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L.L.Jenkovszky, B.Kämpfer, V.M.Sysoev

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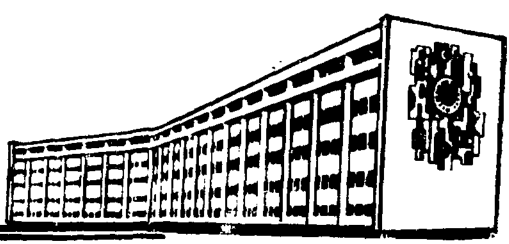

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Инфляция Вселенной и метастабильная кварк-глюонная плазма

В рамках модели Фридмана с уравнением состояния $p(T)=aT^4-AT$ показано, что в состоянии метастабильной кварк-глюонной плазмы Вселенная расширялась экспоненциально. Масштабный фактор на этой стадии увеличился на много порядков.

L.L.Jenkovszky, B.Kämpfer, V.M.Sysoev

Inflating Metastable Quark-Gluon Plasma Universe

We show within the Friedmann model with the equation of state $p(T)=aT^4-AT$ that our universe has expanded exponentially when it was in a metastable quark-gluon plasma state. The scale factor during that epoch increased by many orders of magnitude.

The inflation of the universe is usually associated (see, e.g. [1]) with phase transitions in unified field theories at very early stages of the cosmic evolution. Less familiar is the possibility of a later (mini-) inflation that might have occured [2 -4] prior to the confinement phase transition. The prefix "mini" indicates the modest increase of the scale factor during the latter [2-4] as compared to the former [1]; during the mini-inflation [2-4] the expansion does not reach the exponential regime, typical of a "true" inflation.

In the present paper we show that the latest inflation of the universe, i.e. its expansion during the epoch dominated by the quark-gluon plasma actually might have been as violent as the earlier ones.

To begin with, we remind the reader that in a homogeneous, isotropic and spatially flat universe the evolution of the scale factor R and the energy density ε of a perfect fluid are determined by the Friedmann equation

$$\dot{R} - GR\sqrt{\varepsilon} = 0 \,, \tag{1a}$$

$$\dot{\varepsilon} + 3(\dot{R}/R)(\varepsilon + p) = 0 \,, \tag{1b}$$

where p is the pressure, $G=\sqrt{8\pi/3}/M_p$, $M_p = 1.2\cdot 10^{19}$ GeV. The relation between ε and p is given by the equation of state (EOS). We shall use also the thermodynamic relation

$$\varepsilon = p'(T)T - p(T) \,, \tag{2}$$

where T is the temperature, valid for vanishing chemical potential, $\mu = 0$ (throughout this paper this will be implied).

The cosmic evolution is called to be an inflationary one, if the expansion is accelerated, i.e. $\ddot{R} > 0$. From eqs.(1) it follows

$$\ddot{R} = -G^2 R(\varepsilon + 3p)/2 \,. \tag{3}$$

Hence, the necessary condition for inflation is

$$3p + \varepsilon < 0 \,. \tag{4}$$

Condition (4) can be attained in an extension of the bag EOS including metastable states of supercooled quark-gluon plasma (as well as that of overheated hadron gas) [5] (fig.1).

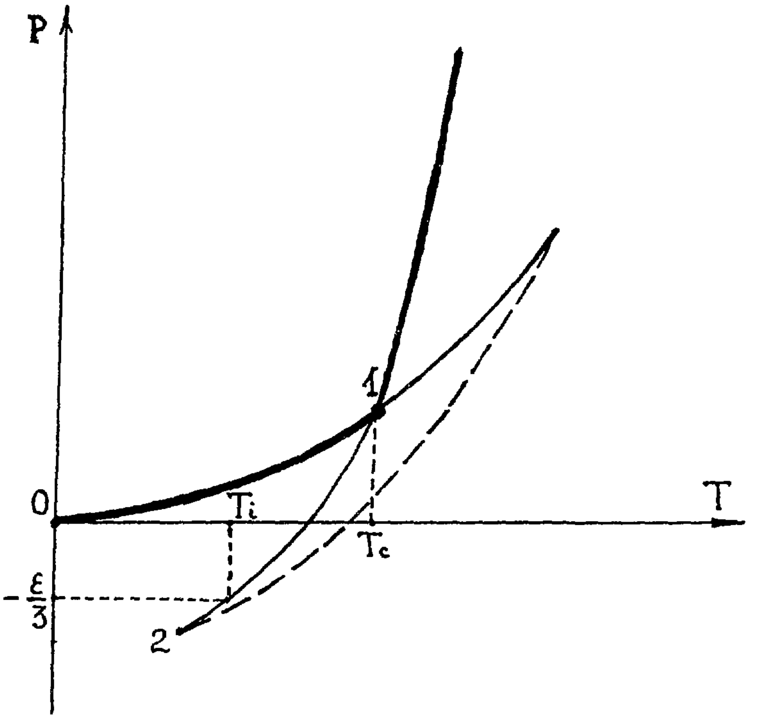

Fig.1. Generalization [5] of the bag EOS that includes metastable states of supercooled quark-gluon plasma (branch 1-2). The point T_i indicates the temperature at which, according to (4), inflation starts (for more details see [4,5]). Point 2 is the spinodal, below which the system becomes unstable.

Another class of EOS realizing metastable states with negative pressure has been derived [6] from the S-matrix formulation of statistical mechanics (for a recent analysis see ref.[7]). Contrary to the familiar picture of fig.1, the latter contains a minimum in the P vs. T plot (fig.2) at finite temperatures. As shown in ref.[8], condition $p'(T)=0$ (which, according to (2) is equivalent to $\varepsilon=-p$) is sufficient to produce inflation.

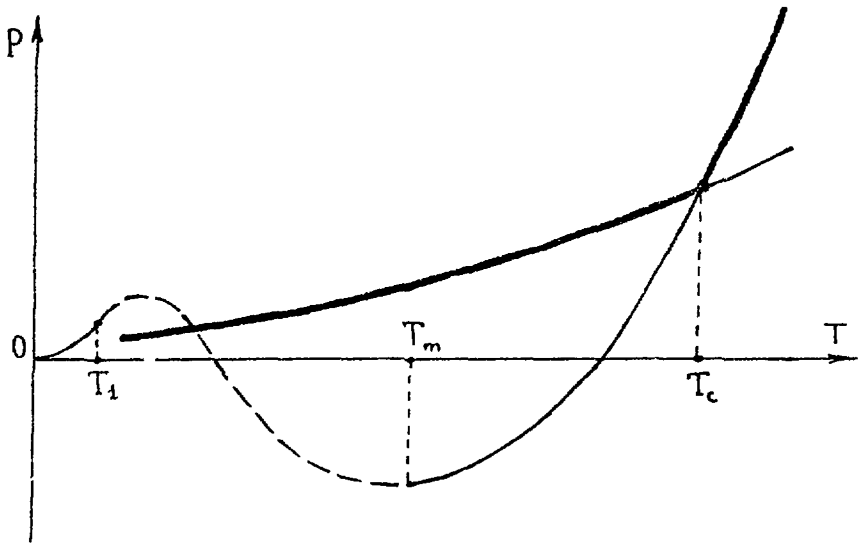

Fig.2. The S-matrix EOS (see [7] and references therein). By supercooling one can penetrate into metastable states with negative pressure and approach the spinodal point at T_m. Note that further cooling will result in a phase transition since the interval $(T_1; T_m)$ is thermodynamically forbidden.

To show this let us rewrite, following ref.[8], eqs.(1a) and (1b) as

$$t = \int \frac{dR}{R\sqrt{G^2\varepsilon}} , \tag{5}$$

$$3 \ln R = -\int \frac{d\varepsilon}{p+\varepsilon} , \tag{6}$$

where $G^2 = 5{,}7 \cdot 10^{-38} \mathrm{GeV}^{-2}$. Eq.(1b) can be integrated and we get by using (2)

$$R^3 = \frac{\mathrm{const}}{p'(T)} , \tag{7}$$

reflecting entropy conservation ($p' = S$ is the entropy density). By inserting (7) into (5) we get for the temporal evolution of the system

$$t = \frac{1}{3} \int_T^\infty \frac{p''(T)\,dT}{p'(T)\,G\sqrt{\varepsilon(T)}} \quad . \tag{8}$$

By expanding $p'(T)$ around $T = T_m$

$$p'(T) \sim T - T_m \tag{9}$$

and inserting (9) into (8) we get an exponential solution for (8).

Paper [8] discusses also other cosmological problems related to inflation, such as the problem of the horizon. Even though EOS of fig.2, giving rise to inflation, is an interesting alternative to that of fig.1, the lack of any direct link to the popular QCD-inspired picture of the hadronic world and large freedom in the numerical values of its parameters (see [7]) require verification of the S-matrix EOS, its relation to current models, as well as its observable consequences.

There is however a class of bag-like EOS sharing the important property of the S-matrix EOS, namely $p'(T) = 0$ (for $T \neq 0$), necessary to produce inflation. It reads [9,10]

$$\begin{aligned} p_q(T) &= a_q T^4 - AT \quad , \\ p_h(T) &= a_h T^4 \quad . \end{aligned} \tag{10}$$

(EOS (10) differs from the conventional bag model by the presence of the term AT instead of the "bag constant" B).

The values of the numerical parameters of (10) are:
$a_q = 1{,}75$, $a_h = 0{,}33$ and $A = (a_q - a_h)T_c^3 = 1{,}14 \cdot 10^{-2}\,\mathrm{Gev}^3$.
With these values we get the characteristic points of the EOS (7): $T_0 = 0{,}89\sqrt[3]{A}$, T_1 (where $p = -\varepsilon/3$) $= 0{,}74\,T_c$, T_m (where $p = -\varepsilon$) $= 0{,}59\,T_c$.

EOS (10) was originally suggested in ref.[9] and rederived in [10]. Its behaviour near the critical point of the phase transition has been discussed recently in ref. [11]. More important from the point of view of a possible inflation is the behaviour of its metastable branch corresponding to the supercooled quark-gluon plasma (interval 1-. in fig.3). As seen from fig.3, the

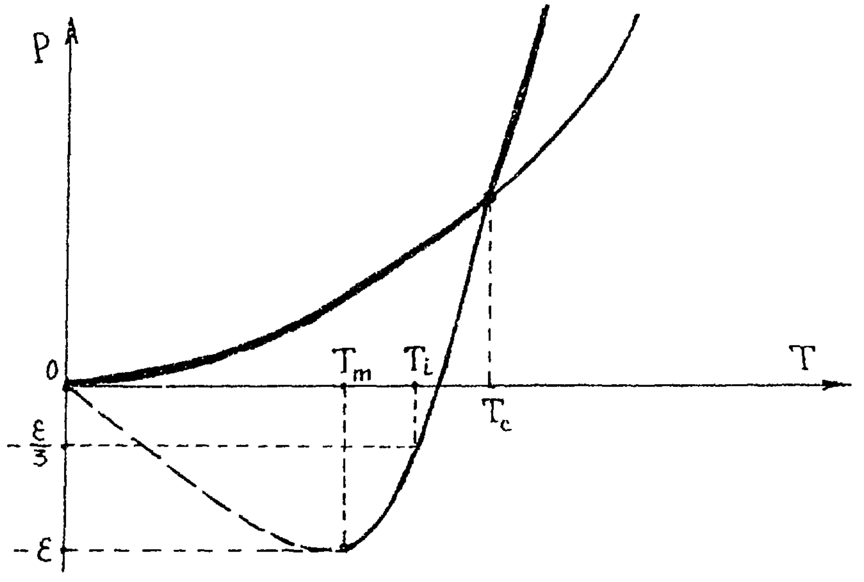

Fig.3. Schematic plot of the EOS (10) of refs.[9,10]. T_i is the temperature where inflation starts and T_m is the spinodal point $(p'(T)=0)$.

metastable branch of EOS (7) has the same qualitative feature $p'(T)=0$ (probably overlooked by other authors, interested in the critical point) as that of fig.2 which, according to (8), can lead to inflation.

To see if this can really happen one has to check the maximal depth of supercooling, i.e. to check whether the system can reach the conditions necessary for inflation before a phase transition will convert the quark-gluon plasma into hadronic gas.

In our previous paper [5] we have treated this problem by means of the so-called Ginzburg parameter, known in the general theory of phase transitions. In view of the large uncertainty in the value of this parameter, here we use a different method to evaluate the nucleation barrier W .

The condition for penetration into the metastable state (i.e. absence of a phase transition) is that [12]

$$G^{*} = W/T > 1 , \qquad (11)$$

where G^* is called the Gibbs parameter.

If the nuclei in the new phase are spherically symmetric [12] then the barrier for the creation of a new phase is

$$W = \frac{16\pi}{3} \frac{\sigma^3}{(\Delta p)^2} , \tag{12}$$

where σ is the interface tension and $\Delta p = |p_h - p_q|$ is the depth of supercooling the plasma (see fig.3). By using (12), the inequality (11) can be rewritten as

$$G^* = \frac{16\pi\sigma^3}{3(\Delta p)^2 T} > 1 . \tag{13}$$

By using the value $\sigma = 0.26T_c^3$ [11], we get for $T = T_m$:

$$G^* = 1{,}13 > 1 , \tag{14}$$

which means that the spinodal point can be safely reached before a phase transition occurs.

Note that deep supercooling in the early universe was possible only because - contrary to conventional media, like water - it was a very pure (clean) system. Small contaminations by, e.g. inclusion of flavour dynamics [13] could slightly modify the above estimate.

Now we integrate the Friedmann equations with the EOS (10). The value of the nucleation barrier established above means that the system will cool down along the quark-gluon branch of EOS (10) up to the spinodal point $T = T_m$.

By inserting $p(T) = a_q T^4 - AT$ into (6) we get

$$\mathcal{E} = \left[(4 + 3R^3 C_1 b)/(4R^3 C_1)\right]^{4/3} , \tag{15}$$

where $b = A/\sqrt[4]{3a_q}$ and C_1 is an integration constant. Further, by inserting (15) into (1a) we get

$$\ln(1+\gamma R^3) + 3\ln(1+U+U^2) - 2\sqrt{3}\,\mathrm{arctg}\frac{2}{\sqrt{3}}(1+U) = 6G\left(\frac{3b}{4}\right)^{2/3} t + C_2 , \tag{16}$$

where C_2 is another integration constant and

$$U = R\sqrt[3]{\gamma/(1+\gamma R^3)} , \qquad \gamma = 3C_1 b/4 .$$

By expressing the integration constant in terms of $R(T_c)$, we get $\left(R(T_c) = R_c \right)$

$$3\ln\frac{R}{R_c}+\ln\left[\left(g+\frac{R_c^3}{R^3}\right)\left(1+U+U^2\right)^3\right]-2\sqrt{3}\,\mathrm{arctg}\frac{2}{\sqrt{3}}(1+U)=6\,G\left(\frac{3}{4}b\right)^{2/3}t+C_2, \quad (17)$$

where $g=a/(3a_q-a_h)$ and a is a_q-a_h .

For large values of the scale factor R eq.(17) reduces to

$$3\ln\frac{R}{R_c}+\ln(27g)-2\sqrt{3}\,\mathrm{arctg}\frac{4}{\sqrt{3}}=6\,G\left(\frac{3}{4}b\right)^{2/3}t+C_2 \quad (18)$$

and we get the exponentially expanding universe

$$R=R_c\exp\left[K(t-t_c)\right], \quad t_c=t(T=T_c), \quad (19)$$

where

$$K=2\,G\left(\frac{3}{4}b\right)^{2/3}\sim 10^5\ \frac{1}{\mathrm{sec.}} \quad (20)$$

The duration of inflation, according to (8), is

$$t=t_0\left[\frac{1}{3}\ln\frac{\sqrt{1+\tau+\tau^2}}{|1-\tau|}+\frac{1}{\sqrt{3}}\,\mathrm{arctg}\,\frac{\tau\sqrt{3}}{2+\tau}-\frac{\pi}{3\sqrt{3}}\right], \quad (21)$$

$$t_0=0{,}875\cdot 10^{-4}\,\mathrm{sec.}, \quad \tau=T/T_m .$$

It can be seen from (21) that the duration of the inflation, i.e. the approach to the spinodal point T_m could be infinitely long. Actually, this process terminates by a phase transition.

To illustrate the increase of the scale factor during our inflation we estimate their ratio at two moments of time, namely $t_1\sim 10^{-5}$ sec. and $t_2\sim 10^{-3}$ sec. We get

$$\frac{R_2}{R_1}\approx 10^{0{,}43K(t_2-t_1)}\sim 10^{20\div 100} .$$

The inflation of the universe under discussion is interesting particularly for two reasons: 1) it is the most important from the point of view of the observations since it has "washed out" the traces of those which occured earlier; 2) EOS (10) can be checked in laboratories, in experiments with heavy ion collisions.

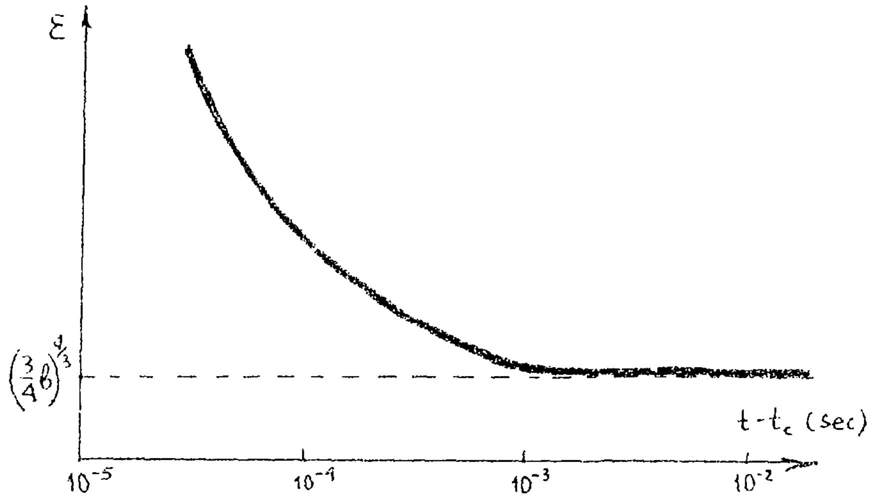

Fig.4. Time dependence of the energy density.

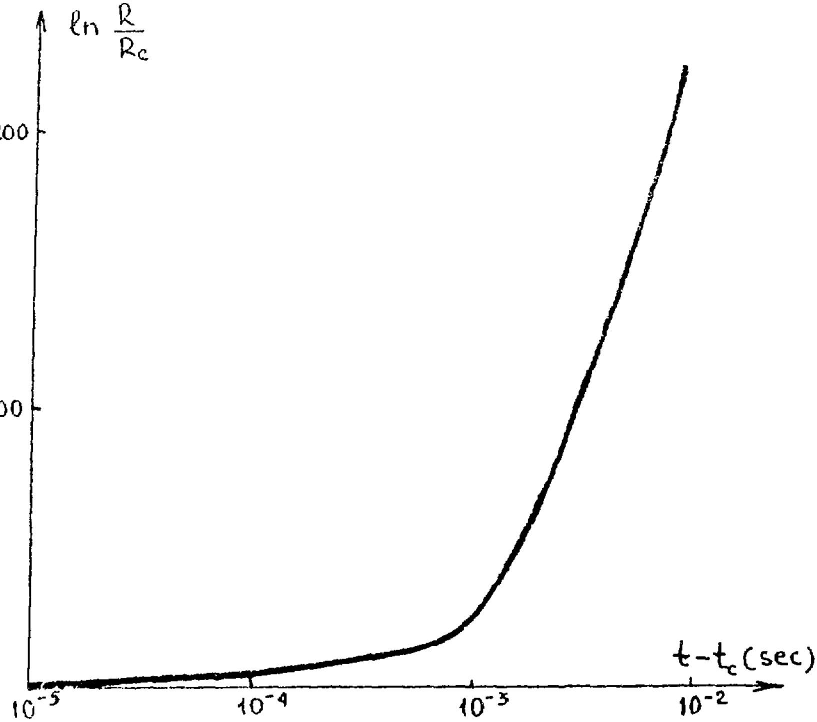

Fig.5. Time dependence of the scale factor.

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