

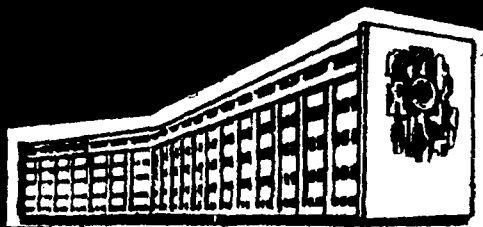
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MUONIC ATOM - LIGHT NUCLEUS
INTERACTION



УДК 539.171, 539.186.2

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Взаимодействие мезоатомов с легкими ядрами

На основе корректного рассмотрения задачи трех кулоновских частиц в аналитическом виде найден эффективный потенциал взаимодействия легкого ядра с двухчастичным атомом на расстояниях, превышающих его борковский радиус. Изучены особенности взаимодействия ядер p , t , ${}^4\text{He}$ и ${}^7\text{Be}$ с мезоатомами μp , μt , $\mu{}^4\text{He}$ и $\mu{}^7\text{Be}$, обусловленные различием масс и зарядов взаимодействующих частиц. Приведены соответствующие потенциалы в допороговой области энергий. Вычислены коэффициенты асимптотической формулы для эффективного потенциала в адиабатическом приближении и с учетом главных внеэнергетических поправок.

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Muonic Atom - Light Nucleus Interaction

The effective potential of the interaction between light nucleus and two particle atom at distances greater than its bohr radius is obtained in the analytic form on the basis of a correct account of three Coulomb particle problem.

Features of the interaction between p , t , ${}^4\text{He}$, ${}^7\text{Be}$ nuclei and muonic atoms μp , μt , $\mu{}^4\text{He}$ and $\mu{}^7\text{Be}$, that arising from the differences in masses and charges of interacting particles, are studied. The corresponding potentials in the pre-threshold energy range are given. The coefficients of the asymptotic formula for the effective potential are calculated in adiabatic approximation and with regard for the main off-shell corrections.

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1. INTRODUCTION

Studies of the negative muon μ^- dynamics in the hydrogen isotope medium with ingredients of light nuclei p, t, ^4He , ^6Li , ^7Li , ^7Be is extremely important due to the problem of the muon catalyzed fusion (see, e.g.^{1,2}). The investigation of the interaction between simplest mesonic atoms and light nuclei in low (heat) energy range is then of special interest. This study can be performed within the model of a three nonrelativistic electrically charged particle system. Since the application of the Faddeev theory to such systems comes across certain difficulties³⁻⁶, the elaboration of the other approaches is still to be actual.

In refs. 8-10) we have developed the method allowing to calculate rigorously the effective interaction potential between a charged particle and a system of two other charged particles under arbitrary ratios between their charges and masses. Here, on the basis of a correct consideration of the three particle problem with pair Coulomb interactions we have obtained the effective interaction potential between light nucleus and a two-particle atom at distances exceeding its Bohr radius. Our effective potential theory is applied to the system of the light nucleus A and mesonic atom μB in the cases, when p, t, ^4He , ^7Be particles are taken as A and B nuclei. Studies of the mentioned above three-particle system allows to extract the peculiarities of the interaction between nucleus A and μB atom as a whole, caused by the differences both in masses and charges of A nucleus and μB atom.

The article is organized as follows. In sect. 2 the theory of the effective potential for three nonrelativistic particles with Coulomb pair interaction is given. The basic equations for determining the interaction potential between the nucleus and two-particle atom using a solution of the integral equation taking rigorously into account the multiparticle dynamics of the system are obtained. In sect. 3 it is shown, that for energies of a relative nucleus - atom motion less than the threshold of an atom decay into two particles, the effective

potential is rigorously local. The explicit expressions for this potential in the adiabatic approximation and with regard for the main nonadiabatic corrections at distances greater than the atom's Bohr radius is written. The effective potential in the asymptotic region of large distances between the nucleus and the atom is obtained in sect. 4. Sect. 5 deals with application of the developed theory to a problem of the interaction between light nuclei and mesonic atoms.

2. THE EFFECTIVE POTENTIAL

Let us consider a nonrelativistic motion of a charged particle A (nucleus) with the mass m_A and charge $Z_A e$ in the electric field of an atom consisting of B nucleus and light particle c with the masses and charges m_B, m_c and $Z_B e, Z_c e$ respectively. Here and in what follows under the cB atom we mean arbitrary hydrogen-like system of two pairwise Coulomb interacting particles. The hamiltonian of the system has the form

$$H = -\Delta_R + h_r + v(R, \xi) \quad , \quad (1)$$

where h_r is the hamiltonian of the cB atom

$$h_r = -\Delta_r - \frac{Z}{r} \quad , \quad (2)$$

whose spectrum of states is well known ¹¹⁾ and $v(R, \xi)$ describes the interaction between A nucleus and particles c and B,

$$v(R, \xi) = 2s_A s_c \left(\left| \frac{Z_A}{Z_B} \right| \frac{1}{|AR - \xi\xi|} - \left| \frac{Z_A}{Z_c} \right| \frac{1}{|AR + \xi\xi|} \right) \quad . \quad (3)$$

We denoted here by

$$\xi = \xi_c - \xi_B, \quad R = \frac{m_c \xi_c + m_B \xi_B}{m_c + m_B} - \xi_A$$

the Jacobi coordinates, $s_1 = \text{sign } Z_1$, $s_c \cdot s_B = -1$. The constants λ, ζ and ξ are the numbers

$$\Lambda = \left(\frac{m}{M} \right)^{1/2}, \quad \xi = \frac{m_B}{m_c + m_B}, \quad \zeta = 1 - \xi, \quad (4)$$

where

$$m = \frac{m_c m_B}{m_c + m_B}, \quad M = \frac{(m_c + m_B) m_A}{m_A + m_B + m_c}$$

are the reduced masses.

The hamiltonian (1) is written in Rydberg energy units $E_0 = \hbar^2/2ma^2$, where $a = a_0/|Z_c Z_B|$, $a_0 = \hbar^2/me^2$ is the Bohr radius of atom cB. Since we are interested in $R > 1$ fm distances, neglecting in (3) by the A-B nuclear interaction is then justified. We note, that the relative coordinates \mathbf{R} and \mathbf{r} in (3) are expressed in the a units. The magnitude E_0 is equal to the cB atom decay threshold energy into particles c and B .

We denote the wave functions of the cB atom as $\langle \mathbf{r} | f \rangle$, where $f = nl$ for all bound states with the main quantum number n and relative angular momentum l ; $f = q$ for the continuous spectrum states of cB-system with momentum q of a relative motion of the particles c and B . The energy spectrum of states is given by the numbers $\epsilon_n = -1/n^2$ and $\epsilon_q = q^2$, so that

$$h_{\mathbf{r}} |f\rangle = \epsilon_f |f\rangle. \quad (5)$$

We shall describe a three-particle system AcB by the Schrödinger equation

$$H|\Psi\rangle = E|\Psi\rangle \quad (6)$$

with the hamiltonian (1) determining its wave function $\langle \mathbf{R}, \mathbf{r} | \Psi \rangle$. Then, using the method developed in refs. ⁸⁻¹⁰) a multiparticle equation (6) can be rigorously reduced to a single-particle one with the effective potential that contains the whole information on multiparticle dynamics of the nonrelativistic system.

Let atom cB before its interaction with nucleus A is in the $|1s\rangle$ -state. Then the total energy of the system is: $E = k^2 - 1$, where k^2 is an energy of the particle A and an atom relative motion. Projecting Eq.(6) onto the initial $|1s\rangle$ state and all others $|f\rangle$ -states with $f \neq 1s$, we get a system of coupling equations for the functions $\langle 1s | \Psi \rangle$ and $\langle f | \Psi \rangle$, that describe the motion of the nucleus A relative to atom cB in $|1s\rangle$ and $|f\rangle$ states,

$$(\Delta_R + k^2) \langle 1s | \Psi \rangle = \langle 1s | v | 1s \rangle \langle 1s | \Psi \rangle + \sum_{f'}' \langle 1s | v | f' \rangle \langle f' | \Psi \rangle, \quad (7)$$

$$(\Delta_R + E - \epsilon_f) \langle f | \Psi \rangle = \langle f | v | 1s \rangle \langle 1s | \Psi \rangle + \sum_{f'}' \langle f | v | f' \rangle \langle f' | \Psi \rangle, \quad f \neq 1s, \quad (8)$$

where the prime on the summation symbol denotes that the term with $f'=1s$ is absent. To exclude the functions $\langle f | \Psi \rangle$ from eq.(7) the solution of eq.(8) relative to $\langle f | \Psi \rangle$, is shown as

$$\langle f | \Psi \rangle = G_o(E - \epsilon_f) \langle f | U | 1s \rangle \langle 1s | \Psi \rangle, \quad (9)$$

where $G_o(E - \epsilon_f) = (E - \epsilon_f + \Delta_R + i0)^{-1}$ is a free two-particle Green's function for the outgoing wave, $\langle f | U | 1s \rangle$ is the matrix element of a new operator U , satisfying the integral equation

$$U = v + vG'(E)U \quad (10)$$

with the truncated three-particle Green's function

$$G'(E) = \sum_f' |f\rangle G_o(E - \epsilon_f) \langle f| \quad (11)$$

Substituting (9) into eq.(7) we find the equation for the function $\langle 1s | \Psi \rangle$

$$\left[\Delta_R + k^2 - \langle 1s | U | 1s \rangle \right] \langle 1s | \Psi \rangle = 0, \quad (12)$$

which coincides in its form with a single-particle Schrodinger equation where as the interaction potential between nucleus A and atom oB the matrix element of the operator U in $|1s\rangle$ states is used. The eq.(10) contains the entire information about the multiparticle dynamics of a three-particle model under consideration. Thus, eq.(10) determines the effective potential $\langle 1s | U | 1s \rangle$ in a rigorous way.

In the approximation taking into account only $|1s\rangle$ -state of the oB atom we find from (10) with account of (3) and the explicit form of the functions $\langle x | 1s \rangle$ ¹¹⁾

$$\langle 1s | U | 1s \rangle \approx \langle 1s | v | 1s \rangle = \frac{2Z_o}{x} - 2Z_A Z_o \left[\left| \frac{Z_A}{Z_B} \right| e^{-\frac{2x}{\xi}} \left(\frac{1}{x} + \frac{1}{\xi} \right) \right] \quad (13)$$

$$- \left| \frac{Z_A}{Z_C} \right| e^{-\frac{2x}{\zeta}} \left(\frac{1}{x} + \frac{1}{\zeta} \right) \Bigg\} ,$$

where we denote $x=AR$. The magnitude α_0 plays the role of an effective charge within the units of energy and length chosen by us. The charge is determined by

$$\alpha_1 = s_A s_C \left[\left| \frac{Z_A}{Z_B} \right| \zeta^1 + (-1)^{l+1} \left| \frac{Z_A}{Z_C} \right| \zeta^1 \right] \quad (14)$$

at $l=0$.

We introduce the parameter $R_0 = N\zeta/2A$, here N is arbitrary integer positive number. Then with accuracy up to the terms of e^{-N} order at $R > R_0$ the potential (13) will be the Coulomb one. For the neutral atom we have $\alpha_0=0$, i.e. in the considered approximation the motion of the nucleus A at the distance $R > R_0$ from an atom is free. According to (13) the effects induced by the atomic structure are exponentially small. In the region $R < R_0$ the interaction (13) is different from the Coulomb one and is finite at the point $R=0$:

$$\lim_{R \rightarrow 0} \langle 1s|U|1s \rangle = 2s_A s_C |Z_A| \begin{cases} \frac{1}{|Z_B|\zeta} - \frac{1}{|Z_B|\zeta} & , \zeta \neq 0 , \\ \frac{1}{|Z_B|} & , \zeta = 0 . \end{cases} \quad (15)$$

This means, that the formulation of the effective potential $\langle 1s|U|1s \rangle$ on the basis of the solution to eq.(10) is more natural comparatively to the approaches of refs. 5,6,8,9,12,13) where the Coulomb interaction between the incident charged particle and c.m. of the two-particle target is extracted artificially, and the theory is constructed proceeding from the difference in Coulomb interactions between pairs of individual particles and the Coulomb interaction with respect to the c.m. of a target. According to eqs.(14) and (10) the potential $\langle 1s|U|1s \rangle$ is valid in the entire region of the variable R .

The main effects associated with the atomic structure are described by the second term in the right hand side of eq.(10). Iterating eq.(10) we show its solution in the form of an

infinite series

$$\begin{aligned}
 U = v - \sum_f v|f\rangle \frac{1}{\epsilon_f + 1 - T_R - i0} \langle f|v \\
 \sum_{f,f'} v|f\rangle \frac{1}{\epsilon_f + 1 - T_R - i0} \langle f|v|f'\rangle \frac{1}{\epsilon_{f'} + 1 - T_R - i0} \langle f'|v \\
 - \sum_{f,f',f''} v|f\rangle \frac{1}{\epsilon_f + 1 - T_R - i0} \langle f|v|f'\rangle \frac{1}{\epsilon_{f'} + 1 - T_R - i0} \\
 \times \langle f''|v|f'\rangle \frac{1}{\epsilon_{f''} + 1 - T_R - i0} \langle f''|v + \dots \dots
 \end{aligned} \quad (16)$$

where $T_R = \Delta_R + k^2$. The account of the atomic particle motion leads to the fact that in the intermediate states the energy of the relative motion of the nucleus A and the atom cB is not conserved. The off-shell energy effects are described by the operator T_R and are taken rigorously into account by the series (16).

3. THE LOW ENERGIES

At small k^2 , when the energy transferred to the atom is small the off-shell energy effects make a small contribution to the potential $\langle 1s|U|1s\rangle$ and can be considered rigorously⁽⁸⁻¹⁰⁾. When $k^2 < 3/4$, i.e. lower than the atom decay threshold into two particles A and B we find from (16)

$$U = \sum_{i=0}^{\infty} U^{(i)} \quad (17)$$

where

$$\begin{aligned}
 U^{(0)} &= v, \\
 U^{(1)} &= - \sum_{n=0}^{\infty} \sum_f v|f\rangle \frac{1}{(\epsilon_f + 1)^{n+1}} T_R^n \langle f|v, \\
 U^{(2)} &= \sum_{m,n=0}^{\infty} \sum_{f,f'} v|f\rangle \frac{T_R^m \langle f|v|f'\rangle T_R^n}{(\epsilon_f + 1)^{m+1} (\epsilon_{f'} + 1)^{n+1}} \langle f'|v, \quad (18)
 \end{aligned}$$

$$U^{(3)} = - \sum_{p,m,n=0}^{\infty} \sum_{f,f',f''} v|f\rangle \frac{T_R^p \langle f|v|f'\rangle T_R^m \langle f'|v|f''\rangle T_R^n}{(\epsilon_f + 1)^{p+1} (\epsilon_{f'} + 1)^{m+1} (\epsilon_{f''} + 1)^{n+1}} \langle f''|v\rangle,$$

etc., where the operators T_R^q act on the all expressions to the right from them and also onto the vector $|1s\rangle\langle 1s|\Psi\rangle$ in accordance with eq.(12). The sum (17) is the expansion into a series in powers of the operator T_R . In zero approximation ($m=0, n=0, p=0$) it transforms into a series of the perturbation theory and at the distances $R > R_0$ corresponds to a local adiabatic potential V^{AD} . Comparing (17) and (16) we can see that the adiabatic approximation corresponds to neglecting the operator T_R in denominators of eq.(16). We have in the adiabatic approximation the potential

$$\begin{aligned} \langle 1s|U|1s\rangle \approx V^{AD}(x) &= \frac{2Z_0}{x} - \alpha_1 \frac{Z_1^2}{x^4} - \alpha_2 \frac{Z_2^2}{x^6} \\ &+ \delta_4^{(00)} \frac{Z_1^2 Z_2^2}{x^7} - (\alpha_3 Z_3^2 + \epsilon_4^{(000)} Z_1^4) \frac{1}{x^8} + \dots \end{aligned} \quad (19)$$

The coefficients α , δ and ϵ correspond to the polarizabilities of hydrogen atom of a different multipole whose order corresponds to the subscript l at Z_l . They are obtained from the following formulae ($x > \xi$):

$$\begin{aligned} \sum_f \frac{\langle 1s|v|f\rangle \langle f|v|1s\rangle}{\epsilon_f + 1} &= \sum_{l=1}^{\infty} Z_l^2 \frac{\alpha_l}{x^{2l+2}}, \\ \sum_{f,f'} \frac{\langle 1s|v|f\rangle \langle f|v|f'\rangle \langle f'|v|1s\rangle}{(\epsilon_f + 1)^{m+1} (\epsilon_{f'} + 1)^{n+1}} &= \sum_{l,l',l''=1}^{\infty} Z_l Z_{l'} Z_{l''} \frac{\delta_{ll'l''}^{(mn)}}{x^{2l+3}}, \\ \sum_{f,f',f''} \frac{\langle 1s|v|f\rangle \langle f|v|f'\rangle \langle f'|v|f''\rangle \langle f''|v|1s\rangle}{(\epsilon_f + 1)^{p+1} (\epsilon_{f'} + 1)^{m+1} (\epsilon_{f''} + 1)^{n+1}} &= \sum_{l,l',l''=1}^{\infty} \sum_{l''',l''''=1}^{\infty} Z_l Z_{l'} Z_{l''} Z_{l'''} Z_{l''''} \frac{\epsilon_{ll'l''l'''}^{(pmn)}}{x^{2l+4}}. \end{aligned} \quad (20)$$

where the terms in which

$$2t = 1+l'+1'', \quad 2s = 1+l'+1''+1''' \quad ; \quad t, s = 2, 3, 4, \dots$$

are nonzero. The formulae for calculation of the parameters α , δ and ε with arbitrary angular momentum l are cumbersome. They are given in ref.¹⁰⁾. For the first several values of l we have

$$\begin{aligned} \alpha_1 &= 9/2, \quad \alpha_2 = 15, \quad \alpha_3 = 525/4, \\ \delta_4^{(00)} &= 213/2, \quad \varepsilon_4^{(000)} = 4329/32, \end{aligned} \quad (21)$$

where we have used the notations

$$\delta_4^{(mn)} \equiv \delta_{112}^{(mn)} + \delta_{121}^{(mn)} + \delta_{211}^{(mn)}, \quad \varepsilon_4^{(pmn)} = \varepsilon_{1111}^{(pmn)}. \quad (22)$$

The distinction of the AcB system from e^-e^-p one is described by the parameters α_1 and Λ , which are equal to one in the last case. So, e.g., the electric dipole polarizability of eB atom induced by the nucleus A is equal to

$$\alpha_E = \frac{9}{2} \alpha_1^2 / \Lambda.$$

Nonadiabatic corrections are due to the off-shell energy effects and are described by the sum terms (7) where m , n , and p are nonzero. Taking into account that the operators T_R^q do not commute with v conveying them to the right¹⁰⁾ the sum (17) can be transformed to the form

$$\langle 1s | U | 1s \rangle = V^{AD}(R) + \sum_{q=0}^{\infty} U_q(R) T_R^q, \quad (23)$$

where in the region $R > R_0$ and the s-wave approximation in a relative motion of the nucleus A and the atom eB the functions $U_q(R)$ are

$$\begin{aligned} U_0(R) &= 2\Lambda \left\{ 2\beta_1 \frac{x^2}{x^5} + 3\beta_2 \frac{x^2}{x^7} + 4\beta_3 \frac{x^2}{x^9} \right. \\ &\quad \left. + (3\sigma_4^{(01)} + 2\tau_4^{(10)}) \frac{x^2 x^2}{x^8} \right\} \frac{\partial}{\partial R} \\ &- 24\Lambda^2 \left\{ \gamma_1 \frac{x^2}{x^6} + 2\gamma_2 \frac{x^2}{x^8} \right\} \frac{\partial^2}{\partial R^2} + \dots, \quad (24) \\ U_1(R) &= -\beta_1 \frac{x^2}{x^4} - \beta_2 \frac{x^2}{x^6} + 6\delta_4^{(01)} \frac{x^2 x^2}{x^7} \end{aligned}$$

$$\begin{aligned}
 & - \left[\beta_3 x_3^2 + 2(3\varepsilon_4^{(001)} + 2\varepsilon_4^{(010)}) x_1^4 \right] \frac{1}{x^8} \\
 & + 4\Lambda \left\{ 2\gamma_1 \frac{x_1^2}{x^5} + 3\gamma_2 \frac{x_2^2}{x^7} - (5\sigma_4^{(02)} + 4\tau_4^{(20)}) \frac{x_1^2 x_2}{x^8} + 4\gamma_3 \frac{x_3^2}{x^9} \right\} \frac{\partial}{\partial R} \\
 & - 72\Lambda^2 \left\{ \omega_1 \frac{x_1^2}{x^6} + 2\omega_2 \frac{x_2^2}{x^8} \right\} \frac{\partial^2}{\partial R^2} + \dots
 \end{aligned}$$

etc. Here we have kept the terms up to x^{-9} , containing the derivatives with respect to R up to the second order. The coefficients β, γ, σ and τ are given by* the relations $\langle x \rangle \xi$:

$$\begin{aligned}
 \sum_f' \frac{\langle 1S|v|f \rangle \langle f|v|1S \rangle}{(\varepsilon_f + 1)^2} &= \sum_{l=1}^{\infty} x_l^2 \frac{\beta_l}{x^{2l+2}}, \\
 \sum_f' \frac{\langle 1S|v|f \rangle \langle f|v|1S \rangle}{(\varepsilon_f + 1)^3} &= \sum_{l=1}^{\infty} x_l^2 \frac{\gamma_l}{x^{2l+2}}, \\
 \sum_f' \frac{\langle 1S|v|f \rangle \langle f|v|1S \rangle}{(\varepsilon_f + 1)^4} &= \sum_{l=1}^{\infty} x_l^2 \frac{\omega_l}{x^{2l+2}}, \quad (25)
 \end{aligned}$$

$$\begin{aligned}
 \sum_{i=1}^3 \sum_{f, f'}' \frac{\langle 1S|v|f \rangle \langle f|v|f' \rangle \partial_1 \langle f'|v|1S \rangle}{(\varepsilon_f + 1)^{m+1} (\varepsilon_{f'} + 1)^{n+1}} \partial_1 \langle 1S|\Psi \rangle_S \\
 = - \sum_{1, 1', 1''=1}^{\infty} \Lambda x_1 x_{1'} x_{1''} \frac{\sigma_{11'1''}^{(mn)}}{x^{2t+4}} \frac{\partial}{\partial R} \langle 1S|\Psi \rangle_S,
 \end{aligned}$$

$$\begin{aligned}
 \sum_{i=1}^3 \sum_{f, f'}' \frac{\langle 1S|v|f \rangle (\partial_1 \langle f|v|f' \rangle) \langle f'|v|1S \rangle}{(\varepsilon_f + 1)^{m+1} (\varepsilon_{f'} + 1)^{n+1}} \partial_1 \langle 1S|\Psi \rangle_S \\
 = - \sum_{1, 1', 1''=1}^{\infty} \Lambda x_1 x_{1'} x_{1''} \frac{\tau_{11'1''}^{(mn)}}{x^{2t+4}} \frac{\partial}{\partial R} \langle 1S|\Psi \rangle_S,
 \end{aligned}$$

where nonzero are the terms in which

* Following the tradition set up in the atomic physics (see, e.g. refs. 14, 15) we keep the Greek letters for the coefficients named $\alpha, \beta, \gamma,$ and δ in eqs. (20) and (25), supplying them, if necessary, with the corresponding sub- and superscripts.

$2t = 1+1'+1''$ at $t = 2, 3, 4, \dots$, $\delta_1 = \partial/\partial R_1$ and R_1 is the i -th Cartesian component of the vector \mathbf{R} ; $\langle 1s|\Psi \rangle_s$ is the s -wave part of the wave function $\langle 1s|\Psi \rangle$. The formulae for the calculations of β , γ , ω , σ and τ with arbitrary orbital momentum values l are given in ref.¹⁰⁾. For the first values of l we have

$$\begin{aligned} \beta_1 &= 43/8, & \beta_2 &= 107/8, & \beta_3 &= 3265/32, \\ \gamma_1 &= 319/48, & \gamma_2 &= 2399/192, & \gamma_3 &= 63901/768, \\ \omega_1 &= 9673/1152, & \omega_2 &= 55631/32256, & \delta_4^{(01)} &= 4313/32, \\ \varepsilon_4^{(001)} &= 244451/1152, & \varepsilon_4^{(010)} &= 39619/256, & & (26) \\ \sigma_4^{(01)} &= 27309/96, & \sigma_4^{(02)} &= 1686401/640, \\ \tau_4^{(10)} &= 5381/16, & \tau_4^{(20)} &= 43667/96, \end{aligned}$$

where, as in (22),

$$\begin{aligned} U_4^{(mn)} &\equiv \sigma_{112}^{(mn)} + \sigma_{121}^{(mn)} + \sigma_{211}^{(mn)}, \\ \tau_4^{(mn)} &\equiv \tau_{112}^{(mn)} + \tau_{121}^{(mn)} + \tau_{211}^{(mn)}. \end{aligned}$$

The coefficients β , γ , ω , σ , and τ , in the same way as α , δ and ε , correspond to the case of e^- scattering by the hydrogen atom.

Substituting the representation of the potential $\langle 1s|U|1s \rangle$ in the form of (23) into eq.(12) we come to an equation nonlinear with respect to the operator T_R , for the function $\langle 1s|\Psi \rangle$,

$$\left\{ \sum_{q=1}^{\infty} (\delta_{q1} - U_q) T_R^q - V^{AD} - U_0 \right\} \langle 1s|\Psi \rangle = 0. \quad (27)$$

To separate in eq.(27) the term corresponding to the effective potential of the interaction between the nucleus A and the atom B in $11s$ -state, this equation should be reduced to the form of the one-particle Schrodinger one. This is easily done in the explicit form for the case of taking into account only the s -wave part of the function $\langle 1s|\Psi \rangle$ and the terms in (27) with $q=1$. Keeping in (27) the terms linear in T_R and taking into account the explicit form (24) of the functions U_0 and U_1 , eq.(27) is transformed into the Schrodinger one

$$\left\{ \frac{d^2}{dR^2} + \frac{2}{R} \frac{d}{dR} + k^2 - V(\Lambda R; k^2) \right\} \Phi(R) = 0. \quad (28)$$

for the function

$$\Phi(R) = \langle 1S | \Psi \rangle e^{-\chi(R; k^2)} \quad (29)$$

and the potential of the form

$$V(x, k^2) = \frac{1}{Q_1^{(0)}(x) + k^2 Q_2^{(6)}(x)} \left\{ V^{AD}(x) + \frac{P_1^{(6)}(x) + k^2 P_2^{(6)}(x)}{Q_1^{(0)}(x) + k^2 Q_2^{(6)}(x)} \right\}, \quad (30)$$

where $Q_1^{(n)}(x)$, $P_1^{(n)}(x)$ are the polynomials beginning from the power n of the variable $1/x$. Up to the terms of the order of x^{-8} they have the form

$$\begin{aligned} Q_1^{(0)}(x) &= 1 + \beta_1 \frac{x_1^2}{x^4} + \left[\beta_2 x_2^2 + 8\Lambda^2 \gamma_1 x_1^2 \right] \frac{1}{x^6} - 6\delta_4^{(01)} \frac{x_1^2 x_2^2}{x^7} \\ &+ \left[\beta_3 x_3^2 + 2(3\epsilon_4^{(001)} + 2\epsilon_4^{(010)}) x_1^4 + 24\Lambda^2 \gamma_2 x_2^2 \right] \frac{1}{x^8} + \dots, \\ Q_2^{(6)}(x) &= 72\Lambda^2 \omega_1 \frac{x_1^2}{x^6} + 144\Lambda^2 \omega_2 \frac{x_2^2}{x^8} + \dots, \\ P_1^{(6)}(x) &= 2\Lambda^2 \beta_1 \frac{x_1^2}{x^6} + 9\Lambda^2 \beta_2 \frac{x_2^2}{x^8} + \dots, \\ P_2^{(6)}(x) &= 12\Lambda^2 \gamma_1 \frac{x_1^2}{x^6} + (7\gamma_2 x_2^2 - 12\omega_1 x_1^2) \frac{6\Lambda^2}{x^8} + \dots \end{aligned} \quad (31)$$

The function $\chi(R; k^2)$ in eq.(29) is equal to

$$\chi(R; k^2) = \int_R^\infty dR \frac{S_1^{(6)}(\Lambda R) + k^2 S_2^{(6)}(\Lambda R)}{Q_1^{(0)}(\Lambda R) + k^2 Q_2^{(6)}(\Lambda R)}, \quad (32)$$

where $S_1^{(6)}(x)$ are the polynomials

$$S_1^{(6)}(x) = 2\Lambda^2\beta_1 \frac{x^2}{x^6} + 3\Lambda^2\beta_2 \frac{x^2}{x^8} + \dots,$$

$$S_2^{(6)}(x) = 4\Lambda^2\gamma_1 \frac{x^2}{x^6} + (\gamma_2 x_2^2 + 12\omega_1 x_1^2) \frac{6\Lambda^2}{x^8} + \dots \quad (33)$$

The main term of the function $\chi(R;k^2)$ behaves as x^{-4} :

$$\chi(R;k^2) = \frac{1}{2} (\beta_1 + 2k^2\gamma_1) \frac{x^{-1}}{4} + \dots \quad (34)$$

With the increasing of R the next terms decrease faster. Therefore, at distances $R \gg R_0$ the functions $\Phi(R)$ and $\langle 1s|\Psi \rangle$ differ small and their asymptotics coincide

$$\Phi(R) \sim \langle 1s|\Psi \rangle \sim \sin(kR + \delta + \sigma_c + \eta_c \ln 2kR)/kR, \quad R \gg R_0, \quad (35)$$

where σ_c is a Coulomb phase, $\eta_c = x_0/\Lambda k$ is a Coulomb parameter, δ is the phase shift induced by non-Coulomb part of the effective potential (30). It follows from (28), (29) and (34) that at distances $R \gg R_0$ the potential (30) is close to the real one describing the interaction between the nucleus A and the atom cB .

The potential (30) is local. This property is a result of expansions (18) showing that in the energy range $k^2 < 3/4$ the effective potential is rigorously local in all orders $U^{(1)}$ of the expansion (17).

4. EFFECTIVE POTENTIAL IN ASYMPTOTIC REGION.

Dividing the polynomials in (30), keeping then the terms linear in k^2 and summands up to the terms of the order x^{-8} we find for the potential $V(x;k^2)$ in asymptotic region $R \gg R_0$, the following formula

$$V(x;k^2) = v^{AD}(x) + v_1^{(5)}(x) + k^2 v_2^{(6)}(x), \quad (36)$$

where $v^{AD}(x)$ is given by the series (19), and

$$v_1^{(5)}(x) = -2\beta_1 \frac{x_0 x_1^2}{x^5} + 6\beta_1 \frac{\Lambda^2 x^2}{x^6} - \left[\beta_2 x_2^2 + 8\Lambda^2 \gamma_1 x_1^2 \right] \frac{2x_0}{x^7} + \left[\alpha_1 \beta_1 x_1^4 + 12\delta_4^{(01)} x_0 x_1^2 x_2 + 3\beta_2 x_2^2 \right] \frac{1}{x^8} + \dots, \quad (37)$$

$$V_2^{(6)}(x) = 20\gamma_1 \frac{\Lambda^2 x^2}{x^6} - 144\omega_1 \Lambda^2 \frac{x_0 x^2}{x^7} + \left[3\gamma_2 x_2^2 + 4\Lambda^2 (\omega_1 x_1^2) \right] \frac{18\Lambda^2}{x^8} + \dots$$

The terms $V_1^{(5)}(x)$ and $V_2^{(6)}(x)$ represent nonadiabatic corrections to the adiabatic potential V^{AD} , which take into account the atomic structure. The formula (36) can be rewritten in the following form which is convenient for comparison of the coefficients at the different degrees of variable $1/x$

$$V(x; k^2) = \frac{a_1}{x} - \frac{a_4}{x^4} - \frac{a_5}{x^5} - \frac{a_6}{x^6} - \frac{a_7}{x^7} - \frac{a_8}{x^8} + \dots + k^2 \left[\frac{b_6}{x^6} - \frac{b_7}{x^7} + \frac{b_8}{x^8} + \dots \right] + \dots, \quad (38)$$

where

$$\begin{aligned} a_1 &= 2x_0, & a_4 &= \alpha_1 x_1^2, & a_5 &= 2\beta_1 x_0 x_1^2, \\ a_6 &= \alpha_2 x_2^2 - 6\beta_1 \Lambda^2 x_1^2, \\ a_7 &= -\delta_4^{(00)} x_1^2 x_2 + (\beta_2 x_2^2 + 8\gamma_1 \Lambda^2 x_1^2) 2x_0, \\ a_8 &= \alpha_3 x_3^2 + (\epsilon_4^{(000)} - \alpha_1 \beta_1) x_1^4 - 12\delta_4^{(01)} x_0 x_1^2 x_2 - 3\beta_2 x_2^2, \\ b_6 &= 20\gamma_1 \Lambda^2 x_1^2, & b_7 &= 144\omega_1 \Lambda^2 x_0 x_1^2, \\ b_8 &= 18\Lambda^2 (4\omega_1 \Lambda^2 x_1^2 + 3\gamma_2 x_2^2). \end{aligned} \quad (39)$$

The formula (38) is valid in the region $x \gg \xi$. For $m_c \ll m_B$ we have $\xi=1$. This proves a natural way of introducing the scale factor Λ in formula (3). The region $x \gg 1$ corresponds to the distances $R \gg a$.

It follows from (14) and (39) that coefficients a_i and b_i depend on the charge of Λ nucleus by the rules:

$$a_i \sim Z_A; \quad a_i \sim Z_A^2, \text{ at } i=4,6; \quad a_i \sim Z_A^3, \text{ at } i=5,7; \quad a_8 \sim c_1 Z_A^2 + c_2 Z_A^4,$$

where c_1 and c_2 are the constants,

$$b_i \sim Z_A^2, \text{ at } i=6,8; \quad b_7 \sim Z_A^3.$$

The dependence $a_4 \sim Z_A^2$ was discussed in ref. (16).

If we restrict ourselves to the terms of the order x^{-6} , for the e⁻H-scattering the potential (38) represents the known Seaton-Steenman potential where, however, $b_6=20\gamma_1$, but not $24\gamma_1$ as

in ref.¹⁴). The coefficient $b_6 = 24\gamma_1$ corresponds to the approximation where the operator T_R is replaced by k^2 , and, besides that, the terms at $\partial/\partial R$ and $\partial^2/\partial R^2$ in the function $U_1(R)$ from (24) are not taken into account. When the terms with derivatives at k^2 and the entire operator T_R in terms without derivatives are taken into account we get¹⁰⁾: $b_6 = 36\gamma_1 \Lambda^2 z_1^2$. The result (39) for b_6 is obtained from eq.(27) in a most successive way. The term $U_1(R)T_R \langle |s| \Psi \rangle$ is calculated with account of the total operator T_R , all terms in $U_1(R)$ of the formulae (24) and up to the terms containing the second derivatives with respect to R . Our calculations demonstrate the sensibility of the coefficient b_6 to the approximation where the term with $q=1$ in (27) is taken into account.

It follows from (38) that if nucleus A interacts with an ion αB , then at the background of the interaction A with a neutral atom in addition to the Coulomb repulsion (attraction) there arises the attraction (repulsion) such as b_7/x^7 with the given energy k^2 . In this case the term a_7/x^7 can describe both the attraction and repulsion due to the parameter values giving by (39) the magnitude and the coefficient sign a_7 .

The account of the nonadiabatic corrections changes the coefficients at x^{-6} , x^{-7} (for the αB ion), x^{-8} . The corrections to the polarization potential

$$V_p(x) = -\alpha_1 \frac{x^2}{x^4} \quad (40)$$

as long as the terms proportional to x^{-2} and x^{-3} do not arise. The formula (30) allows to generalize the potential in such a way that its analytic extension to the small x region including the point $x=0$ become possible. Keeping the terms in (30) up to the order x^{-4} we find

$$V(x; k^2) = \frac{2z_0}{x} - \frac{\alpha_1 z_1^2}{x^4 + \beta_1 z_1^2} \left[1 + \frac{2\beta_1 z_0}{\alpha_1 x} \right] + \dots \quad (41)$$

The second term,

$$\tilde{V}_p(x) = - \frac{\alpha_1 z_1^2}{x^4 + \beta_1 z_1^2}, \quad (42)$$

transforms into the polarization potential (40) at distances

$$x \gg (\beta_1 z_1^2)^{1/4}$$

and can be regarded as its natural generalization which has no nonphysical singularity at $x=0$. The parameters in (42) are not free but are determined, according to (20) and (25), by the dynamics of the three-particle system. The coefficient α_1 is the dipole polarizability, and β_1 corresponds to the main nonadiabatic correction (25) of hydrogen atom.

5. APPLICATION TO MESONIC ATOMS

We apply the theory developed in the previous sections to analyze the light nuclei - mesonic atom interaction. We consider the distances $R \gg a$, and choose the particles p , t , ${}^4\text{He}$ and ${}^7\text{Be}$, that differ between themselves both in masses and charges, as the nuclei A and B . The table 1 gives the parameters a , E_0 and $R_0 = \xi a$ of the considered mesonic atoms. The values for the hydrogen atom are presented for comparison in the last row. The table 2 lists the coefficients a_1 and b_1 of the asymptotic formula (38) for the $A\mu B$ system and for the comparison purposes ones for the interaction of an electron with hydrogen atom. The numerical values of a scale factor Λ which for the considered system is less than one since $m_A \gg m$ are given here too.

Figures 1-4 show the typical pictures of the effective interaction potentials between nuclei A and mesonic atoms μB calculated by eq.(30) as the functions of two variables R and k^2 . The potential V and the energy k^2 of a nucleus A are given in E_0 units, and the distance R in R_0 units taken from Table 1. The comparison of figures and the coefficients given in Table 2 makes it possible to observe how $A-\mu B$ interaction depends on masses and charges of particles A , μ , and B . For the systems shown in figs. 1, 3, 4a, 4d the potential takes its asymptotic form (38) up to 1%

at $R \sim 6R_0$, but in the case of interaction of ${}^7\text{Be}$ with mesonic atoms μt and $\mu{}^4\text{He}$ - at distances $R \sim 9R_0$ with less accuracy ($\sim 10\%$). If the nucleus A interacts with neutral atom the potential is attractive. For such systems the energy dependence k^2 is more essential at distances $R \sim 3R_0$. With the interaction between the nucleus A and positively charged ion μB the potential is repulsive. The "b" and "c" type figures demonstrate than under the given number the dependence of this potential on the cB ion charge value. Compare the figures of different number but with the same letter shows in "a" and "b" cases the dependences of interaction on A mass and in "c" and "d" cases - on its charge also. As it should be the potential value increases with increasing the cB ion charge. The increase in the mass and charge of A leads to the strengthening of the interaction in the considered R and k^2 regions for the given atom μB . Generally the potential V is not a monotone function of R. In $R < 3R_0$ region the formula (30) is inapplicable since the denominator in its right hand side may turn to zero, and the absence of such zero values was assumed by us when converting the $(\delta_{q1} - U_q)$ operator in (27). To find the potential $\langle \psi | U | \psi \rangle$ of eq.(12) in the entire region of variable R the direct solution of eq.(10) is required.

Table 1. Atomic parameters.

Atom	a, Fm	E_0 , keV	R_0 , Fm
μp	284.5	2.516	255.7
μt	265.3	2.698	255.7
$\mu{}^4\text{He}$	131.5	10.888	127.9
$\mu{}^7\text{Be}$	64.96	44.073	63.93
ep	$5.29 \cdot 10^4$	$13.61 \cdot 10^{-3}$	$5.287 \cdot 10^4$

Table 2. The coefficients a, b and Λ from eq.(38) for the $\mu\mu\text{B}$ and $e\mu\text{B}$ systems.

(a) $\mu\mu\text{B}$

Nucl.B	p	t	^4He	^7Be
Λ	0.438	0.379	0.369	0.356
a_1	0.0	0.0	1.0	1.5
a_4	4.5	4.5	1.188	0.309
a_5	0.0	0.0	1.419	0.553
a_6	3.344	8.277	2.182	0.598
a_7	84.942	98.778	18.164	3.633
a_8	154.96	181.75	127.32	25.755
b_6	25.544	19.069	4.783	1.153
b_7	0.0	0.0	21.753	7.869
b_8	104.81	95.713	23.460	5.655

(b) $e\mu\text{B}$

Nucl.B	p	t	^4He	^7Be
Λ	0.353	0.267	0.252	0.230
a_1	0.0	0.0	1.0	1.5
a_4	4.5	4.5	1.188	0.309
a_5	0.0	0.0	1.419	0.553
a_6	5.518	10.608	2.803	0.761
a_7	84.942	98.778	17.14	3.299
a_8	154.96	181.75	127.32	25.755
b_6	16.585	9.462	2.224	0.480
b_7	0.0	0.0	10.115	3.277
b_8	62.967	44.384	10.171	2.194

Table 2. (continuation)
(c) ${}^4\text{He}\mu\text{B}$

Nucl.B	p	t	${}^4\text{He}$	${}^7\text{Be}$
Λ	0.341	0.250	0.233	0.209
a_1	0.0	0.0	2.0	3.0
a_4	18.0	18.0	4.752	1.235
a_5	0.0	0.0	11.351	4.425
a_6	23.146	43.584	11.519	3.124
a_7	679.54	790.23	136.12	25.432
a_8	1953.0	2060.0	1811.4	350.84
b_6	61.910	33.10	7.631	1.589
b_7	0.0	0.0	69.415	21.679
b_8	232.71	153.91	34.586	7.191

(d) ${}^7\text{Be}\mu\text{B}$

Nucl.B	p	t	${}^4\text{He}$	${}^7\text{Be}$
Λ	0.325	0.225	0.206	0.178
a_1	0.0	0.0	4.0	6.0
a_4	72.0	72.0	19.006	4.940
a_5	0.0	0.0	90.807	35.403
a_6	98.233	180.39	47.688	12.921
a_7	5436.3	6321.8	1078.3	199.26
a_8	29142.0	29571.0	28080.0	5368.4
b_6	224.36	107.44	23.875	4.607
b_7	0.0	0.0	434.38	125.72
b_8	832.17	493.87	106.93	20.598

(e) eep

Λ	a_1	a_4	a_5	a_6	a_7	a_8	b_6	b_7	b_8
1.0	0.0	4.5	0.0	-17.283	-106.38	201.88	132.92	0.0	1277.8

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FIGURE CAPTIONS

- Fig.1 The effective potentials of proton p and mesonic atoms μp ("a"), μt ("b"), $\mu^4\text{He}$ ("c"), $\mu^7\text{Be}$ ("d") interaction. Values V and k^2 are in E_0 and R_0 units respectively, which are given in Table 1.
- Fig.2 The effective potentials of triton t and mesonic atoms interaction. See Fig.1 captions.
- Fig.3 The effective potentials of nucleus ^4He and mesonic atoms interaction. See Fig.1 captions.
- Fig.4 The effective potentials of nucleus ^7Be and mesonic atoms interaction. See Fig.1 captions.

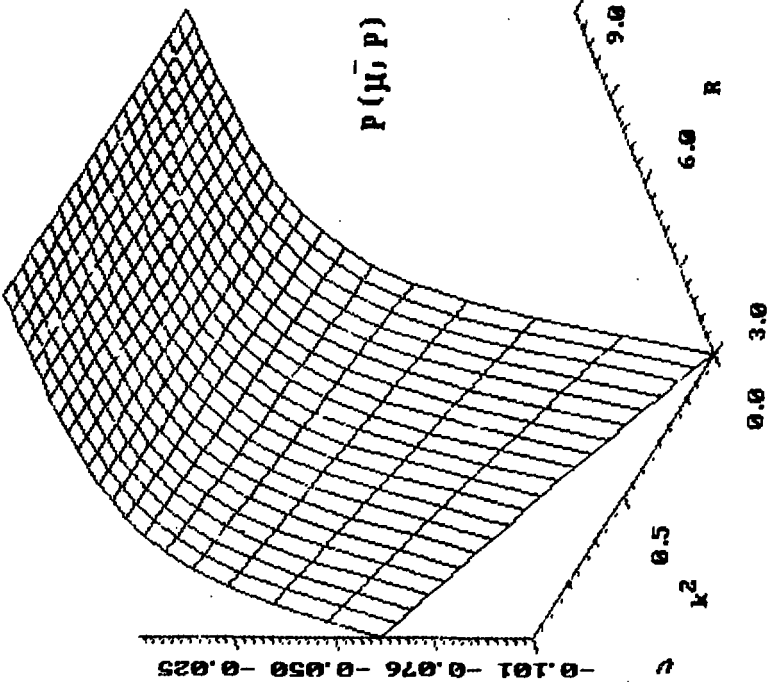


Fig. 1a

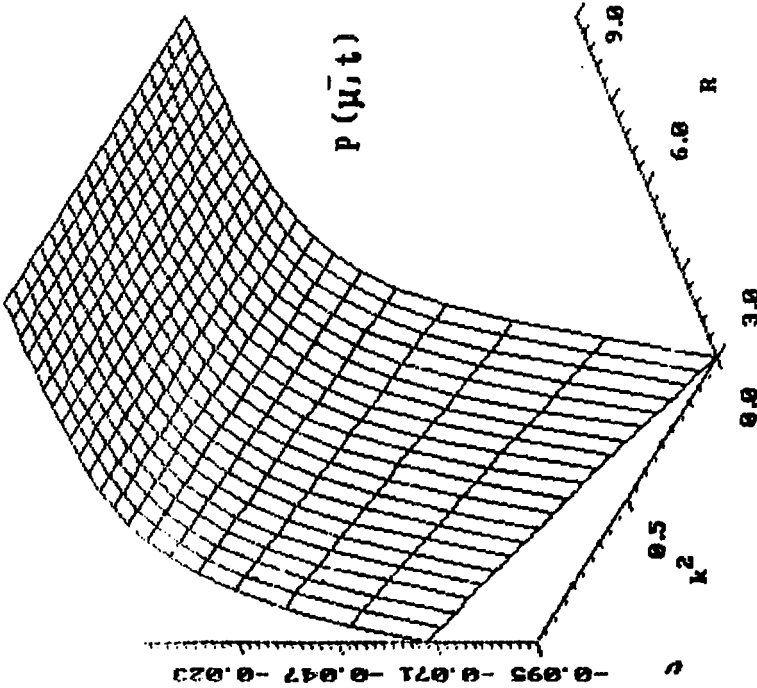
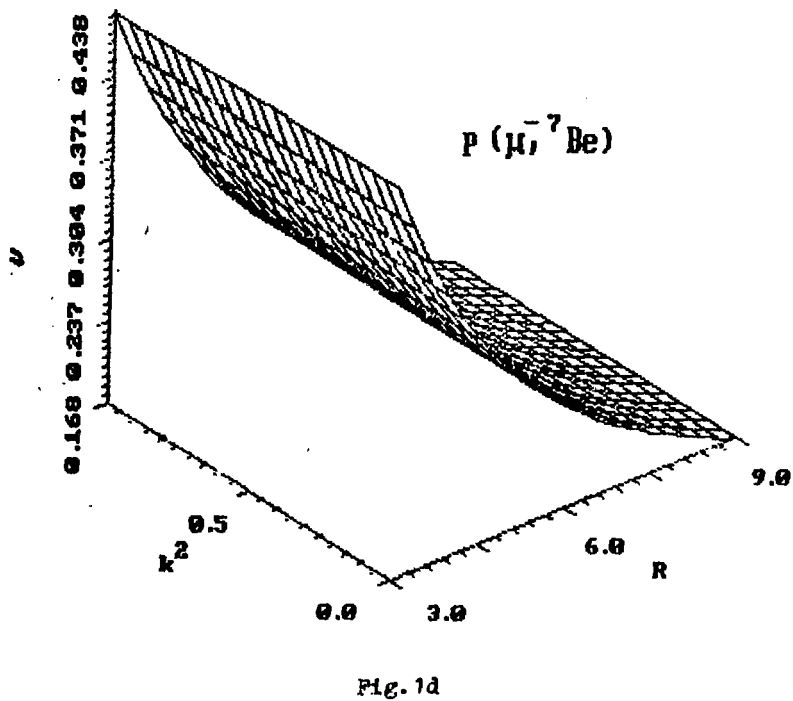
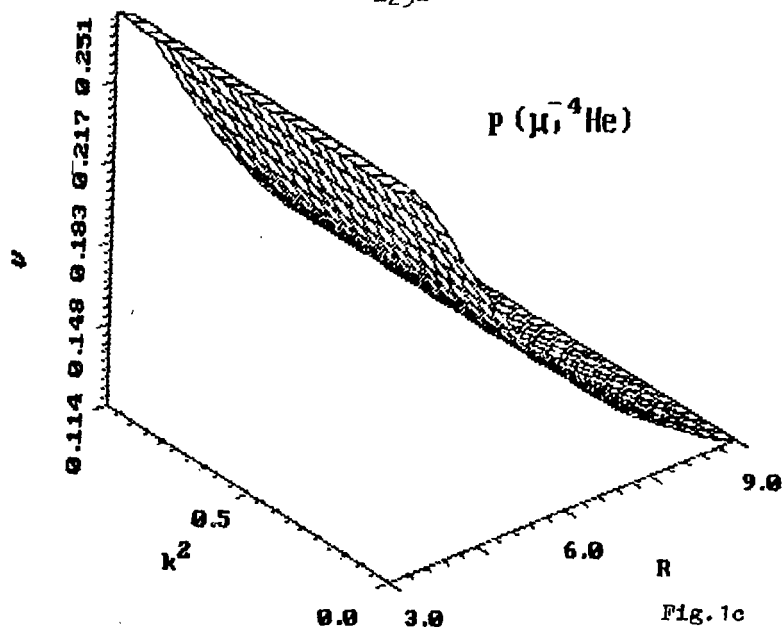


Fig. 1b



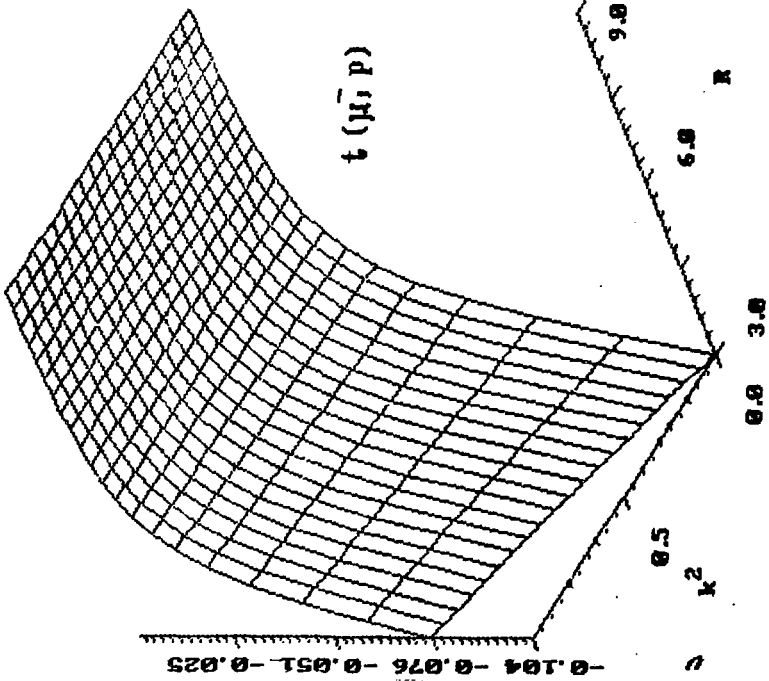


Fig.2a

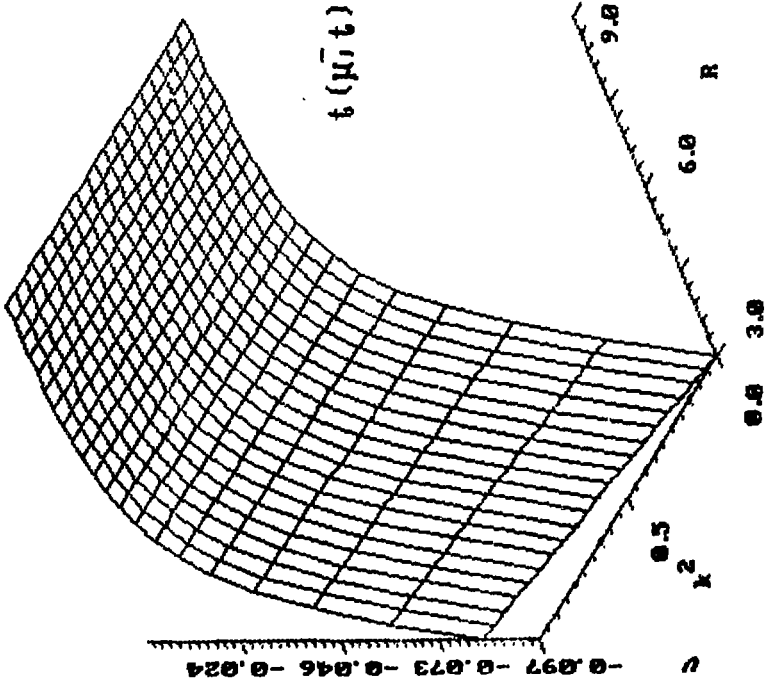
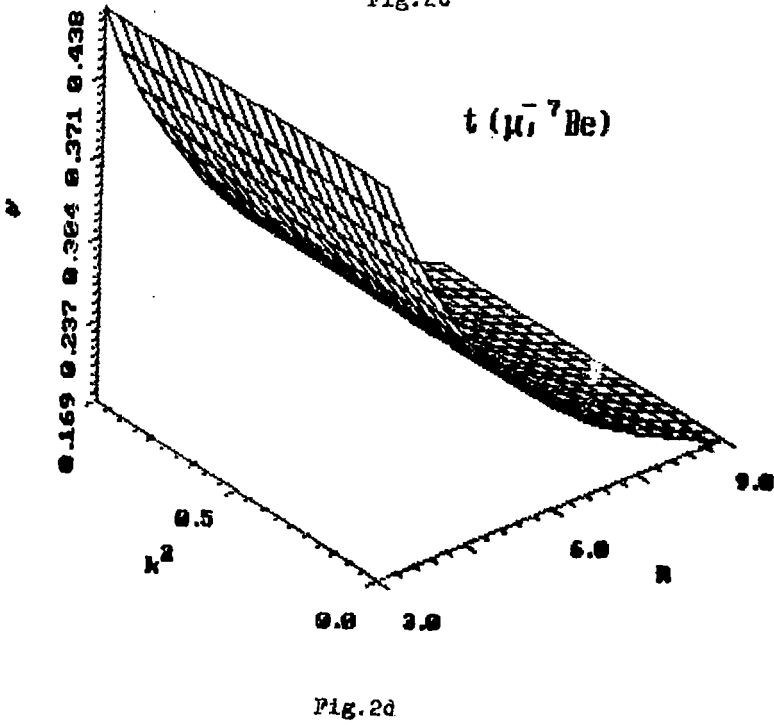
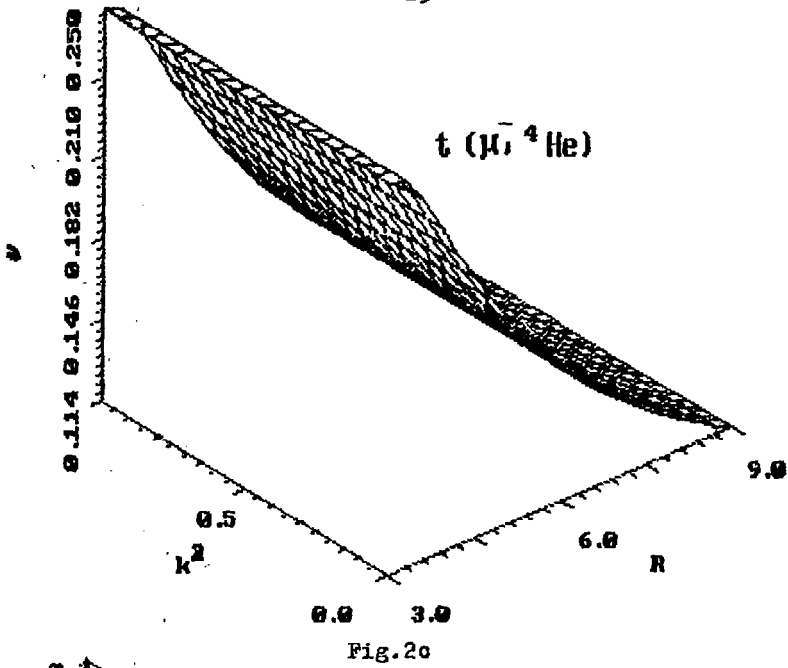


Fig.2b



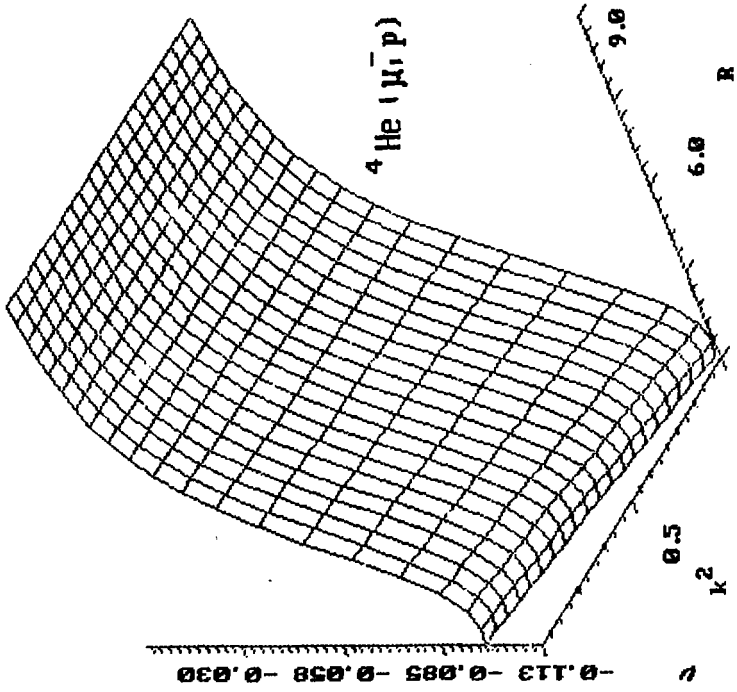


Fig. 3a

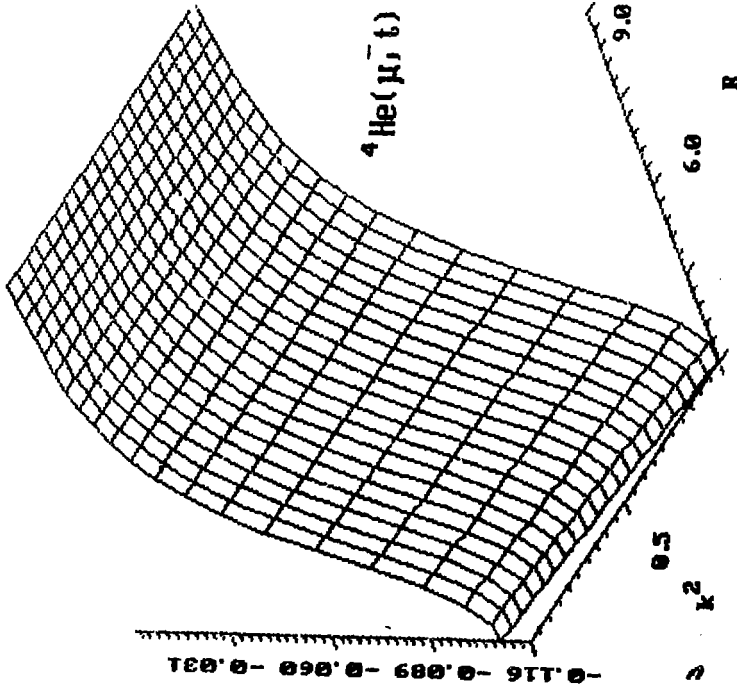


Fig. 3b

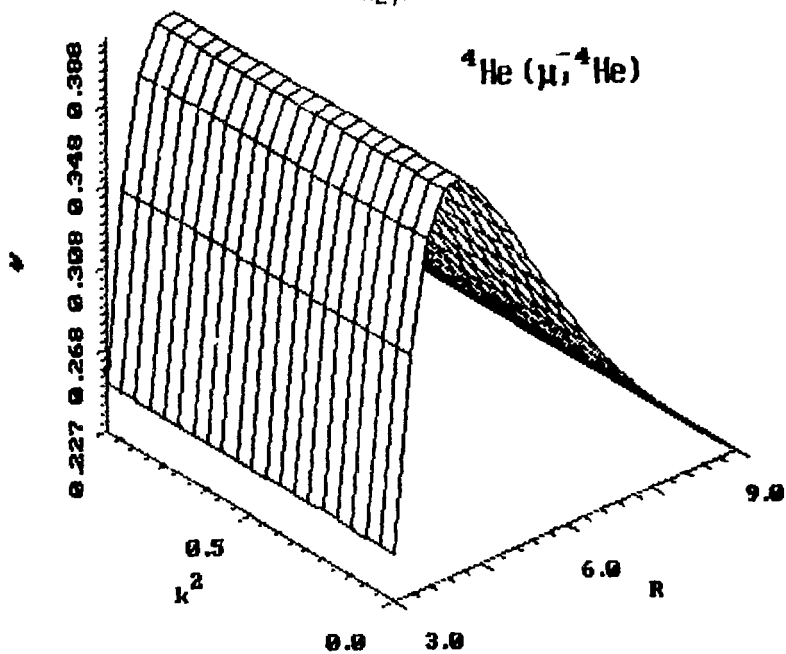


Fig. 3c

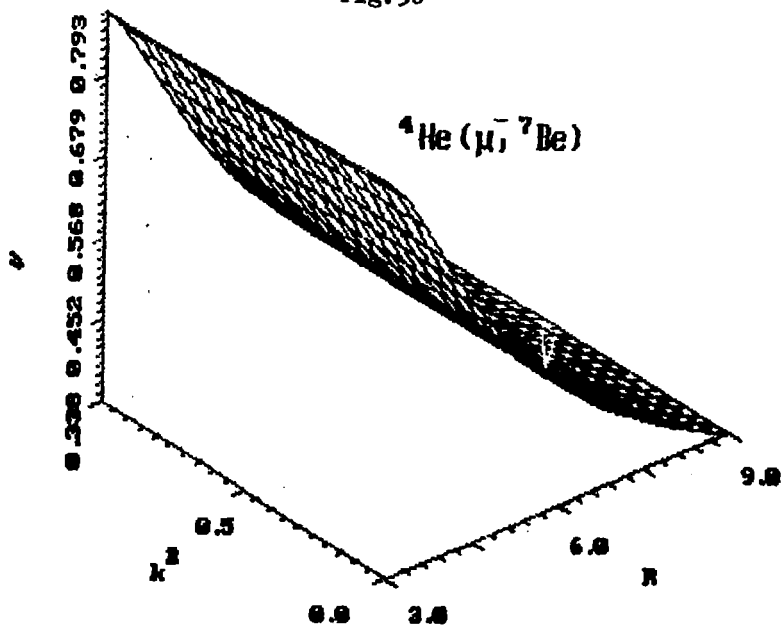


Fig. 3d

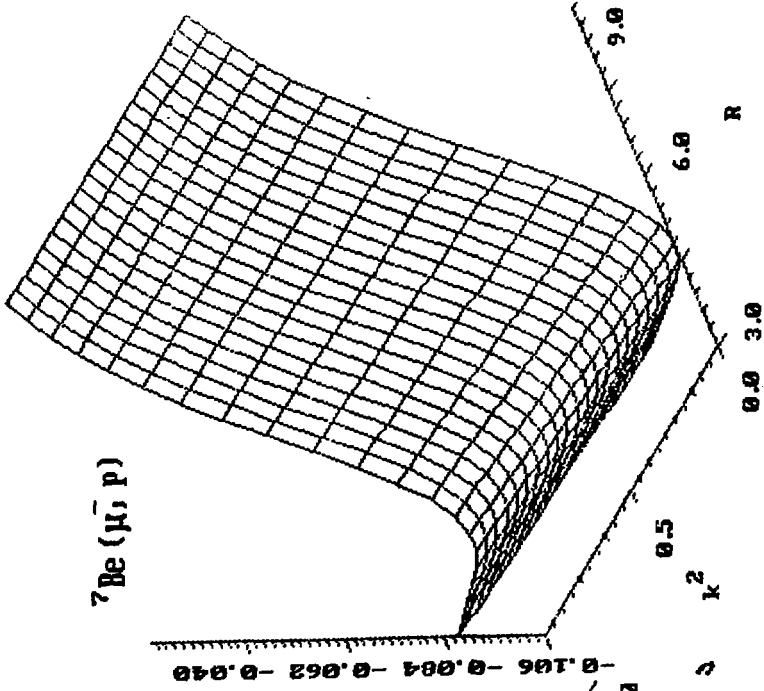


Fig. 4e

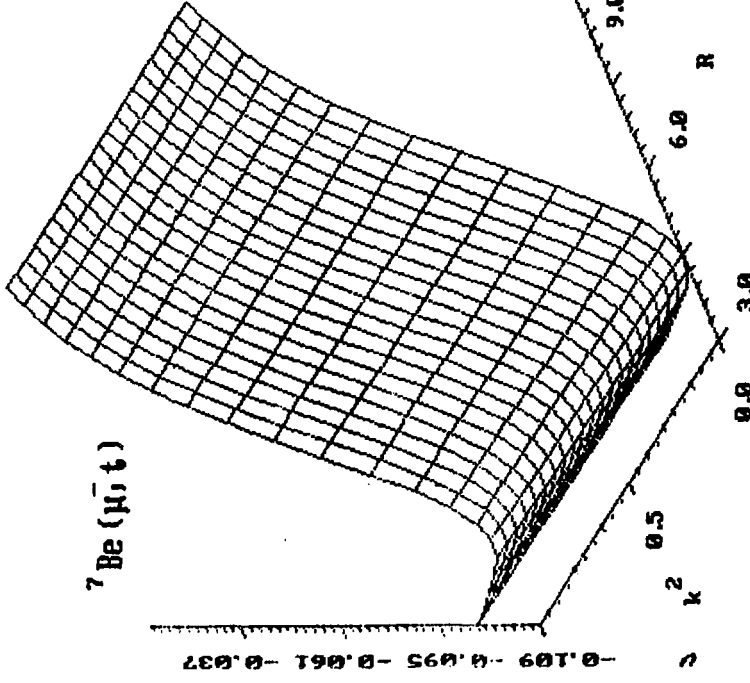
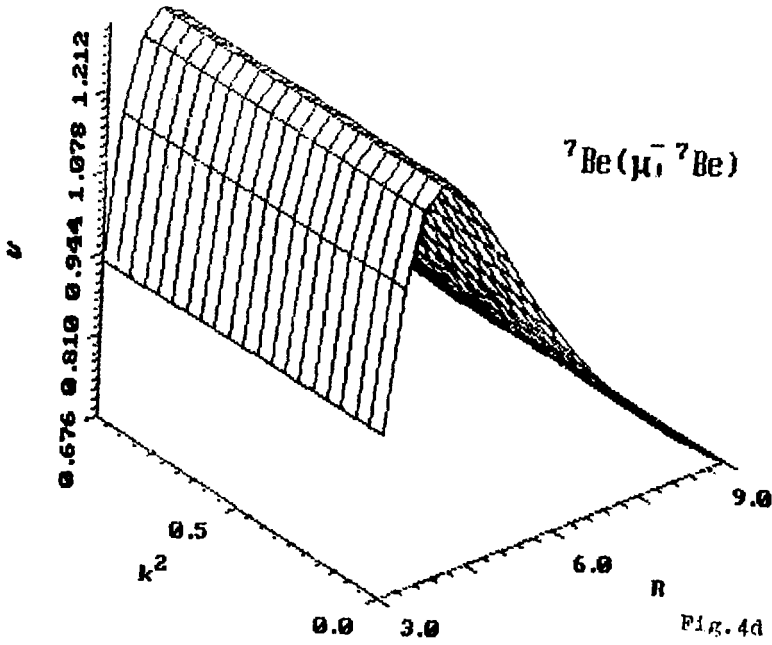
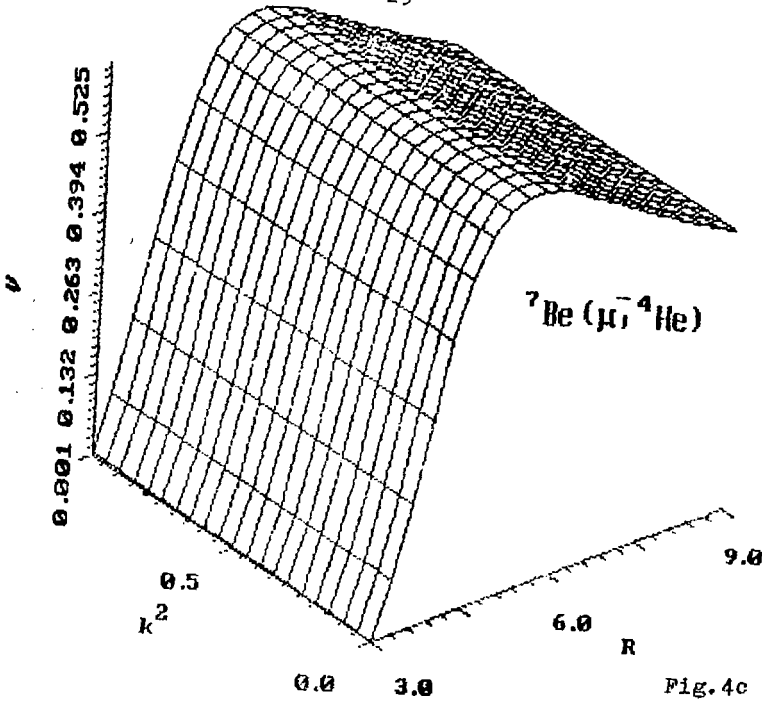


Fig. 4b



10 кон.

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