Conf 911120 -1

ANL-HEP-CP--91-109

DE92 006974

CHIRAL SYMMETRY BREAKING IN GAUGE THEORIES FROM REGGEON DIAGRAM ANALYSIS *

Alan R. White †
High Energy Physics Division
Argonne National Laboratory
Argonne, IL 60439

Abstract

It is argued that reggeon diagrams can be used to study dynamical properties of gauge theories containing a large number of massless fermions. SU(2) gauge theory is studied in detail and it is argued that there is a high-energy solution which is analogous to the solution of the massless Schwinger model. A generalized winding-number condensate produces the massless pseudoscalar spectrum associated with chiral symmetry breaking and a "trivial" S-Matrix.

*Work supported by the U.S. Department of Energy, Division of High Energy Physics, Contract W-31-109-ENG-38.

†Presented at the International Workshop on Electroweak Symmetry Breaking, Hiroshima, Japan, November 1991.

1. Introduction

New techniques for studying the dynamical properties of gauge theories are surely to be welcomed. In this talk I will briefly outline a potentially powerful approach which I hope will become more widely appreciated. I will focus on chiral symmetry breaking as most relevant to the purpose of this meeting, but the close inter-relation with confinement and other dynamical properties will be clear.

Over the years I have gradually discovered[1] how and why the origin of the "Pomeron" in QCD depends intricately on all aspects of the dynamical solution of the theory - including confinement and chiral symmetry breaking, asymptotic freedom and the parton model, the anomaly and the quark sea and even the quark flavor spectrum. More surprisingly, perhaps, I have simultaneously realised that the techniques and concepts developed for the understanding of Regge behavior in QCD can be used to directly study many dynamical properties of general gauge theories. In this talk I should like to present my discussion from this latter point of view.

The basis for my approach is that all Regge limit calculations of massive (spontaneously-broken) non-abelian gauge theories give simple reggeon diagrams exactly as predicted by the general formalism[2] of Analytic Multi-Regge Theory - strongly suggesting that the structure of such theories is closely controlled by analyticity and unitarity in this kinematic regime. A suitable infra-red analysis, as the gauge-boson mass is removed, may then allow us to understand the impact of analyticity and unitarity on the dynamical solution of unbroken gauge theories.

My reggeon diagram infra-red analysis is directly valid when enough massless fermions are present to produce an infra-red fixed-point for the gauge coupling. This analysis also requires the gauge symmetry to be built up step-by-step via a series of infra-red limits. Consequently we obtain information about gauge theories in a very different dynamical situation to most other starting points.

The need for brevity will be unfortunate in that the analysis is heavily dependent on complicated aspects of multi-Regge theory and is also unconventional in many respects e.g. existence of the appropriate massless "pions", rather than a chiral condensate, is used to argue for chiral symmetry breaking. For those who would like to know more, I hope to publish a full-scale developmental review in the next few months.

I shall introduce reggeon diagrams as a very economic means for summarizing perturbative calculations. After emphasizing the power of abstract multi-Regge to generalize the results I shall go on to outline the infra-red analysis needed to obtain results for unbroken gauge theories. Massless quarks are vital to obtain physically sensible results.

2. Gluon Reggeon Diagrams

We consider SU(2) gauge theory, without quarks at first, but with a doublet Higgs field used to give all gluons a mass. There remains a global SU(2) symmetry which we refer to as color (I = 0,1,$\cdots$) in the following. The first result[1] is that (the Fourier transform of) the Regge pole amplitude

$$\Gamma_{1,1}(E,k^2) = \frac{1}{k^2 - M^2} \quad \frac{1}{E - \Delta(k^2)} \tag{1}$$

where E is the (Fourier) conjugate to $\log s$ and

$$\Delta(k^2) = (k^2 + M^2)\frac{g^2}{16\pi^2}\int \frac{d^2p}{[(p-k)^2 + M^2]\,[p^2 + M^2]} \tag{2}$$

completely represents the leading-log result for the Regge limit of elastic scattering amplitudes in which $s \to \infty$, and $t = -k^2$ is finite. In this approximation only amplitudes with $I = 1$ in the t-channel are non-zero. It is also well-known[2] that with the reggeon propagator (1) and an appropriate four-reggeon interaction (which we shall not give explicitly) the series of "bubble" reggeon diagrams completely represents the next-to-leading log results - which give $I = 0$ and $I = 2$ amplitudes.

The most important point for our general purpose is that the success of the reggeon diagram formalism in summarizing leading and next-to-leading log calculations implies that higher-order reggeon diagrams will similarly summarize the contribution of arbitrarily non-leading logs. Furthermore, the structure of such diagrams is essentially determined by the abstract reggeon unitarity equations derived from general multi-Regge theory.[2]

3. Infra-red Divergences

The basic infra-red divergence of the reggeon diagram formalism as $M^2 \to 0$ is that associated with the propagator particle pole i.e.

$$\int \frac{d^2k}{k^2 + M^2} \quad \underset{M^2 \to 0}{\longrightarrow} \quad ln M^2 \delta^2(k) \tag{3}$$

This divergence occurs in the trajectory function $\Delta(k^2)$ given by (2). As a result the divergence actually appears in the denominator of (1) and completely removes the E-dependence of the amplitude - giving zero when the Fourier transform is computed. If this denominator is expanded to reproduce the original perturbation expansion then the result is an exponentiation of infra-red divergences. The Regge pole amplitude of (1) is the simplest example of "exponentiation of infra-red divergences" described by reggeon diagrams.

In general diagrams there is an interplay between the divergence in the trajectory function and divergences in the interaction. This interplay is studied by defining a kernel which includes both. For I = 1,2, $\cdots$ the two-reggeon kernel diverges as $ln\ M^2$, whereas for I = 0 the $ln\ M^2$ divergences cancel and the kernel is a scale-invariant distribution. Combined with the "Ward Identity" zero structure of the four-reggeon interaction at zero transverse momenta, this divergence pattern gives the following (well-known) results[1]

A Only I=0 reggeon diagrams survive as $M^2 \to 0$.

B Off-shell I=0 reggeon amplitudes are finite.

C I=0 particle/reggeon amplitudes are finite.

D I=0 particle/particle amplitudes are *infinite.* Because of the Ward Identity zeroes this divergence does *not exponeniate.*

These results actually imply that scattering amplitudes given by pure gluon reggeon diagrams do not have a sensible zero-mass limit.

4. Massless Quarks

Quarks reggeize[1] in the same way as gluons in a massive gauge theory *but* giving the quarks zero mass does not immediately produce infra-red divergences. In general quark loops contribute to gluon (reggeon) interactions only at (very) non-leading log levels and such interactions can not be distinctively isolated. To calculate their contribution in our formalism we have to be able to isolate the corresponding reggeon diagrams in a well-defined way. In general this requires the quark state to carry flavor quantum numbers. However, in a *non-abelian theory* quark/antiquark reggeon exchange can, after some analysis involving multi-Regge amplitudes,[1] be shown to produce a four reggeized gluon interaction vertex of the form

$$V_Q(q,k,k') = \int d^2p \frac{Tr\left[(-\not{p}+\not{q}+m)\not{k}(-\not{p}-\not{q}+m)\not{k}'\right]}{[(p+q)^2+m^2]\,[(p-q)^2+m^2]} \tag{4}$$

This vertex has some important properties. Note first that

$$V_Q \quad \underset{\substack{m \to 0 \\ q \to 0}}{\sim} \quad 8\pi\left[\frac{2(q\cdot k)(q\cdot k')}{q^2} - k\cdot k'\right] \quad \underset{\substack{q=\pm k \\ q=\pm k'}}{\longrightarrow} \quad 8\pi k\cdot k' \tag{5}$$

Therefore, V_Q does not have the vital "Ward-Identity zero" property discussed above - which we expect to be associated with gauge invariance. In fact this is no different to the conventional Ward-Identity violation from the ultra-violet region of a fermion loop. As we briefly discuss later, our infra-red analysis must involve a transverse momentum cut-off, and so we *can not make a Pauli-Villars subtraction* to restore the Ward Identity property. (This may well be related to Gribov's argument[3] that quark loops should not be Pauli-Villars subtracted initially.)

5. Further Infra-red Analysis and the Anomalous Odderon

Because of the quark-loop modification of the four-reggeon interaction it appears at first sight that if we repeat the infrared analysis leading to the results described in Section 2, then *all* reggeon diagrams exponentiate to zero. Certainly the particle/particle scattering amplitudes which were infinite now vanish.

There remains, however, one form of potential divergence which we have not yet discussed. We recall the scale-invariance of the I=0 kernel noted above. This scale-invariance is responsible for the generation of the Lipatov Pomeron[4] at large transverse momentum but as we now discuss, can also be responsible for an infrared divergence of the form

$$\int \frac{d^2k_1\cdots d^2k_4}{(k_1^2+M^2)\cdots(k_4^2+M^2)} K^{\circ}(k_1,\cdot\cdot,k_4) \quad \underset{M^2 \to 0}{\sim} \quad lnM^2 \tag{6}$$

A particular place where a divergence of this form might be expected to occur is in diagrams where the $I=0$ "soft" glue configuration accompanies an $I=0$ combination of quarks. However, we anticipate that this divergence will, in general, exponentiate via a reggeon interaction involving massless quark loops similar to that of (4). To avoid this, the reggeon interaction must be forbidden for some reason.

Consider now the $I=0$ odd-signature three reggeon state with the quantum numbers of $O = \epsilon_{ijk}A^iA^jA^k$ where A^i is the gluon color matrix. This "anomalous odderon" state carries odd-signature $\tau = -1$, but (in contrast to the state identified[4] as an odderon in perturbative QCD) carries anomalous color charge parity $C = +1$. As a result there must be a helicity transition for it to couple at zero transverse momentum, in violation of the perturbative chiral symmetry of the massless quark theory we are considering.

Since the anomalous odderon kernel is scale-invariant it can produce an infrared divergence of the form (6) accompanying quark states. Indeed if a quark-antiquark pair is involved carrying "non-anomalous" quantum numbers, i.e. $C = +1$, $P = +1$, then there will be no exponentiation. This is because helicity conservation will ensure that the reggeon interaction which would be involved, vanishes at zero transverse momentum.

But if perturbative chiral helicity-conservation persists in all reggeon couplings then the anomalous odderon scaling divergence will simply not couple to any states in the theory. To avoid this, reggeon diagrams must somehow reproduce the helicity transitions of non-perturbative instanton interactions! Indeed a "triangle" diagram formed from *transverse momentum* propagators, could give a (zero transverse momentum) quark mass infra-red divergence violating helicity conservation. Since the ultra-violet triangle anomaly does not occur in any perturbative diagram, its infrared occurrence in reggeon diagrams has to be a consequence of the Regge limit.

6. The Transverse-Momentum Anomaly and the Odderon Condensate

It is very important that the quark transverse momentum triangle diagram we need *does not* occur in a general "triple-Regge" coupling of multigluon exchanges. The multiple discontinuity structure of multi-Regge diagrams determines that it occurs only in a special component[1,2] of the triple-Regge amplitude which does not appear in inclusive cross-sections and more importantly does not appear in the reggeon diagrams describing elastic scattering.

The outcome is that the triangle anomaly does produce a coupling for the "anomalous-odderon" scaling infrared divergence, but *only in very special multi-Regge configurations.* The divergence can not occur in a single reggeon channel but must *span a multi-Regge diagram.* This implies that not only is exponentiation avoided, but in a multi-Regge amplitude there is a distinct divergence associated with each reggeon channel. This divergence therefore selects the "physical" reggeon states and can be viewed as producing a reggeon condensate - that is a zero transverse momentum configuration accompanying stable (against exponentiation) finite momentum (quark) states.

7. Massless Mesons and Chiral Symmetry Breaking

The requirement of stability in the presence of the "condensate" means that the finite momentum quark states (with flavor) must be SU(2) color singlets and carry *normal* color charge parity. At this stage it is important that the "nonsense" nature of reggeon states implies that "mesons" formed this way will be massless as a result of the masslessness of the quarks. [If $j = \alpha_Q(t)$ is the quark trajectory, then[2] $j = 2\alpha_Q(t/4) - 1$ is the trajectory for a two-quark reggeon state. If $\alpha_Q(0) = \frac{1}{2}$ then $2\alpha_Q(0) - 1 = 0$, giving a spin-zero massless state].

Since the anomalous odderon converts a normal parity quark state to "abnormal" parity, the physical massless "meson" states in SU(2) gauge theory are actually *pseudoscalars.* This implies that we have both confinement and *chiral symmetry breaking* in our physical spectrum.

Note that quark/antiquark reggeon states survive (and are converted to even-signature pseudo-scalar Regge pole states by the condensate) because they are infra-red finite. The $I = 0$ gluon states are not infrared finite and so do not survive. Consequently, in this analysis, there is *no Pomeron* in SU(2) gauge theory. There are scattering amplitudes involving meson exchange but asymptotically, since there is no Pomeron, all amplitudes go to zero like a power and the S-Matrix is "trivial"[2] in an extreme sense.

8. Physical Significance of the Condensate

Having argued that the anomalous odderon condensate is responsible for confinement and chiral-symmetry breaking in the low transverse-momentum region of reggeon diagrams, we can understand its physical significance as follows. First we note that the configuration generating the condensate couples only via the quark triangle anomaly (in transverse momentum diagrams). Also the anomalous odderon reggeon state is an angular momentum continuation from states which include those created by the "Winding-Number Operator"

$$K^{o} = -\frac{2}{3}\epsilon_{o\mu\nu\delta}\epsilon^{ijk}A^i_\mu A^j_\nu A^k_\delta + \epsilon_{o\mu\nu\delta}F^i_{\mu\nu}A^i_\delta \qquad (7)$$

Consequently we can view the reggeon condensate as a "Generalized Winding-Number Condensate" which arises from "regularization of the quark sea in the presence of the anomaly". We note that this closely parallels what happens in the Schwinger model.[5] That our low transverse momentum, high-energy, solution of SU(2) gauge theory can be described as a theory of non-interacting pseudoscalar mesons further emphasizes the Schwinger model parallel.

9. Validity of the Analysis

To argue that we can reach a pure gauge theory by an infra-red analysis of a spontaneously-broken gauge theory we must appeal to the lattice gauge principle[1] of *complimentarity.* In our context we interpret this to mean that if we use fundamental representation Higgs scalars and a transverse momentum cut-off then we can smoothly reach the massless theory we want by an infra-red limit from the corresponding massive theory. For an SU(N) theory, decoupling the scalar fields one-by-one builds up the gauge symmetry via the sequence $\rightarrow\ SU(2)\ \rightarrow\ SU(3)\ ...\rightarrow\ SU(N)$.

The infra-red divergences of reggeon diagrams do not incorporate the infra-red growth of the gauge coupling and so the analysis we describe is only immediately valid if we add sufficient massless quarks to the theory to produce an infra-red fixed-point preventing such growth.[1] This is presumably related to the vital role played by massless quarks in the analysis. For gauge groups larger than SU(2) the large number of quarks also implies asymptotic-freedom for some of the scalar fields involved. As a result the transverse momentum cut-off can be removed before the complete gauge symmetry is restored. [Note that in the SU(2) theory we have studied we have effectively performed all manipulations with a transverse cut-off. There is no obvious obstacle to removing it within the high-energy (trivial) S-Matrix!]

10. SU(N) Gauge Theory

In this talk I have concentrated on the analysis of SU(2) gauge theory with many quarks. I have no space to elaborate on results for higher gauge groups except to note that the (no Pomeron) "Schwinger Model solution" of SU(2) theory provides the basis for understanding the emergence of a Pomeron with all the right physical properties in QCD. If the gauge symmetry is increased still further a more complicated Pomeron spectrum emerges. Although I have only very preliminary ideas on how it might work, I believe that the corresponding Super-Critical Pomeron theory may one day provide a basis for understanding the origin of dynamical symmetry-breaking of a unification gauge group when there is a large number of fermions present. (The large number of fermions may, of course, be desirable to produce many of the phenomena currently associated with Walking Technicolor).

References

1. More details, and or relevant references, can be found in A. R. White, *Light Quarks and the Pomeron in QCD*, ANL-HEP-CP-91-54, and Proceedings of the Fourth Blois Workshop on Elastic and Diffractive Scattering, to be published in Nucl. Phys. B (Proc. Suppl.).

2. A. R. White, *Int. J. Mod. Phys.* **A11**, 1859 (1991).

3. V. N. Gribov, *Phys. Lett.* **B194**, 119 (1987).

4. L. N. Lipatov, in *Perturbative QCD* edited by A.H. Mueller, World Scientific (1989).

5. N. S. Manton, *Ann. Phys.* **159**, 220(1985).