CONF-910864--1

DE92 008893

MESON SPECTRA FROM TWO-BODY DIRAC EQUATIONS WITH MINIMAL INTERACTIONS

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Many authors have used two-body relativistic wave equations with spin in nonperturbative numerical quark model calculations of the meson spectrum. Usually, they adopt a truncation of the Bethe-Salpeter equation of QED and/or scalar (exchange) QED and replace the static Coulomb interactions of those field theories with a semiphenomenolgical $Q\bar{Q}$ potential whose insertion in the Breit terms give the corresponding spin corrections. However, the successes of these wave equations in QED have invariably depended on perturbative treatment of the terms in each beyond the Coulomb terms. There have been no successful nonperturbative numerical test of two-body quantum wave equations in (spin-dependent) QED, because in most equations the effective potentials beyond the Coulomb are singular and can only be treated perturbatively. This is a glaring omission that we rectify here[1] for the case of the two-body Dirac equations of constraint dynamics[2]. (Two of us have performed such a test analytically[3] for the equal mass singlet states of two-body Dirac equations by finding a family of exact solutions whose energy spectrum agrees with that of QED through order α^4.) We show here that a nonperturbative numerical treatment of these equations for QED yields the same spectral results as a perturbative treatment of them which in turn agrees with the standard spectral results for positronium and muonium. This establishes that the vector and scalar interaction structures of our equations accurately incorporate field theoretic interactions in a bona fide relativistic wave equation (and increases our confidence in the suitability of such equations for nonperturbative quark-model calculations). The last portion of this work will report recent quark model calculations using these equations with the Adler-Piran static $Q\bar{Q}$ potential.

We begin with two coupled Dirac equations

$$\mathcal{S}_1\psi = \gamma_{51}(\gamma_1\cdot(p_1 - A_1) + m_1 + S_1)\psi = 0 \qquad (1a)$$

$$\mathcal{S}_2\psi = \gamma_{52}(\gamma_2\cdot(p_2 - A_2) + m_2 + S_2)\psi = 0 \qquad (1b)$$

that contain a constituent minimal interaction structure. $\mathcal{S}_1$ and $\mathcal{S}_2$ must be compatible in the sense that $[\mathcal{S}_1, \mathcal{S}_2]\psi = 0$. This condition requires that the potentials be given in terms of[2] five invariant functions G, E_1, E_2, M_1, M_2 such that

$$A_1^\mu = [(\epsilon_1 - E_1) - i\frac{G}{2}\gamma_2\cdot(\frac{\partial E_1}{E_2} + \partial lnG)\gamma_2\cdot\hat{P}]\hat{P}^\mu + (1-G)p^\mu - \frac{i}{2}\partial G\cdot\gamma_2\gamma_2^\mu \quad (2a)$$

$$A_2^\mu = [(\epsilon_2 - E_2) + i\frac{G}{2}\gamma_1\cdot(\frac{\partial E_2}{E_1} + \partial lnG)\gamma_1\cdot\hat{P}]\hat{P}^\mu - (1-G)p^\mu + \frac{i}{2}\partial G\cdot\gamma_1\gamma_1^\mu, \quad (2b)$$

$$S_1 = M_1 - m_1 - \frac{i}{2}G\gamma_2\cdot\frac{\partial M_1}{M_2} \qquad (3a)$$

$$S_2 = M_2 - m_2 - \frac{i}{2} G \gamma_1 \cdot \frac{\partial M_2}{M_1}. \qquad (3b)$$

where $P = p_1 + p_2, w = \sqrt{-P^2}$, $\equiv P/w. \epsilon_1 + \epsilon_2 = w$ with $\epsilon_1 = (w^2 + m_1^2 - m_2^2)/2w$, $\epsilon_2 = (w^2 + m_2^2 - m_1^2)/2w. p = (\epsilon_2 p_1 - \epsilon_1 p_2)/w$. The compatibility condition further requires that E_1, E_2, G, M_1 and M_2 depend only on the transverse projection $x_\perp^\mu = (g^{\mu\nu} + \hat{P}^\mu \hat{P}^\nu)(x_1 - x_2)_\nu$ of the relative separation and that M_1, M_2, E_1, E_2, and G depend on only three independent invariant functions S, V, and $\mathcal{A}$ which generate scalar, time-like vector, and electromagnetic-like vector interactions. We choose these relations to be

$$E_i^2(\mathcal{A}, V) = G^2((\epsilon_i - \mathcal{A})^2 - 2\epsilon_w V - V^2). \qquad (4)$$

$$G^2 = \frac{1}{(1 - 2\mathcal{A}/w)}. \qquad (5)$$

$$M_i^2(\mathcal{A}, S) = m_i^2 + G^2(2m_w S - S^2) \qquad (6)$$

so that the spin independent interactions of our equations take the minimal Todorov form $(\vec{p}^2 + 2\epsilon_w \mathcal{A} - \mathcal{A}^2 + 2\epsilon_2 V - V^2 + 2m_w S + S^2) = b^2(w)$ in order that the resulting equations possess the correct semirelativistic ($O(1/c^2)$) dynamics beyond the nonrelativistic limit. (In Eqs.(4-6), $m_w = m_1 m_2/w$ and $\epsilon_w = (w^2 - m_1^2 - m_2^2)/2w$ are the relativistic reduced mass and energy of the fictitious particle of relative motion and satisfy the mass-shell condition $\epsilon_w^2 - m_w^2 = w^4 - 2w^2(m_1^2 + m_2^2) + (m_1^2 - m_2^2)^2)/4w^2 \equiv b^2(w)$.) To relate our equations to quantum field theory in the most compact fashion we choose an equivalent invariant hyperbolic parameterization

$$M_1 = m_1\ chL + m_2 shL \qquad (7a)$$

$$M_2 = m_2\ chL + m_1\ shL \qquad (7b)$$

$$E_1 = \epsilon_1\ chJ + \epsilon_2 shJ \qquad (7c)$$

$$E_2 = \epsilon_2\ chJ + \epsilon_1\ shJ \qquad (7d)$$

$$G = e^{\mathcal{G}}. \qquad (7e)$$

$L(x_\perp), J(x_\perp)$, and $\mathcal{G}(x_\perp)$ generate scalar, time-like vector and space-like vector interactions respectively. In terms of these variables, each Dirac equation in the pair takes the form[4] $[(\gamma_i \cdot p_i + m_i)ch(\Delta) + (-\gamma_j \cdot p_j + m_j)sh(\Delta)]\psi = 0$, $i \neq j = 1, 2$ and $\Delta = \frac{1}{2}(\gamma_1 \cdot \hat{P}\gamma_2 \cdot \hat{P} J(x_\perp) + 1_1 1_2 L(x_\perp) + \gamma_{1\perp} \cdot \gamma_{2\perp} \mathcal{G}(x_\perp))$.

The Dirac wave function has sixteen components - four four-component spinors. By squaring the two-body Dirac equations we obtain a single second order Schrödinger-like equation whose upper-upper component is

$$\{p^2 + 2m_w S - S^2 - 2\epsilon_w \mathcal{A} - \mathcal{A}^2 + 2\epsilon_w V - V^2 + \frac{1}{4}(\partial J - \partial L)^2$$

$$+ iln'\chi_1 \hat{r} \cdot p - \frac{ln'\chi_1}{r} L \cdot \sigma_1 + iln'\chi_2 \hat{r} \cdot p - \frac{ln'\chi_2}{r} L \cdot \sigma_2$$

$$-\frac{1}{2}\partial^2 lnG + \frac{3}{4}(ln'G)^2 + \frac{1}{2}ln'\chi_1\chi_2 ln'G)$$

$$+[\frac{1}{3}\partial^2 lnG - \frac{1}{2}(ln'G)^2 - \frac{1}{3}ln'\chi_1\chi_2 ln'G]\sigma_1\cdot\sigma_2$$

$$[-\frac{1}{6}(ln''G - \frac{ln'G}{r}) + \frac{1}{6}ln'\chi_1\chi_2 ln'G]S_T\}w_1$$

$$+\{[+\frac{1}{6}ln'\chi_1\chi_2(J-L)' + \frac{1}{2}ln'G(J-L)' - \frac{1}{6}\partial^2(J-L)]\sigma_1\cdot\sigma_2$$

$$[-\frac{1}{6}ln'\chi_1\chi_2(J-L)' - \frac{1}{6}((J-L)'' - \frac{(J-L)'}{r})]S_T\}w_4 = b^2(w)w_1 \qquad (8a)$$

where w_1 and w_4 are the upper-upper and lower-lower wave functions and $\chi_i = (E_i - M_i)/G$. This couples to a corresponding equation Eq.(8b) with w_1, w_4 interchanged and $\chi_i \to \bar{\chi}_i = (E_i - M_i)/G$.

For electrodynamics we extract $\mathcal{A}$ from perturbative quantum field theory by comparing the square of the two-body Dirac operators in the weak potential limit ($\mathcal{A} << m_1 + m_2$) with Sazdjian's quantum mechanical transform of the Bethe-Salpeter equation[5]. Both take the form $(\vec{p}^2 + \Phi_w(\vec{r},\vec{p}))w_w = b^2(w)w_w$. To lowest order $\Phi_w(\vec{r},\vec{p}) = \Phi_w^{(1)}(\vec{r},\vec{p}) = -T_w^{(1)}$ with $T_w^{(1)}$ the Born amplitude. For electromagnetic interactions the minimal interaction structure of our constraint equations yields $\mathcal{A}^{(1)} = e_1e_2/4\pi r$. At this order we take $\mathcal{A}(r) = \mathcal{A}^{(1)}(r)$, and $S(r) = \mathcal{V}(r) = 0$. Then the weak potential form of the Eq.(8a) becomes

$$\{p^2 + 2\epsilon_w\mathcal{A} - \mathcal{A}^2 - \frac{1}{2}\partial^2\frac{\mathcal{A}}{M} + i\frac{\mathcal{A}'(-1 - M/2\mu)}{M}\hat{r}\cdot p$$

$$+\frac{\mathcal{A}'}{M}(1 + \frac{1}{2}\frac{m_2}{m_1})\frac{L\cdot\sigma_1}{r} + \frac{\mathcal{A}'}{M}(1 + \frac{1}{2}\frac{m_1}{m_2})\frac{L\cdot\sigma_2}{r}$$

$$+\frac{1}{3}\frac{\partial^2\mathcal{A}}{M}\sigma_1\cdot\sigma_2 - \frac{1}{6}\frac{(\mathcal{A}'' - \frac{\mathcal{A}'}{r})}{M}S_T\}w_1 = b^2(w)w_1. \qquad (9)$$

where $M \equiv m_1 + m_2$. Eq.(9) produces the correct spectrum through order α^4. We give the nonperturbative numerical results for electromagnetic interactions for positronium (not including the effects of the annihilation diagram) with $\mathcal{A}(r) = \mathcal{A}^{(1)}(r)$ and $S(r) = \mathcal{V}(r) = 0$ used in the unapproximated Eqs.(8a-b) in the table below along with the perturbative results and the difference over $\mu\alpha^4$.

l	s	j	n	N_c		perturbative	numerical	diff/$\frac{\mu\alpha^4}{n^3}$
0	0	0	1	1		-6.8033256279	-6.8032861579	5.45E-02
0	0	0	1	2	*	-6.8033256279	-6.8033256719	-6.08E-05
0	1	1	1	1		-6.8028426132	-6.8028074990	4.84E-02
0	1	1	1	2	*	-6.8028426132	-6.8028082195	4.75E-02
0	1	1	1	2	#	-6.8028426132	-6.8028239499	2.58E-02
0	1	1	1	4		-6.8028426132	-6.8028426636	-6.97E-05

Note that for the $^1S_0$ state, inclusion of the coupling ($N_c = 2$) to the lower-lower component of the wave function is crucial in order to obtain agreement with the perturbative results within an error of $\mu\alpha^6$. Likewise to obtain agreement for the $^3S_1$ states one must keep this coupling along with the tensor coupling (four coupled equations, $N_c = 4$). As we shall see, the corresponding strong potential relativistic structure of the coupling has striking consequences in calculations of the light quark mesons.

We apply the two-body Dirac equations to meson spectroscopy using the Adler-Piran static $Q\bar{Q}$ potential[6]: $V(r) = \Lambda(U(\Lambda r) + U_0)$, depending on the two parameters, Λ and U_0. The short and long distance forms for $U(\Lambda r)$ (analytically derived) agree with known lattice and continuum field theory results: $\Lambda U(\Lambda r << 1) \rightarrow 8\pi/(27 r ln\Lambda r)$ and $\Lambda U(\Lambda r >> 1) \rightarrow \Lambda^2 Qr + 2/3Q^{3/2}(16\pi\Lambda^2/9)^{1/2} ln(\Lambda r)$ ($Q = \sqrt{4/3}$). We use this static potential $V(r)$ to construct the invariants $\mathcal{A}, \mathcal{V}$, and S which generate the 16x16 quasipotential $\Phi(\mathcal{A}, \mathcal{V}, S)$ in Eq.(8). We choose the confining part of the nonrelativistic potential to be half scalar and half time-like four vector $S = \mathcal{V} = \frac{1}{2}\Lambda U(\Lambda r >> 1)$ with remainder the electromagnetic-like $\mathcal{A} = V - S - \mathcal{V}$. (This division is essential to maintain an effective linear confinement potential as well as to cancel strong long distance spin-orbit forces which would partially invert the light quark meson spin-orbit multiplets.) We take these as invariant functions of $r \equiv \sqrt{x_\perp^2}$. In our quark model calculation, we use the same $\Phi(\mathcal{A}, \mathcal{V}, S)$ for $\bar{Q}Q, \bar{Q}q$, and $\bar{q}q$ mesons.

We perform a best χ^2 fit to the entire meson spectrum. With quark mass values of $m_b = 4.8945, m_c = 1.5187, m_s = 0.2212, m_u = 0.09113$ GeV, and potential energy parameters $\Lambda = 0.2241, \Lambda U_0 = 1.8627$ GeV we obtain the values listed in the table abbreviated by $AP4$, the 4 designating that the full coupling involves the solution of up to 4 simultaneous Schrödinger-like equations. We find that we must use the fully coupled system of equations to obtain these results or else one obtains such poor results (not given in the table) as 0.785,0.982 GeV for the $K - K^*$ mesons and 0.817,1.002 GeV for the $\pi - \rho$ mesons. The principal defects of our tabulated results are that the masses of the radial and orbital excited states of the light mesons are too large and the LS spittings of the charmonium system is too small. The fit improves (see the column abbreviated APT) when we include chromodynamic corrections not present in the Adler-Piran potential. In work to be submitted for publication, Van Alstine and Crater show that the collective minimal electromagnetic structure $p^2 + 2\epsilon_w\mathcal{A} - \mathcal{A}^2 = \epsilon_w^2 - m_w^2$ must be replaced by $p^2 + 2\epsilon_w\mathcal{A} - 17\mathcal{A}^2/8 = \epsilon_w^2 - m_w^2$. With this change, the best χ^2 fit improves from 597 to 416. The mass and potential parameters change to $m_b = 4.9637, m_c = 1.6061, m_s = 0.4001, m_u = 0.1536, \Lambda = 0.2066$, and $\Lambda U_0 = 1.6614$ GeV. Although these parameters give an excellent fit, the legitimacy of this approach ultimately depends on our demonstration here that a nonperturbative treatment of our equations for QED reproduces standard spectral results. Competing approaches to meson spectroscopy that use relativistic two-body wave equations must also demonstrate this ability.

1. This paper summarizes a detailed treatment to be submitted for publication.
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MESON MASS FIT PRODUCED BY COVARIANT GENERALIZATION OF THE ADLER PIRAN STATIC $Q\bar{Q}$ POTENTIAL

NAME	EXP.	AP	APT
$\Upsilon : b\bar{b}\ 1^3S_1$	9.460	9.457	9.452
$\Upsilon : b\bar{b}\ 1^3P_0$	9.860	9.852	9.858
$\Upsilon : b\bar{b}\ 1^3P_1$	9.892	9.884	9.904
$\Upsilon : b\bar{b}\ 1^3P_2$	9.913	9.906	9.930
$\Upsilon : b\bar{b}\ 2^3S_1$	10.023	10.022	10.023
$\Upsilon : b\bar{b}\ 2^3P_0$	10.235	10.236	10.225
$\Upsilon : b\bar{b}\ 2^3P_1$	10.255	10.258	10.255
$\Upsilon : b\bar{b}\ 2^3P_1$	10.269	10.274	10.273
$\Upsilon : b\bar{b}\ 3^3S_1$	10.356	10.367	10.349
$\Upsilon : b\bar{b}\ 4^3S_1$	10.577	10.633	10.660
$\Upsilon : b\bar{b}\ 5^3S_1$	10.860	10.859	10.852
$\Upsilon : b\bar{b}\ 6^3S_1$	11.019	11.062	11.025
$B : b\bar{u}\ 1^1S_0$	5.278	5.285	5.226
$B^* : b\bar{u}\ 1^3S_1$	5.325	5.318	5.286
$\eta_c : c\bar{c}\ 1^1S_0$	2.980	3.017	2.965
$\psi : c\bar{c}\ 1^3S_1$	3.097	3.108	3.126
$\chi_0 : c\bar{c}\ 1^3P_0$	3.415	3.426	3.420
$\chi_1 : c\bar{c}\ 1^3P_1$	3.511	3.492	3.506
$\chi_2 : c\bar{c}\ 1^3P_2$	3.556	3.535	3.553
$\eta_c : c\bar{c}\ 2^1S_0$	3.594	3.625	3.587
$\psi : c\bar{c}'\ 2^3S_1$	3.686	3.685	3.671
$\psi : c\bar{c}\ 1^3D_1$	3.770	3.794	3.790
$\psi : c\bar{c}\ 3^3S_1$	4.030	4.106	4.130
$\psi : c\bar{c}\ 2^3D_1$	4.159	4.177	4.130
$\psi : c\bar{c}\ 3^3D_1$	4.415	4.509	4.422

NAME	EXP.	AP	APT
$D : c\bar{u}\ 1^1S_0$	1.864	1.902	1.823
$D^* : c\bar{u}\ 1^3S_1$	2.007	1.994	1.971
$D^* : c\bar{u}\ 1^3P_1$	2.422	2.376	2.371
$D_s : c\bar{s}\ 1^1S_0$	1.969	1.974	1.962
$D_s^* : c\bar{s}\ 1^3S_1$	2.113	2.069	2.124
$K : s\bar{u}\ 1^1S_0$	0.494	0.506	0.474
$K^* : s\bar{u}\ 1^3S_1$	0.892	0.897	0.866
$K_1 : s\bar{u}\ 1^1P_1$	1.270	1.418	1.327
$K_1 : s\bar{u}\ 1^3P_1$	1.400	1.421	1.316
$K_2^* : s\bar{u}\ 1^3P_2$	1.426	1.565	1.424
$K^* : s\bar{u}\ 1^3D_1$	1.717	1.902	1.701
$K_2 : s\bar{u}\ 1^3D_2$	1.770	1.943	1.759
$K_3^* : s\bar{u}\ 1^3D_3$	1.776	1.965	1.821
$\phi : s\bar{s}\ 1^3S_1$	1.019	0.973	1.044
$f_1 : s\bar{s}\ 1^3P_1$	1.422	1.492	1.458
$f_2' : s\bar{s}\ 1^3P_2$	1.527	1.573	1.559
$\phi : s\bar{s}\ 2^3S_1$	1.685	1.830	1.814
$\phi : s\bar{s}\ 1^3D_3$	1.853	2.005	1.936
$\pi : u\bar{u}\ 1^1S_0$	0.135	0.122	0.161
$\rho : u\bar{u}\ 1^3S_1$	0.770	0.831	0.701
$b_1 : u\bar{u}\ 1^1P_1$	1.233	1.349	1.171
$a_1 : u\bar{u}\ 1^3P_1$	1.260	1.463	1.222
$c_2 : u\bar{u}\ 1^3P_2$	1.318	1.483	1.317
$\pi : u\bar{u}\ 2^1S_0$	1.300	1.643	1.339
$\rho : u\bar{u}\ 2^3S_1$	1.578	1.806	1.649
$\pi_2 : u\bar{u}\ 1^1D_2$	1.665	1.899	1.667
$\rho_3 : u\bar{u}\ 1^3D_3$	1.691	1.936	1.729
χ^2		597	416