This is the stationary solution to the regular piece of the axial-vector vertex (2.17) in the chiral limit. The stationary solution (2.25) can obviously be obtained from the general solution (2.21) setting

$$R_6(t) = R_{11}(t) = 0, \tag{2.26}$$

so that the stationary solution is completely determined. Let us use this solution in the next section in order to find the exact expression for the pion decay constant in current algebra representation.

3. Pion decay constant in current algebra representation.

In the CA (for excellent reviews on current algebra see Refs. 9) the pion decay constant F_π is defined as

$$\langle 0|J_{5\mu}^i(0)|\pi^j(q)\rangle = iF_\pi q_\mu \delta^{ij} \tag{3.1}$$

(The normalization $F_\pi = 93.3MeV$ is used; however, see [10]). Clearly, this matrix element can be written in terms of the pion- quark-antiquark proper vertex (2.15) and quark propagators as

$$iF_\pi q_\mu \delta^{ij} = \int \frac{d^4p}{(2\pi)^4} Tr\Big\{\Big(\frac{\lambda^i}{2}\Big)\gamma_5\gamma_\mu S(p+q)G_5^j(p+q,p)S(p)\Big\}. \tag{3.2}$$

Here and below, a trace over the Dirac and colour indices is understood. To get the expression for F_π one has to differentiate both sides of eq. (3.2) with respect to the external momentum q_ν and then set $q_\nu = 0$. This is shown in Fig.1. Let us denote the first term in Fig.1 by $D^{ij}_{\mu\nu}(a)$ and the second one by $D^{ij}_{\mu\nu}(b)$, respectively. By taking into account the BS pion wave function at zero momentum transfer ($q = 0$) (2.16) and also (2.2) with the substitution $p \to p + q$, the expression for the first term can easily be evaluated. Going over the Euclidean space ($d^4p \to id^4p$, $p^2 \to -p^2$) and using dimensional variables (2.20), the final result of the evalution of the first term can be written down as

$$D(a) \equiv \delta^{ij} g_{\mu\nu} D^{ij}_{\mu\nu}(a) = \frac{i}{F_\pi} \frac{12\pi^2}{(2\pi)^4} \mu^2 \int_0^\infty dt\, t\overline{B}(t)\{-[AB + \frac{1}{2}t(A'B - AB')]\}, \qquad (3.3)$$

Here and below $A = A(t)$, $B = B(t)$ and $\overline{B}(t)$ is given by (2.22-2.23). Note that this term does not depend explicitly on the regular piece of the axial-vector vertex (2.27).

The second term in Fig.1 can be calculated in the same way. Nevertheless, the final result takes a more complicated form since the dependence on the regular piece of the axial-vector appears. Omitting all intermediate algebraic manipulations we can write the final result as

$$\begin{aligned} D(b) \equiv \delta^{ij} g_{\mu\nu} D^{ij}_{\mu\nu}(b) = \frac{i}{F_\pi} \frac{12\pi^2}{(2\pi)^4} \mu^2 \int_0^\infty dt\, t\{\frac{3}{4} tABG_4(t) - \\ - \frac{1}{8} tE[G_3(t) - 2\overline{A}'(t)] - \frac{1}{4}[G_1(t) + \overline{A}(t)](E - 3B^2)\}, \end{aligned} \qquad (3.4)$$

where $\overline{A}(t)$ and $E \equiv E(t)$ are shown in eqs. (2.22-2.23). Summing now the two terms (3.3) and (3.4)

$$iF_\pi = D(a) + D(b) \qquad (3.5)$$

one finally obtains the exact expression for the pion decay constant in the CA representation. Using the general, chiral solution (2.21) for the regular piece of the axial-vector vertex, the following result is obtained:

$$F^2 = \frac{12\pi^2}{(2\pi)^4} \mu^2 \int_0^\infty dt\, t\overline{B}(t)\{-[AB + \frac{1}{2}t(A'B - AB')] - \frac{3}{4} tABR_{11}(t) + \frac{1}{4} R_6(t)(E - 3B^2)\}, \qquad (3.6)$$

where we denote the pion decay constant F_π in the chiral limit by F in accordance with Refs. 11. This is the general, chiral expression for the pion decay constant in the CA representation. Obviously, it is necessary to choose some Ansatzs for the arbitrary form factors $R_6(t)$ and $R_{11}(t)$ in order to calculate the pion decay constant within this expression. For example, in our previous paper [12], assuming the close relationship between the regular pieces at zero momentum transfer of the vector and axial-vector vertices in the chiral limit, we were able to choose some definite expressions for these form factors. We obtained a very good numerical result for the pion decay constant in this case.

As mentioned above, in order to obtain the stationary solution from the general, chiral solution (3.6), it is necessary to put $R_6(t) = R_{11}(t) = 0$, so that one obtains

$$F^2 = \frac{12\pi^2}{(2\pi)^4}\mu^2 \int_0^\infty dt\, t\overline{B}(t)\{-[AB + \frac{1}{2}t(A'B - AB')]\}, \tag{3.7}$$

Obviously, this expression is only the result of the first diagram in Fig.1, since the contribution from the second diagram completely disappears, because of the condition of stationarity (2.24). This is the chiral, stationary solution for the pion decay constant in the CA representation. The advantage of this expression is that it does not depend on arbitrary form factors and is completely expressed only in terms of the corresponding quark propagator (2.2) Euclidean scalar functions $A(t)$ and $B(t)$. Let us also express this solution in terms of the inverse quark propagator (2.3), Euclidean scalar functions $\overline{A}(t)$ and $\overline{B}(t)$

$$F^2 = \frac{12\pi^2}{(2\pi)^4}\mu^2 \int_0^\infty dt\, t\frac{\overline{B}(t)}{\overline{E}^2(t)}\{-[\overline{A}\overline{B} + \frac{1}{2}t(\overline{A}'\overline{B} - \overline{A}\overline{B}')]\}, \tag{3.8}$$

where $\overline{A} \equiv \overline{A}(t)$ and $\overline{B} \equiv \overline{B}(t)$ are given by (2.22) and

$$\overline{E}(t) = t\overline{A}^2(t) + \overline{B}^2(t). \tag{3.9}$$

Let us now express the stationary, chiral formulae for the pion decay constant (3.8) in terms of the Pagels-Stokar (PS) [13] and Cornwall's [14] variables. the inverse quark propagator in their variables is

$$S^{-1}(p) = \hat{p} - \Sigma(p^2) \tag{3.10}$$

Writing $M(p^2)$ instead of $\Sigma(p^2)$, one obtains the Cornwall parametrization. Comparing this expression with our parametrization for the inverse quark propagator (2.3), one gets

$$\overline{A}(-p^2) = 1, \qquad \Sigma(p^2) = \overline{B}(-p^2). \tag{3.11}$$

(Note that we include the factor $(-i)$ into the definition of the quark propagator). In Euclidean space, (3.11) should read

$$\overline{A}(p^2) = -1, \qquad \overline{B}(p^2) = -\Sigma(-p^2). \tag{3.12}$$

Setting formally $\mu^2 = 1$, so that $t = p^2$ and substituting relations (3.12) into the chiral, stationary solution (3.8), one finally obtains

$$F^2 = \frac{3}{(2\pi)^2}\int_0^\infty dp^2\, p^2 \frac{\Sigma(-p^2)}{\{p^2 + \Sigma^2(-p^2)\}^2}\{\Sigma(-p^2) - \frac{1}{2}p^2\frac{d}{dp^2}\Sigma(-p^2)\}. \tag{3.13}$$

It was precisely this expression that was obtained by Pagels and Stokar, and Cornwall (by substitution $\Sigma(-p^2) \to M(-p^2)$ in Ref.13 and Ref.14, respectively (see also [2]). Within our terminology, this is the chiral, stationary solution for the pion decay constant in the CA representation, expressed by means of the the Pagels-Stokar-Cornwall's (PSC) variables. For this reason, we propose to call eq. (3.13) shortly as the PSC expression for the pion decay constant in chiral QCD. It seems to us that Cornwall was the first to obtain this expression, correctly starting from the CA representation for the pion decay constant. Pagels and Stokar obtained this expression, starting (as they thought) from the Jackiw-Johnson (JJ) representation for the pion decay constant (see next section).

4. Pion decay constant in the Jackiw-Johnson representation

In QCD, there is an exact representation for the pion decay constant F_π, derived by Jackiw and Johnson (JJ) in Ref.6 (see also Refs. 15 and 16). This representation is free of overlapping divergences but requires explicit investigation of the corresponding BS scattering kernel. JJ representation for the pion decay constant is shown in Fig.2. Obviously the contribution from the second diagram in Fig.2 must in each case be calculated separately. For example, the derivative of the BS kernel in the ladder approximation vanishes so that the second diagram does not contribute at all in this case. At the same time, the contribution from the first diagram in Fig.2 can be investigated completely.

Let us denote the first term in Fig.2 by $D^{ij}_{\mu\nu}(a)$ as in the previous section. The analytical expression for $D^{ij}_{\mu\nu}(a)$ can be written down as follows

$$D^{ij}_{\mu\nu}(a) = \int \frac{d^4p}{(2\pi)^4} Tr\{\Gamma^{iR}_{5\mu}(p,p)\left[\frac{\partial}{\partial q_\nu}S(p+q)\right]G^j_5(p,p)S(p)\}_{q_\nu=0}. \tag{4.1}$$

By taking into account the BS pion wave function (2.16) and the regular piece of the axial-vector vertex (2.17) at zero momentum transfer and also (2.2) with the substitution $p \to p+q$, this expression can easily be evaluated. Going then over the Euclidean space and using dimensionless form factors and variables (2.20), the final result can be written as

$$
\begin{aligned}
D(a) = \delta^{ij} g_{\mu\nu} D^{ij}_{\mu\nu}(a) = \frac{i}{F_\pi} \frac{12\pi^2}{(2\pi)^4} \mu^2 \int_0^\infty & dt\, t\overline{B}(t) \\
& \{-G_1(t)[AB + \frac{1}{2}t(A'B - AB')] \\
& - tG_2(t)[\frac{1}{4}A^2 + \frac{1}{2}(tAA' + BB')] \\
& + tG_3(t)[\frac{1}{4}AB + \frac{1}{2}t(A'B - AB')] \\
& - tG_4(t)[A^2 + \frac{1}{2}(tAA' + BB')]\},
\end{aligned} \tag{4.2}
$$

For the sake of convenience, let us recall that $\overline{B}(t)$ is shown in eqs. (2.22-2.23), $A \equiv A(t)$, $B \equiv B(t)$, and the primes denote the differentiation with respect to the argument. This is the most general expression for the first diagram in Fig.2. Substituting into eq. (4.2) the solution (2.21) for the corresponding form factors, one obtains the general, chiral expression for the first diagram in Fig.2 in the JJ representation.

From now on, let us restrict ourselves to the ladder approximation for the BS kernel in the second diagram in Fig.2. As was mentioned above, in this case the second diagram does not contribute at all to the expression for the pion decay constant. For this reason, $iF_\pi = D(a)$ simply, and substituting into eq. (4.2) the chiral, stationary system (2.25), one obtains

$$
\begin{aligned}
F^2 = \frac{12\pi^2}{(2\pi)^4} \mu^2 \int_0^\infty dt\, t\overline{B}(t) \{ & \overline{A}(t)[AB + \frac{1}{2}t(A'B - B'A)] \\
& - 2t\overline{B}'(t)[\frac{1}{4}A^2 + \frac{1}{2}(tAA' + BB')] \\
& + 2t\overline{A}'(t)[\frac{1}{4}AB + \frac{1}{2}t(A'B - B'A)]\},
\end{aligned} \tag{4.3}
$$

This is the chiral, stationary expression for the pion decay constant in the JJ representation. Using relations (2.22) it can easily be expressed in terms of the quark propagator Euclidean scalar functions $A(t)$ and $B(t)$ only. Using the same relations, let us express

(4.3) in terms of the inverse quark propagator scalar functions $\overline{A}(t)$ and $\overline{B}(t)$

$$F^2 = \frac{12\pi^2}{(2\pi)^4}\mu^2 \int_0^\infty dt\, t \frac{\overline{B}(t)}{\overline{E}^2(t)} \{(\overline{A} + 2t\overline{A}')[\overline{A}\,\overline{B} + \frac{1}{2}t(\overline{A}'\overline{B} - \overline{B}'\overline{A})] + \frac{1}{2}t[\overline{B}'\overline{E}' - 3\overline{B}\,\overline{A}\,\overline{A}']\}, \tag{4.4}$$

where $\overline{E}(t)$ is given by (3.9).

Let us express this stationary, chiral solution for the pion decay constant in the JJ representation in terms of the PSC variables (3.10) and (3.12). Similarly to (3.13), one finally obtains

$$F^2 = \frac{3}{(2\pi)^2} \int_0^\infty dp^2\, p^2 \frac{\Sigma^2(-p^2)}{\{p^2 + \Sigma^2(-p^2)\}^2} \{1 + p^2 (\frac{d}{dp^2}\Sigma(-p^2))^2\}. \tag{4.5}$$

So that within the JJ representation and in the PSC variables this very expression should be obtained, not expression (3.13). In order to point this out, let us write down explicitly the chiral, stationary system (2.25) in the PSC variables

$$\begin{aligned} G_1(p^2) &= 1 \\ G_2(p^2) &= 2\Sigma'(-p^2) \\ G_3(p^2) &= G_4(p^2) = 0, \end{aligned} \tag{4.6}$$

where now the prime denotes the differentiation with respect to the dimensional Euclidean momentum variable ($-p^2$). This is the most simple, nontrivial solution to the regular piece of the axial-vector vertex at zero momentum transfer. In derivation of their expression Pagels and Stokar neglected the form factor $G_2(p^2)$ by saying formally it is of order g^2 in the coupling constant. In this case the regular piece of the axial-vector vertex (2.17) simply becomes the point like vertex. Obviously within this formal assumption (plus also the ladder approximation for the BS kernel) the JJ representation simply reduces to the CA representation, i.e., they have actually analysed the first diagram in Fig.1, and not the first diagram in Fig.2. For precisely this reason, they obtained expression (3.13) and not expression (4.5). In general, the form factor $G_2(p^2)$ can never be neglected. Indeed, this form factor is equal to zero in two cases only: 1) when $\Sigma(-p^2) = constant$, but it means in the PSC variables that the current "bare" mass of a single quark is not equal to zero. This is in contradiction with initial chiral condition. 2) when $\Sigma(-p^2)$ is equal to zero identically, but in this case there is no dynamically chiral symmetry breaking. This trivial case is also not acceptable. Thus form factor $G_2(p^2)$ is never equal to zero within the DCSB and in this case one certainly arrives at expression (4.5) within the JJ representation in the PSC variables. Generally speaking, the form factors $G_j(t)$ $(j = 2,3,4)$, in particular the form factor $G_2(t)$ in the PSC variables, can never be neglected in any order of perturbation theory. Perturbative arguments are not suitable for analyzing such nonperturbative phenomenon as DCSB within the corresponding axial WT identity.

5. Renormalization

In this section, let us make a few remarks on the renormalization ofthe expressions obtained for the pion decay constant. As it is well known, due to the local, bilinear structure of the quark fields for the axial-vector current $J^i_{5\mu}$, the corresponding vertex part $\Gamma^i_{5\mu}$, defined as

$$S(p)\Gamma^i_{5\mu}(p',p)S(p') = -\int d^4x d^4y\, e^{ipx} e^{-ip'y} \langle 0|Tq(x)J^i_{5\mu}(0)q(y)|0\rangle \tag{5.1}$$

is multiplicatively renormalizable. Following the standard Dyson procedure, we can write

$$\begin{aligned} \Gamma^i_{5\mu}(p,p') &= Z_A^{-1}\tilde{\Gamma}^i_{5\mu}(p,p') \\ S(p) &= Z_2\tilde{S}(p). \end{aligned} \tag{5.2}$$

Here and below all the renormalized quantities denoted by tilde are finite (cutoff-independent), while Z_A and Z_2 are cutoff-dependent renormalization constants. Note that in the chiral limit $m_0 = 0$ and in the absence of the chiral anomaly (non-singlet channel), the divergence of the axial-vector current $J^i_{5\mu}$ is conserved, i.e., $\partial_\mu J^i_{5\mu} = 0$ [17]. Substituting eq. (5.2) into the initial WT identity (2.1) and taking a variation with respect to the cutoff, one obtains that the ratio, defined as $g_A = Z_A/Z_2$, is cutoff-independent, and hence finite. This is the content of the Preparata-Weisberger (PW) theorem [17,18]. According to the PW theorem, multiplication by the wave function renormalization constant Z_2, would make $\Gamma^i_{5\mu}$ finite. In other words, the regular piece of the axial-vector vertex $\Gamma^{iR}_{5\mu}(p,p')$ and the bound-state amplitude $G^i_5(p,p')$ each renormalize like the inverse quark propagator, i.e.,

$$\begin{aligned} \Gamma^{iR}_{5\mu}(p,p') &= Z_2^{-1}\tilde{\Gamma}^{iR}_{5\mu}(p,p') \\ G^i_5(p,p') &= Z_2^{-1}\tilde{G}^i_5(p,p') \end{aligned} \tag{5.3}$$

From the renormalized integral equation for the bound-state amplitude $\tilde{G}^i_5(p,p')$, it follows that the corresponding two-particle irreducible (2PI) quark-antiquark BS scattering kernel, is renormalized according to

$$\tilde{K}(p,p') = Z_2^2 K(p,p'). \tag{5.4}$$

Using these relations (namely eqs. (5.3-5.4) and the second of eqs.(5.2)), one obtains that all expressions for the pion decay constant in the JJ representation are invariant under renormalization. Therefore finite, renormalized quantities $\tilde{S}$, $\tilde{G}^i_5$, $\tilde{\Gamma}^{iR}_{5\mu}$ and $\tilde{K}$ can be used in evaluating the decay constant within this representation. At the same time, in the CA representation all the expressions for the pion decay constant should finally be multiplied by the wave function renormalization constant Z_2. In the most simple way, this can be seen from the diagrams in Fig.1 for the CA representation and diagrams in Fig.2 for the JJ representation. Concluding these general remarks, let us note that in order to calculate the pion decay constant numerically in either of these representations it is worth performing the regularization and renormalization program separately in each concrete case.

6. Conclusions

In this paper we have investigated flavour non-singlet, chiral axial-vector WT identity in the framework of DCSB. Introducing the most general expression (2.7) for the corresponding axial-vector vertex, we obtained four important relations (2.8-2.11) between various arbitrary (at this stage) form factors. This made it possible to decompose the axial-vector vertex into two parts in a self-consistent way (2.12). The pole term at $q^2 = 0$ (momentum transfer) with the residue proportional to the bound-state amplitude and the corresponding regular piece, depend on four arbitrary form factors. In order to determine these arbitrary form factors, we propose to use the condition of stationarity (2.24) for the residue at pole $q^2 = 0$ of the axial-vector vertex. In other words, we were able to find the bound-state amplitude and the regular piece of the initial axial vertex in terms of the quark propagator variables only. Therefore we have resolved the old standing problem of the dependence of the bound-state amplitudes and the regular pieces of the axial verticies on arbitrary form factors within the corresponding WT identities.

For this reason, we have found the exact expressions for the pion decay constant in the CA representation (eqs. (3.7) and (3.8)) as well as in the JJ representation (eqs. (4.3) and (4.5)). These expressions make it possible to calculate one of the most important physical parameters by integrating only over the quark propagators' variables. Thus the chiral QCD dynamics on the microscopic (quark) level provides full information for the pion semileptonic weak decays. We have obtained a new expression for the pion decay constant in the PSC variables within the JJ representation (4.5). It would be interesting to compare numerically the expressions for the pion decay constant in both representations. For the PSC variables, namely eqs. (3.13) and (4.5), it can be done, for example, in dual QCD [19], or within the approach of Ref.20. The more general expressions, namely eqs. (3.7) or (3.8) and eqs.(4.3) or (4.4) respectively, can be numerically compared within our dynamical quark propagator approach to QCD at large distances [21]. This numerical program will be performed elsewhere. The generalization on the nonchiral limit ($m_0 \neq 0$) will not be difficult, but at the same time, it is not straightforward. For this reason it requires separate consideration.

It would also be of great interest to compare our expression of pion decay constant for finite temperatures (4.5) with the PCS expression (3.13) for finite temperature and density obtained by Barducci et al. [22].

Let us underline again the advantage of each of the two representations. The expressions in the JJ representation are free of overlapping divergences and invariant under the renormalization, whereas the expressions in the CA representation do not explicitly depend on the BS scattering kernel. Evidently, there is no hope for an exact treatment of the BS scattering kernel. Concluding our paper, let us make one essential remark. We could express the observed physical quantities, such as pion decay constant, in terms of unobserved quark propagator variables only. Moreover, we could find many other physical parameters, such as meson form factors, their charge radii and so on, in the same way. Thus, we are able to answer to the question of how to describe the real world of observed hadrons in terms of unobserved quarks and gluons.

The authors would like to thank K. Szegő, A. Frenkel, J. Révai, K. Tóth, P. Hraskó and other members of the Theoretical Department of the RMKI for their constant interest, valuable discussions and support.

References.

[1] Y.Nambu and G.Jona-Lasinio,Phys.Rev.**22**(1961)124,345.
B.A.Arbuzov,R.N.Faystov and A.N.Tavkhelidze,
Dokl.Acad.Nauk SSSR,**139**(1961)345.
M.Gell-Mann and M.Levy,Nuovo Cim.**16**(1960)705.

[2] A.Barducci,R.Casalbuoni,S.De Curtis,D.Dominici and R.Gatto,Phys.Rev.**D38**(1988)238.

[3] M.E.Peskin,SLAC report,SLAC-PUB-2021,1982T.

[4] K.Lane,Phys.Rev.**D10**(1974)2605
S.-H.H.Tye,E.Tomboulis and E.C.Poggio,Phys.Rev.**D11**(1975)2839
E.J.Eichten and F.L.Feinberg,Phys.Rev.**D10**(1974)3254

[5] G.'t Hooft,in Resent developments in gauge theories,
ed.G.'t Hooft et al (Plenum Press,NY,1980)

[6] R.Jackiw and K.Johnson,Phys Rev.**D8**(1973)2386

[7] J.Berstein,Elementary particles and their currents
(W.H.Freeman and Comp.,San-Francisco and London,1968).

[8] M.L.Goldberger and D.E.Soper,Phys.Rev.**D14**(1976)1117

[9] S.Adler and R.Dashen,Currents Algebras (W.A.Benjamin,NY,1968)
B.Renner,Current Algebras and Their Applications,(Pergamon Press,London,1968)

[10] B.R.Holstein,Phys.Lett.**B244**(1990) 83

[11] P.Gerber and H.Leutwyler,Nucl.Phys.**B321**(1989)387
J.Gasser and H.Leutwyler,Nucl.Phys.**B250**(1985)517

[12] V.Sh.Gogohia,Gy.Kluge and B.A.Magradze,Phys.Lett. **B244**(1990) 68

[13] H.Pagels and S.Stokar,Phys.Rev.**D20**(1979)2947

[14] J.M.Cornwall,Phys.Rev.**D22**(1980)1452

[15] P.Langacker and H.Pegels,Phys.Rev.**D9**(1974)3413

[16] H.Pagels,Phys.Rev.**D19**(1979)3080

[17] S.Adler,Phys.Rev.**177**(1968)2426

[18] G.Preparata and W.I.Weisberger,Phys.Rev.**175**(1968)1965

[19] M.Baker,J.S.Ball and F.Zachariasen,Phys.Rev. **D38**(1988)1926
G.Krein and A.G.Williams,Phys.Rev.**D43**(1991)3541

[20] K.-I.Aoki,M.Bando,T.Kugo,M.G.Mitchard and H.Nakatani
Prog.Theor.Phys.**84**(1990)683

[21] V.Sh.Gogohia,Phys.Rev.**D40**(1989)4157;
Phys.Rev.**D41**(1990)3279E.

[22] A.Barducci,R.Casalbuoni,S.De Curtis,R.Gatto and G.Pettini,
Phys.Rev.**D42**(1990)1757;
Phys.Lett.**B240**(1990)429.

Figure Captions.

Fig.1. Exact expression for the pion decay constant F_π in the current algebra. Here G_5^j, S and $J_{5\mu}^i$ are the pion-quark bound-state wave function, the quark propagator and the axial-vector current, respectively. The slash denotes differentiation with respect to momentum q_ν and setting $q_\nu = 0$.

Fig.2. Exact expression for the pion decay constant F_π in the Jackiw-Johnson (JJ) representation. Here G_5^j and S are the pion-quark bound-state wave function and the quark propagator, respectively. K and $\Gamma_{5\mu}^{iR}$ are the BS scaterring kernel and the regular piece of the corresponding vertex, respectively. The slash denotes differentiation with respect to momentum q_ν and setting $q_\nu = 0$ as in Fig.1.

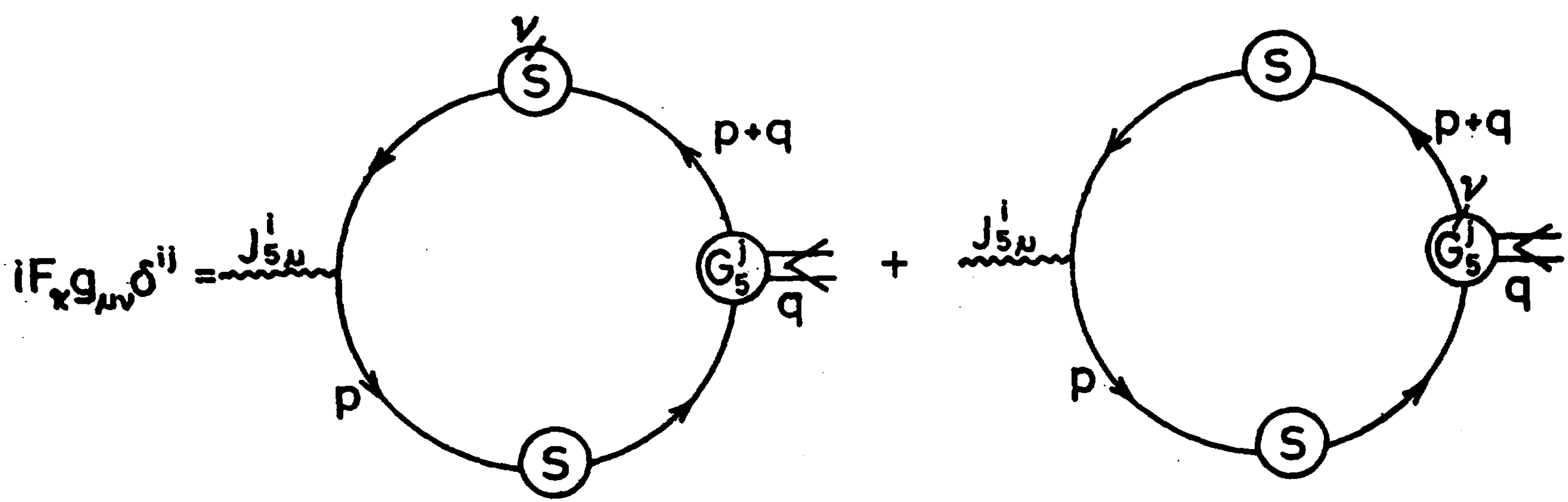

Fig. 1

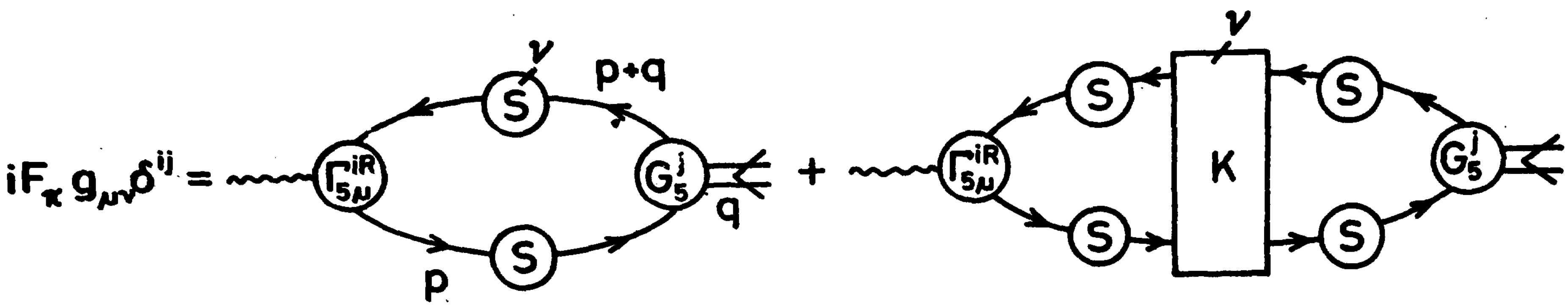

Fig. 2.

The issues of the KFKI preprint/report series are classified as follows:

- **A.** Particle and Nuclear Physics
- **B.** General Relativity and Gravitation
- **C.** Cosmic Rays and Space Research
- **D.** Fusion and Plasma Physics
- **E.** Solid State Physics
- **F.** Semiconductor and Bubble Memory Physics and Technology
- **G.** Nuclear Reactor Physics and Technology
- **H.** Laboratory, Biomedical and Nuclear Reactor Electronics
- **I.** Mechanical, Precision Mechanical and Nuclear Engineering
- **J.** Analytical and Physical Chemistry
- **K.** Health Physics
- **L.** Vibration Analysis, CAD, CAM
- **M.** Hardware and Software Development, Computer Applications, Programming
- **N.** Computer Design, CAMAC, Computer Controlled Measurements

The complete series or issues discussing one or more of the subjects can be ordered; institutions are kindly requested to contact the KFKI Library, individuals the authors.

Title and classification of the issues published this year:

KFKI-1991-01/M A. Ambrozy: Experience during applying the ORACLE 4GL system (in Hungarian)

KFKI-1991-02/M S. Dibuz Wagner: A knowledge base for protocols

KFKI-1991-03/G Gy. Ézsöl et al.: Adaptation of RELAP5/MOD2 code: Analysis of IAEA SPE 3 (in Hungarian)

KFKI-1991-04/G Gy. Ézsöl et al.: IAEA SPE 3. Valuation of measurement results (in Hungarian)

KFKI-1991-05/G A. Takács et al.: SMABRE - as the dynamical model of the primary circuit for the code KARATE (in Hungarian)

KFKI-1991-06/A J. Balog et al.: Locality problem for the Liouville field

KFKI-1991-07/E N. Éber et al.: Linear electromechanical effect in a polymeric ferroelectric liquid crystal

KFKI-1991-08/C B. Lukács: Prices and sunspots

KFKI-1991-09/E M.V. Kozlovsky et al.: A chiral side chain polymer with layered structure

KFKI-1991-10/G P. Vértes: Calculation of transmission and other functionals from evaluated data in ENDF format by means of personal computers

KFKI-1991-11/E F. Tóth: Non destructive testing methods (in Hungarian)

KFKI-1991-12/H Vera Jánossy et al.: Multichannel recordings and data analysis from spinal cord explants and cell cultures

KFKI-1991-13/G Z. Szatmáry: User's manual of program RFIT. Part 1. General description

KFKI-1991-14/G Z. Szatmáry: User's manual of program RFIT. Part 2. Parameter estimation

KFKI-1991-15/G Z. Szatmáry: User's manual of program RFIT. Part 3. The data files

KFKI-1991-16/G Z. Szatmáry: User's manual of program RFIT. Part 4. Statistical analysis. Task PLOT

KFKI-1991-17/A T. Haugset et al.: Is the breakup time a free parameter when describing heavy ion collisions?

KFKI-1991-18/G Margit Telbisz: SNARE -- A response matrix based few-group nonlinear reactor code in hexagonal geometry

KFKI-1991-19/B G. Paál et al.: Inflation and compactification from galaxy redshifts?

KFKI-1991-20/G L. Perneczky: ISP 27. Pre-test calculations for BETHSY SB LOCA test with RELAP5/RMA code (in Hungarian)

KFKI-1991-21/A V.Sh. Gogohia et al.: Phenomenological dynamics in QCD at large distances

KFKI-1991-22/C M.I. Verigin et al.: The dependence of the Martian magnetopause and bow shock on solar wind ram pressure according to Phobos 2/TAUS ion spectrometer measurements

KFKI-1991-23/A V.Sh. Gogohia et al.: Dynamical chiral symmetry breaking and pion decay constant

Kiadja a Központi Fizikai Kutató Intézet
Felelős kiadó: Szegő Károly
Szakmai lektor: Frenkel Andor
Nyelvi lektor: Sebestyén Ákos
Példányszám: 354 Törzsszám: 91-181
Készült a KFKI sokszorosító üzemében
Felelős vezető: Gonda Péter
Budapest, 1991. augusztus hó