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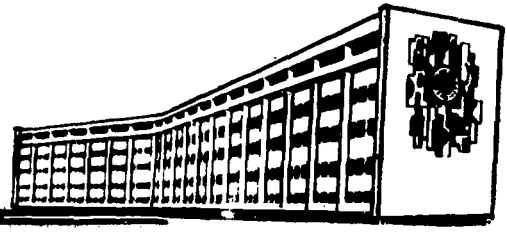
EFFECTIVE HAMILTONIAN WITHIN THE MICROSCOPIC UNITARY NUCLEAR MODEL
II. THE STUDY OF sd-NUCLEI WITH TWO VALENCE PARTICLES



КИЕВ

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В.И.Авраменко, А.Л.Блохин

Эффективный гамильтониан микроскопической унитарной модели ядра. II. Исследование sd-ядер с двумя валентными нуклонами

В рамках микроскопической версии унитарной коллективной модели с горизонтальным смешиванием построен эффективный гамильтониан ядер ^{18}O и ^{18}Ne. Алгебраическая структура полученного гамильтониана сходна с гамильтонианами известных феноменологических моделей, содержащими SU(3) - смешивающие слагаемые, которые требуются для описания переходных вращательно-колебательных спектров. Проведена диагонализация гамильтониана, учитывающего центральное ядерное и кулоновское взаимодействия, на базисе трех неприводимых представлений группы SU(3) с двумя орбитальными схемами Юнга.

Effective Hamiltonian Within the Microscopic Unitary Nuclear Model. II. The Study of sd-Nuclei With Two Valence Particles

V.I.Avramenko, A.L.Blokhin

Within the microscopic version of the unitary collective model with the horizontal mixing the effective Hamiltonian for ^{18}O and ^{18}Ne nuclei is constructed. The algebraic structure of the Hamiltonian is compared to the familiar phenomenological ones with the SU(3)-mixing terms which describe the coupled rotational and vibrational spectra. The Hamiltonian, including central nuclear and Coulomb interaction, is diagonalized on the basis of three SU(3) irreducible representations with two orbital symmetries.

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EFFECTIVE HAMILTONIAN WITHIN THE MICROSCOPIC UNITARY NUCLEAR MODEL
II. THE STUDY OF sd-NUCLEI WITH TWO VALENCE PARTICLES

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I. INTRODUCTION

In recent years the problem of fully microscopic formulation of familiar semiphenomenological algebraic models, describing nuclear collective motion, was contributed by many authors. The determination of the collective potential occured to be the most difficult part of the stated problem, and various approaches, both approximate and exact, were put forward for its solution [1-4]. In the previous paper of this series [5] (hereafter it would be referred to as I) we proposed a technique of restoring an effective Hamiltonian (EH) of the unitary collective model from the microscopic Hamiltonian matrix elements between the coherent states of separate or mixed SU(3) irreducible representations (irreps). The concrete expressions are obtained for the standard central nucleon-nucleon interaction with Gaussian radial dependence.

The aim of the present work is to calculate the rotational spectrum and E2 transition rates for almost magic light nuclei, starting from an extended axial unitary EH derived in Sec.IV of the paper I. The mentioned extension implies using the collective model with horizontal mixing [6] which, contrary to the conventional unitary model [7] (and, what is more, the symplectic one [8]), distinguishes between nucleons with different spin and isospin projection. The space of extended unitary model states is defined by the direct product of partial SU(3) irreps characterizing each of four nucleon spin-isospin subsystems. The subsequent expansion of the extended model basis into SU(3) irreps due to the group chain

$$\bigotimes_{(\sigma\tau)} SU^{(\sigma\tau)}(3) \supset SU(3)$$

where $(\sigma\tau)$ are the spin and isospin projection values, exposes several SU(3) irreps which belong to the same oscillator shell. The necessity of intrashell (or, in another words, horizontal) mixing for, at least, low-medium weight nuclei, is firmly stated after the semimicroscopic shell-model calculations initiated by Elliott [7] and Arima with collaborators [9-11]. The interacting boson model (IBM) [12-13] enabled parametrizing the spectra of medium-weight and heavy nuclei by accounting the horizontal mixing through the dynamical symmetry extension to the SU(6) group.

We proceed with studying the simplest case when the model

with horizontal mixing is applicable, namely, a light nucleus with two valence particles. Such a system possesses the SU(3)⊗ SU(3) dynamical symmetry which is temptatively close to the SU(6) one of the IBM. In Sec.II the generators of partial SU(3) algebras are expressed in terms of the dynamical variables of two dot rotors. The structure of involved basis functions in their general from and in the particular case of sd-shell is analysed in Sec.III. The EHs of the sd-shell nuclei with two valence neutrons and protons are deduced in Sec.IV from the appropriate results of Ref.11 and compared with the familiar phenomenological Hamiltonians. The calculated spectra and transition rates are discussed in Sec.V. The last section contains the concluding remarks.

II. REALIZATION OF $su^{(1)}(3) \oplus su^{(2)}(3)$ ALGEBRA

Within the extended unitary model, the state of every subsystem with a fixed spin-isospin projection of nucleons is set on an SU(3) irrep basis. Closed shells, being the SU(3) scalars, produce no influence on the unitary labelling, therefore, to choose the necessary basis, one takes into account only the valence nucleons. Two valence nucleons with equal charge as a rule carry on different spin projections. Consequently, in the case concerned, each of the open shell particles determines the SU(3) quantum numbers of the corresponding subsystem; henceforth the latter are distinguished by an uppex, namely (1) or (2). We suppose that the open shells possess the main quantum number equal n for both the subsystems; so they are described by the symmetric (n0) irreps (in the Elliott notation).

Restricting the study by the symmetric irreps, one may realize the SU(3) generators in terms of a single vector. Following Eq.(19) from paper I, the $su^{(1)}(3) \oplus su^{(2)}(3)$ algebra is expressed in terms of two operator-valued matrices

$$\mathcal{B}^{(s)} = \| \mathcal{B}^{(s)}_{ij} \| = \| (U^{(s)})_i \left(\frac{\partial}{\partial U^{(s)}} \right)_j - \frac{1}{3} U^{(s)} \frac{\partial}{\partial U^{(s)}} \delta_{ij} \|, \quad (1)$$
$$i,j = \overline{1,3} \quad , \quad s = 1,2,$$

where the matrix notation taken over the Cartesian indices. The commutation relations

$$[\mathcal{B}^{(s)}_{ij}, \mathcal{B}^{(r)}_{k\ell}] = (\delta_{jk}\mathcal{B}^{(s)}_{i\ell} - \delta_{i\ell}\mathcal{B}^{(s)}_{kj})\delta_{sr} \qquad (2)$$

are easily verified. The matrices $\mathcal{B}^{(1)}$ and $\mathcal{B}^{(2)}$ are allowed to act on the n-fold outer product $T^{n,n}$ of a reducible second-order tensor

$$T^{1,1} = \vec{u}^{(1)} \otimes \vec{u}^{(2)} \qquad (3)$$

To simplify the realization of infinitesimal operators and the structure of basis functions, one often reduces a consideration to a selected coordinate system. Say, the algebra of rigid rotor $[R^5]$ SO(3) group in the quadrupole tensor proper system retains a nontrivial dependence only on the orbital momentum components [14]. An analogous representation holds for the SU(3) through the contraction possibility onto the $[R^5]$ SO(3) [15-18]. The subsequent narration is also connected with a specific coordinate system but the latter is adapted for two dot rotors.

Let us normalize the three-dimensional vectors $\vec{u}^{(1)}$ and $\vec{u}^{(2)}$

$$\vec{e}^{(1)} = \frac{\vec{u}^{(1)}}{|\vec{u}^{(1)}|}, \quad \vec{e}^{(2)} = \frac{\vec{u}^{(2)}}{|\vec{u}^{(2)}|}, \quad r^{(1)} = |\vec{u}^{(1)}|, r^{(2)} = |\vec{u}^{(2)}| \qquad (4)$$

and utilize $\vec{e}^{(1)}$ and $\vec{e}^{(2)}$ to construct an orthonormalized reper

$$\vec{\xi} = \frac{\vec{e}^{(1)} + \vec{e}^{(2)}}{|\vec{e}^{(1)} + \vec{e}^{(2)}|}, \quad \vec{\zeta} = \frac{\vec{e}^{(1)} \times \vec{e}^{(2)}}{|\vec{e}^{(1)} \times \vec{e}^{(2)}|}, \quad \vec{\eta} = \frac{\vec{e}^{(1)} - \vec{e}^{(2)}}{|\vec{e}^{(1)} - \vec{e}^{(2)}|}. \qquad (5)$$

A three-parametric orthogonal block matrix $d = (\vec{\xi}, \vec{\zeta}, \vec{\eta})$, orienting the reper (5) with respect to the laboratory system, together with an angular variable

$$\vartheta = \arccos t, \quad t = \vec{e}^{(1)} \cdot \vec{e}^{(2)} \qquad (6)$$

parametrize the subgroup $SO(3) \otimes SO_{\vartheta}(2)$ or the direct produot $SO^{(1)}(3) \otimes SO^{(2)}(3)$. (The mapping is bijective as the dot rotor needs two coordinates only).

By using definitions (4) and (5), one deduces that

$$\frac{\partial}{\partial \vec{u}^{(1)}} = d\,\vec{\Lambda}^{(1)}, \qquad (7.a)$$

$$\vec{\Lambda}^{(i)} = \begin{pmatrix} \cos\frac{\vartheta}{2}\frac{\partial}{\partial r^{(i)}} - \frac{1}{r^{(i)}}\sin\frac{\vartheta}{2}\left(\frac{\partial}{\partial\vartheta} + \frac{i}{2}\hat{L}_3\right) \\ -\sin\frac{\vartheta}{2}\frac{\partial}{\partial r^{(i)}} - \frac{1}{r^{(i)}}\cos\frac{\vartheta}{2}\left(\frac{\partial}{\partial\vartheta} + \frac{i}{2}\hat{L}_3\right) \\ \frac{i}{r^{(i)}\sin\vartheta}\left(\cos\frac{\vartheta}{2}\hat{L}_1 + \sin\frac{\vartheta}{2}\hat{L}_2\right) \end{pmatrix}, \qquad (7.b)$$

where $\hat{L}_1, \hat{L}_2, \hat{L}_3$ are the Cartesian components of the orbital angular momentum vector $\vec{L}$ due to the SO(3) group:

$$\vec{L} = \vec{L}^{(1)} + \vec{L}^{(2)} \qquad (8.a)$$

$$\vec{L}^{(s)} = -i\vec{u}^{(s)} \times \frac{\partial}{\partial\vec{u}^{(s)}} \qquad (8.b)$$

To rewrite Eq.(7) for the second subsystem, one has to replace by $r^{(1)}$ and $r^{(2)}$, ϑ by $-\vartheta$.

At this step one introduces matrices $\beta^{(1)}$ and $\beta^{(2)}$ of partial generators in the proper coordinate system of the reper (5):

$$\beta^{(s)} = d^T \mathcal{B}^{(s)} d \qquad (9.a)$$

and, vice versa,

$$\mathcal{B}^{(s)} = d\,\beta^{(s)} d^T, \qquad (9.b)$$

where the uppex T implies the transposition. By comparing Eqs.(1),(4)-(7) with (9.b), the following realization for $\beta^{(s)}$ matrix elements is obtained:

$$\begin{aligned}
\beta^{(s)}_{11} &= \frac{n}{2}(1+\cos\vartheta) - \frac{1}{2}\sin\vartheta\frac{\partial}{\partial\vartheta} \mp \frac{i}{4}\sin\vartheta\hat{L}_3 - \frac{n}{3}\,, \\
\beta^{(s)}_{12} &= \mp\frac{n}{2}\sin\vartheta \mp \frac{1}{2}(1+\cos\vartheta)\frac{\partial}{\partial\vartheta} - \frac{i}{4}(1+\cos\vartheta)\hat{L}_3\,, \\
\beta^{(s)}_{21} &= \mp\frac{n}{2}\sin\vartheta \pm \frac{1}{2}(1-\cos\vartheta)\frac{\partial}{\partial\vartheta} + \frac{i}{4}(1-\cos\vartheta)\hat{L}_3\,, \\
\beta^{(s)}_{22} &= \frac{n}{2}(1-\cos\vartheta) + \frac{1}{2}\sin\vartheta\frac{\partial}{\partial\vartheta} \pm \frac{i}{4}\sin\vartheta\hat{L}_3 - \frac{n}{3}\,, \\
\beta^{(s)}_{13} &= \pm\frac{i}{2}\frac{1+\cos\vartheta}{\sin\vartheta}\hat{L}_1 + \frac{i}{2}\hat{L}_2\,, \quad \beta^{(s)}_{23} = -\frac{i}{2}\hat{L}_1 \mp \frac{i}{2}\frac{\sin\vartheta}{1+\cos\vartheta}\hat{L}_2\,, \\
\beta^{(s)}_{31} &= \beta^{(s)}_{32} = 0\,, \quad \beta^{(s)}_{33} = -\frac{n}{3}\,,
\end{aligned} \qquad (10)$$

where the upper sign conforms with $\beta^{(1)}$ and the lower one - with $\beta^{(2)}$. Also the use is made of the fact that operators $u^{(s)}\frac{\partial}{\partial u^{(s)}}$, $s=1,2$, have the same eigenvalue n , when acting on the tensor $T^{n,n}$. To select the $SO^{(1)}(3)$ and $SO^{(2)}(3)$ subalgebras, one

has to antisymmetrize the matrices (10)

$$\ell_1^{(s)} = -\left(\tfrac{1}{2}\hat{L}_1 \pm \tfrac{1}{2}\tan\tfrac{\nu}{2}\hat{L}_2\right),$$
$$\ell_2^{(s)} = -\left(\tfrac{1}{2}\hat{L}_2 \pm \tfrac{1}{2}\cot\tfrac{\nu}{2}\hat{L}_1\right), \qquad (11)$$
$$\ell_3^{(s)} = -\left(\tfrac{1}{2}\hat{L}_3 \pm \frac{\partial}{\partial\nu}\right)$$

Note that the operators $\ell^{(s)}$ differ in sign from $\vec{L}^{(s)}$ of Eq. (8). The reason is that these operators are defined in a rotating coordinate system, in contrast to the usual orbital momentum in the laboratory system.

III. BASIS FUNCTIONS AND ENVELOPING ALGEBRA

To provide a direct comparison with the conventional SU(3) model, it is expedient to make use of the irrep basis functions of the $SU(3) \subset SU^{(1)}(3) \otimes SU^{(2)}(3)$ group, e.g. the eigenfunctions of the second order Casimir operator [7]

$$C_{32} = Tr\left(\beta^{(1)} + \beta^{(2)}\right)^2 \qquad (12)$$

Within the realization due to Eq.(9.b) one passes from (12) to

$$C_{32} = \sum_{i,j=1}^{3} \beta_{ij}\beta_{ji} - (i-t)\beta_{11} - (1+t)\beta_{22} + \frac{2}{3}n, \qquad (13.a)$$

$$\beta_{ij} = \beta_{ij}^{(1)} + \beta_{ij}^{(2)}, \qquad (13.b)$$

where t is a variable of Eq.(6). The last three terms appear in the right-hand side of (13.a) as a consequence of non-commutability of the angular momentum L and the rotational matrix d. By substituting Eq.(10) in (13), one realizes the Casimir operator in terms of the SU(3) quantum number, orbital momentum algebra and additional variable t :

$$C_{32} = 2(1-t^2)\left[(1-t^2)\frac{\partial^2}{\partial t^2} + (2n-1)t\frac{\partial}{\partial t} - n(n+4) + 2n\frac{n+2}{1-t^2} - \frac{1}{4}\hat{L}_3^2\right] - \frac{4}{3}n^2. \qquad (14)$$

The eigenvalues of operator (14) are defined by the familiar formula

$$g_2(\lambda\mu) = \lambda^2 + \lambda\mu + \mu^2 + 3\lambda + 3\mu, \qquad (15)$$

whereas the quantum numbers of the admissible SU(3) irreps con-

form with the following expansion:

$$(n0)\otimes(n0) = \sum_{\mu=0}^{n} (2n-2\mu,\mu) \tag{16}$$

Supposing the rotational invariance of nuclear Hamiltonian, one demands the desired basis functions to be the simultaneous eigenfunctions of G_2, L^2 and L_3^2. The separation of variables as follows

$$\Phi_K^{\lambda\mu}(t)\,\mathcal{D}^L_{mK}(\alpha) \tag{17}$$

where K is an eigenvalue of L_3, $\mathcal{D}^L_{mK}(\alpha)$ is a spherical Wigner function, and Elliott quantum numbers $(\lambda\mu)$ undergo the relation $\lambda+2\mu=2n$ with a subsequent substitution

$$\Phi_K^{\lambda\mu}(t) = (1-t^2)^{\mu/2}\,\chi_K^{\lambda}(t) \quad , \quad K=\pm\lambda,\pm(\lambda-2),\dots,0, \tag{18}$$

leads to the equation

$$\left[(1-t^2)\frac{\partial^2}{\partial t^2} + (\lambda-1)t\frac{\partial}{\partial t} - \frac{1}{4}(\lambda^2-K^2)\right]\chi_K^{\lambda}(t) = 0 \tag{19.a}$$

The latter has a polynomial hypergeometric solution

$$\chi_K^{\lambda} = {}_2F_1\left(-\frac{\lambda-K}{2}, -\frac{\lambda+K}{2}, -\frac{\lambda-1}{2}; \frac{1+t}{2}\right) \tag{19.b}$$

symmetric with respect to the sign of K. The functions $\Phi_K^{\lambda\mu}(t)$ satisfy orthogonality conditions with appropriate weight function along the imaginary axis of the complex t plane. They admit the quadratic hypergeometric transformation [19], resulting in functions

$$\varphi_K^{\lambda\mu}(t) = (1-t^2)^{\mu/2}\begin{cases} {}_2F_1\left(-\frac{\lambda-K}{4}, -\frac{\lambda+K}{4}, \frac{1}{2}; t^2\right) & \text{for even } \frac{\lambda-K}{2} \\ t\,{}_2F_1\left(-\frac{\lambda-K}{4}, -\frac{\lambda+K}{4}, \frac{3}{2}; t^2\right) & \text{for odd } \frac{\lambda-K}{2} \end{cases} \tag{20}$$

with some other normalization whose parity in t equals $(-1)^{\frac{\lambda-K}{2}}$.

In concrete applications instead of (17) one utilizes somewhat modified basis functions. Firstly, the rotational factor is usually taken in the form of

$$\mathcal{D}^{L}_{m\kappa\pm}(\alpha) = \frac{1}{\sqrt{2(1+\delta_{\kappa 0})}}\left[\mathcal{D}^{L}_{m\kappa}(\alpha) + (-1)^{\lambda+\mu+L}\mathcal{D}^{L}_{m-\kappa}(\alpha)\right], \qquad (21)$$

adapted to the $\mathcal{D}_2$ symmetry subgroup of the rigid top quadrupole tensor R^5 [7,17]. And then, at fixed $SU(3)\supset SO(3)$ quantum numbers, an additional summation over the index κ must be carried out. Really, our choice of the proper coordinate system makes the orbital momentum projection on the ζ axis to accept $\lambda+1$ values (see Eq.(18)), whereas the Elliott rules [7] prescribe the number of min $(\lambda,\mu)+1$. To avoid such a discrepancy, one has to replace the functions

$$\varphi^{\lambda\mu}_{\kappa}(t)\,\mathcal{D}^{L}_{m\kappa\pm}(\alpha), \qquad (22)$$

generally speaking, by their linear combinations compatible with the $SU(3)\supset SO(3)$ reduction conditions, say, due to the basis of Elliott or Bargmann and Moshinsky [20].

For example, in the case of $n=2$ (i.e. two particles in the sd-shell) the Elliott rules show that every admissible set of quantum numbers λ,μ and L is supplied by a single basis function (provided one neglects the trivial phase distinction between the states with opposite values of the orbital momentum intrinsic projection). And indeed, the functions (22), being gathered in the following linear combinations

$$\begin{aligned}
&(\lambda\mu)=(40),\\
L=0:\ &\frac{1}{3\sqrt{5}}\,(1+2t^2),\\
L=2:\ &\sqrt{\frac{2}{21}}\left[(1+2t^2)\mathcal{D}^{2}_{m0}(\alpha)-3\sqrt{3}\,t\,\mathcal{D}^{2}_{m2+}(\alpha)\right],\\
L=4:\ &\frac{1}{\sqrt{210}}\left[(1+2t^2)\mathcal{D}^{4}_{m0}(\alpha)-2\sqrt{5}\,t\,\mathcal{D}^{4}_{m2+}(\alpha)+\sqrt{35}\,\mathcal{D}^{4}_{m4+}(\alpha)\right],\\
&(\lambda\mu)=(21),\\
L=1:\ &\sqrt{\frac{4}{5}}\,\sqrt{1-t^2}\,t\,\mathcal{D}^{1}_{m0}(\alpha),\\
L=2:\ &\sqrt{\frac{2}{3}}\,\sqrt{1-t^2}\,\mathcal{D}^{2}_{m2-}(\alpha),\\
L=3:\ &\frac{1}{\sqrt{5}}\,\sqrt{1-t^2}\left[t\mathcal{D}^{3}_{m0}(\alpha)-\sqrt{\frac{5}{3}}\,\mathcal{D}^{3}_{m2+}(\alpha)\right],
\end{aligned} \qquad (23)$$

$$(\lambda\mu) = (02)$$

$$L = 0 \qquad \frac{1}{3}(1 - t^2)$$

$$L = 2 \qquad \frac{\sqrt{2}}{3}(1 - t^2)\mathcal{D}^2_{m0}(d)$$

form a space, locked with respect to the action of operators (10).

The symmetry of basis states (22) determines the preferable structure of scalar operators on the enveloping algebra of the $SU^{(1)}(3) \otimes SU^{(2)}(3)$ group, in terms of which the EH is expressed.

It is expedient to separate the operators, possessing the dynamical SU(3) symmetry, from the ones, mixing the SU(3) irreps. The axial EH of the SU(3) model needs only four independent operators, namely, the squared orbital momentum $\tilde{L}^2$, the Casimir operators G_2 (see Eqs.(12) and (15)) and

$$G_3 = Tr(\mathcal{B}^{(1)} + \mathcal{B}^{(2)})^3 \qquad (24.a)$$

and the Bargmann-Moshinsky operator Q,

$$Q = L^T Q L, \qquad (24.b)$$

where the quadrupole matrix Q is the symmetrized form of $\mathcal{B}^{(1)} + \mathcal{B}^{(2)}$ matrix. The rotational invariants of higher order on the SU(3) generators at any rate contain a non-axial part.

As for the mixing terms, the structure of the system under study permits choosing them symmetric with respect to the mutual permutation of partial generators, $\mathcal{B}^{(1)} \leftrightarrow \mathcal{B}^{(2)}$. Besides, the direct construction of the EH, according to the results of Ref. 1, proves (see the next Section) that they arise in the polynomial form on the partial orbital momenta $\tilde{L}^{(1)}$ and $\tilde{L}^{(2)}$ only. The latter condition noticeably simplifies producing the representation of mixing operators on the basis (22) (cf. Eqs.(10) and (11)).

IV. EFFECTIVE HAMILTONIAN FOR ^{18}O and ^{18}Ne

The EH for the almost magic light and low-medium weight nuclei, containing up to four nucleons over the closed shell, was

restored in Sec.IV of Ref.5 from the microscopic Hamiltonian matrix elements between the many-body oscillator functions which transformed by the direct product of symmetric SU(3) irreps. In the present Section this result is specified to study the simplest nucleon system where the account of horizontal mixing is necessary, namely, with two sd-particles of the same charge. The corresponding shell-model space is divided into three SU(3) irreps with two space symmetries. The central interaction mixes (40) and (20) irreps with the orbital Young pattern [2]; the admixture of the spatially antisymmetric (21) irrep, produced by the spin-dependent forces, is expected to be negligible for ^{18}O and ^{18}Ne.

The kinetic energy of A-particle shell-model system is determined by the whole number N of oscillator quanta:

$$T = \frac{\hbar^2}{2mr_0^2}\left[N + \frac{3}{2}(A-1)\right], \qquad (25)$$

where m is the average nucleon mass and $r_0 = \sqrt{\hbar/m\omega}$ is the oscillator radius. In the present case A=18 and N=16 .

The central nucleon interaction is supposed to carry on Gaussian radial dependence

$$U_C = \sum_{i<j\le A} \sum_{S,T=0}^{1} V_{2S+1,2T+1} \exp\left[-\frac{(1-z)}{2zr_0^2}(r_i-r_j)^2\right] \mathcal{P}^S \mathcal{P}^T \qquad (26)$$

where $V_{2S+1,2T+1}$ are the spin-isospin potential magnitudes, $\mathcal{P}^S$ and $\mathcal{P}^T$ are the nucleon pair spin and isospin projectors.

By applying Eqs.(28)-(32) from paper I, one derives an analytic expression for the effective operator generated by (26):

$$U_C = U^C_{shell} + U^C_{coupling} + U^C_{extra} \qquad (27.a)$$

$$U^C_{shell} = \frac{9V_{33} + 3V_{13} + 3V_{31} + V_{11}}{2} z^{3/2}\left[16 - 12(1-z) + \frac{15}{4}(1-z)^2\right] \qquad (27.b)$$

$$+ \frac{-9V_{33} + 3V_{13} + 3V_{31} - V_{11}}{2} z^{3/2}\left[16 - 12(1+z) + \frac{3}{4}(5 + 6z + 5z^2)\right],$$

$$U^C_{coupling} = \frac{9V_{33} + 3V_{13} + 3V_{31} + V_{11}}{2} z^{3/2}\left[1 + \frac{1}{2}z + \frac{5}{2}z^2 + \right.$$

$$\left. + \frac{1}{32}(1-z)^2(1+7z)\left(10 - \frac{1}{2}\vec{L}^2 + \vec{L}^{(1)}\cdot\vec{L}^{(2)}\right) + \right.$$

$$+\frac{-9V_{33}+3V_{13}+3V_{31}-V_{11}}{2}\,z^{3/2}\Big[1-z+\frac{1}{32}(1-z)^2(1+7z)\cdot \qquad (27.c)$$
$$\times\Big(10-\frac{1}{2}\vec{L}^2+\vec{L}^{(1)}\cdot\vec{L}^{(2)}\Big)\Big],$$

$$U^c_{extra}=\frac{1}{2}(V_{33}+V_{13})\,z^{3/2}\varphi(z)+\frac{1}{2}(-V_{33}+V_{13})\,z^{3/2}\varphi(z)P,$$

$$\varphi(z)=1-2(1-z)-\frac{5}{12}(1-z)^2+\frac{35}{108}(1-z)^3+\frac{2375}{576}(1-z)^4$$

$$-\frac{1}{16}(1-z)^2\Big[1-\frac{7}{6}(1-z)+\frac{19}{16}(1-z)^2\Big]\vec{L}^2-\frac{1}{8}(1-z)^2\Big[1-\frac{103}{16}(1-z)^2\Big]\vec{L}^{(1)}\cdot\vec{L}^{(2)}$$

$$+\frac{1}{4}(1-z)^2\Big[1-\frac{1}{12}(1-z)-\frac{165}{64}(1-z)^2\Big]G_2-$$

$$-\frac{1}{48}(1-z)^3\Big[1-\frac{3}{8}(1-z)\Big](4G_3+3Q)+$$

$$+\frac{3}{128}(1-z)^4\Big[\Big(G_2-\vec{L}^{(1)}\cdot\vec{L}^{(2)}\Big)^2+\frac{1}{2}\big(\vec{L}^{(1)}\big)^2\big(\vec{L}^{(2)}\big)^2\Big],$$

where subscripts "shell", "coupling" and "extra" imply the interaction of closed-shell nucleons among themselves, with extra nucleons and the interaction between extra nucleons respectively. The operator

$$P=\frac{1}{16}G_2^2-\frac{1}{12}G_3-\frac{7}{6}G_2-\frac{167}{27} \qquad (27.f)$$

is responsible for the permutation of subsystems (1) and (2). For the present case it merely coincides with the Majorana operator (i.e. the sum over all two-body permutations) [21] and, consequently, accepts only two eigenvalues: 1 for spatially symmetric states and - 1 for the antisymmetric ones.

By making use of integral representation

$$\frac{1}{X}=\frac{1}{\sqrt{2\pi}}\int_0^\infty \exp\Big(-\frac{1-z}{2z}X^2\Big)\frac{dz}{\sqrt{1-z}} \qquad (28)$$

and Eq.(27), we derive an effective operator of Coulomb interaction

$$U^{coul}_{shell}=\frac{83}{2}\frac{e^2}{\sqrt{2\pi}r_0}, \qquad (29.a)$$

$$U^{coul}_{coupling} = \frac{e^2}{\sqrt{2\pi}}\frac{1}{r_0} * \begin{cases} 0 & \text{for } {}^{18}O, \\ 20 + \frac{\gamma}{40}\left(10 - \frac{1}{2}\vec{L}^2 + \vec{L}^{(1)}\cdot\vec{L}^{(2)}\right) & \text{for } {}^{18}Ne, \end{cases} \tag{29.b}$$

$$U^{coul}_{extra} = \frac{e^2}{\sqrt{2\pi}}\frac{1}{r_0} * \begin{cases} 0 & \text{for } {}^{18}O, \\ \int_0^1 \frac{dz}{\sqrt{1-z}}\varphi(z)(2-P) & \text{for } {}^{18}Ne. \end{cases} \tag{29.c}$$

The entire EH is gathered from Eqs.(25)-(29):

$$\mathcal{H} = T + U_c + U_{coul} \tag{30}$$

Leaving the microscopic calculation till the subsequent Section, we proceed with some remarks concerning the collective features, inherent to EH(30).

As one could expect, the obtained EH contains the $SU(3) \supset SO(3)$ integrity basis operators $\vec{L}^2, G_2, G_3$ and Q which are necessary for parametrizing the collective spectral bands of axial nuclei within the SU(3) model [18,22]. In fact, these operators compose the Bohr-Mottelson-Frankfurt Hamiltonian in its projection on the lowest SU(3) irreps [23].

An appearance of the Majorana operator (we retain the symbol P for it) and the mixing ones is also worth noting. Within the phenomenological models the Majorana operator usually is used to expose the ordering of collective bands with different space symmetry on the energy scale. For example, Banerjee et al.[24] combined it with the SU(3) quadrupole-quadrupole force:

$$U = aP + b\,\mathrm{Tr}\,Q^2 \tag{31}$$

to reproduce the sequence of levels in the sd-shell spectra. In realistic situation the parameter a has a negative value, lowering in energy the states of maximum orbital symmetry. The SU(3) - mixing potential which goes as

$$U = a + bP + C\,\vec{L}^{(1)}\cdot\vec{L}^{(2)} \tag{32}$$

for the two-nucleon system, is sufficient to explain p-shell spectra, according to Elliott [25].

To perform an accurate comparison of the phenomenological potential with the realistic one, one should firstly note that the potentials (31) and (32) are the second-order terms in the

long-range perturbation series with the parameter $(R/\mu)^2$ where R is the characteristic interparticle distance and μ is the interaction range [26]. At the same time one realizes by analysing Eqs.(26) and (27) that the obtained potential is expanded in powers of

$$1-z = \frac{r_0^2}{r_0^2 + \frac{1}{2}\mu^2} \tag{33}$$

with μ as a radius of Gaussian function (26). Naturally, $1-z$ is identified with a renormalized long-range expansion parameter. All the operators of (31) and (32) can be found in the second-order perturbation terms of Eq.(27).

And what is more, the EH(30) is suitable for parametrizing the coupled rotational and vibrational spectra, which is exposed when restricting oneself to a fixed orbital symmetry; then the EH, cut at the mentioned second-order term, takes a form of

$$\mathcal{H} = a + bG_2 + c\vec{L}^2 - 2d\vec{L}^{(1)}\vec{L}^{(2)}. \tag{34}$$

By expanding the right-hand side of Eq.(14) in powers of the angle $\vartheta = \arccos t$ and reducing the ground state energy to nought, Eq.(34) is rewritten as follows

$$\mathcal{H} = (c-d)\vec{L}^2 + d[(\vec{L}^{(1)})^2 + (\vec{L}^{(2)})^2] + \frac{a}{2}\vartheta^2 + \ldots \tag{35}$$

Eq.(35) is identical with the accuracy of $\vec{L}^2$ to the two-rotor model Hamiltonian of de Franceschi et al. [27]. These authors introduced the quadratic potential in the angle ϑ between the rotors with orbital momenta $\vec{L}^{(1)}$ and $\vec{L}^{(2)}$ to couple the vibrational degrees of freedom to the rotational one, in accordance with the experiment for the low-medium weight and heavier nuclei.

V. NUMERICAL RESULTS

The diagonalization of collective Hamiltonian (25)-(30) is performed on the basis functions (23) with the semirealistic two-Gaussian nucleon interactions V1 by Volkov [28] and BB1 by Brink and Boecker [29]. The oscillator radii r_0 are chosen to fit the observed mean-square size, namely 1.77 Fm for ^{18}O and 1.724 Fm for ^{18}Ne. The data concerning both the predicted and experimental spectra are situated in Figs.1 and 2 and Tables 1

and 2 . The one-irrep Sp(2,R) calculations by Ovcharenko et al. [30] and the three-irrep SU(3) ones without mixing are also placed to form a quantitative estimate on the role of vertical and horizontal mixing effects.

The ground state energies, obtained with the V1 potential, (-119,7 MeV for ^{18}O and - 115,5 MeV for ^{18}Ne) are in satisfactory accordance with the experimental values [31] (-139,8 MeV and -132,1 MeV respectively). The BB1 potential yields the ground states too high in energy (-91,5 MeV and - 81,4 MeV) and, besides, inverts the ordering of 4_1^+ and 2_2^+ states with respect to 2_1^+ and 0_2^+ ones. The reason lines in the competition of two tendencies which determine the rotational spectrum, these are the preferable increase in orbital momentum for free extra nucleons and the quadrupole-quadrupole residual interaction, generating the "normal" $L(L+1)$ sequence. It can be noted that the horizontal mixing contributes on the whole to the second tendency.

The rotational 1^+, 2^+ and 3^+ levels, produced by the (21) irrep, correspond to the states of valence nucleons with unit spin and are separated by 8-10 MeV distance from the ground state. To identify them quantitatively with the experimental levels, 0_3^+ and so on, the spin-orbit and tensor interactions must be taken into consideration.

The expansion coefficients $C^{(\lambda\mu)}$ with respect to the $SU(3) \supset SO(3)$ basis for ^{18}Ne eigenfunctions and the E2 transition rates for the same nucleus are situated in Tables 3 and 4 . Note that the account of horizontal mixing is rather essential; thus, the weight of dominant (40) irrep in the structure of 0_1^+ state equals 76 percent for the V1 potential and 53 per cent for the BB1 one. Nevertheless, the predicted intensities of electric quadrupole transitions are too small within the microscopic unitary model. The agreement with experimental data can be improved by extending the model to the symplectic one, i.e. by the account of collective excitation contribution.

VI. CONCLUSION

In the scope of the fully microscopic unitary model with horizontal mixing we studied the low-energy structure of nuclei

with two-valence particles. The approach is based on the construction of effective Hamiltonian with the SU(3) ⊗ SU(3) dynamical symmetry which is equivalent to the microscopic Hamiltonian with respect to the action on the basis functions.

The concrete study is accomplished for the sd-shell nuclei with two extra particles of equal charge, i.e. ^{18}O and ^{18}Ne. The microscopic potential energy operator was supposed to consist of the central internucleon and Coulomb force. The algebraic structure of the obtained effective Hamiltonian occurred to correlate with the familiar phenomenological Hamiltonians, pertinent for the nuclei with coupled rotational and vibrational spectra. The predicted low-energy spectrum and corresponding E2 transition rates are in the qualitative agreement with the observed values.

Our calculations testify to the necessity of developing the discussed scheme in several aspects. First of all, the model needs to incorporate the dynamical deformation effects, originated from the collective excitations. Such a passage to the symplectic model with horizontal mixing may sufficiently decrease all energy levels, starting from the ground state. Then, leaving the energy region where the maximum orbital symmetry is dominant, the spin-dependent nuclear interactions are to be taken into account. (In odd and odd-odd nuclei whose spectrum is crucial to these forces, one has to involve them from the very beginning. The SU(3)-mixing Hamiltonian terms are therefore produced both by the nuclear and Coulomb potential). At the same time, ascending by the energy scale, the model basis is to be extended by the irreps due to particle-hole excitations provided they come rather low in energy [32].

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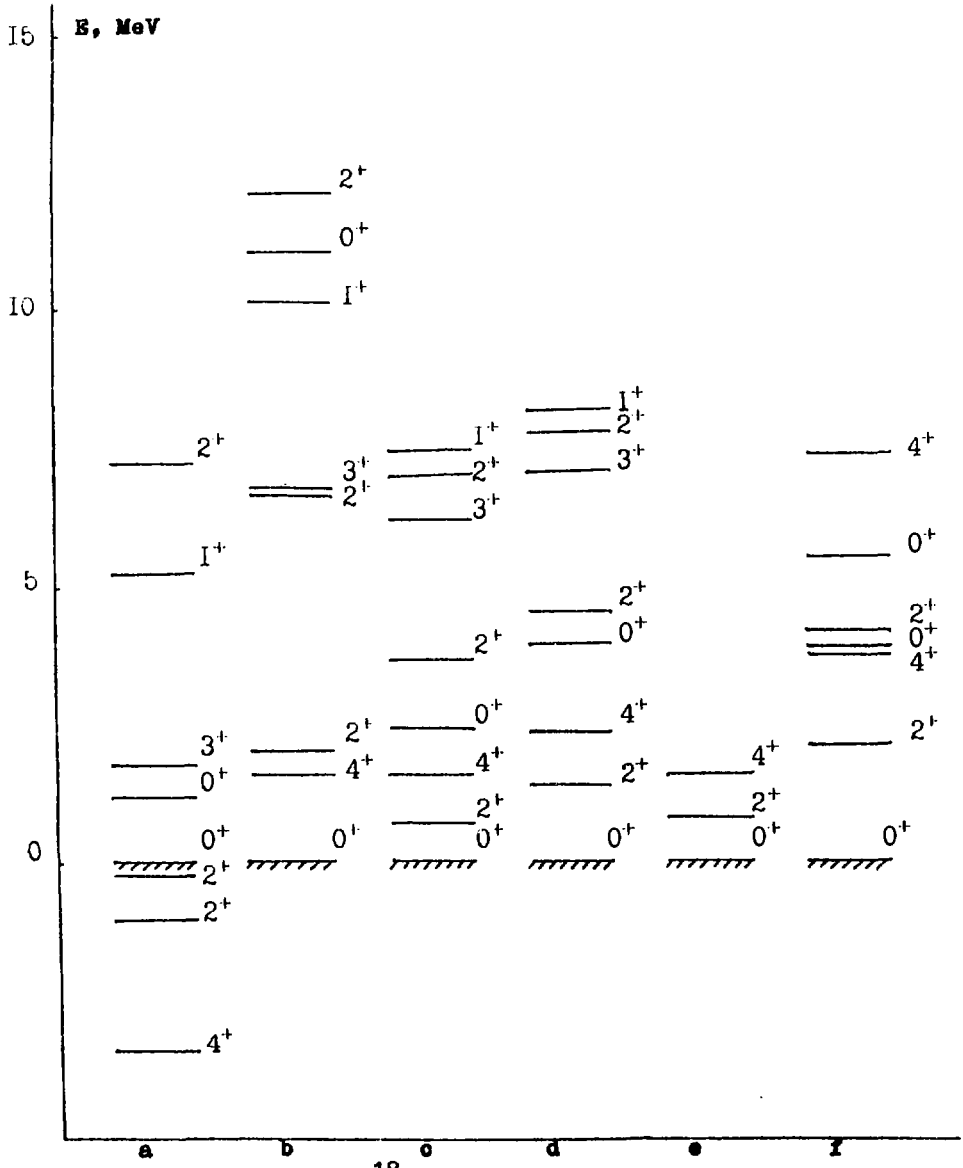

Fig.1. The spectrum of ^{18}O: a) BB1 + Coulomb force without mixing; b) the same force with mixing; c) V1 + Coulomb force without mixing; d) the same force with mixing; e) BB1 + Coulomb force, the one-irrep Sp(2,R) calculation [30]; f) experiment [31]. The energies of excited levels are counted from the 0^+_1 state.

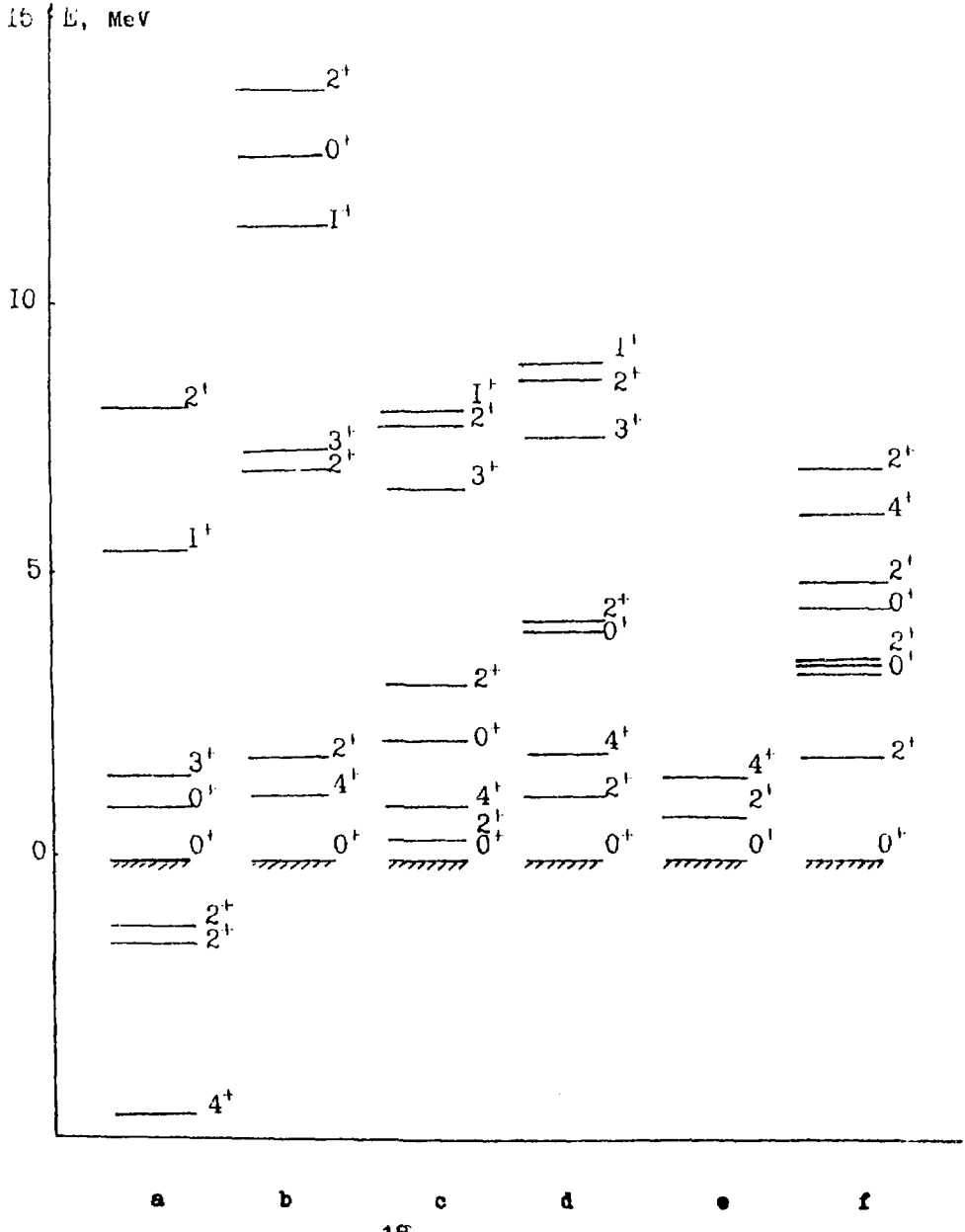

Fig.2. The spectrum of ^{16}Ne. The notations are identical to Fig.1.

Table 1. The spectrum of ^{18}O (MeV units): BB - BB1 + Coulomb force without mixing; BB, h.m. - BB1 + Coulomb force with mixing (h.m.=horizontal mixing); V and V,h.m. - the same for V1 + Coulomb force; BB, v.m. - BB1 + Coulomb force, the one-irrep Sp(2,R- calculation 30 ; exp-experimental data 31 .

Level	BB	BB, h.m.	V	V, h.m.	BB, v.m.	exp.
0^+_1	-86.62	-91.48	-118.98	-119.70	-94.48	-139.81
2^+_1	-1.07	1.92	0.64	1.21	0.86	1.95
4^+_1	-3.36	1.50	1.48	2.20	1.63	3.55
0^+_2	1.17	10.91	2.29	3.43		3.63
2^+_2	-0.25	6.49	3.55	4.42		3.92
1^+	5.13	10.00	7.29	8.01		
2^+	7.08	11.95	6.83	7.55		
3^+	1.70	6.57	6.12	6.84		

Table 2. The spectrum of ^{18}Ne. The notations are identical to Table 1.

Level	BB	BB, h.m.	V	V, h.m.	BB, v.m.	exp.
0^+_1	-75.55	-81.43	-114.58	-115.52	-87.67	-132.14
2^+_1	-1.64	1.82	0.37	1.13	-0.86	1.89
4^+_1	-4.68	1.20	0.98	1.93	1.63	3.38
0^+_2	0.88	12.64	2.17	4.06		3.58
2^+_2	-1.33	6.97	3.12	4.25		3.62
1^+	5.47	11.36	7.99	8.94		
2^+	8.01	13.89	7.71	8.65		
3^+	1.47	7.36	6.62	7.56		

Table 3. The expansion coefficients $C^{(\lambda\mu)}$ of ^{18}Ne states with respect to $(\lambda\mu)$ irreps.

Level	V		BB	
	$C^{(40)}$	$C^{(02)}$	$C^{(40)}$	$C^{(02)}$
0^+_1	0.876	0.[illegible]	0.7312	0.6821
2^+_1	0.9698	0.2435	0.7283	0.685
4^+	1	0	1	0
0^+_2	-0.482	0.876	-0.682	0.7312
2^+_2	-0.2434	0.9696	-0.685	0.728

Table 4. The reduced rates of E2 transition in ^{18}Ne (e^2Fm^4 units); the other notations are identical to Table 1.

Transition	BB, v.m.	c.s.	V, h.m.	BB, h.m.	exp.
$2^+_1 - 0^+_1$	74.15	19.68	15.16	8.17	50.93-5.2
$4^+_1 - 2^+_1$	62.86	16.08	17.00	9.59	25.5±3.4
$2^+_2 - 0^+_2$		1.12	2.02	6.96	
$0^+_2 - 2^+_1$		0	2.93	3.13	14.56±7.28
$2^+_2 - 0^+_1$		0	0.16	2.43	1.89±0.95

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