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**FRONT-END ELECTRONICS FOR H.E.P.**

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## FRONT-END ELECTRONICS FOR H.E.P.

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### ABSTRACT

A simplified description of the front-end electronics used for High Energy Physics Detectors is given. A brief analysis of the speed limitation due to the time necessary for the detector charge transfer is given, which depends as well of the detector behaviour as of the preamplifier configuration. A description of the sample electronic circuits like differentiation, integration, pole zero circuit and preamplifier are given. Noise analysis is carried out to derive the relations for the equivalent noise signal for the measuring device with some description of practical noise measuring. The shaping of the signals to obtain an optimization for the noise is considered and some hints for shaping amplifier design, with a description of the noise weighting function for normal and time variant shaping are given.

### DETECTION OF CHARGED PARTICLES

A charged particle going through any material will have interactions<sup>1)</sup>. Ionization and excitation are produced directly by the interaction of the particle electromagnetic field with electrons of the detecting medium. *The resultant ionization is distributed along the path of the particle.*

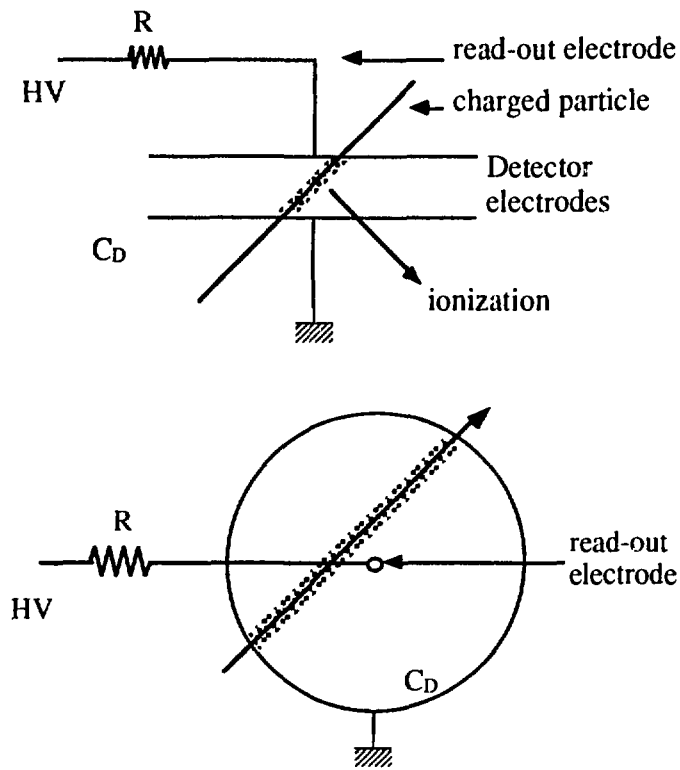
A typical particle energy is in order of *few GeV* while the energy loss in the detector can be *below the MeV*. In most detectors the total ionization, proportional to the energy loss, is collected using an externally applied electrical field. Sometimes an amplification process by avalanche in high electrical field is used.

Examples of detectors :

- proportional chamber (drift chamber)
- liquid argon chamber
- semiconductor
- spark chambers

- bubble chambers
- Cerenkov
- etc.

Almost all detectors provide a certain amount of charge into an output electrode (Fig. 1).



**Fig. 1 - Detector configurations**

The read-out electrode represents a capacitance to ground. *In the detector, the particle energy (or a part of it), is converted to an amount of charge,  $Q_s$  which we want to read and evaluate .*

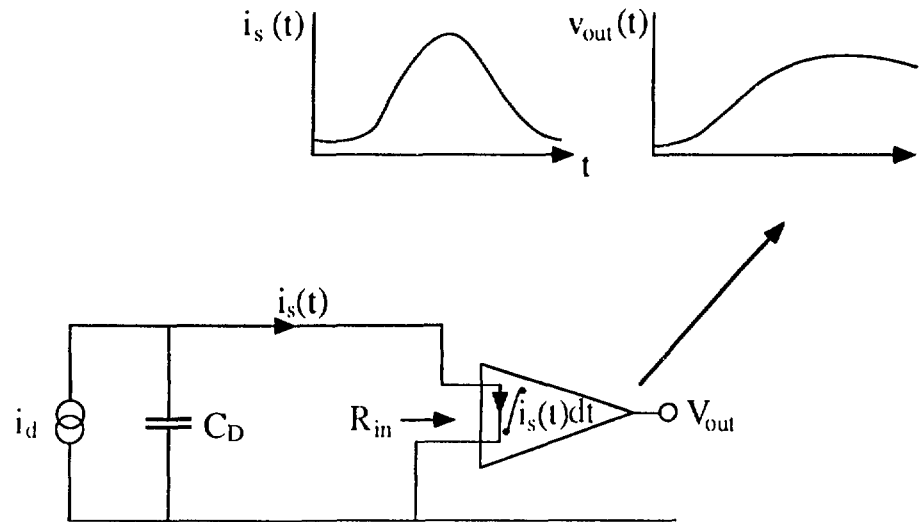
Almost all detectors can be considered as capacitive sources of current. By discharging the detector, and by integrating this discharging current we can evaluate  $Q_s$ , the charge created by the incident particle. Hence, *to measure the charge  $Q_s$ , a low input impedance preamplifier is used, in which the detector is discharged.*

The charge measurement can be done, also, by using a voltage amplifier and by measuring the voltage on the detector capacitance. This method has the disadvantages to give a measure which is a function of  $C_D$  and to introduce a higher electronic noise.

**Speed limitations**

The preamplifier output is proportional to the detector current integral, which is the signal charge  $Q_s$  : Fig. 2

$$V_{out} \equiv Q_s = \int i_s(t) dt \quad (1)$$

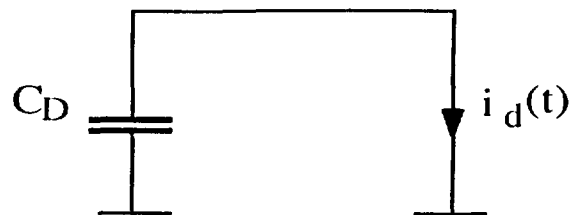


**Fig. 2 - Detector equivalent circuit with front-end amplification**

The signal current shape is defined by two processes :

- > the charge collection process in the detector —>  $i_d(t)$
- > the input time constant of the system,  $R_{in} C_{in} = \tau_{in}$

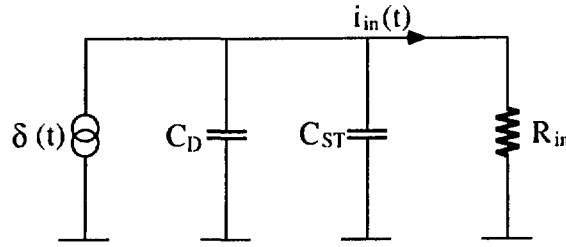
a) To define  $i_d(t)$  the short circuit detector current due to an event has to be measured (Fig. 3).



**Fig. 3 - The short circuit detector current**

On a short circuited detector, the short circuit current shape is defined by the detector collection process.

b) To define  $i_{in}(t)$  the circuit of Fig. 4 can be used :



**Fig. 4 - Equivalent circuit defining the current integrated on the front-end preamplifier**

$$i_{in}(t) = (1/C_{in}) \cdot e^{-t/\tau_{in}} \quad (2)$$

where  $C_{in} = C_D + C_{ST}$

$i_{in}(t)$  is defined by a  $\delta(t)$  driven input circuit configuration where  $R_{in}$  and  $C_{ST}$  are the preamplifier equivalent parameters.

c)  $i_s(t)$  will be the current integrated by the charge preamplifier.

$$i_s(t) dt = i_d(t) * i_{in}(t) dt \quad (3)$$

where \* means convolution integral.

Examples :

– for proportional wire chamber detector :

$$i_d(t) = i_m / (1 + t/t_0) \quad (4)$$

$t_0$  and  $i_m$  depend on the gas characteristics, ion mobility and detector geometry.

– for a liquid argon detector :

$$i_d(t) = i_m [1 - (t/t_d)] \quad (5)$$

$i_m$  and  $t_d$  depend on the detector geometry on the electron mobility.

The rise time of the output signal is limited by the detector current duration and by the preamplifier input time constant. To obtain an output corresponding to the *total* created charge  $Q_s$ , the integration must last as long as the duration of the signal current  $i_s(t)$ .

*That is a limitation of the maximum read-out speed (the minimum shaping time).*

### Noise limitations

In the majority of detectors the signal produced by the detected particles is relatively small and is contaminated by noise generated by the detector itself and by the amplifier.

It is important to know :

- what is the minimum signal which we can detect and measure ?
- what electronics to choose ? (Front Amplifying element, and filtering).

If the created charge is small, the electronic noise will limit the minimum sensitivity of the system.

Hence, we will use a detector with intrinsic gain like the proportional wire chamber, where by accelerating the initial charges (in a strong field) a proportional multiplication is obtained so that the integral of the output current is  $10^4$  to  $10^5$  times larger than the initial charge.

### ELECTRONICS

We will present a general description of electronics units used in the HEP instrumentation. A discussion of the "input-output" signals of every element will be presented with a minimum circuit analysis.

#### Differentiation circuit

It consists of a capacitor and a resistor : Fig. 5

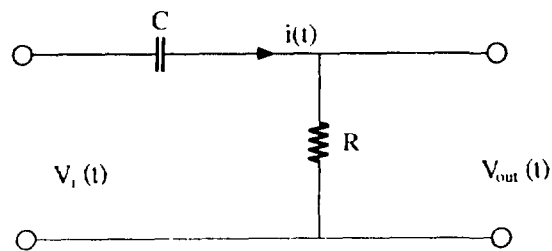


Fig. 5 - Differentiation circuit.

The step response of the differentiation circuit is given on Fig. 6

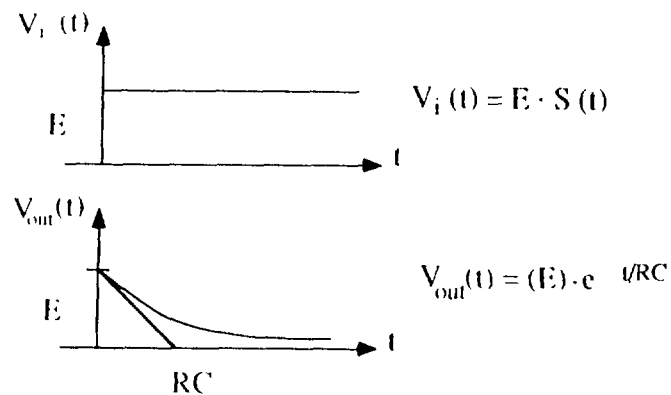
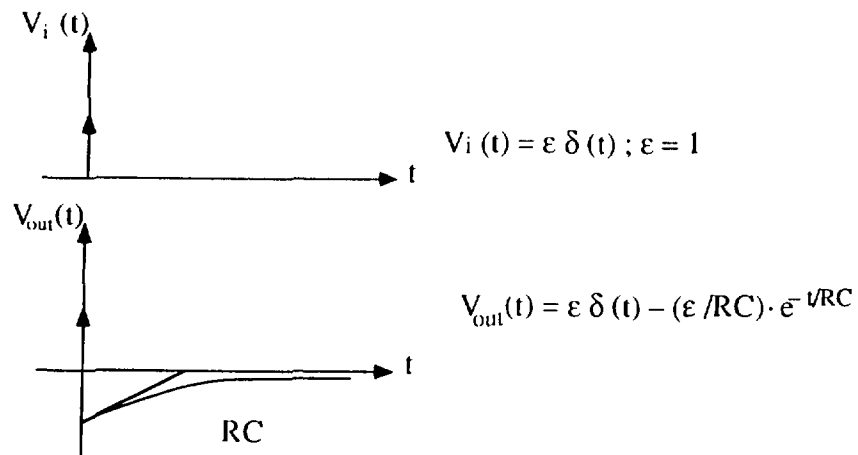


Fig. 6 - The step response of the differentiation circuit.

The impulse response of the differentiation circuit is given on Fig. 7.

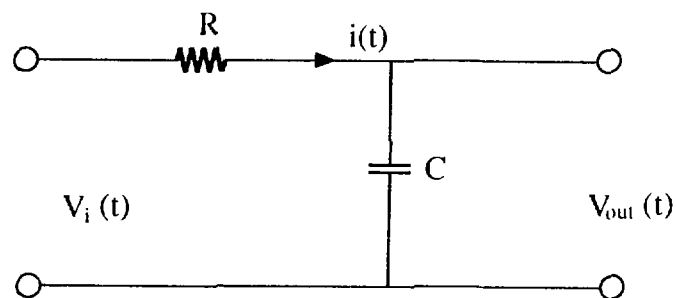


**Fig. 7 - The impulse response of the differentiation circuit.**

The impulse response can be intuitively obtained by supposing at  $t=0 \rightarrow$  the capacitance as a short circuit. Hence the output is equal to the input. Once the capacitor is charged, and from the charge balance law the negative part is deducted. The differentiation is operational for  $RC \ll$  of the inspected time interval.

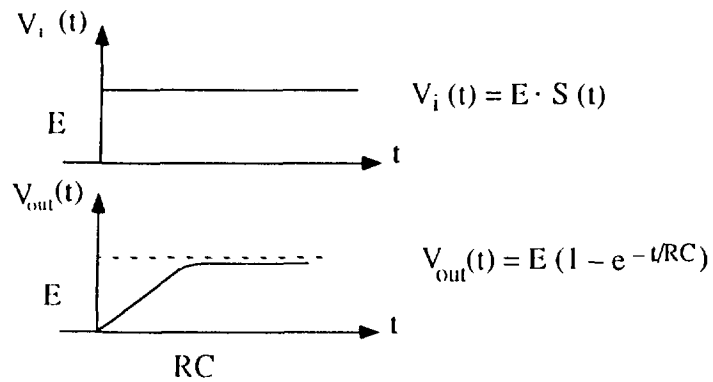
**Integration circuit (Fig. 8)**

It consists of a resistor and a capacitor :



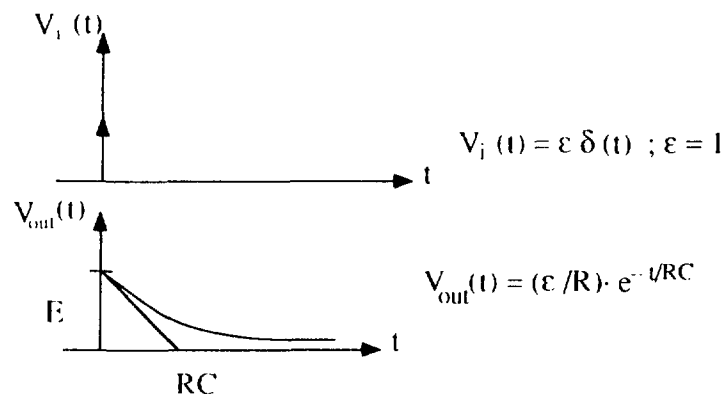
**Fig. 8 - Integration circuit.**

The step response of the integration circuit is given on Fig. 9.



**Fig. 9 - The step response of the integration circuit.**

The impulse response of the integration circuit is given on Fig. 10.



**Fig. 10 - The impulse response of the integration circuit.**

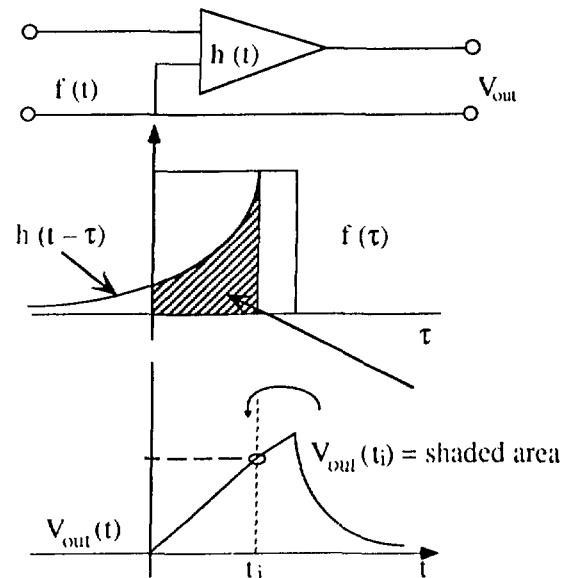
The integration is operational for  $RC \gg$  than the inspected time interval.

### **The impulse response and the time convolution method<sup>21</sup>**

The response of a circuit to an arbitrary input function can be found by using the method of convolution.



Knowing the impulse response,  $h(t)$ , the response to  $f(t)$  of the circuit will be :



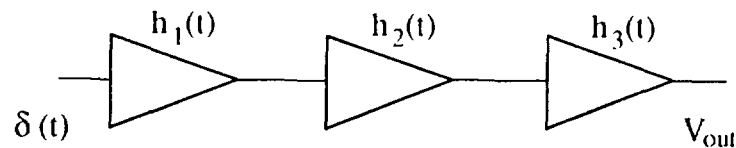
**Fig. 11 - The convolution integral graphic representation**

$$V_{out}(t) = \int_{-\infty}^{+\infty} f(\tau) \cdot h(t - \tau) d\tau \quad (6)$$

The example is given for a simple function of  $f(t)$  —> a pulse. The convolution integral will be a simple integration of  $h(t - \tau)$  over the interval of the input pulse (Fig. 11).

For non-rectangular pulse inputs, the value of the integral is no longer given by the area of overlap!

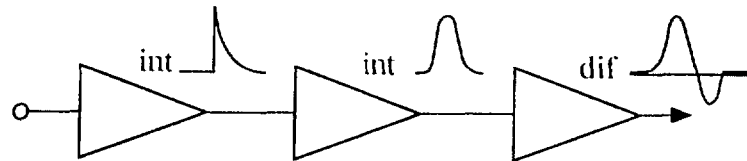
The output of several cascaded circuits will be given by successive convolutions of the impulse response of each circuit with the corresponding input waveform (Fig. 12).



$$V_{out} = [h_1(t) * h_2(t)] * h_3(t) \quad (7)$$

**Fig. 12 - The impulse response of the three-stage circuit.**

Example : Fig. 13



**Fig. 13 - The impulse response after two integrations and a differentiation circuit.**

### **THE PREAMPLIFIER**

The purpose of the preamplifier is :

- to provide a fast charge transfer from the detector.
- to optimize the coupling between the detector and the rest of the read-out.
- to minimize the electronics noise by introducing a first stage low noise gain.
- to minimize the crosstalk at the detector output (for multi-electrode detectors).
- to assure output driving capabilities for signal transmission through a cable.

We will consider :

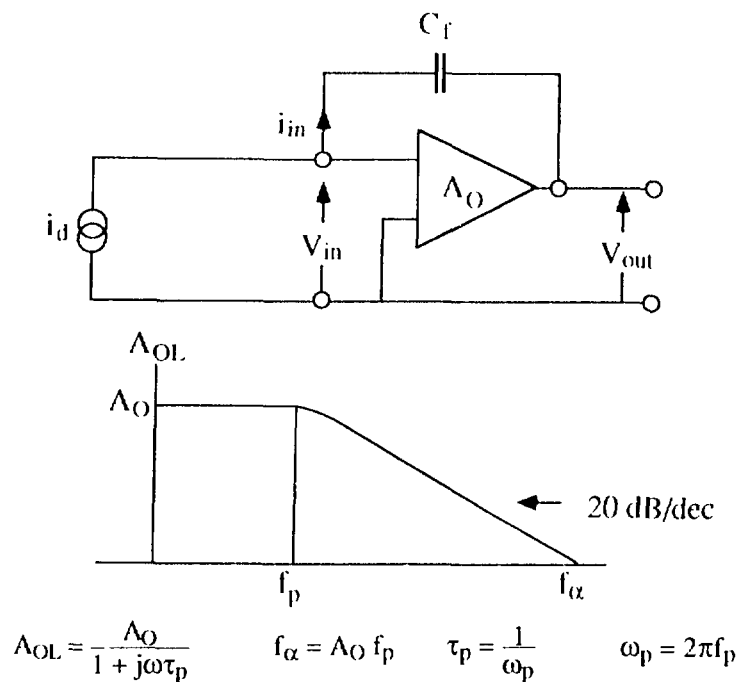
- a) - the input impedance,  $R_{in}$  and the input time constant  $\tau_{in}$  which is determinant for the *charge transfer speed* and for the *crosstalk (in multi-electrode detectors)*.
- b) - the noise sources of the preamplifier, which is determinant for the *minimum sensivity (minimum signal charge detection)* of the read-out system.

There is two basic configurations used as low input impedance charge sensitive preamplifiers :

- the feedback preamplifier
- the grounded base preamplifier

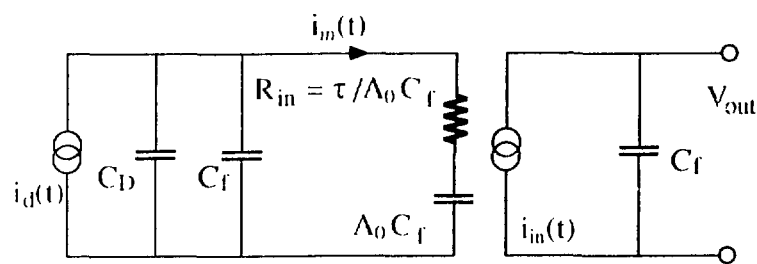
### The feedback preamplifier

The basic schematic is given on Fig. 14.



**Fig. 14 - The feedback preamplifier and parameters definition.**

The equivalent scheme of which is given on Fig. 15.



**Fig. 15 - Equivalent schematic of the feedback preamplifier.**

The input time constant will be :

$$\tau_{in} = \frac{\tau_p}{A_O C_f} \cdot \frac{A_O C_f (C_f + C_D)}{A_O C_f + C_f + C_D} \quad (8)$$

In practice

$$A_0 C_f \gg C_D$$

$$\tau_{in} = \frac{\tau_p}{A_0 C_f} \cdot (C_f + C_D) = R_{in} (C_f + C_D) \quad (9)$$

Where

$$R_{in} = \tau_p / A_0 C_f \quad \tau_p \text{ the dominant pole.}$$

The output voltage is given by :

$$V_{out}(t) = \frac{1}{C_f} \int i_{in} dt = \frac{1}{C_f} \int \frac{I}{(C_D + C_f)} \cdot \frac{1}{R_{in}} e^{-t/\tau_{in}} dt = \frac{1}{C_f} (1 - e^{-t/\tau_{in}}) \quad (10)$$

The gain of such amplifier is given in [V/As] and is given by :

$$G [V/As] = V_{out} / Q_s \cong 1/C_f \quad [\text{for } A_0 C_f \gg C_D] \quad (10. a)$$

Let us take as example the schematic of a charge amplifier : Fig. 16

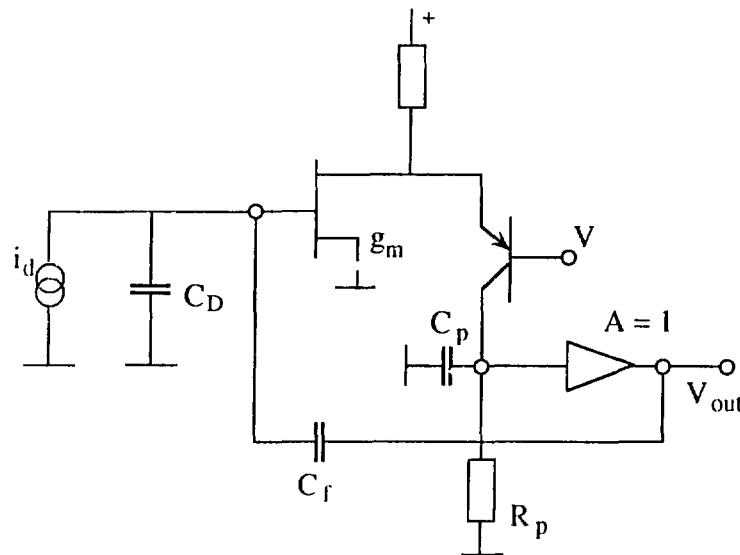


Fig. 16 - Schematic of a cascade charge amplifier.

The dominant pole

$$\tau_p = R_p C_p \quad (10. b)$$

$C_p$  - the total capacitance at the node.

The DC gain

$$A_0 = g_m \cdot R_p \quad (11)$$

Hence

$$R_{in} = \frac{\tau_p}{\Lambda_0 C_f} = \frac{1}{g_m} \cdot \frac{C_p}{C_f} \quad (12)$$

$$\tau_{in} = \frac{1}{g_m} \cdot \frac{C_p}{C_f} (C_D + C_f) \quad (13)$$

### The grounded base configuration

This configuration is used with bipolar transistors, for detectors with high capacitance and when fast response (short shaping time) is needed.

The schematic of such a preamplifier is given on Fig. 17.

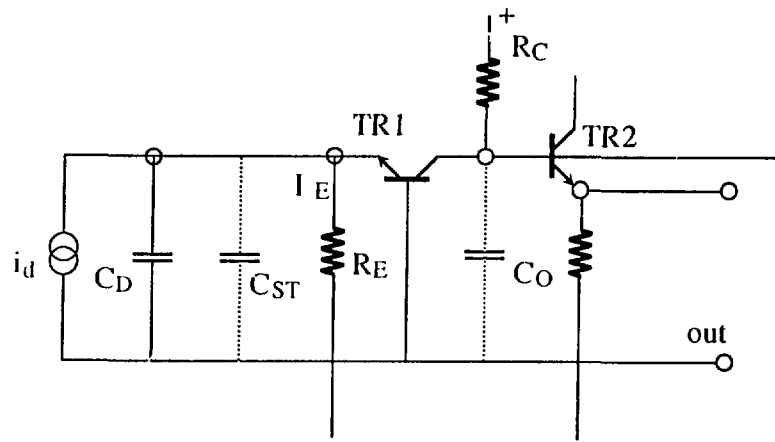


Fig. 17 - The grounded base preamplifier.

The input impedance of this configuration is given :

$$R_{in} = 26 / I_E \text{ [mA]} \quad (14)$$

With a current of 1mA  $R_{in} = 26 \Omega$

The input time constant :

$$\tau_{in} = R_{in} \cdot (C_D + C_{ST}) \quad (15)$$

where  $C_{ST}$  is relatively low, depending of the choice of the first transistor

$$C_{ST} \cong 2 - 10 \text{ pF}$$

The charge gain depends on  $C_O$  which includes all stray capacitances on this node

$$G = 1 / C_O \text{ [V/As]} \quad (16)$$

The noise performances of such preamplifier are worst due to the noise contributions of RC, RL, and the base current of TR1 and TR2.

The input resistance of this preamplifier is in order of 25–100  $\Omega$  depending of the collector current. This is convenient when using the preamplifier as a termination of 50  $\Omega$  or 100  $\Omega$  strip line cable used for signal transmission from the detector electrode to the preamplifier (Fig. 18).

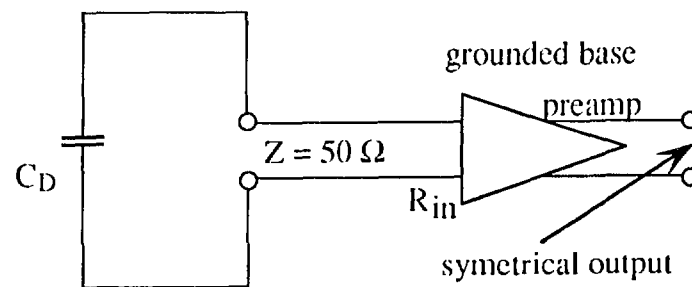


Fig. 18 - Transmission line by the preamplifier input resistance.

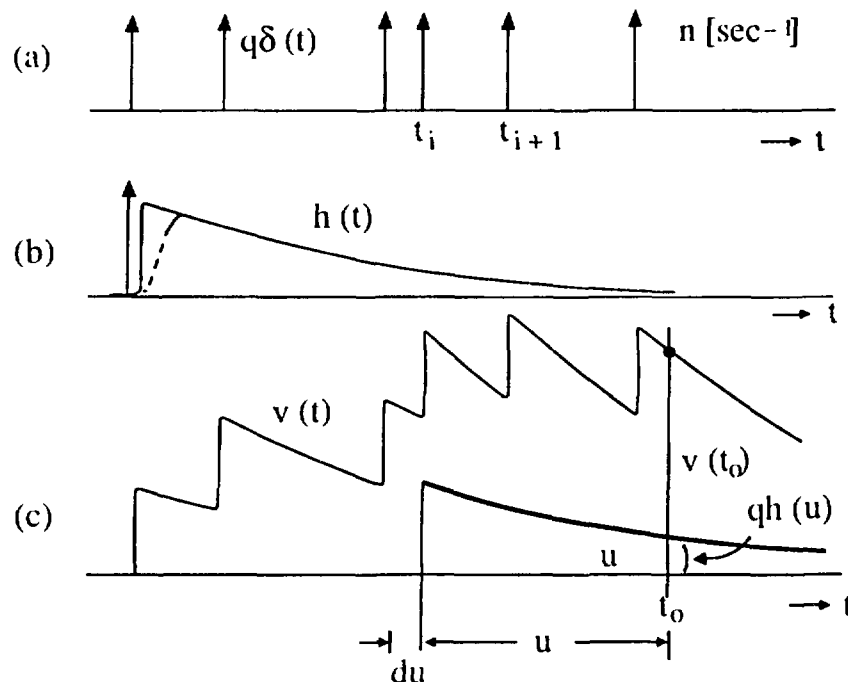
#### NOISE VARIANCE AND EQUIVALENT NOISE SIGNAL

The basic of a noise process can be represented as a sequence of randomly generated elementary impulses with POISSON distribution in time.

Such a random sequence arises, for example, in electronic emission from a thermionic cathode (or photocathode) or in any charge carrier generation where charge carriers with thermal (Boltzman) distribution of energy have to cross a potential barrier.

Each carrier induces a charge  $q$  in the collector (or sensing electrode).

Individual impulses may or may not be separated in time depending on their width and rate of occurrence  $n$ .



**Fig. 19 - Charge amplifier output for a random sequence of impulses at the input.**

When the impulse response of the device, with which we are looking on the noise signal, is much longer than  $1/n$ , the characteristic noise waveforms (those we observe on an oscilloscope) are produced as a superposition of responses to individual impulses<sup>31</sup>. This is illustrated on the Fig. 19.

$$h(t) = CU(t) \cdot e^{-\lambda t} \quad (17)$$

where  $U(t)$  is the unit step function,  $C$  is a constant.

For a signal measurement, we are interested in the *measurement error* due to the noise at the observation time  $t_0$  when the signal is observed. (The signal is not shown in the figure).

The amplitude of the noise at  $t_0$  is the result of many randomly generated noise impulses.

To calculate the *noise variance* we will use the Campbell's theorem, which states that *the sum of mean square contributions* of all preceding impulses equals the variance.

From fig. 19 one can state that the *number of impulses* in the interval  $du$  is a random variable with a Poisson distribution.

The mean and the variance of this variable are equal to  $n \cdot du$  (this is true for a Poisson distribution).

Hence, the noise amplitude at  $t_0$  due to noise impulses generated at  $du$  will be :  $n \cdot du \cdot q \cdot h(u)$  and this amplitude is a random variable with a mean square value equal to :

$$d(\sigma^2) = n \cdot du \cdot [q \cdot h(u)]^2 = n \cdot [q \cdot h(u)]^2 \cdot du \quad (18)$$

(Supposing that the noise impulses are at both polarities, the mean output value equals 0).  
By adding up all the previous history we obtain :

$$\sigma^2 = nq^2 \int h^2(u) du \quad (19)$$

We have made use of the assumption that the random process is stationary so that  $\sigma^2$  is independent of the time  $t_0$  of the measurement.

The integration has to be carried out for all non-zero values of the impulse response  $h(u)$ , independently of the origin of the time variable  $u$ .

The noise variance is determined by the noise process, the rate of impulses  $n$ , and their area  $q$  (charge) and by the impulse response  $h(t)$ , (the frequency band of the measuring system).

The result for the variance  $\sigma^2$  has been derived without any reference to noise description in the *frequency domain* in terms of the *noise power spectrum* and *spectral density*.

The impulse response  $h(t)$  is a description of the measurement system in the *time domain*. The total "energy" of the system response is *independent* of whether the output is analyzed in terms of *time* or *frequency* so that we can write :

$$\int_{-\infty}^{+\infty} h^2(t) dt = \frac{1}{2\pi} \int_{-\infty}^{+\infty} |H(\omega)|^2 d\omega \quad (20)$$

This equality follows from Parseval's theorem. Since  $|H(\omega)|^2$  is an even function (and with  $\omega = 2\pi f$ ) :

$$\int_{-\infty}^{+\infty} h^2(t) dt = 2 \int_0^{+\infty} |H(\omega)|^2 df \quad (21)$$



The noise variance is then :

$$\sigma^2 = 2 nq^2 \int_0^{\infty} |H(\omega)|^2 d\omega \quad (22)$$

where  $H(\omega)$  is the system transfer function.

This relation represents an integrated power spectrum at the output of the system. Since the output power spectrum is :

$$W_{\text{out}}(\omega) = W_{\text{in}}(\omega) |H(\omega)|^2 \quad (23)$$

The input power spectral density is clearly

$$W_{\text{in}}(\omega) = 2 nq^2 = W_0 \quad (24)$$

Since the noise spectrum of a random sequence of impulses is white, the subsequent physical system upon which the noise acts determines the noise spectrum at the output of the system.

The original white spectrum  $W_0$  is transformed into  $W_{\text{out}}(\omega)$  by the system transfer function  $H(\omega)$ .

The impulse response  $h(t)$  of the system is the Fourier transform of the transfer function  $H(\omega)$  and each elementary impulse is transformed into a pulse of shape  $h(t)$ .

The term "power" refers to the amplitude squared of any variable observed (current, voltage or charge).

Thus we can use power spectral density for white noise in calculations of the noise variance from the system *time response*  $h(t)$  and eq. (19) became :

$$\sigma^2 = \frac{1}{2} W_0 \int h^2(t) dt \quad (25)$$

which gives the variance in  $[V^2]$  at the output of the system, obtained in the "*time domain*". The variance obtained using the "*frequency domain*" will be

$$\sigma^2 = W_0 \int_0^{\infty} |H(\omega)|^2 d\omega \quad (26)$$

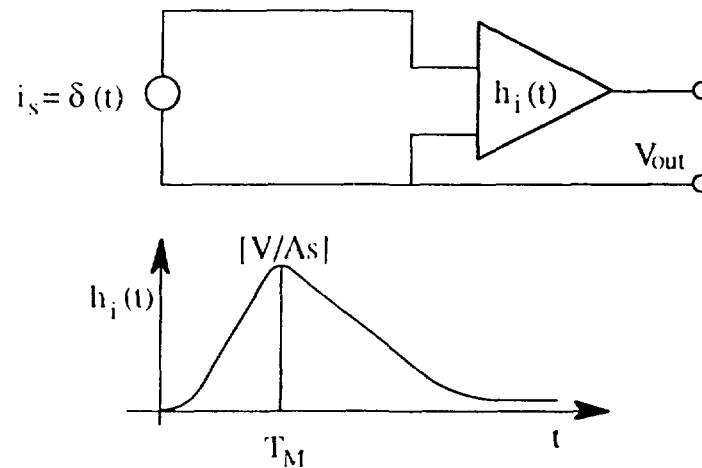
The spectral density  $W_0$  can be :

$$1 - W_0 = \bar{i}_n^2 = 2qI \text{ for shot noise}$$

$$2 - W_0 = \bar{i}_n^2 = \frac{4kT}{R} \text{ for current thermal noise}$$

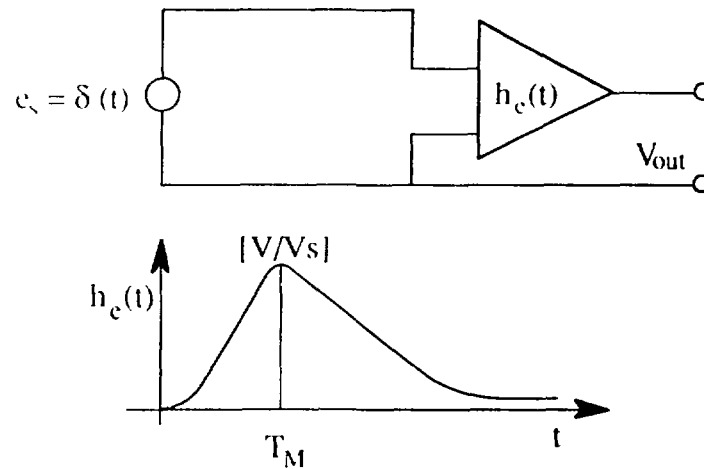
$$3 - W_0 = \bar{e}_n^2 = 4kTR \text{ for voltage thermal noise}$$

For cases 1 and 2, the impulse response  $h(t)$ , eq. (25) is  $h_i(t)$  as defined in Fig. 20.



**Fig. 20 - The current impulse response**

For case 3 the voltage impulse response is used (Fig. 21).



**Fig. 21 - The voltage impulse response.**

### Equivalent noise signal

The E.N.S. is the signal giving at the output of the system a response of amplitude equal to the rms (root mean square) noise voltage<sup>41</sup>.

The signal can be defined by :

- the signal amplitude

(generally when the signal length is larger than the amplifier impulse response).

- the signal integral

(generally when the signal is short compared to the amplifier impulse response).

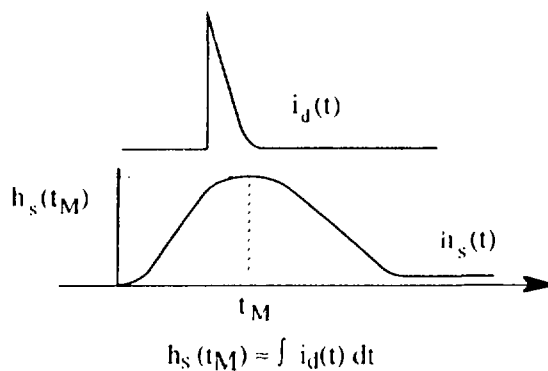
The units of the E.N.S. will be those of the signal units (signal amplitude or signal integral).

For detectors used in Nuclear Physics, the signal is the charge furnished by the detector in "electrons". In this case we define the *Equivalent Noise Charge*. Any measuring system is defined by its impulse response :  $h_i(t)$ . When measuring current impulses, the impulse response to a :

$$i(t) = \delta(t) \quad (27)$$

will be :  $h_i(t)$  (Fig. 20).

If the detector current  $i_d(t)$  is not of  $\delta(t)$  shape, the output amplitude will be proportional to the integral of  $i_d(t)$  as long as the duration of the detector current is small compared to that of the impulse response (Fig. 22).



**Fig. 22 - The response to  $i_d(t)$ .**

$$h_s(t) = i_d(t) * h_i(t) \quad (28)$$

and for  $i_d(t)$  much shorter than  $h_i(t)$

$$h_s(t) = [\int i_d(t) dt] \cdot h_i(t) \quad (29)$$

a - **Equivalent Noise Charge : ENC**

To obtain the *ENC for a NOISE SOURCE*, we have, first to calculate the variance  $\sigma^2$  for that noise source

$$\sigma^2 = \frac{1}{2} \bar{i}_n^2 \int h_{in}^2(t) dt \quad [V^2] \quad (30)$$

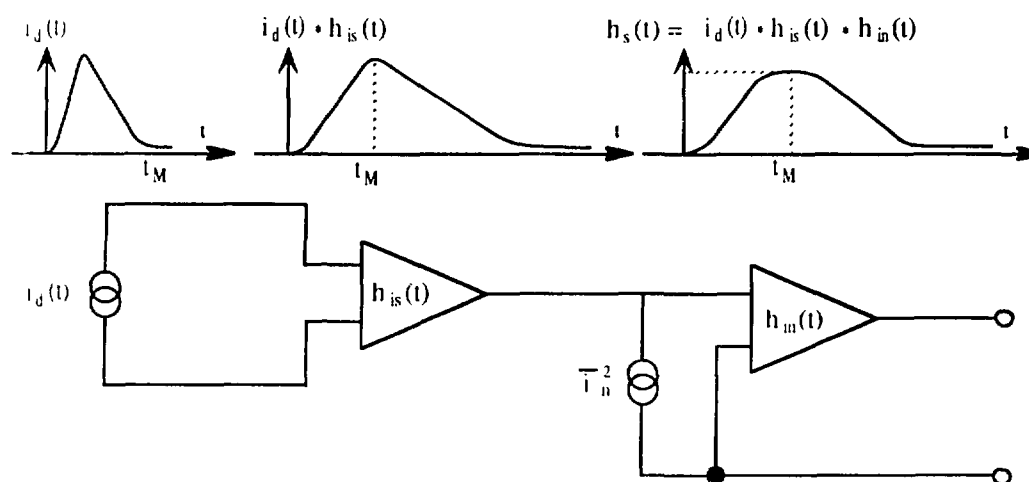


Fig. 23 - Signal and noise response for a general case ENC calculation.

For the noise the impulse response is :  $h_{in}(t)$

For the signal the response is :

$$\begin{aligned} V_{s \text{ out}} = h_s(t) &= i_d(t) * h_{is}(t) * h_{in}(t) \\ &\approx [\int i_d(t) dt] \cdot [h_{is}(t) * h_{in}(t)] \end{aligned} \quad (31)$$

Then the ENC will be :

$$ENC^2 = \frac{(1/2) \bar{i}_n^2 \int h_{in}^2(t) dt}{[h_s(t_M)]^2} [As]^2 \quad (32)$$

where  $h_s(t_M)$  is the maximum of the signal output, for a signal of unity charge.

For a voltage NOISE SOURCE we have to replace  $\bar{i}_n^2 \rightarrow \bar{e}_n^2$  and  $h_{in}(t)$  by  $h_{en}(t)$ ,  $h_{en}(t)$  : the voltage response.

**b - Equivalent Noise Amplitude**

When  $i_d(t)$  is much longer than  $h_i(t)$  and has unity amplitude

$$h_s(t) = i_d(t) * h_i(t) \\ \approx i_d(t) \cdot \int h_i(t) dt \quad (33)$$

and the maximum signal output will be equal to the integral of  $h_i(t)$

$$ENA = \frac{\sigma^2}{[h_s(t)]_{\max}^2} \quad [A^2] \quad (34)$$

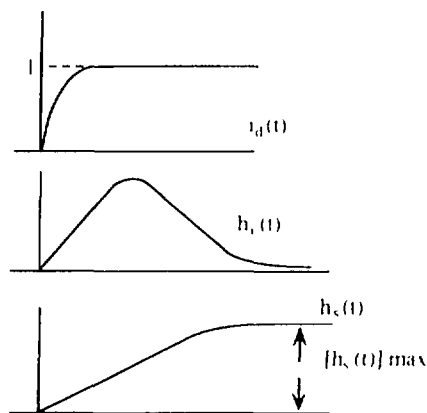


Fig. 24 - The signal response for a very long signal.

**Noise in charge amplifiers**

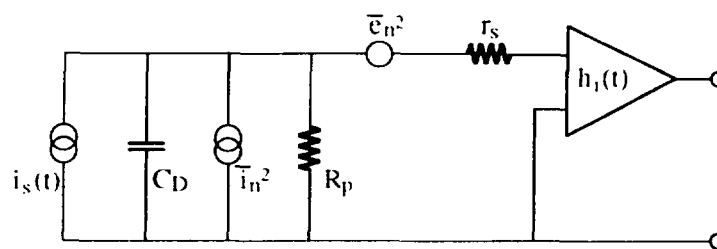


Fig. 25 - Noise sources for a charge preamplifier.

By supposing that the input resistance of the charge amplifier is very low,  $Z_{in} \rightarrow 0$  and  $r_s \rightarrow 0$ , the response to  $i_s(t)$  will be the impulse response  $h_i(t)$ , of the amplifier (with  $C_D$  and  $R_p$  included) (Fig. 25).

The noise due to the parallel noise sources at the amplifier input is defined by  $i_n^2$  parallel noise.

The noise due to the serie input sources is defined by  $e_n^2$  serie noise.

a - **Parallel noise :**

$$\sigma^2 = \frac{1}{2} \frac{4kT}{R_p} \int h_i^2(t) dt \quad (35)$$

where  $h_i(t) = h_{in}(t)$  – the signal impulse response = the noise impulse response

$$\begin{aligned} ENC_p^2 &= \frac{1}{2} \frac{4kT}{R_p} \frac{\int h_i^2(t) dt}{h_i(t_M)^2} \\ &= \frac{1}{2} \frac{4kT}{R_p} \int h_{iN}^2(t) dt \end{aligned} \quad (36)$$

where  $h_{iN}(t)$  is normalized impulse response (unity amplitude).

b - **Series noise**

To calculate the variance due to the series noise source we need to know the voltage response of the amplifier, due to the series noise voltage source (Fig. 26).

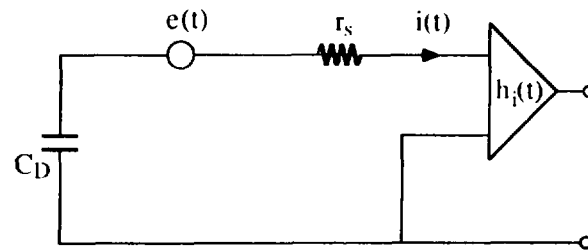


Fig. 26 - Serie noise response definition.

The response to  $e(t)$  will be :

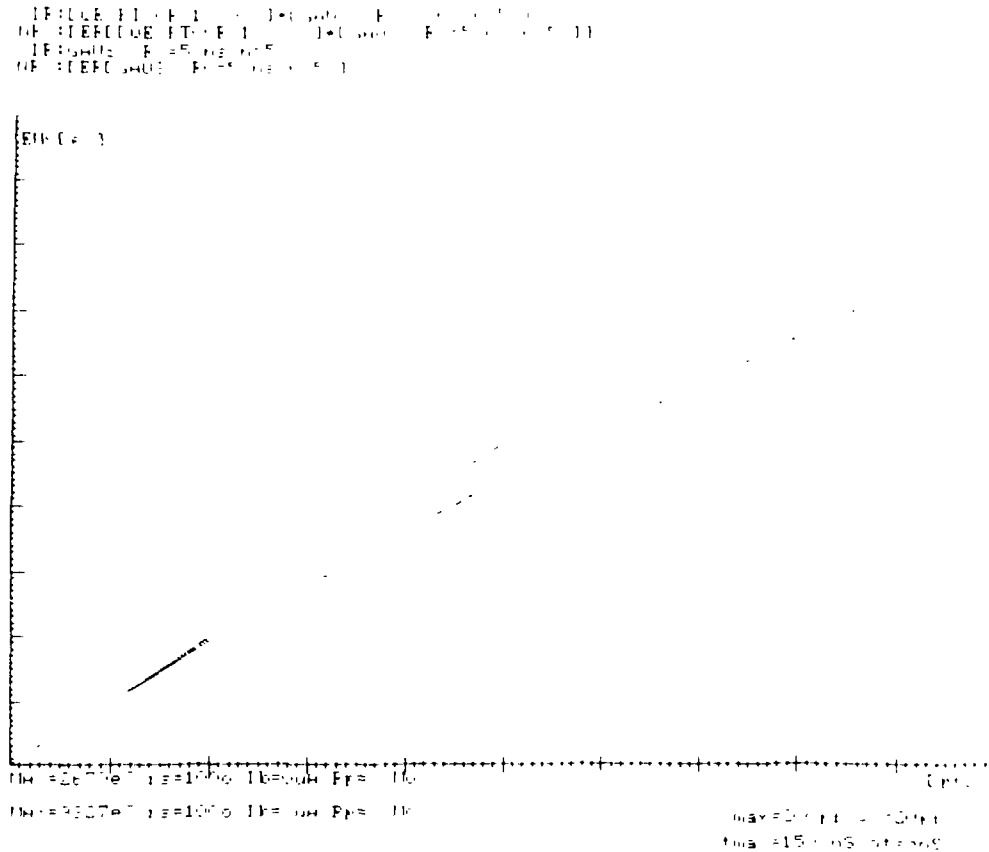
$$\begin{aligned} h_{cn}(t) &= i(t) * h_i(t) \\ &= C_D [d e(t)/dt] * h_i(t) \\ &= C_D \delta'(t) * h_i(t) \\ &= C_D h'_i(t) \end{aligned} \quad (37)$$

The  $ENC^2$  will be

$$\begin{aligned} ENC_s^2 &= \frac{\frac{1}{2} e_n^2 \int [h'_i(t)]^2 dt}{[h_i(t_M)]^2} \\ &= \frac{1}{2} (e_n C_D) \int [h'_{iN}(t)]^2 dt \end{aligned} \quad (38)$$

The impulse response  $h_s(t)$ , is defined by a circuit including  $R_p$  and  $C_D$ . Hence,  $h_s(t) = f(C_D)$ . For small values of  $C_D$  we can suppose that  $h_s(t) = h_i(t) \neq f(C_D)$  and the serie noise (i.e the  $ENC_s^2$ ) will be a linear function of  $C_D$ .

But for large value of  $C_D \rightarrow$  the effect of  $C_D$  on  $h_i(t)$  is significant. We can see this influence of  $C_D$  on  $h_i(t)$  and on  $ENC^2$  with the simulated responses for  $ENC_s^2$  calculations (Fig. 27).



**Fig. 27 - ENC's simulation with and without the influence of  $C_D$**

The total noise for a charge amplifier is :

$$\begin{aligned}
 ENC^2 &= \frac{1}{2} i_n^2 \int h_i^2(t) dt + \frac{1}{2} (e_n C_D)^2 \int [h_i(t)]^2 dt \\
 &= \frac{1}{2} i_n^2 a_{F2} \tau_F + \frac{1}{2} (e_n C_D)^2 a_{F1} / \tau_F
 \end{aligned} \tag{39}$$

where :  $a_{F1}$  and  $a_{F2}$  are shape constants factor of the impulse response  $\tau_F$  is a time constant

factor of the impulse response. So that :

$$ENC^2 = 2kTr_s C_D^2 \cdot \frac{1}{\tau_F} \left[ a_{F1} + \left( \frac{\tau_F}{\tau_c} \right)^2 a_{F2} \right] \quad (40)$$

where

$$\tau_c = C_D \sqrt{r_s R_p} \quad \text{the "noise corner time constant"}$$

$$r_s = \bar{e}_n^2 / 4kT$$

$$R_p = 4kT / \bar{i}_n^2$$

For  $\tau_c = \tau_F \sqrt{\frac{a_{F2}}{a_{F1}}}$  serie and parallel noise sources contributions are equal and this is optimal condition for NOISE.

For given  $r_s$  and  $R_p$ ,  $\tau_F$  is chosen so that  $\tau_F = \tau_c \sqrt{\frac{a_{F1}}{a_{F2}}}$  for optimum noise performances.

It can be shown that it exists an optimal shape (a cusp) for minimum noise. But for practical circuits the semigaussian shaping is used for the shaping of the detector signals, which gives noise close to the optimum.

The serie noise can be also optimized by matching the preamplifier input capacitance to the detector capacitance  $C_D$ .

### Noise in large band amplifiers

The noise in a large band amplifier is due to the serie resistance at the input. A large band amplifier is driven by a voltage source signal (Fig. 28).

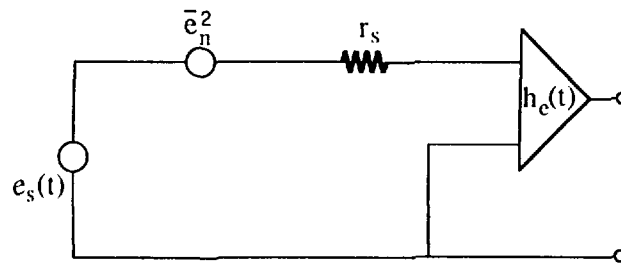


Fig. 28 - Noise in voltage amplifier.

For a long signal (unit step)

$$e_s(t) = S(t) \quad (41)$$



The output will be :

$$h_S(t) = S(t) * h_e(t) = \int h_e(t) dt \quad (42)$$

where  $h_e(t)$  is the impulse response.

$$\sigma^2 = \frac{1}{2} \bar{e}_n^2 \int h_e^2(t) dt \quad (43)$$

$$ENA^2 = \frac{\frac{1}{2} \bar{e}_n^2 \int h_e^2(t) dt}{\left[ \int h_e(t) dt \right]^2} \quad (44)$$

$$ENA^2 = \frac{\frac{1}{2} \bar{e}_n^2 a_{F2} \tau_F}{(a_{F3} \tau_F)^2} = \frac{1}{2} \frac{a_{F2}}{a_{F3}^2} \bar{e}_n^2 / \tau_F \quad (45)$$

The equivalent noise voltage at the input is proportional with the amplifier band-pass. For a triangular response with peaking time  $t_M$  :

$$a_{F2} = 2/3 \quad \tau_F = t_M$$

$$a_{F3} = 1$$

$$ENA^2 = \bar{e}_n^2 / 3t_M \quad (46)$$

### Measuring the ENC

#### 1. By measurement of the correlation function with an oscilloscope<sup>51</sup>

The correlation function can be interpreted as the *mean* of the conditional probability density function that the noise will have value  $V(\tau)$  at time  $t = \tau$  if it had value  $V_{TR}$  at time  $t = 0$ , where  $V_{TR}$  is the oscilloscope triggering level at  $t = 0$  ( $t$  means triggering time).

The maximum  $V_{TR}$  at which the triggering density goes to *ZERO* will correspond approximately to something between 3 and 4  $\sigma$ . Fig. 29.

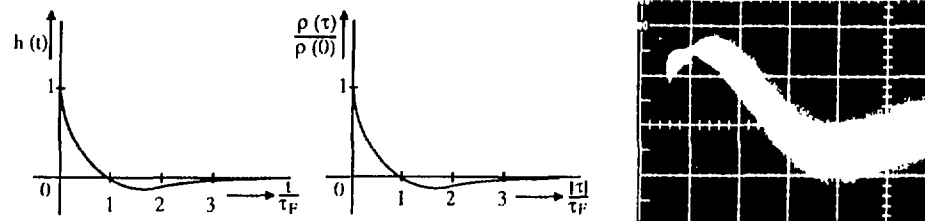


Fig. 29 - Correlation function for  $CR = RC = \tau_F$  filter

2. **The most precise noise measurement is done by a Multichannel Analyzer.**

Let us suppose that the amplitude distribution of the output noise is given by  $f_1(V)$ , in general a Gaussian. This distribution can be obtained by sampling the output signal with the MCA. For an input signal.

$$i_s(t) = Q \delta(t) \quad (47)$$

The mean value of the obtained distribution will be  $N = \alpha \cdot Q$  and the variance of the distribution will be a measure for the noise.

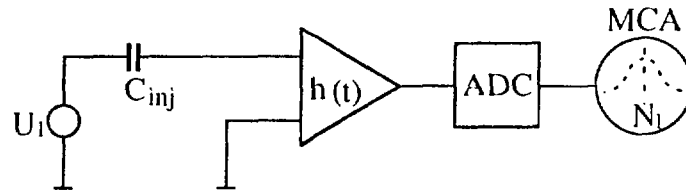


Fig. 30 - Noise measurement with a multichannel analyzer.

$$\sigma = \text{FWHM} / 2.36 [\text{ch}] \quad (48)$$

To obtain the Equivalent Noise Charge we have to divide the result for  $\sigma$ , by  $\alpha$ , the calibration constant

$$\text{ENC} = \text{FWHM} / (2.36 \alpha) \quad (49)$$

FWHM is the full width half maximum of the obtained distribution. To obtain  $\alpha$ , we inject a known charge through  $C_{inj}$ :

$$Q_1 = C_{inj} U_1 = N_1 / \alpha \quad (50)$$

where  $N_1$  - the channel number on the MCA. A second measure is done with:

$$Q_2 = C_{inj} U_2 = N_2 / \alpha \quad (51)$$

$$\text{and : } \alpha = \frac{N_2 - N_1}{C_{inj} (U_2 - U_1)} \text{ Channel/As} \quad (52)$$

Note : When using a QVT as a MCA, mode Q, the integration time of the QVT gate will modify the impulse response of the measuring device if  $\tau_F \approx T_g$  ( $T_g$  - the gate width of the QVT).

### SHAPING

Pulse shaping is important for the preamplifier signal in order to obtain noise performances close to the optimum.

It has been shown that the gaussian shaping has favorable features when charge is measured. Hence, the design of a filter amplifier with an impulse response approaching a gaussian shape will be considered. The amplifier output is an exponential function with  $\tau_f \approx R_f C_f$  of the preamplifier. To drive the shaping amplifier we need to add a "pulse-zero" cancellation circuit so that to replace the long exponential by a shorter one corresponding to the integration time constant of the shaping amplifier (Fig. 31).

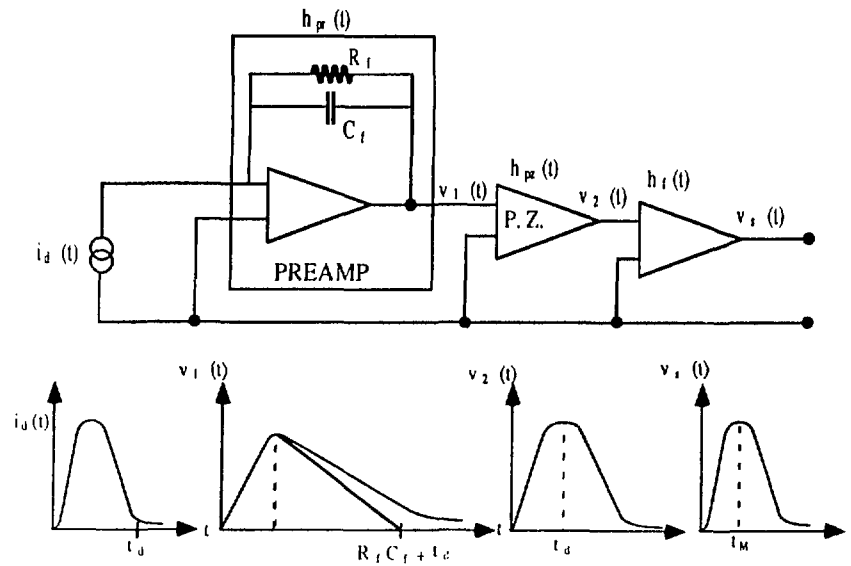


Fig. 31 - An analog chain for detector signal read-out.

$$v_s = i_d(t) * h_{pr}(t) * h_{pz}(t) * h_f(t) \quad (53)$$

$h_f(t)$  is an  $(n - 1)$  integrations circuit with integration time constant  $t_0$ . The first integration being defined in the p - z circuit.

From the pole-zero cancellation circuit (Fig. 32) we can obtain :

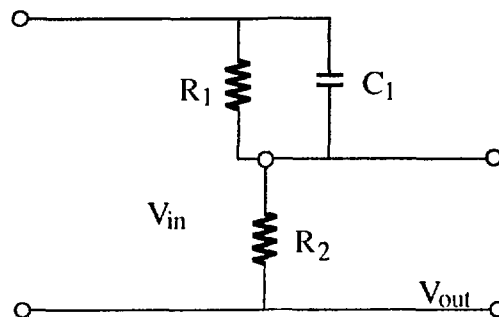


Fig. 32 - The pole-zero (p-z) cancellation circuit.

$$\frac{V_{out}}{V_{in}} = \frac{p + 1/\tau_1}{p + 1/\tau_0} \quad (54)$$

$$\tau_1 = R_1 C_1 \quad (55)$$

$$\tau_0 = \frac{R_1 R_2}{R_1 + R_2} \cdot C_1 \quad (56)$$

and we can see that :

$$\tau_0 < \tau_1 \quad (57)$$

When driving this circuit with an exponential shape  $\rightarrow e^{-t/\tau_f}$

$$V_{out} = \frac{1}{p + 1/\tau_f} \cdot \frac{p + 1/R_1 C_1}{p + \frac{1}{C_1} \left( \frac{1}{R_1} + \frac{1}{R_2} \right)} \quad (58)$$

and with

$$\tau_f = R_1 C_1 \quad (59)$$

$$V_{out} = K \cdot e^{-t/\tau_0} \quad (60)$$

The output of the measuring system will be :

$$V_{out} = K e^{-t/\tau_0} * e^{-t/\tau_0} * e^{-t/\tau_0} \dots * e^{-t/\tau_0} \quad (61)$$

n time integration will give n time convolutions and

$$V_{out} = \frac{1}{n!} \left( \frac{1}{\tau} \right)^n e^{-t/\tau_0} \quad (62)$$

which is a semi gaussian response.

The integrations can be done by "active filter integrators" (fig. 34). When realizing active filters, each section gives two integrations. The impulse response of an active integrating filter of n sections is given by <sup>6)</sup>:

$$H(p) = \frac{K_0}{p + 1/\tau_0} \cdot \frac{K_1 \omega_{oi}^2}{p^2 + \alpha_1 \omega_{oi} p + \omega_{oi}^2} \dots \frac{K_i \omega_{oi}^2}{p^2 + \alpha_i \omega_{oi} p + \omega_{oi}^2} \quad (63)$$

On table 1, the values for  $\omega_{oi}$ ,  $\alpha_i$  and  $\tau_0$  are given as a function of  $\sigma$  ( $\sigma \approx t_M/2$  where  $t_M$  - peaking time of the "Gaussian" response) for filters up to 3 stages - 6 or 7 integrations (the first integration is that obtained by the pole-zero circuit).

TABLE 1

	n = 3	n = 4	n = 5	n = 6	n = 7
$\sigma/\tau_0$	1.26		1.48		1.66
$\sigma\omega\omega_1$	1.39	1.39	1.54	1.58	1.70
$\alpha_1$	1.65	1.94	1.84	1.97	1.91
$\sigma\omega\omega_2$		1.59	1.77	1.68	1.82
$\alpha_2$		1.49	1.36	1.73	1.64
$\sigma\omega\omega_3$				1.95	2.11
$\alpha_3$				1.26	1.17

Example: we desire an 5 integration filter : n = 5 (Fig. 23).



Fig. 33 - 5 integration filter

$$\tau_0 = \frac{\sigma}{1.48} = \frac{t_M}{2 \times 1.48} \quad (64)$$

where  $t_M$  is the desired peaking time,  $\tau_0$  is the final constant for the "pole-zero cancellation" circuit.

$$\omega_{01} = 1.54/\sigma = (1.54 \times 2) / t_M \quad (65)$$

$$\alpha_1 = 1.84 \quad (66)$$

for the S1 stage

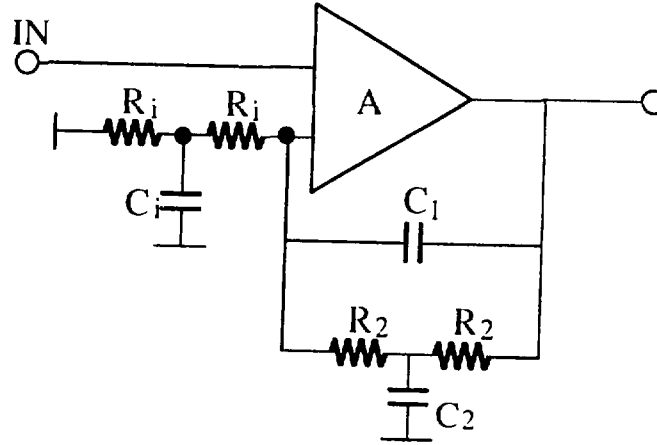
and

$$\omega_{02} = (1.77 \times 2) / t_M \quad (67)$$

$$\alpha_2 = 1.36 \quad (68)$$

for the S2 stage

**Operational amplifier filter configurations for S1 and S2**



**Fig. 34 - Operational amplifier filter configurations**

The impulse response :

$$H(p) = \frac{2}{R_i C_1 C_1} \cdot \frac{1}{p^2 + \frac{2}{R_2 C_2} p + \frac{1}{R_2^2 C_1 C_2}} \quad (69)$$

with  $R_i C_i = R_2 C_2$  (70)

and by choosing  $C_1$  to be enough large to make the strays capacitance negligible we have from Table 1.

$$C_2 = \frac{4}{\alpha_i^2} C_1 \quad ; \quad R_2 = \frac{\alpha_i}{2\omega_{oi} C_1} \quad (71)$$

$$R_i = R_2 / G \quad \text{and} \quad C_i = R_2 C_2 / R_i \quad (72)$$

where  $G$  is the desired DC Gain.

Using the values obtained for  $\omega_{01}$ ,  $\omega_{02}$ ,  $\alpha_1$  and  $\alpha_2$  we can define  $C_2$ ,  $C_1$ ,  $R_2$ ,  $R_i$  and  $C_i$  for S1 and S2 stages of our example.

The shaping can also be done by delay line (Fig. 35)

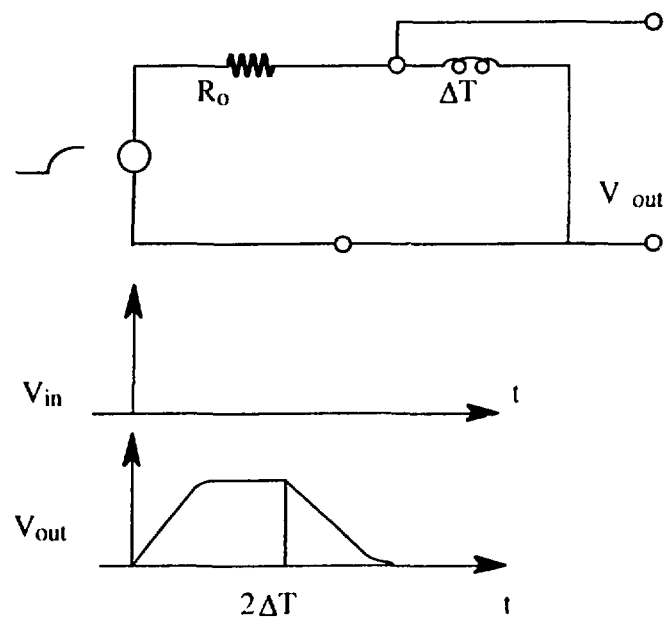


Fig. 35 - Delay line unipolar shaping.

An example of a Si. Det. read-out channel amplifier with delay line shaping is given on Fig. 36.

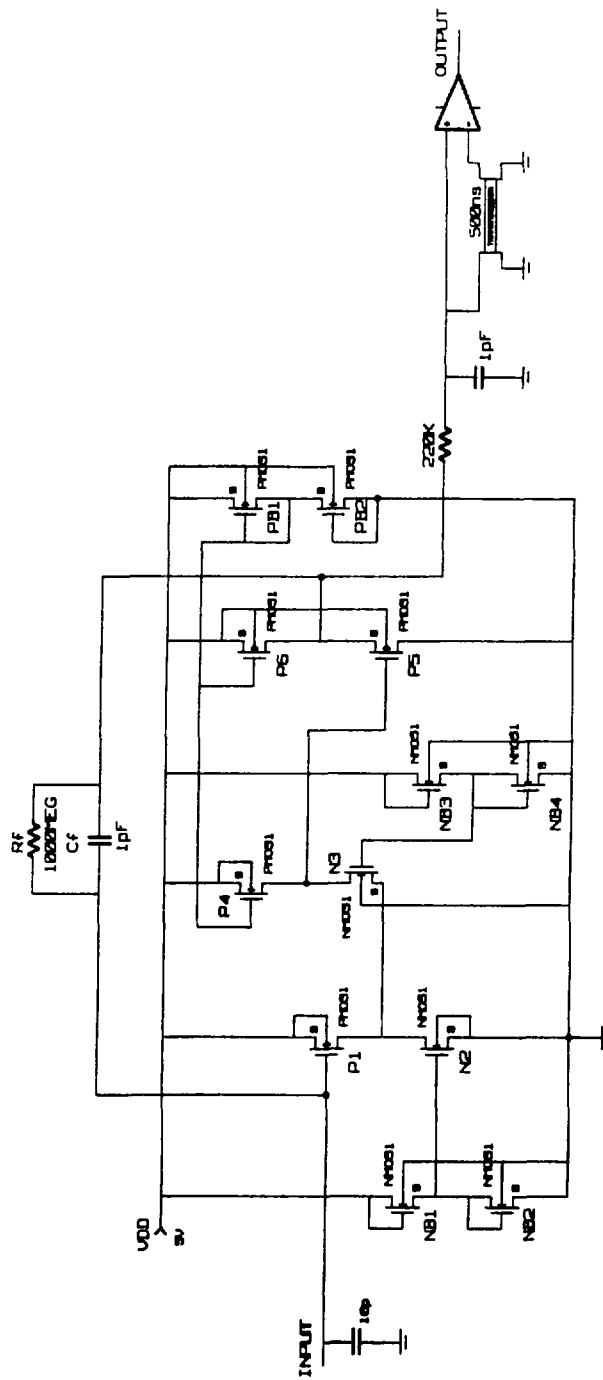
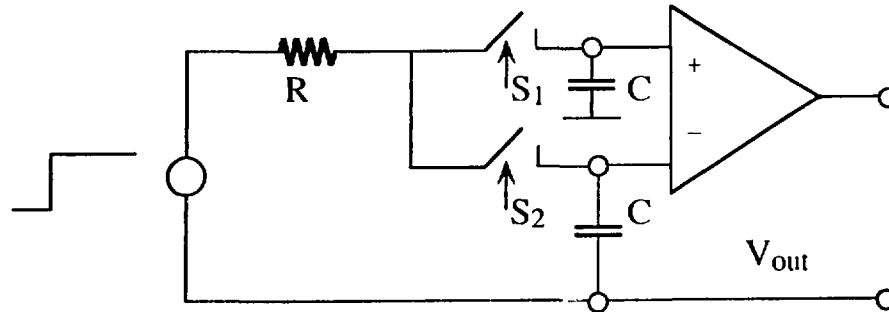


Fig. 36 - Si. Det. read-out with delay line shaping



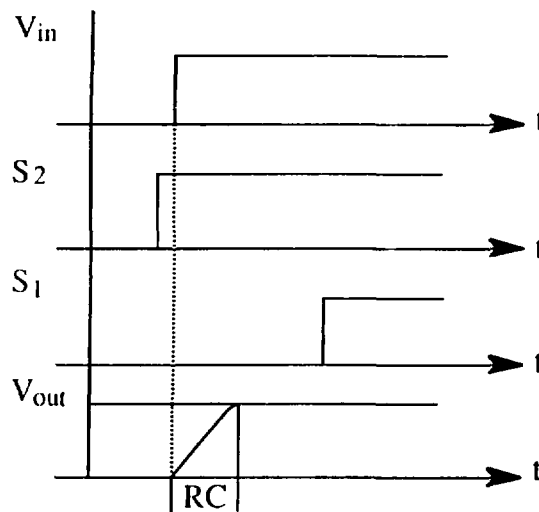
Similar shaping can be done by double correlated sampling (Fig. 37).



**Fig. 37 - Double correlated sampling method for shaping amplifier.**

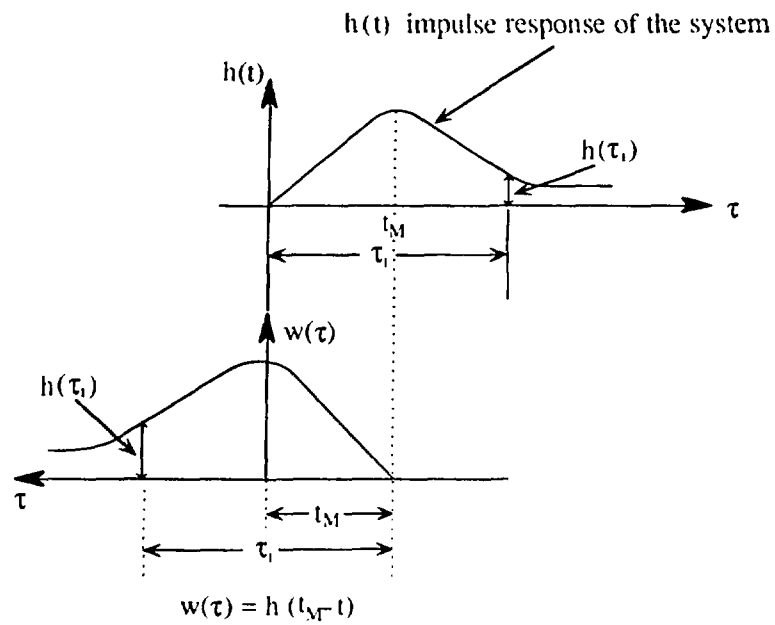
This is a "time variant" filter and the signal impulse response will be different than that one for the noise.

Signal response (Fig. 38).



**Fig. 38 - The signal response for a double correlated sampling.**

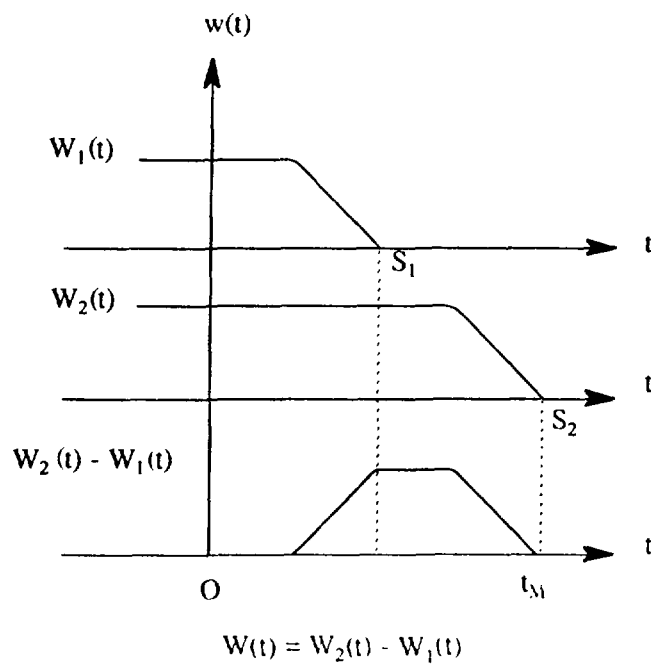
The noise response will be defined by introducing the "NOISE WEIGHTING FUNCTION",  $w(\tau)$ , which gives the contribution to the output voltage at time  $t_M$  (the measuring time) of a noise  $\delta(t)$  current arriving a time  $\tau_i$  before  $t_M$  - Fig. 39.



**Fig. 39 - The weighting function definition.**

In the case of double correlated sampling the weighting function for noise will be

**Fig. 40**



**Fig. 40 - The noise weighting function for the double correlated sampling method.**

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