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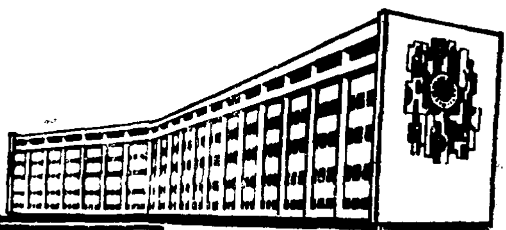
EFFECTIVE HAMILTONIAN WITHIN THE MICROSCOPIC UNITARY NUCLEAR MODEL.

1. COHERENT-STATE CONSTRICTION TECHNIQUE FOR MICROSCOPIC HAMILTONIAN



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Г.Ф.Филиппов, А.Л.Блохин

Эффективный гамильтониан микроскопической унитарной модели ядра. I. Метод сужения микроскопического гамильтониана на когерентных состояниях

Предложен метод проектирования микроскопического ядерного гамильтониана на обертывающую алгебру группы SU(3). В развиваемом подходе эффективный гамильтониан восстанавливается по матричным элементам на когерентных состояниях неприводимых представлений SU(3) Получены явные результаты для околомагических ядер на базисе нескольких представлений и для произвольных ядер на одном представлении.

G.F.Filippov, A.L.Blokhin

Effective Hamiltonian Within the Microscopic Unitary Nuclear Model. I. Coherent-State Constriction Technique for Microscopic Hamiltonian

A technique of projecting the microscopic nuclear Hamiltonian on the SU(3)-group enveloping algebra is developed. The approach proposed is based on the effective Hamiltonian restored from the matrix elements between the coherent states of the SU(3) irreducible representations. The technique is displayed for almost magic nuclei within the mixed representation basis, and for arbitrary nuclei within the single representation.

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1. COHERENT-STATE CONSTRICTION TECHNIQUE FOR MICROSCOPIC HAMILTONIAN

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1. INTRODUCTION

The appearance of Elliott's SU(3) model [1] stimulated the development of an algebraic approach in nuclear structure theory. This initiated precise semicroscopic shell-model calculations of light nuclei [2-4] utilizing an SU(3)-basis in parallel with a phenomenological description of medium-weight and heavy nuclei, based on the SU(3) enveloping algebra Hamiltonian with coefficients fitted to experimental spectra [5-8]. The subsequent extension of dynamical symmetry to the U(6) one of the interacting boson model [9,10] and to the Sp(6,R) one of the collective model [11-16] formed an algebraic base for a consistent theory of complementary intrinsic and collective motion.

So long as an effective Hamiltonian (EH) of many-body problem is expressed via single-particle coordinates combined in Lie algebra generators, one may connect the algebraic formulation of the model with its microscopic treatment. However such EHs as a rigid rotor one by Ui [17] or its extensions to the three-dimensional collective model [18] and further to the symplectic model [19], retain their phenomenological essence until the researcher finds a direct way to derive the values of EH parameters from the constants of nucleon interaction and several basic assumptions on the structure of a nucleus.

To realize an indispensable microscopic description, one makes use of appropriate techniques in the coordinate space. By starting from the standard realistic or effective internucleon potentials, e.g. with Gaussian radial dependence [20,21], it seems natural to construct the microscopic Hamiltonian matrix between the basis states of the irreducible representations (irreps) in the coordinate form. In the case of the symplectic model the approaches mentioned were elaborated by means of both the generator function method [12,22,23] and direct routines. (In practice, as one can note, the latter need a sufficient restriction on the initial Hamiltonian [14] or on the utilized basis volume [24], and so they don't provide a complete solution of the problem stated).

The purpose of the present paper is to bridge the gap between the fully microscopic and algebraic formulation of the collective nuclear models. The passage from one to the other is based on the possibility of restoring the EH from an explicit analytic expression for the microscopic Hamiltonian matrix elements between the coherent states of the corresponding Lie group. This technique was displayed by Filippov and Avramenko in the lowest-order of perturbation theory for an arbitrary SU(3) irrep [25], and by Vasilevsky and Filippov for closed shells in the Sp(6,R) model [26]. Here we consider a general approach to the standard SU(3) model and an almost-magic-nuclei case with the SU(3) configuration mixing in the valence shell. The EHs, thus obtained, are entirely equivalent to their microscopic propotype with respect to the action on the basis functions involved.

Sec.II contains a brief discussion of the algebraic structure of EH, characteristic for the collective nuclear model. In Sec.III, some necessary results for the microscopic Hamiltonian matrix in the SU(3) basis are presented. The technique of restoring the EH is developed in Sec.IV for the symmetric SU(3) irreps and their direct products, and in Sec.V for the arbitrary irreps. In the concluding Section we indicate a feasible way to generalize the present approach to the symplectic collective model and announce the contents of our next paper.

II. PHENOMENOLOGICAL EFFECTIVE HAMILTONIAN

Let us denote the SU(3) algebra in the oscillator second quantization picture [1,3]. In a Cartesian representation, for a A-nucleon translationally invariant system, the creation and anhilation operators are defined as follows:

$$a^+_{nj} = \frac{1}{\sqrt{2}}\,\frac{x_{nj} - i r_0^2 p_{nj}}{r_0}, \quad n=\overline{1,A-1}$$
$$a_{nj} = \frac{1}{\sqrt{2}}\,\frac{x_{nj} + i r_0^2 p_{nj}}{r_0}, \quad j=\overline{1,3}, \tag{1}$$

where the Jacobi coordinates x_{nj} and the momenta p_{nj} obey the Heisenberg-Weyl commutation relations

$$[p_{ni}, x_{mj}] = \delta_{nm}\,\delta_{ij}\,\hbar \tag{2}$$

and $r_0 = \sqrt{\hbar/m\omega}$ is the oscillator radius.

The nine bilinear operators

$$A_{ij} = \frac{1}{2} \sum_n (a^+_{ni} a_{nj} + a_{nj} a^+_{ni}) \tag{3.a}$$

which commute, as one sees from Eqs.(1) and (2), according to

$$[A_{ij}, A_{kl}] = \delta_{jk} A_{il} - \delta_{li} A_{kj}, \tag{3.b}$$

generate an algebra of the U(3) group. By subtracting the diagonal elements

$$B = A - \frac{1}{3} H_0, \tag{4}$$

where E is a three-dimensional unit matrix, and $H_0 = Tr\, A$ is the oscillator Hamiltonian,the SU(3) generators are obtained. (We use the matrix notation with respect to Cartesian indices).

The problem of algebraic formulation of the phenomenological EH has its general solution in integrity basis terms. For the SU(3) enveloping algebra, it was shown by Judd et al. [27] that rotational invariants form the integrity basis as follows

$$G_2,\ G_3,\ \vec{L}^2,\ \Omega = \vec{L}^T Q \vec{L},\ \Omega_J = \vec{L}^T Q Q \vec{L} \tag{5.a}$$

with an auxiliary invariant $[\Omega, \Omega_J]$, where G_2 and G_3 are Casimir operators:

$$G_2 = Tr\, B^2, \tag{5.b}$$

$$G_3 = Tr\, B^3, \tag{5.c}$$

$\vec{L}$ is the orbital angular momentum vector

$$\vec{L} = \begin{pmatrix} L_1 \\ L_2 \\ L_3 \end{pmatrix},\ L_j = -i \sum_{kl} \varepsilon_{jkl} A_{kl} \tag{5.d}$$

and Q is the symmetric quadrupole tensor

$$Q = \frac{1}{2} (B + B^T) \tag{5.e}$$

(The uppex T denotes a transposition).

By choosing the EH as a series in powers of the operators (5.a), it is possible to produce a satisfactory phenomenological description of the low-lying spectrum region of both the light [6,7,28] and heavy nuclei [8,29]. As a rule, the order of EH with respect to the SU(3) generators is taken equal to four. Obviously, the order of the ab initio unitary EH is restricted only by the shell structure of a real nucleus, so for medium-weight and heavy nuclei the above figure can be surpassed. But the high-order coefficients occur to be linked with the low-order ones by a set of rigorous relations. The latter are not defined in the phenomenological scheme.

For the simpler case of an axial nucleus, the EH is constructed on the basis of quadrupole tensor rotational invariants of the second and third order:

$$H_{coll} = H_0 + c_2 \mathrm{Tr}\, Q^2 + c_3 \mathrm{Tr}\, Q^3 + c_4 \mathrm{Tr}^2 Q^2 + \dots \qquad (6)$$

Eq.(6) defines the familiar Bohr-Mottelson-Frankfurt Hamiltonian [30]. Being restricted to a single $(\lambda 0)$ SU(3) irrep (henceforth the Elliott's notation is used), it transforms into a simple EH of the rigid-rotor type. The latter statement is easily confirmed by comparing the identities

$$\mathrm{Tr}\, Q^2 = G_2 - \frac{1}{2} \vec{L}^2, \qquad (7.a)$$

$$\mathrm{Tr}\, Q^3 = G_3 - \frac{3}{2} G_2 + \frac{3}{4} \Omega \qquad (7.b)$$

with Eqs. [31]

$$G_2 \,|\, (\lambda 0) L M 0 \rangle = \frac{2}{3} \lambda(\lambda+3) \,|\, (\lambda 0) L M 0 \rangle, \qquad (8.a)$$

$$\Omega \,|\, (\lambda 0) L M 0 \rangle = -\frac{1}{6} (3+2\lambda) L(L+1) \,|\, (\lambda 0) L M 0 \rangle \qquad (8.b)$$

where $|(\lambda\mu) L M K\rangle$ are the functions of Elliott SU(3) $\supset$ SO(3) basis.

The real spectra of medium-weight and heavy nuclei are essentially more complicate than those predicted by means of conventional collective models utilizing the one-irrep basis of dynamical group. The necessity of taking a mixed representation into account was confirmed in early semimicroscopic SU(3) calculations by Elliott [1] and Arima with collaborators [2,4], who fixed single-particle energies at experimental values. But this empiric substitution, as one can note, has no group-theoretical equivalent. An alternative way, introduced by Otsuka et al. [32] in application to the interacting boson model, was based on the dynamical symmetry extension. The IBM-2, accounting for the distinction between proton and neutron subsystems, was followed by the isospin-conserving version, IBM-3, of Elliott and White [33]. Charge extensions of the Sp(6,R) collective dynamical symmetry to Sp(12,R) by Georgieva et al.[34] and to the inhomogeneous Sp(6,R) group by Quesne[35] were also proposed to incorporate electric dipole excitations. It is clear that, by making use of the extended collective models, one deals with mixed Sp(6,R) irreps. As for the phenomenological EH, such extensions result in an exorbitant increase in the number of adjustable parameters and, therefore, in uncertainty of their fitting.

The approach developed below provides simultaneous determination of the EH algebraic structure and the microscopic values of the parameters. A concrete realization is accomplished for the unitary (intrinsic) submodel of the microscopic symplectic nuclear model with horizontal mixing [36]. The latter assumes that the single-particle subspace is amplified compared with the conventional Sp(6,R) model [11,30], to distinguish among nucleons both with respect to the spin and isospin projection values. Thus, the model discussed operates with various intrashell transitions, or, from the viewpoint of partition labelling, with the horizontal mixing of intrinsic U(3) irreps.

The dynamical algebra of the unitary model with horizontal mixing is settled by means of direct sum $\bigoplus_{(\sigma\tau)} SU^{(\sigma\tau)}(3)$ with $\sigma,\tau \in \{-\frac{1}{2},\frac{1}{2}\}$. Corresponding operators $A_{ij}^{(\sigma\tau)}$ are obtained by analogy to (3.a), if the summation over n is restricted by nucleons with fixed values of σ and τ. One can calculate the integrity basis for the model, generalizing (5) to different subsystems. Naturally, a complete set of the basis operators is too large to be used for unambiguous phenomenological EH.

III. GENERATING KERNEL OF THE NUCLEAR HAMILTONIAN

A starting point of our analysis is to determine a way of restoring EH from the analytic expression of microscopic Hamiltonian between the algebraic model coherent states. Note that the term "coherent state" used henceforth implies not the general form of this object resulting from the action of arbitrary group operator on the lowest-weight state, but a shortened, or vector, coherent state as a generator function for the model basis states [37].

One usually defines the coherent image of a ket $|\psi\rangle$ state as a scalar product

$$\psi(u) = \langle u|\psi\rangle \tag{9.a}$$

with the coherent bra, denoted by a set of generator coordinates u. An operator F, which acts in the space of $|\psi\rangle$ states, is mapped on a corresponding constriction $\Gamma(F)$ in the coherent-state realization:

$$\langle u|F|\psi\rangle = \Gamma(F)\psi(u) \tag{9.b}$$

Obviously, the operator $\Gamma(F)$ is a generator coordinate representation of an effective operator for F. By choosing $|\psi\rangle$ in the form of a coherent ket $|v\rangle$ with an independent set of generator coordinates, and identifying F with the microscopic Hamiltonian H_{micr}, one finds that

$$\langle u|H_{micr}|v\rangle = \mathcal{H}(u)\langle u|v\rangle \tag{10}$$

where the notation $\mathcal{H}(u)$ symbolizes the u-dependence of the EH representation. Eq.(10) points out a way of solving the problem stated.

In the scope of conventional and extended symplectic nuclear models the microscopic Hamiltonian matrix elements between the coherent states were calculated recently for arbitrary representations [36]. Now we present these results restricted to the intrinsic subspace, e.g. to the lowest SU(3) irreps.

As it was noted in the previous Section, the present version of the extended collective model treats the system of A nucleons as divided into four interacting subsystems. Each of the subsystems contains nucleons with a fixed spin-isospin projection $(\sigma\tau)$ and, consequently, is described by one Slater determinant.

Let us apply the Cartesian notation to the occupied single-particle oscillator orbitals: $\vec{n}^{(\sigma\tau)} = \{n_1^{(\sigma\tau)}, n_2^{(\sigma\tau)}, n_3^{(\sigma\tau)}\}$. By summing the numbers of quanta of the occupied orbitals:

$$\sum_{n_i^{(\sigma\tau)}} n_i^{(\sigma\tau)} = f_i^{(\sigma\tau)}, \quad i = \overline{1,3}, \tag{11}$$

one obtains a partition $[\vec{f}^{(\sigma\tau)}]=[f_1^{(\sigma\tau)}, f_2^{(\sigma\tau)}, f_3^{(\sigma\tau)}]$ which labels the U(3) irrep for the $(\sigma\tau)$ subsystem basis states. The corresponding SU(3) irrep is labelled by $(\lambda^{(\sigma\tau)}, \mu^{(\sigma\tau)}) =$ $= (f_1^{(\sigma\tau)}-f_2^{(\sigma\tau)}, f_2^{(\sigma\tau)}-f_3^{(\sigma\tau)})$ Elliott quantum numbers.

Since the SU(3) representation is spanned by rotation of the lowest-weight state [1], one can identify the generator coordinates of the $(\sigma\tau)$ subsystem with the parameters of orthogonal matrix $u^{(\sigma\tau)}$ of the Cartesian axes orientation. If one defines three orthonormal vectors composing the latter matrix as a block $(\vec{u}_1^{(\sigma\tau)}, \vec{u}_2^{(\sigma\tau)}, \vec{u}_3^{(\sigma\tau)})=u^{(\sigma\tau)}$, then the coherent state overlap is calculated by generalized Elliott's equation

$$\langle u|v\rangle = \prod_{(\sigma\tau)} (\vec{u}_1^{(\sigma\tau)}\cdot\vec{v}_1^{(\sigma\tau)})^{\lambda^{(\sigma\tau)}}(\vec{u}_3^{(\sigma\tau)}\cdot\vec{v}_3^{(\sigma\tau)})^{\mu^{(\sigma\tau)}}. \qquad (12)$$

The mean value of kinetic energy between the unitary model basis functions is merely a constant calculated from the many-body oscillator virial theorem:

$$T=\frac{\hbar^2}{2mr_0^2}\left\{\sum_{i=1}^{3}\sum_{(\sigma\tau)} f_i^{(\sigma\tau)}+\frac{3}{2}(A-1)\right\}, \qquad (13)$$

where m is the average nucleon mass.

All non-trivial operator contributions to the EH are originated by the potential energy. For simplicity, we consider only the case of two-body central Wigner internucleon potential of the Gaussian type:

$$U=W_0\sum_{i<j\le A}\exp\left[-\frac{\gamma(\vec{r}_i-\vec{r}_j)^2}{2r_0^2}\right]. \qquad (14)$$

The chosen radial dependence is easily generalized by summing or integrating over γ with an appropriate weight function. The generalization to the spin and isospin-depending potential magnitude also causes no difficulties.

The potential energy operator (14) matrix element between the coherent states used, or, what is the same, the generating kernel for the potential energy matrix has the following representation:

$$\langle u|U|v\rangle = W_0\langle u|v\rangle\cdot$$
$$\cdot\sum_{(\sigma\tau)}\sum_{(\sigma'\tau')}\left[K^{+}_{(\sigma\tau,\sigma'\tau')}(\gamma)-K^{-}_{(\sigma\tau,\sigma'\tau')}(\gamma)\right], \qquad (15.a)$$

where the integrals of direct (+) and exchange (-) interaction are determined by the equations:

$$K^{\pm}_{(\sigma\tau,\sigma'\tau')}(\gamma)=\sum_{\vec{n}^{(\sigma\tau)}}\sum_{\vec{n}^{(\sigma'\tau')}} L_{n_1^{(\sigma\tau)}}\left(-\frac{1}{3}\frac{\partial}{\partial H}-\frac{\partial}{\partial h_\lambda}\right)\times \tag{15.b}$$

$$\times L_{n_2^{(\sigma\tau)}}\left(-\frac{1}{3}\frac{\partial}{\partial H}+\frac{\partial}{\partial h_\lambda}-\frac{\partial}{\partial h_\mu}\right)L_{n_3^{(\sigma\tau)}}\left(-\frac{1}{3}\frac{\partial}{\partial H}+\frac{\partial}{\partial h_\mu}\right)\times$$

$$\times L_{n_1^{(\sigma'\tau')}}\left(-\frac{1}{3}\frac{\partial}{\partial H}-\frac{\partial}{\partial h'_\lambda}\right)L_{n_2^{(\sigma'\tau')}}\left(-\frac{1}{3}\frac{\partial}{\partial H}+\frac{\partial}{\partial h'_\lambda}-\frac{\partial}{\partial h'_\mu}\right)\times$$

$$\times L_{n_3^{(\sigma'\tau')}}\left(-\frac{1}{3}\frac{\partial}{\partial H}+\frac{\partial}{\partial h'_\mu}\right)\det{}^{-1/2}\left(\varkappa^{\pm}(\gamma)\right)\Big|_{\substack{H=1,\\ h_\lambda=h'_\lambda=0,\ h_\mu=h'_\mu=0}},$$

$$\varkappa^{+}(\gamma)=E+\frac{\gamma}{2}\left[R_{(\sigma\tau,\sigma'\tau')}+R^{T}_{(\sigma\tau,\sigma'\tau')}\right], \tag{15.c}$$

$$\varkappa^{-}(\gamma)=R_{(\sigma\tau,\sigma'\tau')}R^{T}_{(\sigma\tau,\sigma'\tau')}+\frac{\gamma}{2}\left[R_{(\sigma\tau,\sigma'\tau')}+R^{T}_{(\sigma\tau,\sigma'\tau')}\right], \tag{15.d}$$

$$R_{(\sigma\tau,\sigma'\tau')}=HE+h_\lambda\left[\frac{\vec{u}_1^{(\sigma\tau)}(\vec{v}_1^{(\sigma\tau)})^T}{(\vec{u}_1^{(\sigma\tau)}\cdot\vec{v}_1^{(\sigma\tau)})}-\frac{1}{3}E\right]+ \tag{15.e}$$

$$+h'_\lambda\left[\frac{\vec{u}_1^{(\sigma'\tau')}(\vec{v}_1^{(\sigma'\tau')})^T}{(\vec{u}_1^{(\sigma'\tau')}\cdot\vec{v}_1^{(\sigma'\tau')})}-\frac{1}{3}E\right]-h_\mu\left[\frac{\vec{v}_3^{(\sigma\tau)}(\vec{u}_3^{(\sigma\tau)})^T}{(\vec{u}_3^{(\sigma\tau)}\cdot\vec{v}_3^{(\sigma\tau)})}-\frac{1}{3}E\right]-$$

$$-h'_\mu\left[\frac{\vec{v}_3^{(\sigma'\tau')}(\vec{u}_3^{(\sigma'\tau')})^T}{(\vec{u}_3^{(\sigma'\tau')}\cdot\vec{v}_3^{(\sigma'\tau')})}-\frac{1}{3}E\right]$$

and $L_n(x)$ are Laguerre polynomials.

The limit of conventional SU(3) model is reached in Eqs.(12), (13) and (15) by eliminating the difference between subsystems with respect to the generator coordinates:

$$\sum_{(\sigma\tau)}\lambda^{(\sigma\tau)}=\lambda,\ \sum_{(\sigma\tau)}\mu^{(\sigma\tau)}=\mu\ ,\ u^{(\sigma\tau)}=u. \tag{16}$$

IV. EFFECTIVE HAMILTONIAN OF ALMOST MAGIC NUCLEI

The simplest case to apply the unitary model is an almost magic nucleon system (up to four nucleons in the valence shell). In the conventional SU(3) model the situation is described by a symmetric $(\lambda 0)$ irrep with a prolate density distribution.

Let us consider the part of the generating kernel (15) which is responsible for the direct interaction. From Eqs.(15) and (16) and the condition for valence nucleon quanta to occupy only the first Cartesian axis one concludes that

$$\langle u|U|v\rangle^{+} = W_0 \, (\vec{u}_1 \vec{v}_1)^{\lambda} \times$$

$$\times \sum_{(\sigma\tau)} \sum_{(\sigma'\tau')} \left\{ L^{3}_{N^{(\sigma\tau)}}\left(-\frac{\partial}{\partial H}\right) + \mathrm{sign}(\lambda^{(\sigma\tau)}) L_{\lambda^{(\sigma\tau)}}\left(-\frac{\partial}{\partial h}\right)\right\} \times \tag{17.a}$$

$$\times \left\{ L^{3}_{N^{(\sigma'\tau')}}\left(-\frac{\partial}{\partial H}\right) + \mathrm{sign}(\lambda^{(\sigma'\tau')}) L_{\lambda^{(\sigma'\tau')}}\left(-\frac{\partial}{\partial h}\right)\right\} \times$$

$$\times \det{}^{-1/2}\left(\varkappa^{+}(\gamma)\right)\Big|_{H=1,\, h=0},$$

$$\varkappa^{+}(\gamma) = (1+\gamma H)E + \frac{1}{2}\gamma h \, \frac{\vec{u}_1 \vec{v}_1^{\,T} + \vec{v}_1 \vec{u}_1^{\,T}}{(\vec{u}_1 \vec{v}_1)}, \tag{17.b}$$

where $N^{(\sigma\tau)}$ is the number of the upper closed shell. Here the summation rules for generalized Laguerre polynomials [38] were used:

$$\sum_{\substack{k_1, k_2, k_3 \geq 0 \\ k_1+k_2+k_3=n}} L_{k_1}(x_1) L_{k_2}(x_2) L_{k_3}(x_3) = L_n^2(x_1+x_2+x_3), \tag{18.a}$$

$$\sum_{n=0}^{N} L_n^{\alpha}(x) = L_N^{\alpha+1}(x). \tag{18.b}$$

Following the logic of Eq.(10), we have to determine an operator which acts on the coherent state overlap $(\vec{u}_1 \vec{v}_1)^{\lambda}$ and yields the right-hand side of Eq.(17.a), and express this operator in terms of the SU(3) enveloping algebra.

It is convenient to introduce an operator-valued matrix

$$\mathcal{A} = \| \mathcal{A}_{ij} \| = \| (\vec{u}_1)_i \left(\frac{\partial}{\partial \vec{u}_1}\right)_j \|, \quad i, j = \overline{1,3}, \tag{19}$$

which coincides with the coherent-state representation of a matrix of the U(3) generators with on accuracy of an additional constant operator. By contracting it with an auxiliary three-dimensional vector $\vec{y}$ to a quadratic form $\vec{y}^T A\vec{y} = (\vec{y}\cdot\vec{u}_1)(\vec{y}\cdot\frac{\partial}{\partial\vec{u}_1})$ and studying its reiterative action on the overlap, one obtains

$$\exp\left(-\frac{\alpha}{\vec{y}^2}\,\vec{y}^T A\vec{y}\right)(\vec{u}_1\vec{v}_1)^\lambda = \tag{20}$$
$$=\left[(\vec{u}_1\vec{v}_1) - \frac{1-e^{-\alpha}}{\vec{y}^2}(\vec{y}\,\vec{u}_1)(\vec{y}\,\vec{v}_1)\right]^\lambda .$$

Eq.(20) is based on the identities

$$\vec{y}^2(\vec{u}_1\vec{v}_1) = (\vec{y}\times\vec{u}_1)(\vec{y}\times\vec{v}_1) + (\vec{y}\cdot\vec{u}_1)(\vec{y}\cdot\vec{v}_1), \tag{21.a}$$

$$(\vec{y}^T A\vec{y})^n(\vec{y}\cdot\vec{u}_1)(\vec{y}\,\vec{v}_1) = (\vec{y}^2)^n(\vec{y}\cdot\vec{u}_1)(\vec{y}\cdot\vec{v}_1), \tag{21.b}$$

$$(\vec{y}^T A\vec{y})^n(\vec{y}\times\vec{u}_1)(\vec{y}\times\vec{v}_1) = \delta_{n0}(\vec{y}\times\vec{u}_1)\cdot(\vec{y}\times\vec{v}_1). \tag{21.c}$$

Simultaneously one makes use of a three-dimensional Poisson integral to write the following expansion

$$\det{}^{-1/2}\left[(1+\gamma H)E + \frac{1}{2}\gamma h\frac{\vec{u}_1\vec{v}_1^T + \vec{v}_1\vec{u}_1^T}{(\vec{u}_1\cdot\vec{v}_1)}\right](\vec{u}_1\vec{v}_1)^\lambda =$$
$$= \frac{1}{\pi^{3/2}}\int d\vec{y}\,\exp[-(1+\gamma H)\vec{y}^2]\sum_{p=0}^{\lambda}\frac{(-\gamma h)^p}{p!}[(\vec{y}\cdot\vec{u}_1)(\vec{y}\cdot\vec{v}_1)]^k(\vec{u}_1\vec{v}_1)^{\lambda-k} \tag{22}$$
$$+ O(h^{\lambda+1}).$$

The higher powers of h are omitted, as they give no contribution to the right-hand side of Eq.(17.a). A substitution of the relation

$$\frac{\lambda!}{(\lambda-p)!}[(\vec{y}\,\vec{u}_1)(\vec{y}\,\vec{v}_1)]^p(\vec{u}_1\vec{v}_1)^{\lambda-p} = \tag{23}$$
$$= \sum_{q=0}^{p}(\vec{y}^2)^p S_p^{(q)}(\vec{y}^T A\vec{y})^q(\vec{u}_1\vec{v}_1)^\lambda,\ p\le\lambda,$$

which is a reversed Eq.(20), into expansion (22), leads to the equality

$$\det{}^{-1/2}\left[(1+\gamma H)E + \frac{1}{2}\gamma h\frac{\vec{u}_1\vec{v}_1^T + \vec{v}_1\cdot\vec{u}_1^T}{(\vec{u}_1\cdot\vec{v}_1)}\right](\vec{u}_1\vec{v}_1)^\lambda =$$
$$= \frac{1}{\lambda!}\sum_{p=0}^{\lambda}\sum_{q=0}^{p}\frac{(\lambda-p)!}{p!}S_p^{(q)}h^p\left(\frac{\partial}{\partial H}\right)^{p-q}\left(\frac{\partial}{\partial\xi}\right)^q \det{}^{-1/2}[(1+\gamma H)E+ \tag{24}$$
$$+\frac{1}{2}\gamma\xi(A+A^T)](\vec{u}_1\cdot\vec{v}_1)^\lambda\Big|_{\xi=0} + O(h^{\lambda+1})$$

where $S_p^{(q)} = \frac{1}{q!}\left(\frac{\partial}{\partial x}\right)^p \ln^q(1+x)$ are Stirling numbers of the first kind [39].

In the investigated limit the exchange integral is also calculated much easier than the general expression (15). A more detailed analysis [36] proves that the result may be obtained from (17.a) by replacing the matrix $\varkappa^+(\gamma)$ with

$$\varkappa^-(\gamma) = \left(1+\frac{\gamma}{H}\right)E - \frac{1}{2}\gamma\frac{h}{H(H+h)}\,\frac{\vec{u}_1\vec{v}_1^{\,T}+\vec{v}_1\vec{u}_1^{\,T}}{(\vec{u}_1\cdot\vec{v}_1)}. \qquad (25)$$

Summing up Eqs.(13) and (17)-(25), we restore the SU(3) EH on an $(\lambda 0)$ irrep:

$$\begin{aligned}\mathcal{H} = {} & \frac{\hbar^2}{2mr_0^2}\left\{\sum_{(\sigma\tau)}\left[\frac{(N^{(\sigma\tau)}+3)!}{(N^{(\sigma\tau)}-1)!\,3!}+\lambda^{(\sigma\tau)}\right]+\frac{3}{2}(A-1)\right\}+ \\ & + W_0\sum_{(\sigma\tau)}\sum_{(\sigma'\tau')}\left\{L^3_{N^{(\sigma\tau)}}\left(-\frac{\partial}{\partial H}\right)+\operatorname{sign}(\lambda^{(\sigma\tau)})L_{\lambda^{(\sigma\tau)}}\left(-\frac{\partial}{\partial h}\right)\right\}\times \qquad (26) \\ & \times\left\{L^3_{N^{(\sigma'\tau')}}\left(-\frac{\partial}{\partial H}\right)+\operatorname{sign}(\lambda^{(\sigma'\tau')})L_{\lambda^{(\sigma'\tau')}}\left(-\frac{\partial}{\partial h}\right)\right\}\times \\ & \times\frac{1}{\lambda!}\sum_{p=0}^{\lambda}\sum_{q=0}^{p}\frac{(\lambda-p)!}{p!}S_p^{(q)}\left(\frac{\partial}{\partial\xi}\right)^q\left\{h^p\left(\frac{\partial}{\partial H}\right)^{p-q}\det{}^{-1/2}\left[(1+\gamma H+\right.\right. \\ & \left.+\frac{1}{3}\gamma\lambda\xi)E+\gamma\xi Q\right]+\frac{(-1)^q}{H^2(H+h)}\left(\frac{h}{H(H+h)}\right)^p\left(H^2\frac{\partial}{\partial H}\right)^{p-q}\det{}^{-1/2}\left[\left(1+\frac{\gamma}{H}+\right.\right. \\ & \left.\left.\left.+\frac{1}{3}\gamma\lambda\xi\right)E+\gamma\xi Q\right]\right\}\Big|_{H=1,h=0,\xi=0},\end{aligned}$$

where the traceless part of $\frac{1}{2}(A+A^T)$ is identified with the quadrupole tensor operator.

To expose the structure of EH (26) in terms of scalar operators, one has to determine the meaning of operator-valued symbol det $(xE+yQ)$ with real (or complex) parameters X and y. It is expedient to make use of *Newton formula* expressing the characteristic polynomial of a matrix through the trace of its powers. In the case of a three-dimensional symmetric traceless matrix, one finds

$$\det(xE+yQ) = x^3 - \frac{1}{2}xy^2\,\mathrm{Tr}\,Q^2 + \frac{1}{3}y^3\,\mathrm{Tr}\,Q^3. \qquad (27)$$

Eqs.(26) and (27) show that the EH on a symmetric SU(3) irrep occurs to be a generalization of the Bohr-Mottelson-Frankfurt axial Hamiltonian (6) to higher powers of the SU(3) generators. So, within the conventional SU(3) model, the almost magic nucleus, composed of particles interacting via central forces (14), behaves like a spherical rotor with a nonlinear Hamiltonian [25].

The same nucleon system in the framework of a unitary model with hirizontal mixing needs a more detailed consideration. Here we divide the interaction operator (14) into a sum of three parts, namely, the interaction of closed shells, first, between themselves, then, with valence nucleons, and, finally, the interaction of valence nucleons between themselves,

$$\mathcal{U} = \mathcal{U}_{shell} + \mathcal{U}_{coupling} + \mathcal{U}_{extra} . \tag{28}$$

The "pure" contribution of closed shells coincides with the same term of the SU(3) model and turns out to be a constant. One selects this term from Eq.(26) by annihilating the parameter h .

The coupling term may also be calculated on the basis of Eq.(26). Since the difference between the conventional and the extended model results from the Cartesian axes orientation, the corresponding effective operator is written as follows:

$$\begin{aligned}\mathcal{U}_{coupling} = W_0 \sum_{(\sigma\tau)} \sum_{(\sigma'\tau')} \Big\langle & L^3_{N^{(\sigma\tau)}}\left(-\frac{\partial}{\partial H}\right) \mathrm{sign}\,(\lambda^{(\sigma'\tau')}) \times \\ \times\, & L_{\lambda^{(\sigma'\tau')}}\left(-\frac{\partial}{\partial h}\right) \sum_{p=0}^{\lambda^{(\sigma'\tau')}} \frac{(\lambda^{(\sigma'\tau')}-p)!}{\lambda^{(\sigma'\tau')}!\,p!}\, S_p^{(q)}\left(\frac{\partial}{\partial \xi}\right)^q \Big\{ h^p \left(\frac{\partial}{\partial H}\right)^{p-q} \times \\ \times\, & \det{}^{-1/2}\left[\left(1+\gamma H+\tfrac{1}{3}\gamma\lambda^{(\sigma'\tau')}\xi\right)E+\gamma\xi Q^{(\sigma'\tau')}\right] + (-1)^q \left(\frac{h}{H(H+h)}\right)^p \times \\ \times\, & \left(H^2\frac{\partial}{\partial H}\right)^{p-q} \det{}^{-1/2}\left[\left(1+\frac{\gamma}{H}+\tfrac{1}{3}\gamma\lambda^{(\sigma'\tau')}\xi\right)E+\gamma\xi Q^{(\sigma'\tau')}\right] \Big\} + \\ + & (\sigma\tau)\leftrightarrow(\sigma'\tau') \Big\rangle \Big|_{H=1,\,h=0,\,\xi=0} ,\end{aligned} \tag{29}$$

where $Q^{(\sigma\tau)}$ are the partial quadrupole tensors.

Contrary to the coupling term within the SU(3) model, the operator (29) is a polynomial of the squared partial orbital momenta, and hence, it mixes the SU(3) irreps. The presence of mixing terms in the EH is necessary to describe not only the rotational type of nuclear spectra, which is characteristic for the conventional model, but also the intermediate rotor-vibrator type [9,10,30].

The effective direct interaction between valence nucleons, as one can see from Eqs.(15) and (23) (the latter must be generalized to distinguish between subsystems), gets the form

$$\mathcal{U}^{+}_{extra} = W_0 \sum_{(\sigma\tau)} \sum_{(\sigma'\tau')} \mathrm{sign}(\lambda^{(\sigma\tau)})\, \mathrm{sign}(\lambda^{(\sigma'\tau')})\, \varphi_{(\sigma\tau,\sigma'\tau')},$$

$$\varphi_{(\sigma\tau,\sigma'\tau')} = \sum_{p=0}^{\lambda^{(\sigma\tau)}} \sum_{q=0}^{p} \sum_{p'=0}^{\lambda^{(\sigma'\tau')}} \sum_{q'=0}^{p'} \frac{1}{(p!p'!)^2} S_p^{(q)} S_{p'}^{(q')} \times \tag{30.a}$$

$$\times \left(\frac{\partial}{\partial H}\right)^{p+p'-q-q'} \left(\frac{\partial}{\partial \xi}\right)^{q} \left(\frac{\partial}{\partial \xi'}\right)^{q'} \det{}^{-1/2}\Big[\big(1+\gamma H + \tfrac{1}{3}\gamma\lambda^{(\sigma\tau)}\xi +$$

$$+ \tfrac{1}{3}\gamma\lambda^{(\sigma'\tau')}\xi'\big)E + \gamma\xi Q^{(\sigma\tau)} + \gamma\xi' Q^{(\sigma'\tau')}\Big]\Big|_{H=1,\, h=h'=0,\, \xi=\xi'=0.} \tag{30.b}$$

where we used the identity

$$L_\lambda\left(-\frac{\partial}{\partial h}\right) h^p \Big|_{h=0} = \frac{\lambda!}{p!(\lambda-p)!}.$$

The operator-valued determinant, given in (30.b), has an expansion in terms of partial generators:

$$\det(xE+yQ+y'Q') = x^3 - \tfrac{1}{2}x\Big[y^2\big(G_2-\tfrac{1}{2}\vec{L}^2\big) + 2yy'\big(\mathrm{Tr}\,BB' -$$

$$- \tfrac{1}{2}(\vec{L}\cdot\vec{L}')\big) + (y')^2\big(G_2' - \tfrac{1}{2}\vec{L}'^2\big) + \tfrac{1}{3}\Big[y^3\big(G_3 - \tfrac{3}{2}G_2 + \tfrac{3}{4}\vec{L}^T Q\vec{L}\big) + \tag{31}$$

$$+ 3y^2y'\big(\mathrm{Tr}\,BB'B + \tfrac{3}{2}\mathrm{Tr}\,BB' + \tfrac{1}{2}\vec{L}^T Q\vec{L}' + \tfrac{1}{4}\vec{L}^T Q'\vec{L}\big) + 3y(y')^2 \times$$

$$\times \big(\mathrm{Tr}\,B'BB' + \tfrac{3}{2}\mathrm{Tr}\,BB' + \tfrac{1}{2}\vec{L}'^T Q'\vec{L} + \tfrac{1}{4}\vec{L}'^T Q\vec{L}'\big) + y^3\big(G_3' - \tfrac{3}{2}G_2' +$$

$$+ \tfrac{3}{4}\vec{L}'^T Q'\vec{L}'\big)\Big]\Big].$$

By comparing Eqs.(15.c) and (15.d), one realizes that the derivation of effective exchange interaction needs, generally speaking, some other technique. However, at the beginning of p-, sd-, and pf- shells, when the values of $N^{(\sigma\tau)}$ coincide for all the sybsystems and the values of $\lambda^{(\sigma\tau)}$ equal either N+1 or zero, this contribution is expressed via the previous operators:

$$\mathcal{U}^{-}_{extra} = W_0 \sum_{(\sigma\tau)} \sum_{(\sigma'\tau')} \mathrm{sign}(\lambda^{(\sigma\tau)})\, \mathrm{sign}(\lambda^{(\sigma'\tau')})\, \varphi_{(\sigma\tau,\sigma'\tau')}\, P_{(\sigma\tau,\sigma'\tau')} \tag{32.a}$$

where $P_{(\sigma\tau,\sigma'\tau')}$ are the permutation operators for the subsystems,

$$P_{(\sigma\tau,\sigma'\tau')} \left(\vec{u}_1^{(\sigma\tau)} \cdot \vec{v}_1^{(\sigma\tau)}\right)^{N+1} \left(\vec{u}_1^{(\sigma'\tau')} \cdot \vec{v}_1^{(\sigma'\tau')}\right)^{N+1} =$$

$$= \left(\vec{u}_1^{(\sigma'\tau')} \cdot \vec{v}_1^{(\sigma\tau)}\right)^{N+1} \left(\vec{u}_1^{(\sigma\tau)} \cdot \vec{v}_1^{(\sigma'\tau')}\right)^{N+1} \tag{32.b}$$

The operators $P_{(\sigma\tau,\sigma'\tau')}$ are invariant with respect to the transformations of an SU(3) subgroup of a direct product $SU^{(\sigma\tau)}(3) \otimes SU^{(\sigma'\tau')}(3)$ group, and therefore they can be expressed through the Casimir operators of the subgroup indicated. Instead of treating these permutations as SU(3)-symmetric ones, they are classified according to the S(2) group of permutations between the subsystems. So the operators $P_{(\sigma\tau,\sigma'\tau')}$ may take only two eigenvalues, 1 or -1 .

In contrast to the SU(3) EH (26) of an almost magic nucleon system composed only of the powers of the SO(3) Casimir operator, the extended model EH (28)-(32) involves both the SU(3) integrity basis (5.a) (except Ω_J , the Judd operator) and a set of SU(3)-mixing operators. One may interpret the extended model with the $\underset{(\sigma\tau)}{\otimes} SU^{(\sigma\tau)}(3)$-dynamical symmetry as a submodel of the SU(6) $\otimes$ SU(6) IBM-2 [32] and, thus, use the above formulae for the microscopic calculation of the EH parameters. However, for strongly deformed nuclei one cannot expect a good agreement with the phenomenological data as a result of the renormalization effect caused by collective excitations [40].

V. EFFECTIVE HAMILTONIAN FOR ARBITRARY REPRESENTATION

In the present section we generalize the approach of sec.IV to an arbitrary $(\lambda\mu)$ irrep of the SU(3) group, compatible with the one-determinant structure of its coherent state.

The generating kernel for the direct interaction matrix is derived by simplifying Eq.(15):

$$\langle u|U|v\rangle^{+} = W_0 (\vec{u}_1 \vec{v}_1)^{\lambda} (\vec{u}_3\cdot\vec{v}_3)^{\mu} \times$$

$$\times \sum_{(\sigma\tau)} \sum_{(\sigma'\tau')} \mathcal{L}_{\vec{n}(\sigma\tau)}\left(-\frac{\partial}{\partial H}, -\frac{\partial}{\partial h_\lambda}, -\frac{\partial}{\partial h_\mu}\right) \mathcal{L}_{\vec{n}(\sigma'\tau')}\left(-\frac{\partial}{\partial H}, -\frac{\partial}{\partial h_\lambda}, -\frac{\partial}{\partial h_\mu}\right) \times \tag{33.a}$$

$$\times \det{}^{1/2}\Big[(1+\gamma H - \tfrac{1}{3}\gamma h_\lambda + \tfrac{1}{3}\gamma h_\mu)E + \tfrac{1}{2}\gamma h_\lambda \frac{\vec{u}_1\vec{v}_1^T + \vec{v}_1\vec{u}_1^T}{(\vec{u}_1\cdot\vec{v}_1)} -$$

$$- \tfrac{1}{2}\gamma h_\mu \frac{\vec{v}_3\vec{u}_3^T + \vec{u}_3\vec{v}_3^T}{(\vec{u}_3\cdot\vec{v}_3)}\Big]\Big|_{H=1,\, h_\lambda = h_\mu = 0} ,$$

$$\mathcal{L}_{\vec{n}(\sigma\tau)}\left(-\frac{\partial}{\partial H}, -\frac{\partial}{\partial h_\lambda}, -\frac{\partial}{\partial h_\mu}\right) = L_{n_1(\sigma\tau)}\left(-\frac{1}{3}\frac{\partial}{\partial H} - \frac{\partial}{\partial h_\mu}\right) \times \tag{33.b}$$

$$\times L_{n_2(\sigma\tau)}\left(-\frac{1}{3}\frac{\partial}{\partial H} + \frac{\partial}{\partial h_\lambda} - \frac{\partial}{\partial h_\mu}\right) L_{n_3(\sigma\tau)}\left(-\frac{1}{3}\frac{\partial}{\partial H} + \frac{\partial}{\partial h_\mu}\right).$$

By analogy to Eqs.(20) and (22), one obtains a series

$$\det{}^{-1/2}\Big[(1+\gamma H - \tfrac{1}{3}\gamma h_\lambda + \tfrac{1}{3}\gamma h_\mu)E + \tfrac{1}{2}\gamma h_\lambda \frac{\vec{u}_1\vec{v}_1^T + \vec{v}_1\vec{u}_1^T}{(\vec{u}_1\vec{v}_1)} -$$

$$- \tfrac{1}{2}\gamma h_\mu \frac{\vec{v}_3\vec{u}_3^T + \vec{u}_3\vec{v}_3^T}{(\vec{u}_3\cdot\vec{v}_3)}\Big] (\vec{u}_1\cdot\vec{v}_1)^{\lambda} (\vec{u}_3\cdot\vec{v}_3)^{\mu} = \int \frac{1}{\pi^{3/2}} d\vec{y}\, \exp\Big[-(1+ \tag{34.a}$$

$$+\gamma H + \tfrac{1}{3}\gamma h_\mu - \tfrac{1}{3}\gamma h_\lambda)\vec{y}^{\,2}\Big] \sum_{\ell=0}^{\lambda} \sum_{m=0}^{\mu} \frac{(\lambda-\ell)!}{\lambda!} \frac{(\mu-m)!}{m!} \frac{(-\gamma h_\lambda \vec{y}^{\,2})^{\ell}}{\ell!} \frac{(\gamma h_\mu \vec{y}^{\,2})^{m}}{m!} \times$$

$$\times \left(\frac{\partial}{\partial x}\right)^{\ell} \left(\frac{\partial}{\partial z}\right)^{m} \exp\left\{\frac{1}{\vec{y}^{\,2}} \ln(1+x)\, \vec{y}^{\,T}(A_\lambda - A_\mu)\vec{y}\right\} \exp\Big\{\frac{1}{\vec{y}^{\,2}} \ln[(1+x)(1+z)] \cdot$$

$$\cdot \vec{y}^{\,T} A_\mu \vec{y}\Big\} (\vec{u}_1\cdot\vec{v}_1)^{\lambda} (\vec{u}_3\cdot\vec{v}_3)^{\mu}\Big|_{x=z=0} + o(h_\lambda{}^{\lambda}, h_\mu{}^{\mu}),$$

where

$$A_\lambda = \|(A_\lambda)_{ij}\| = \|(\vec{u}_1)_i \left(\frac{\partial}{\partial \vec{u}_1}\right)_j\| , \tag{34.b}$$

$$A_\mu = \|(A_\mu)_{ij}\| = \|(\vec{u}_3)_j \left(\frac{\partial}{\partial \vec{u}_3}\right)_i\| ,\ i,j = \overline{1,3} \tag{34.c}$$

are partial operator-valued matrices such as (19). The matrix of the $SU(3)$ generators equals

$$A = A_\lambda + E\,\mathrm{Tr}\, A_\mu - A_\mu \tag{35}$$

with an accuracy of an additional constant; within the $(\lambda\mu)$ irrep, one may replace $\mathrm{Tr}\, A_\mu$ with its eigenvalue μ. The

definition (35) is obtained by excluding the dependent vector $\vec{u}_2$ from the sum over three Cartesian axes.

Since the EH must be constructed on the SU(3) enveloping algebra, the factor on the right-hand side of Eq.(34.a), depending on the partial matrix A_μ , is to be replaced by an equivalent term, depending on the matrix A . According to Eq.(A.13) from the Appendix, an appropriate equivalence relation has the form

$$\exp\left\{\frac{1}{\bar{y}^2}\ln[(1+x)(1+z)]\,\vec{y}^T A_\mu \vec{y}\right\}(\vec{u}_1\cdot\vec{v}_1)^\lambda(\vec{u}_3\cdot\vec{v}_3)^\mu =$$

$$=\sum_{k=0}^{\lambda+\mu+1}(x+z+xz)^k \mathcal{F}_k\left(\frac{1}{\bar{y}^2}\vec{y}^T(A^2-\mu(\mu+1)E)\vec{y},\frac{1}{\bar{y}^2}\vec{y}(A-\mu E)\vec{y},1\right)\times \tag{36.a}$$

$$\times(\vec{u}_1\cdot\vec{v}_1)^\lambda(\vec{u}_3\cdot\vec{v}_3)^\mu+O\left((x+z)^{\lambda+\mu+2}\ln(x+z)\right),$$

where

$$\mathcal{F}_k(\omega,\varrho,\chi)=\begin{cases}1, & k=0,\\ \prod_{s=0}^{k-1}\dfrac{\omega+(s-\mu-1)\varrho+s(\lambda+\mu+1+s)\chi}{(s+1)(\lambda+\mu+1\ \ s)}, & k>0.\end{cases} \tag{36.b}$$

By substituting (36) in (34) and utilizing the identity

$$\left(\frac{\partial}{\partial x}\right)^\ell\left(\frac{\partial}{\partial z}\right)^m\exp[\varrho\ln(1+x)]\sum_{k\geq 0}(x+z+xz)^k\mathcal{F}_k(\omega,\varrho,1)\Big|_{x=z=0}=$$

$$=\sum_{n=0}^{\ell}\sum_{k=0}^{\ell-n}\frac{(m+k)!\,\ell!}{k!\,(\ell-k)!}S_{\ell-k}^{(n)}(\varrho+m)^n\mathcal{F}_k(\omega,\varrho,1),$$

one derives that

$$\det{}^{-1/2}\Big[\left(1+\gamma H-\tfrac{1}{3}\gamma h_\lambda+\tfrac{1}{3}\gamma h_\mu\right)E+\tfrac{1}{2}\gamma h_\lambda\frac{\vec{u}_1\vec{v}_1^T+\vec{v}_1\vec{u}_1^T}{(\vec{u}_1\vec{v}_1)}-$$

$$-\tfrac{1}{2}\gamma h_\mu\frac{\vec{v}_3\vec{u}_3^T+\vec{u}_3\vec{v}_3^T}{(\vec{u}_3\cdot\vec{v}_3)}\Big](\vec{u}_1\cdot\vec{v}_1)^\lambda(\vec{u}_3\cdot\vec{v}_3)^\mu=\sum_{\ell=0}^{\lambda}\sum_{m=0}^{\mu}\frac{(\lambda-\ell)!}{\lambda!}\frac{(\mu-m)!}{\mu!}\times$$

$$\times\frac{h_\lambda^\ell}{\ell!}\cdot\frac{(-h_\mu)^m}{m!}\exp\left[\tfrac{1}{3}(h_\lambda-h_\mu)\frac{\partial}{\partial H}\right]\sum_{n=0}^{\ell}\sum_{k=0}^{\ell-n}\frac{(m+k)!\,\ell!}{k!\,(\ell-k)!}S_{\ell-k}^{(n)}\times \tag{37}$$

$$\times\left(\frac{\partial}{\partial H}\right)^{\ell+m-n-k}\left(\frac{\partial}{\partial\xi}+\frac{\partial}{\partial H}\right)^n\mathcal{F}_k\left(\frac{\partial}{\partial\eta},\frac{\partial}{\partial\xi},\frac{\partial}{\partial H}\right).$$

$$\times \det{}^{1/2}\Big[(1+\gamma H)E+\delta\xi\Big(\frac{A+A^{T}}{2}-\mu E\Big)+\gamma_{\eta}\Big(\frac{A^{2}+(A^{T})^{2}}{2}-\mu(\mu+1)E\Big)\Big]\times$$

$$\times(\vec{u}_1\vec{v}_1)^{\lambda}(\vec{u}_3\vec{v}_3)^{\mu}\Big|_{\xi=\eta=0}+o(h_{\lambda}^{\lambda},h_{\mu}^{\mu}).$$

A substitution of Eq.(37) into (33) determines the result

$$\mathcal{U}^{+}=W_0\sum_{(\sigma\tau)}\sum_{(\sigma'\tau')}\mathcal{L}_{\vec{n}(\sigma\tau)}\Big(-\frac{\partial}{\partial H},-\frac{\partial}{\partial h_{\lambda}},-\frac{\partial}{\partial h_{\mu}}\Big)\times$$

$$\times\mathcal{L}_{\vec{n}'(\sigma'\tau')}\Big(-\frac{\partial}{\partial H},-\frac{\partial}{\partial h_{\lambda}},-\frac{\partial}{\partial h_{\mu}}\Big)\sum_{l=0}^{\lambda}\sum_{m=0}^{\mu}\frac{(\lambda-l)!}{\lambda!}\frac{(\mu-m)!}{\mu!}\frac{h_{\lambda}^{l}}{l!}\frac{(-h_{\mu})^{m}}{m!}\times$$

$$\times\exp\Big[\frac{1}{3}(h_{\lambda}-h_{\mu})\frac{\partial}{\partial H}\Big]\sum_{n=0}^{l}\sum_{k=0}^{l-n}\frac{(m+k)!\,l!}{k!\,(l-k)!}\,S_{l-k}^{(n)}\Big(\frac{\partial}{\partial H}\Big)^{l+m-n-k}\times$$

$$\times\Big(\frac{\partial}{\partial\xi}+\frac{\partial}{\partial H}\Big)^{n}\mathcal{F}_{k}\Big(\frac{\partial}{\partial\eta},\frac{\partial}{\partial\xi},\frac{\partial}{\partial H}\Big)\det{}^{-1/2}\Big\{\Big[1+\gamma H+\frac{1}{3}\gamma(\lambda-\mu)\xi+ \tag{38}$$

$$+\frac{1}{3}\gamma(\lambda^{2}-\mu^{2}+2\lambda\mu+2\lambda-\mu)\eta\Big]E+\gamma\Big[\xi+\frac{2}{3}(\lambda+2\mu)\eta\Big]Q+$$

$$+\gamma_{\eta}\Big[\frac{B^{2}+(B^{2})^{T}}{2}-\frac{1}{3}G_{2}E\Big]\Big\}\Big|_{H=1,\,h_{\lambda}=h_{\mu}=0,\,\xi=\eta=0}.$$

Within the operator-valued matrix, under the determinant sign we selected a traceless part, taking into account that the eigenvalue of the Casimir operator G_2 on the $(\lambda\mu)$ irrep equals

$$g_2(\lambda\mu)=\frac{2}{3}(\lambda^{2}+\mu^{2}+\lambda\mu+3\lambda+3\mu).$$

Note that the indicated determinant has the sixth order in the SU(3) generators, therefore it contains the whole SU(3)$\supset$ SO(3) integrity basis (5.a) with the auxiliary commutator $[\Omega,\Omega_J]$ of Bargmann-Moshinsky and Judd operators. However, we guess that the usage of a simple operator-determinant expansion like (27) without the transfer to integrity basis is more preferable for the subsequent multiparameter differentiation.

An effective operator for the exchange interaction, as one can see from Eq.(15), is obtained with the help of the following replacement after the summation sign over l and m on the right-hand side of Eq.(38):

$$H \to H' = \frac{1}{3}\left[\left(H + \frac{2}{3}h_\lambda + \frac{1}{3}h_\mu\right)^{-1} + \left(H - \frac{1}{3}h_\lambda + \frac{1}{3}h_\mu\right)^{-1} + \left(H - \frac{1}{3}h_\lambda - \frac{2}{3}h_\mu\right)^{-1} \right],$$
$$h_\lambda \to h_\lambda' = -h_\lambda\left[\left(H + \frac{2}{3}h_\lambda + \frac{1}{3}h_\mu\right)\left(H - \frac{1}{3}h_\lambda + \frac{1}{3}h_\mu\right)\right]^{-1},$$
$$h_\mu \to h_\mu' = -h_\mu\left[\left(H - \frac{1}{3}h_\lambda - \frac{2}{3}h_\mu\right)\left(H - \frac{1}{3}h_\lambda + \frac{1}{3}h_\mu\right)\right]^{-1}. \tag{39}$$

VI. CONCLUSION

Starting from the analytical expressions for the matrix elements of the microscopic nuclear Hamiltonian with two-body central interaction between the coherent states of both the conventional and extended unitary models, we have restored the equivalent *EH* and determined the microscopic values of its parameters. The obtained representations of the EH contain the dependence on the $SU(3)$ generators in the form of an operator-valued factor. The latter occurs to be an inversed square root of a determinant of the matrix, whose elements are the linear functions of the generators, if the axial almost magic nucleus is described, and the quadratic functions for an arbitrary non-axial nucleus. The determinant form of the operator-valued factor allows one to analyze the structure of the basis of invariant operators by means of standard linear algebra methods.

Similarly to the phenomenological EHs, the Hamiltonians, restored from the microscopic unitary model matrix elements, also have a finite order in the Lie algebra generators. However, in the case investigated this restriction is generated by the internal property of the model, namely, the finite volume of the shell-model basis. To remove the restriction, one can generalize the present approach to a symplectic nuclear model. The necessary matrix elements on the infinite basis of separate and mixed Sp(6,R) irreps are previously calculated [36]. The corresponding coherent-state representations of the Sp(6,R) generators are also discussed by different authors [26,30,37].

The result of Sec.IV, dealing with an axial nucleus within the extended unitary model, is applicable for studying the low-lying spectral region. A diagonalization of the axial EH with semirealistic internucleon forces on the basis of three irreps of the SU(3) group will be utilized to calculate the spectrum of sd-shell nuclei with two valence nucleons in our next paper.

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APPENDIX: REITERATED ACTION OF OPERATOR-VALUED QUADRATIC FORMS ON AN SU(3) REPRESENTATION BASIS

As shown in sec.V, the independent occupation of the first and third Cartesian axes of the proper coordinate system of a non-axial nucleus by oscillator quanta causes a splitting of the SU(3) generators (see Eqs.(34) and (35)). But the microscopic physical operators and, consequently the effective operators are, by all means, free of that dependence and may consist only of full unsplitted Lie algebra generators.

Linear operations with the operator-valued matrices $\mathcal{A}_\lambda$ and $\mathcal{A}_\mu$ yield the only unsplitted matrix, namely, $\mathcal{A}=\mathcal{A}_\lambda+\mu E-\mathcal{A}_\mu$. Therefore, we have to choose a nonlinear equivalent transformation, replacing the "extra" matrix, say, $\mathcal{A}_\mu$. For example, by making use of definitions (34.b) and (34.c), one gets the relation

$$[\mathcal{A}^2-\mu(\mu+1)E](\vec{u}_1\cdot\vec{v}_1)^\lambda(\vec{u}_3\cdot\vec{v}_3)^\mu=[(\lambda+2\mu+2)\mathcal{A}_\lambda-\\-(\mu+1)\mathcal{A}_\mu](\vec{u}_1\cdot\vec{v}_1)^\lambda(\vec{u}_3\cdot\vec{v}_3)^\mu. \tag{A1}$$

In contrast to the identity (35), the equality (A1) is justified only in projections on the SU(3) coherent state overlap. To study the reiterated action of partial operators $\mathcal{A}_\lambda$ and $\mathcal{A}_\mu$, one has to develop a general algorithm of the equivalent replacement.

Following the line of sec.IV and V, we work with operator-valued quadratic forms

$$\omega=\vec{e}^{\,T}[\mathcal{A}^2-\mu(\mu+1)E]\vec{e},\ \vec{\nu}=\tfrac{1}{2}\vec{e}^{\,T}(\mathcal{A}-\mu E)\vec{e},$$

$$\varkappa=\tfrac{1}{2}\vec{e}^{\,T}(\mathcal{A}_\lambda+\mathcal{A}_\mu)\vec{e}, \tag{A2.a}$$

obeying the commutation rules

$$[\omega,\nu]=0,\ [\nu,\varkappa]=0,\ [\omega,\varkappa]\neq 0, \tag{A2.b}$$

where $\vec{e}$ is the normalized three-dimensional vector obtained from an arbitrary vector $\vec{y}$,

$$\vec{e} = \frac{\vec{y}}{\sqrt{\vec{y}^2}}. \tag{A2.c}$$

The action of operators (A2.a) is defined on the functional space with $(\lambda+1)(\mu+1)$ basis functions, e.g.

$$(\vec{u}_1\cdot\vec{v}_1)^{\lambda-\ell}[(\vec{e}\,\vec{u}_1)(\vec{e}\cdot\vec{v}_1)]^{\ell}(\vec{u}_3\cdot\vec{v}_3)^{\mu-m}[(\vec{e}\cdot\vec{u}_3)(\vec{e}\cdot\vec{v}_3)]^{m}, \quad \ell=\overline{0,\lambda}\,,\quad m=\overline{0,\mu}\,. \tag{A3}$$

The latter statement is proved by a direct construction:

$$\begin{aligned}&\omega(\vec{u}_1\vec{v}_1)^{\lambda-\ell}[(\vec{e}\cdot\vec{u}_1)(\vec{e}\cdot\vec{v}_1)]^{\ell}(\vec{u}_3\cdot\vec{v}_3)^{\mu-m}[(\vec{e}\cdot\vec{u}_3)(\vec{e}\cdot\vec{v}_3)]^{m}=\\ &=[(\lambda+2\mu+2-m)(\nu+\varkappa)+(\mu+1+\ell)(\nu-\varkappa)+\ell m]\times\\ &\times(\vec{u}_1\cdot\vec{v}_1)^{\lambda-\ell}[(\vec{e}\,\vec{u}_1)(\vec{e}\cdot\vec{v}_1)]^{\ell}(\vec{u}_3\cdot\vec{v}_3)^{\mu-m}[(\vec{e}\cdot\vec{u}_3)(\vec{e}\cdot\vec{v}_3)]^{m}.\end{aligned} \tag{A4}$$

Note that the functional space mentioned is spanned by a generator function

$$\begin{aligned}&\exp[-x(\nu+\varkappa)-z(\nu-\varkappa)](\vec{u}_1\cdot\vec{v}_1)^{\lambda}(\vec{u}_3\cdot\vec{v}_3)^{\mu}=\\ &=[(\vec{u}_1\cdot\vec{v}_1)+(e^{-x}-1)(\vec{e}\cdot\vec{u}_1)(\vec{e}\cdot\vec{v})]^{\lambda}[(u_3\cdot v_3)+(e^{z}-1)(\vec{e}\cdot\vec{u}_3)(\vec{e}\cdot\vec{v}_3)]^{\mu}.\end{aligned} \tag{A5}$$

As is seen, Eq.(A5) generalizes Eq.(20). By using Eqs.(A4) and (A5), one considers the action of operator ω on the generator function

$$\begin{aligned}&\omega\exp[-x(\nu+\varkappa)-z(\nu-\varkappa)](\vec{u}_1\vec{v}_1)^{\lambda}(\vec{u}_3\cdot\vec{v}_3)^{\mu}=\\ &=\Big\{(\lambda+3\mu+3)\nu+(\lambda+\mu+1)\varkappa-(\nu+\varkappa)(1-e^{-z})\frac{\partial}{\partial z}+\\ &+(\nu-\varkappa)(e^{x}-1)\frac{\partial}{\partial x}+(e^{x}-1)(1-e^{-z})\frac{\partial^2}{\partial x\partial z}\Big\}\times\\ &\times[(\vec{u}_1\vec{v}_1)+(e^{-x}-1)(\vec{e}\cdot\vec{u}_1)(\vec{e}\cdot\vec{v}_1)]^{\lambda}[(\vec{u}_3\cdot\vec{v}_3)+(e^{z}-1)(\vec{e}\cdot\vec{u}_3)(\vec{e}\cdot\vec{v}_3)]^{\mu}.\end{aligned} \tag{A6}$$

The right-hand side of Eq.(A6) is written in a mixed representation of the generator coordinates x and z and the commuting operators ν and $\varkappa$. Returning to the operator representation of the generator function, one derives the same representation for ω:

$$\omega=(\lambda+3\mu+3)\nu+(\lambda+\mu+1)\varkappa+(\nu^2-\varkappa^2)\left(1-e^{-\partial/\partial\varkappa}\right). \tag{A7}$$

Now we must inverse Eq.(A7), to obtain a formula of the reiterated action of operator $\varkappa$ in terms of operators ω and ν. For this purpose, let us compare two different expressions defining the action of a commutator $[\omega, e^{-\beta\varkappa}]$ with arbitrary parameter β on the coherent state overlap. Firstly, by making use of Eq.(A7), we conclude that

$$[\omega, e^{-\beta\varkappa}] = (e^{\beta}-1)(\varkappa^2-\nu^2)e^{-\beta\varkappa}e^{-\partial/\partial\varkappa},$$

and, therefore,

$$[\omega, e^{-\beta\varkappa}](\vec{u}_1\cdot\vec{v}_1)^{\lambda}(\vec{u}_3\cdot\vec{v}_3)^{\mu} = (e^{\beta}-1)\left(\frac{\partial^2}{\partial\beta^2}-\nu^2\right)e^{-\beta\varkappa}\times \quad \text{(A8)}$$
$$\times(\vec{u}_1\cdot\vec{v}_1)^{\lambda}(\vec{u}_3\cdot\vec{v}_3)^{\mu}.$$

On the other hand,

$$(\omega e^{-\beta\varkappa} - e^{-\beta\varkappa}\omega)(\vec{u}_1\vec{v}_1)^{\lambda}(\vec{u}_3\vec{v}_3)^{\mu} = \{\omega-(\lambda+3\mu+3)\nu +$$
$$+(\lambda+\mu+1)\frac{\partial}{\partial\beta}\}e^{-\beta\varkappa}(\vec{u}_1\vec{v}_1)^{\lambda}(\vec{u}_3\vec{v}_3)^{\mu}. \quad \text{(A9)}$$

Comparing Eqs.(A8) and (A9), we get a differential equation

$$\{(e^{\beta}-1)\left(\frac{d^2}{d\beta^2}-\nu^2\right)-\left[(\lambda+\mu+1)\frac{d}{d\beta}+\omega-(\lambda+3\mu+3)\nu\right]\}K(\beta;\omega,\nu)=0, \quad \text{(A10.a)}$$

where the unknown function $K(\beta;\omega,\nu)$ is determined as follows

$$K(\beta;\omega,\nu)(\vec{u}_1\vec{v}_1)^{\lambda}(\vec{u}_3\cdot\vec{v}_3)^{\mu} = e^{-\beta\varkappa}(\vec{u}_1\vec{v}_1)^{\lambda}(\vec{u}_3\cdot\vec{v}_3)^{\mu}. \quad \text{(A10.b)}$$

By changing to a new function

$$N(\beta;\omega,\nu) = e^{\beta\nu}K(\beta;\omega,\nu) \quad \text{(A11)}$$

and a new variable

$$g = 1-e^{-\beta}, \quad \text{(A12.a)}$$

Eq.(A10.a) is transformed into a hypergeometric equation

$$\{g(g-1)\frac{d^2}{dg^2}+[(2\nu-\lambda-\mu)g+\lambda+\mu+1]\frac{d}{d\beta}+$$
$$+[\omega-2(\lambda+2\mu+2)\nu]\}N(g;\omega,\nu)=0. \quad \text{(A12.b)}$$

If one determines a formal hypergeometric series, concording with Eq.(A12.b), the upper parameters are found to be equal to $a+\nu$ and $b+\nu$, where

$$a+b=-(\lambda+\mu+1)\ ,\ ab=\omega-(\lambda+3\mu+3)\nu-\nu^2,$$

and the lower parameter equals $-(\lambda+\mu+1)$. The latter condition demands choosing a degenerate solution of the hypergeometric equation. Hence the transformation sought for takes the form

$$\exp\left\{-\frac{\beta}{\vec{y}^2}\vec{y}^{\,T}\Lambda_\mu\vec{y}\right\}(\vec{u}_1\vec{v}_1)^\lambda(\vec{u}_3\vec{v}_3)^\mu=\sum_{k=0}^{\lambda+\mu+1}\left(\frac{e^{-\beta}-1}{\vec{y}^2}\right)^k\frac{(\lambda+\mu+1-k)!}{(\lambda+\mu+1)!\,k!}\times$$

$$\times\prod_{n=0}^{k-1}\left\{\vec{y}^{\,T}\left[\Lambda^2-\mu(\mu+1)E+(n-\mu-1)(\Lambda-\mu E)+n(\lambda+\mu+1+n)E\right]\vec{y}\right\}\times \tag{A13}$$

$$\times(\vec{u}_1\vec{v}_1)^\lambda(\vec{u}_3\cdot\vec{v}_3)^\mu+O(\beta^{\lambda+\mu+2}\ln\beta).$$

The neglected singular term $O(\beta^{\lambda+\mu+2}\ln\beta)$ has no influence on the subsequent results, because one never needs acting more than $\lambda+\mu$ times by the splitted SU(3) generators (see Eq. (34.a)).

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