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BLOOD FLOW IN CURVED PIPE WITH RADIATIVE HEAT TRANSFER

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ABSTRACT

Blood flow in a curved pipe such as the aorta is modelled in this study. The aorta is modelled as a curved pipe of slowly varying cross–section. Asymptotic series expansions about a small parameter δ, which is a measure of the curvature ratio is employed to obtain the velocity and temperature distributions. The study simulates the effect of radio–frequency heating, for instance during physiotherapy, on the flow of blood in the cardiovascular system assuming an external constant pressure gradient; and our results agree very well with results obtained by Pedley [9].

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** Regretfully, Professor A.R. Bestman passed away before the publication of this report.

1. Introduction

In an earlier study Bestman [1], looked at the distensibility of the aorta under pulsatile flow situation. Anatomically the vena cava is more distensible than the aorta but the aorta still distends enough to warrant apprasial. The primary objective of that study was the steady flow in a curved pipe of varying section subjected to a constant pressure at the inlet.

Actually, the character of steady flow in a curved pipe has been of broad interest both theoretically and experimentally. The first theoretical breakthrough was made by Dean [2], [3]. Dean's analysis was restricted to small values of the Dean's parameter D. Since then the analysis has been extended to cover the whole range of values of Dean's parameter.

From the physiological standpoint, the flow through a curved pipe is paramount in understanding pathological situations in the cardiovascular system. In the large blood vessels it is a good approximation to consider whole blood as a Newtonian viscous fluid. However in a lot of other physiological flow situations it is necessary to consider blood as a suspension of blood cells in plasma. Kaimal [7], [8] has studied such a situation in an axisymmetric tube of slowly varying radius and in a circular tube undergoing peristalsis under the long wavelength approximation, but now it is fairly unanimously agreed that the best model is the rheological effect rather than the suspension model.

In Section 2 of the study the governing equations are presented in non-dimensional form. This is followed in Section 3 by the determination of the solutions for the leading approximations obtained by setting δ, the curvature ratio, equal to zero . In Section 4 higher approximate solutions are deduced followed by a brief numerical discussion of the results obtained in Section 5.

2. Mathematical Formulation

The velocity components in Dean's coordinate system, figure 1a, (r , ϕ, θ) is given by u , v , w.

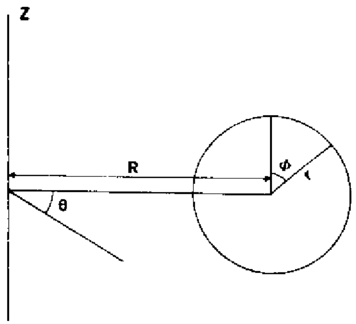

figure 1a Dean's coordinate system.

Following Bestman [1], the non-dimensional equations of continuity, momentum and energy which we propose here are

$$\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{\delta usin\phi}{1+\delta r.sin\phi}+\frac{1}{r}\frac{\partial v}{\partial \phi}+\frac{\delta v.cos\phi}{1+\delta r.sin\phi}+\frac{1}{1+\delta r.sin\phi}\frac{\partial w}{\partial \theta}=0$$

$$\delta^2(u\frac{\partial u}{\partial r}+\frac{v}{r}\frac{\partial u}{\partial \phi}+\frac{1}{1+\delta r.sin\phi}w\frac{\partial u}{\partial \theta}-\frac{v^2}{r})-\frac{\delta w.sin\phi}{1+\delta r.sin\phi}=-\frac{\partial p}{\partial r}-$$
$$-\delta\{(\frac{1}{r}\frac{\partial}{\partial \phi}+\frac{\delta.cos\phi}{1+\delta r.sin\phi})(\frac{\partial v}{\partial r}-\frac{1}{r}v-\frac{1}{r}\frac{\partial u}{\partial \phi})-\frac{\delta^2}{(1+\delta r.sin\phi)}\frac{\partial^2 u}{\partial \theta^2}+$$
$$+\frac{1}{1+\delta r.sin\phi}(\frac{\partial^2 w}{\partial r\partial \theta}+\frac{\delta.sin\phi}{1+\delta r.sin\phi}\frac{\partial w}{\partial \theta})\}-\delta G_r\Theta$$

$$\delta^2(u\frac{\partial v}{\partial r}+\frac{v}{r}\frac{\partial v}{\partial \phi}+\frac{1}{1+\delta r.sin\phi}w\frac{\partial v}{\partial \theta}+\frac{uv}{r})-\frac{\delta w^2.cos\phi}{1+\delta r.sin\phi}=-\frac{1}{r}\frac{\partial p}{\partial \phi}+$$
$$+\delta(\frac{\partial}{\partial r}+\frac{\delta.sin\phi}{1+\delta.sin\phi})(\frac{\partial v}{\partial r}+\frac{v}{r}-\frac{1}{r}\frac{\partial u}{\partial \phi})+\frac{delta^2}{(1+\delta r.sin\phi)^2}\frac{\partial^2 v}{\partial \theta^2}-$$
$$-\frac{1}{1+\delta r.sin\phi}(\frac{1}{r}\frac{\partial^2 w}{\partial \theta\partial \phi}+\frac{\delta.cos\phi}{1+\delta r.sin\phi}\frac{\partial w}{\partial \theta})+\delta G_r\Theta$$

$$\delta(u\frac{\partial w}{\partial r}+\frac{v}{r}\frac{\partial w}{\partial \phi}+\frac{\delta.u.w.sin\phi}{1+\delta r.sin\phi}+\frac{\delta.u.w.cos\phi}{1+\delta r.sin\phi}+\frac{1}{1+\delta r.sin\phi}w\frac{\partial w}{\partial \theta})=$$
$$-\frac{\delta}{1+\delta r.sin\phi}\frac{\partial p}{\partial \theta}+(\frac{\partial}{\partial r}+\frac{1}{r})(\frac{\partial w}{\partial r}+\frac{\delta w.sin\phi}{1+\delta r.sin\phi})+\frac{1}{r^2}\frac{\partial^2 w}{\partial \phi^2}++\frac{1}{r}\frac{\partial}{\partial \phi}(\frac{\delta w.sin\phi}{1+\delta r.sin\phi})-$$
$$(\frac{\partial}{\partial r}+\frac{1}{r})(\frac{\delta}{1+\delta r.sin\phi}\frac{\partial}{\partial \phi})-\frac{1}{r}\frac{\partial}{\partial \phi}(\frac{\delta}{1+\delta r.sin\phi}\frac{\partial v}{\partial \phi}) \qquad (2.1a,b,c,d)$$

where $\delta=\frac{a_0}{R}$, is the curvature ratio; a_0 in this case is a typical radius. The pipe is coiled in a circle of radius R.

The boundary conditions are the pressure condition and the condition of no-slip at the pipe wall. In non-dimensional form they are

$$p(r,\phi)=k$$

$$\theta.=u.=v.=0.=w \quad on \quad r=1 \qquad (2.2a,b)$$

A supplementary equation which has been found useful is obtained by eliminating the pressure gradient between (2.1 b , c) and it is

$$\delta\{\frac{1}{r}\frac{\partial}{\partial r}[r(u\frac{\partial v}{\partial r}+\frac{v}{r}\frac{\partial v}{\partial \phi}+\frac{\delta}{1+\delta r.sin\phi}w\frac{\partial v}{\partial \theta}+\frac{uv}{r})]-$$
$$-\frac{1}{r}\frac{\partial}{\partial \phi}(u\frac{\partial u}{\partial r}+\frac{v}{r}\frac{\partial u}{\partial \phi}+\frac{\delta}{1+\delta r.sin\phi}w\frac{\partial u}{\partial \theta}-\frac{v^2}{r})\}-\delta[\frac{1}{r}\frac{\partial}{\partial r}(\frac{rw^2.cos\phi}{1+\delta r.sin\phi})-$$
$$-\frac{1}{r}\frac{\partial}{\partial \phi}(\frac{w^2.sin\phi}{1+\delta r.sin\phi})]=\frac{1}{r}\frac{\partial}{\partial r}\{r[(\frac{\partial}{\partial r}+\frac{\delta.sin\phi}{1+\delta r.sin\phi})(\frac{1}{r}\frac{\partial}{\partial r}(rv)-\frac{1}{r}\frac{\partial u}{\partial \phi})+\frac{\delta}{(1+\delta r.sin\phi)^2}\frac{\partial^2 v}{\partial \theta^2}-$$
$$-\frac{1}{1+\delta r.sin\phi}(\frac{1}{r}\frac{\partial^2 w}{\partial \theta\partial \phi}+\frac{\delta cos\phi}{1+\delta r.sin\phi}\frac{\partial w}{\partial \theta})]\}+\frac{1}{r}\frac{\partial}{\partial \phi}[(\frac{1}{r}\partial\partial\phi+\frac{\delta cos\phi}{1+\delta.sin\phi})(\frac{1}{r}\frac{\partial}{\partial r}(rv)-\frac{1}{r}\frac{\partial u}{\partial \phi})-$$
$$-\frac{\delta^2}{(1+\delta r.sin\phi)^2}\frac{\partial^2}{\partial \theta^2}+\frac{1}{1+\delta r.sin\phi}(\frac{\partial^2 w}{\partial r\partial \theta}+\frac{\delta sin\phi}{1+\delta r.sin\phi}\frac{\partial w}{\partial \theta})]+$$
$$+G_r[\frac{\partial \Theta}{\partial r}sin\phi+\frac{1}{r}\frac{\partial \Theta}{\partial \phi}cos\phi] \qquad (2.3)$$

We assume blood is an optically thin fluid and also assume the blood temperature is not much different from the temperature of the blood vessel in which case we invoke the optically thin limit of the differential equation for the radiative flux as given in Ogulu and Bestman [4]. The condition on the temperature is therefore

$$r. = 0, \quad \Theta < \infty; \quad r = 1 \qquad \Theta = 1 \tag{2.4}$$

and the Radiative heat transfer equation is

$$\delta P_r(u\frac{\partial\Theta}{\partial r} + \frac{v}{r}\frac{\partial\Theta}{\partial\phi} + \frac{\delta}{1+\delta r.sin\phi}w\frac{\partial\Theta}{\partial\theta}) = \nabla^2\Theta - R_a\Theta + R_a \tag{2.5}$$

where

$$\nabla q_R = R_a(\Theta^4 - \Theta_\infty^4) \tag{2.6}$$

Θ_∞ is linearized to 1. The statement of the problem is now complete.

3. Basic Approximate solutions.

On account of the non-linearity of our leading equations we seek asymptotic expansions in the spirit of Ogulu and Bestman [4]. For the velocity components we write

$$u = u^{(0)}(r,\phi,\theta) + \delta u^{(1)} + \ldots\ldots\ldots\ldots\ldots$$

$$v = v^{(0)}(r,\phi,\theta) + \delta v^{(1)} + \ldots\ldots\ldots\ldots\ldots$$

$$w = w^{(0)}(r,\theta) + \delta w^{(1)}(r,\phi,\theta) + \ldots\ldots\ldots\ldots\ldots$$

For the pressure we write

$$\frac{1}{\delta}p^{(0)}(r,\theta) + p^{(1)}(r,\theta\phi) + \ldots\ldots\ldots\ldots\ldots$$

and for the temperature we have

$$\theta = \theta^{(0)}(r,\theta) + \delta\theta^{(1)}(r,\phi,\theta) + \ldots. \tag{3.1a, b, c, d, e}$$

Substituting (3.1) in (2.1 a, d), (2.3) and (2.5) , the order 0(1) problem is

$$\frac{\partial u^{(0)}}{\partial r} + \frac{u^{(0}}{r} + \frac{1}{r}\frac{\partial v^{(0)}}{\partial\phi} = 0$$

$$-\frac{\partial p^{(0)}}{\partial\theta} + \frac{\partial^2 w^{(0)}}{\partial r^2} + \frac{1}{r}\frac{\partial w^{(0)}}{\partial r} = 0$$

$$\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial}{\partial r})(\frac{1}{r}\frac{\partial}{\partial r}(rv^{(0)}) - \frac{1}{r}\frac{\partial u^{(0)}}{\partial\phi}) + \frac{1}{r}\frac{\partial}{\partial\phi}(\frac{1}{r}\frac{\partial}{\partial\phi})(\frac{1}{r}\frac{\partial}{\partial r}(rv^{(0)}) - \frac{1}{r}\frac{\partial u^{(0)}}{\partial\phi})+$$

$$+G_r\frac{\partial\Theta^{(0)}}{\partial r}sin\phi$$

$$\nabla^2\Theta^{(0)} - R_a\Theta^{(0)} + R_a = 0 \tag{3.2a, b, c, d}$$

where $\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\phi^2}$

From (3.2d) we obtain the temperature as

$$\theta^{(0)} = A.I_0(R_a r) - 1 \tag{3.3}$$

$I_0(\alpha r)$ is Bessel function of the first kind of order zero, $\alpha = R_a^{\frac{1}{2}}$, A is an arbitrary constant obtained on imposition of the boundary condition $\theta^{(0)} = \theta_w....on....r = 1$ as

$$A = \frac{\theta_w + 1}{I_0(R_a^{\frac{1}{2}})} \tag{3.4}$$

From (3.2b) we obtain the axial velocity as

$$w^{(0)} = \frac{k}{4}(r^2 - 1) \tag{3.5}$$

where k is the constant blood pressure in the left ventricle of the heart for a healthy person.

The $u^{(0)}$ and $v^{(0)}$ components of the velocity are obtained by solving (3.2a) and (3.2c) simultaneously, the results are

$$U^{(0)} = B + \frac{A_1 r^2}{8} - \frac{A.G_r I_1(R_1^{\frac{1}{2}}r)}{R_a^{\frac{3}{2}}}$$

and

$$V^{(0)} = -B - \frac{3A_1 r^2}{8} + A.G_r I_1(R_a^{\frac{1}{2}} r) \qquad (3.6a,b)$$

The arbitrary constants A_1 and B are obtained on imposition of the boundary condition $V^{(0)} = 0 = U^{(0)}....on....r = 1$ as

$$A_1 = \frac{2AG_r I_1(R_a^{\frac{1}{2}})}{R_a}\{\frac{1}{R_a} - 1\}$$

and

$$B = A.G_r I_1(R_a^{\frac{1}{2}})\{\frac{1}{R_a^{\frac{3}{2}}} - \frac{1}{4R_a^{\frac{1}{2}}} + \frac{1}{4}\} \qquad (3.7a,b)$$

The basic approximate solutions are now complete.

4. Higher Approximations.

If we continue the expansion started in (3.1) we find that the governing equations for the order $0(\delta)$ problem may be put in the convenient form

$$\frac{\partial u^{(1)}}{\partial r} + \frac{u^{(1)}}{r} + u^{(0)}.sin\phi + \frac{1}{r}\frac{\partial v^{(1)}}{\partial \phi} + v^{(0)}.cos\phi + \frac{\partial w^{(0)}}{\partial \theta} = 0$$

$$u^{(0)}\frac{\partial w^{(0)}}{\partial r} + w^{(0)}\frac{\partial w^{(0)}}{\partial \theta} = -\frac{\partial p^{(1)}}{\partial \theta} + (\frac{\partial}{\partial r} + \frac{1}{r})(\frac{\partial w^{(1)}}{\partial r} + w^{(0)}.sin\phi) + \frac{1}{r^2}\frac{\partial^2 w^{(1)}}{\partial \phi^2} - (\frac{\partial}{\partial r} + \frac{1}{r})\{\frac{\partial u^{(0)}}{\partial \phi}\}$$

$$\frac{1}{r}\frac{\partial}{\partial r}[r(u^{(0)}\frac{\partial v^{(0)}}{\partial r} + \frac{v^{(0)}}{r}\frac{\partial v^{(0)}}{\partial \phi} + \frac{u^{(0)}v^{(0)}}{r})] - \frac{1}{r}\frac{\partial}{\partial \phi}(u^{(0)}\frac{\partial u^{(0)}}{\partial r} + \frac{v^{(0)}}{r}\frac{\partial u^{(0)}}{\partial \phi} + \frac{v^{(0)^2}}{r}) -$$

$$-\frac{1}{r}\frac{\partial}{\partial r}(rw^{(0)}cos\phi) = \frac{1}{r}\frac{\partial}{\partial \phi}(w^{(0)^2}sin\phi) + \frac{1}{r}\frac{\partial}{\partial r}\{r[(\frac{\partial}{\partial r})(\frac{1}{r}\frac{\partial}{\partial r}(rv^{(1)}) -$$

$$-\frac{1}{r}\frac{\partial u^{(1)}}{\partial \phi}) + sin\phi(\frac{1}{r}\frac{\partial}{\partial r}(rv^{(0)} - \frac{1}{r}\frac{\partial u^{(0)}}{\partial \phi}) - (\frac{1}{r}\frac{\partial^2}{\partial \phi \partial \theta})]\} +$$

$$+\frac{1}{r}\frac{\partial}{\partial \phi}[(\frac{1}{r}\frac{\partial}{\partial \phi})(\frac{1}{r}\frac{\partial}{\partial r}(rv^{(1)}) - \frac{1}{r}\frac{\partial u^{(1)}}{\partial \phi}) + cos\phi(\frac{1}{r}\frac{\partial}{\partial r}(rv^{(0)}) - \frac{1}{r}\frac{\partial u^{(0)}}{\partial \phi}) +$$

$$+\frac{\partial^2 w^{(1)}}{\partial r \partial \theta} + G_r(\frac{\partial \Theta^{(1)}}{\partial r}.sin\phi + \frac{1}{r}\partial\frac{\Theta^{(1)}}{\partial \phi}cos\phi)$$

$$P_r u^{(0)}\frac{\partial \Theta^{(0)}}{\partial r} = \nabla^2 \Theta^{(1)} - R_a \Theta^{(1)} \qquad (4.1a,b,c,d)$$

where

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \phi^2}$$

The boundary conditions are

$$U^{(1)} = V^{(1)} = 0 = \Theta^{(1)} \quad on \quad r = 1 \qquad (4.2)$$

To solve (4.1) we write

$$\Theta^{(1)} = \Theta_0^{(1)} + \Theta_1^{(1)}$$

and

$$w^1 = w_0^{(1)} + w_1^{(1)} + w_2^{(1)}$$

After some algebra it is possible to show that

$$\Theta_0^{(1)} = 0$$

and

$$\Theta_1^{(1)} = M.I_1(R_a^{\frac{1}{2}} r) + \frac{A.A_1 P_r R_a^{\frac{3}{2}} r^3}{8} - \frac{A.A_1 P_r R_a r^3}{16} + A.A_1 P_r r - \frac{A.BP_r^2 R_a r}{2} + \frac{a^2 P_r G_r R_a r}{4} \qquad (4.3a,b)$$

while

$$w_0^{(1)} = \frac{p_\theta^{(1)}}{4}(r^2 - 1)$$

where $(p_\theta^{(1)} = \frac{dp^{(1)}}{d\theta}$

$$w_1^{(1)} = -\frac{kr^3}{12} + \frac{kr}{4} - \frac{A_1 r^3}{24} - Br + \frac{AG_r r}{R_a^2} + Y$$

$$w_2^{(1)} = const.r + \frac{kBr^3}{16} + \frac{A_1 kr^5}{384} - \frac{AkG_r I_1(R_a^{\frac{1}{2}} r)}{2R_a^{\frac{5}{4}}}$$

where

$$const. = -\frac{Bk}{16} - \frac{A_1 k}{384} + \frac{AkG_r I_1(R_a^{\frac{1}{2}} r)}{2R_a^{\frac{5}{4}}}$$

Details of the method of solution are given in Ogulu [5]. The arbitrary constants M and Y follow on imposition of the appropriate boundary condition and rejection of unbounded quantities. They are

$$M = \frac{1}{2I_1(R_a^{\frac{1}{2}})}\{\frac{AA_1P_rR_a}{8} - \frac{AA_1P_rR_a^{\frac{3}{2}}}{4} - 2AA_1P_r + ABP_rR_a^2 - \frac{A^2P_rG_rR_a}{2}\}$$

$$Y = B + \frac{A_1}{24} + \frac{k}{6} - \frac{AG_r}{16R_a^2}$$

We continue the solution of (4.1 a, c). We write

$$u^{(1)} = U_0^{(1)}(r) + U_1^{(1)}(r)sin\phi + U_2^{(1)}(r)cos2\phi + U_3^{(1)}(r)sin2\phi$$

$$v^{(1)} = V_0^{(1)}(r) + V_1^{(1)}(r)sin\phi + V_3^{(1)}(r)sin2\phi$$

$$v^{(0)} = V^{(0)}sin\phi$$

$$u^{(0)} = U^{(0)}cos\phi \qquad (4.4a, b, c, d)$$

Substitute (4.4) in (4.1 a, c) to obtain respectively

$$-\frac{dw_0^{(1)}}{d\theta} - 2(U^{(0)} + V^{(0)})sin2\phi = \frac{1}{r}\frac{d}{dr}(rU^{(1)}) + (\frac{1}{r}\frac{d}{dr}(U_1^{(1)}) - \frac{1}{r}V_1^{(1)})sin\phi +$$
$$+(\frac{1}{r}\frac{d}{dr}(rU_2^{(1)}) + \frac{2}{r}V_1^{(1)})cos2\phi + (\frac{1}{r}\frac{d}{dr}(rU_3^{(1)}) - \frac{2}{r}V_3^{(1)})sin2\phi$$

and

$$\frac{1}{r}\frac{d}{dr}[r(U^{(0)}\frac{dV^{(0)}}{dr} + \frac{V^{(0)^2}}{r} + \frac{U^{(0)}V^{(0)}}{r})]\frac{sin2\phi}{2} - \frac{1}{r}(U^{(0)}\frac{dU^{(0)}}{dr})sin2\phi -$$
$$-\frac{1}{r}(\frac{V^{(0)}U^{(0)}}{r} + \frac{V^{(0)^2}}{r})sin2\phi - [\frac{1}{r}\frac{d}{dr}(rw^{(0)^2}) - \frac{w^{(0)^2}}{r}]cos\phi -$$
$$-\frac{1}{2r}\frac{d}{dr}[\frac{d}{dr}(rV^{(0)}) + \frac{1}{r}U^{(0)}] - \frac{1}{r}\frac{d}{dr}(rV^{(0)}) + \frac{1}{r}U^{(0)}]\frac{cos2\phi}{2} -$$
$$-\frac{1}{r}[\frac{1}{r}\frac{d}{dr}(rV^{(0)}) + \frac{1}{r}U^{(0)}]cos\phi - G_r(\frac{d\Theta^{(1)}}{dr} - \frac{1}{r}\Theta^{(1)})\frac{sin2\phi}{2} =$$
$$= \frac{1}{r}\frac{d}{dr}\{r[\frac{d}{dr}(\frac{1}{r}\frac{d}{dr}(rV_1^{(1)}))]\} + \frac{1}{r}\frac{d}{dr}\{r[\frac{d}{dr}(\frac{1}{r}\frac{d}{dr}(rV_1^{(1)}) - \frac{1}{r}U_1^{(1)})]\}cos\phi -$$
$$-\frac{1}{r^2}[\frac{1}{r}\frac{d}{dr}(rV_1^{(1)}) - \frac{1}{r}U_1^{(1)}]cos\phi + \frac{1}{r}\frac{d}{dr}\{r[\frac{d}{dr}(\frac{1}{r}\frac{d}{dr}(rV_2^{(1)}) + \frac{2}{r}U_2^{(1)})]\}sin2\phi -$$
$$-\frac{4}{r^2}[\frac{1}{r}\frac{d}{dr}(rV_2^{(1)}) + \frac{2}{r}U_2^{(1)}]sin2\phi + \frac{1}{r}\frac{d}{dr}\{r[\frac{d}{dr}(\frac{1}{r}\frac{d}{dr}(rV_3^{(1)} - \frac{2}{r}U_3^{(1)})]\}cos2\phi -$$
$$-\frac{4}{r^2}[\frac{1}{r}\frac{d}{dr}(rV_3^{(1)}) - \frac{2}{r}U_3^{(1)}]cos2\phi \qquad (4.5a, b)$$

From (4.5 a) we have

$$\frac{1}{r}\frac{d}{dr}(rU_0^{(1)}) = -\frac{dw_0^{(1)}}{d\theta} \qquad (4.6)$$

On integration (4.6) yields, for bounded solution on $r = 0$,

$$U_0^{(1)} = \frac{p_{\theta\theta}^{(1)}}{16}(r^3 - 2r)$$

and subsequently $p_{\theta\theta}^{(1)} = 0$ in which case $p_\theta^{(1)}$ is constant.

Next from (4.5 a, b) we take

$$\frac{1}{r}\frac{d}{dr}\{r[\frac{d}{dr}(\frac{1}{r}\frac{d}{dr}(rV_0^{(1)}))]\} = -\frac{1}{2r}\frac{d}{dr}[\frac{1}{r}\frac{d}{dr}(rV^{(0)} + \frac{1}{r}U^{(0)}] \qquad (4.7)$$

Straight forward integration of (4.7) after substitution for $V^{(0)}$ and $U^{(0)}$; and appealling to Abramowitz and Stegun [6] for the limiting form for small arguments, and keeping only finite terms, yields

$$V_0^{(1)} = \frac{A_1r^2}{6} - \frac{AG_r}{2R_a}$$

For the other velocity components, say, $U_1^{(1)}$ and $V_1^{(1)}$, we obtain by simultaneous solution of (4.5 a, b) for the components. The results are

$$U_1^{(1)} = F_1 - \frac{H.r^2}{8} + \frac{K^2.r^6}{4608} - \frac{K^2.r^4}{768}$$

and

$$V_1^{(1)} = F_1 - \frac{H.r}{4} + \frac{7K^2.R^6}{4608} - \frac{K^2.r^4}{768} \qquad (4.8a, b)$$

where F_1 and H are constants obtained on imposition of (4.2) as

$$F_1 = \frac{K^2}{201}$$

and

$$H = \frac{721K^2}{13848};$$

$$U_2^{(1)} = M_3 r + \frac{M_2 r^3}{15} + \frac{9A_1 B r^4}{1536} + \frac{A_1^2 r^5}{11264} - \frac{ABG_r r^4}{96} - \frac{9AA_1 G_r r^5}{5600 R_a^{\frac{1}{2}}} + \frac{A^2 G_r^2 r^4}{512 R_a} -$$

$$-\frac{A_1 B r^3}{120} + \frac{A_1 r^4}{512} - \frac{AG_r r^3}{30 R_a} + \frac{A_1 B r^3}{45} - \frac{AA_1 G_r r^1}{6144 R_a^{\frac{1}{2}}} + \frac{A^2 G_r^2 R_a r^5}{2000} + \frac{MG_r R_a^{\frac{1}{2}} r^3}{90} +$$

$$+\frac{AA_1 G_r P_r R_a^{\frac{3}{2}} r^5}{2000} - \frac{ABG_r P_r R_a^2 r^3}{90} + \frac{A^2 G_r P_r R_a r^3}{180} - \frac{AA_1 G_r P_r R_a r^5}{4200} + \frac{AA_1 G_r P_r r^3}{45} +$$

$$+\frac{AA_1 G_r r^3}{120 R_a^{\frac{3}{7}}} - \frac{AA_1 G_r r^3}{120}$$

and

$$V_1^{(1)} = -M_3 r - \frac{2M_2 r^3}{15} - \frac{90 A_1 B r^4}{1536} + \frac{3A_1^2 r^5}{11264} - \frac{21 A_1^2 r^6}{4096} + \frac{5ABG_r r^4}{192} +$$

$$+\frac{27 AA_1 G_r r^5}{5600 R_a^{\frac{1}{2}}} - \frac{5A^2 G_r^2 r^4}{R_a} + \frac{A_1 B r^3}{60} - \frac{A_1 r^4}{1024} + \frac{AG_r r^3}{30 R_a} - \frac{2A_1 B r^3}{45} + \frac{5AA_1 G_r r^4}{12288 R_a^{\frac{1}{2}}} -$$

$$-\frac{A^2 G_r^2 R_a r^5}{700} - \frac{MG_r R_a^{\frac{1}{2}} r^3}{45} - \frac{AA_1 G_r P_r R_a^{\frac{3}{2}} r^5}{700} + \frac{ABG_r P_r R_a^2 r^3}{45} - \frac{A^2 G_r P_r R_a r^3}{90} +$$

$$+\frac{AA_1 G_r P_r R_a r^5}{1400} - \frac{2AA_1 G_r P_r r^3}{45} - \frac{AA_1 G_r r^3}{60 R_a^{\frac{3}{2}}} + \frac{AA_1 G_r r^3}{60} \qquad (4.9a, b)$$

where M_2 and M_3 are arbitrary functions given by

$$M_2 = \frac{15ABG_r}{64} + \frac{27AA_1 G_r}{R_a^{\frac{1}{2}}} - \frac{45A^2 G_r}{512 R_a} + \frac{AB}{8} + \frac{15A_1}{1024} - \frac{A_1 B}{3} + \frac{35AA_1 G_r}{273} -$$

$$-\frac{A^2 G_r^2 R_a}{70} + \frac{ABG_r P_r R_a^2}{6} - \frac{AG_r P_r R_a}{12} - \frac{AA_1 G_r P_r R_a}{140} - \frac{AA_1 G_r}{8 R_a^{\frac{3}{7}}} - \frac{4A_1^2}{43}$$

and

$$M_3 = -\frac{9A_1^2}{2048} + \frac{3A_1 B}{512} + \frac{ABG_r}{192} + \frac{9AA_1 G_r}{5600 R_a^{\frac{1}{7}}} + \frac{A^2 G_r^2}{128 R_a} + \frac{A_1}{3072} + \frac{2AG_r}{15 R_a} +$$

$$+\frac{AA_1 G_r}{12238 R_a} - \frac{A^2 G_r^2 R_a}{2100} - AA_1 G_r P_r R_a^{\frac{3}{2}}$$

Finally from (4.5 a, b) we take

$$\frac{1}{r}\frac{d}{dr}\{r[\frac{d}{dr}(\frac{1}{r}\frac{d}{dr}(rV_3^{(1)}) - \frac{2}{r}U_3^{(1)})]\} - \frac{4}{r^2}[\frac{1}{r}\frac{d}{dr}(rV_3^{(1)}) - \frac{2}{r}U_3^{(1)}] =$$

$$= \frac{1}{2r}\frac{d}{dr}[\frac{1}{r}\frac{d}{dr}(rV^{(0)}) + \frac{1}{r}U^{(0)}] - \frac{1}{r}\{\frac{1}{r}\frac{d}{dr}(rV^{(0)} + \frac{1}{r}U^{(0)}] \qquad (4.10)$$

Following the previous procedure it is easy to deduce that

$$U_3^{(1)} = L_1 r + \frac{2Lr^3}{15} + \frac{2A_1 r^3}{45} - \frac{AG_r^3}{90 R_a^{\frac{1}{2}}}$$

and

$$V_3^{(1)} = L_1 r + \frac{4Lr^3}{15} + \frac{4A_1 r^3}{45} - \frac{AG_r r^3}{45 R_a^{\frac{1}{2}}} \qquad (4.11a, b)$$

where L and L_1 are arbitrary constants given by

$$L = \frac{A_1}{3} + \frac{AG_r}{45 R_a^{\frac{1}{2}}}$$

and

$$L_1 = -\frac{4A_1}{45} - \frac{AG_r}{45 R_a^{\frac{1}{2}}}$$

5. Discussion.

In the last four sections we have formulated and solved approximately for the velocity components and temperature for flow in a rigid curved pipe of circular cross-section such as the aorta. Though the aorta distends to some extent, Bestman [1], we have assumed a tube of constant radius. In systemic circulation the nutrients are convected by the blood plasma in the axial direction so our discussion of the velocity distribution will be limited to the axial component of the velocity only. The other components of the velocity can be discussed similarly.

δ in this discussion will be taken as 0.001 . We also take ϕ in this numerical discussion as zero for simplicity. Actually other values of ϕ make no appreciable change to the results. The other parameters of the problem are kept the same as in Ogulu and Bestman [4].

Figure 1 depicts the axial velocity distribution for two values of the radiation parameter, R_a. $r = 0$ is along the axis of the tube; that is to say there is symmetry about the axis of the tube. It can be seen from figure 1 that as the radiation parameter increases the axial flow velocity also increases. This is in agreement with the results of Ogulu and Bestman [4] for flow in a straight tube of circular section of slowly varying radius. Here we notice a slight modification of the velocity profile as a result of coriolis acceleration associated with circular motion. The velocity profile as depicted in figure 1 is in complete agreement with the results obtained by Pedley [9] who was interested in the effect of uniform curvature on blood flow in a curved pipe.

Figure 2 shows the temperature distribution for flow in a curved pipe for two different values of the radiation parameter. It obvious from figure 2 that the difference between the axial temperature and the wall temperature of the blood vessel decreases with increase in the radiation parameter, but the axial temperature is always less than the temperature at the wall even for large values of the radiation parameter. See Ogulu and Bestman [4], [5].

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References

[1] Bestman, A. R.,"Flow in curved pipes of slowly varying cross-section:1" Modelling Simulation and Control, B , AMSE press vol 2 no. 21985 p 51 - 64 1985.

[2] Dean, W. R., "Notes on the motion of fluid in curved pipes," Phil. Mag. 5 1928.

[3] Dean, W. R., "The Streamline motion of fluid in a curved pipe," Phil. Mag. 5 1928.

[4] Ogulu, A. , Bestman, A. R., "Deep Heat muscle Treatment : A Mathematical Model I". This Journal.

[5] Ogulu, A. , Bestman, A. R., "Deep Heat Muscle Treatment : A Mathematical Model II". This Journal.

[6] Abramowitz, M., Stegun, I. A., Handbook of Mathematical Functions Dover Publications Inc. New York. 1965.

[7] Kaimal, M. R., "Low Reynolds number flow of a suspension in slowly varying axisymmetric tubes" Int. J. Engng. Sci. vol 17 p 615 - 624 1979.

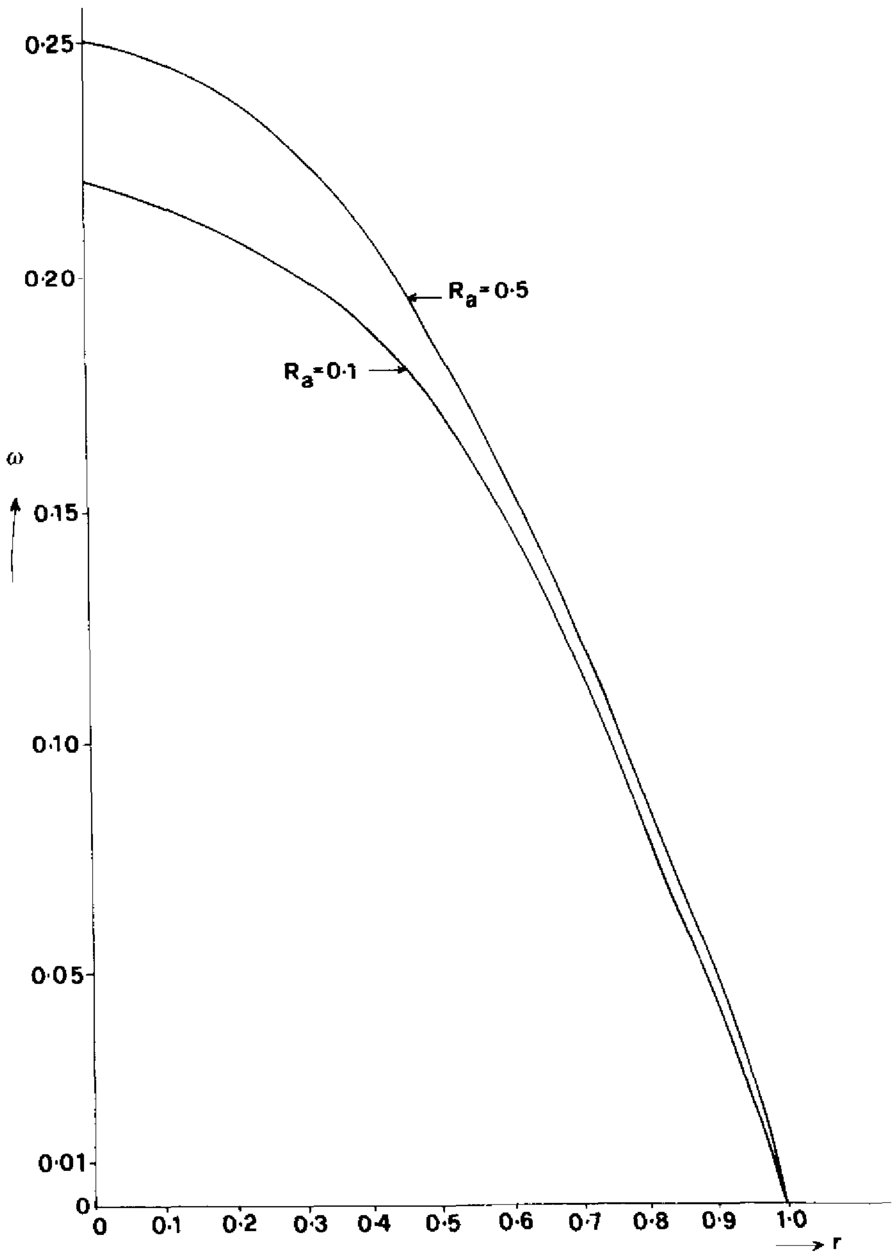
[8] Kaimal, M. R., "Peristaltic pumping of Newtonian fluid with particles suspended in it at low Reynoids number under long wavelength Approximation" J. Appl. Mech. ASME vol 45 p32 - 36 1978.

[9] Pedley, T. J., Fluid dynamics of large blood vessel. Cambridge Press 1980.

FIGURE CAPTIONS

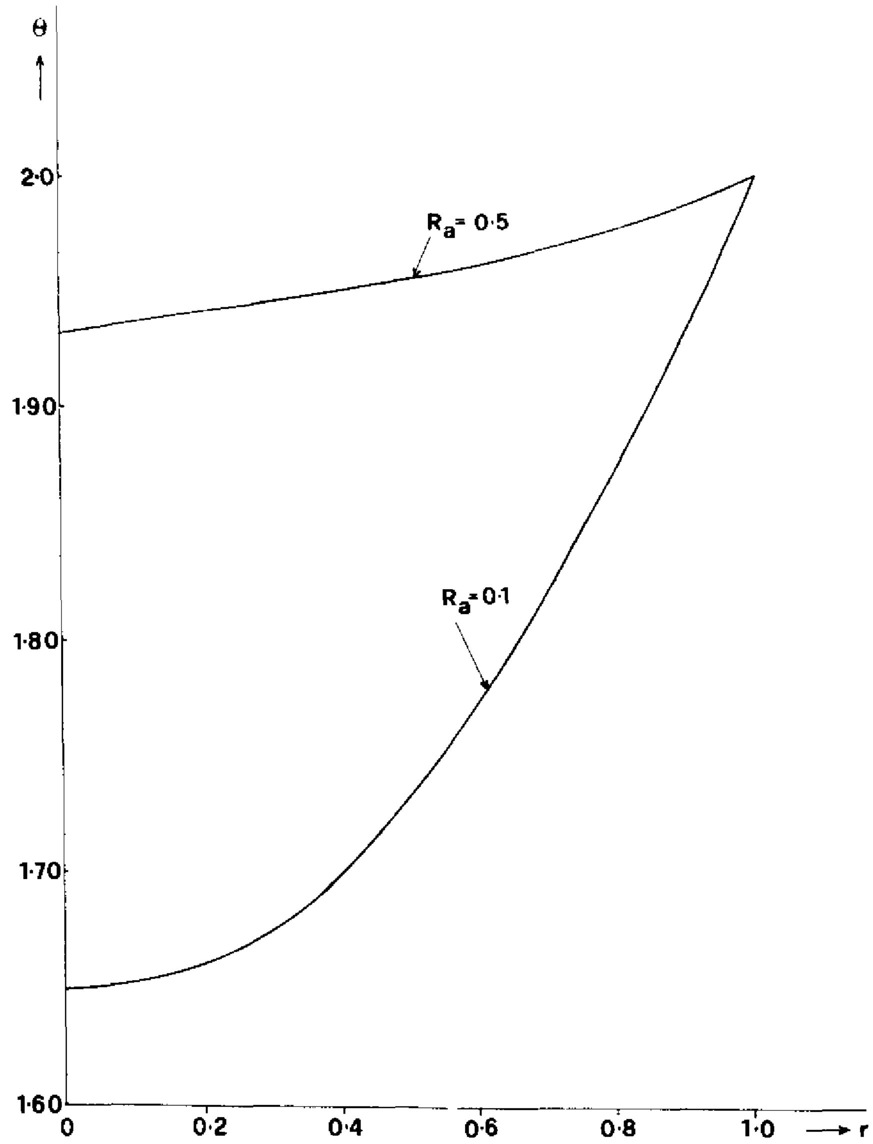
Fig.1 Velocity profile flow in curved pipe Ra = 0.1, 0.5.

Fig.2 Temperature distribution flow in curved pipes Ra = 0.1, 0.5.



0·25
0·20
0·15
0·10
0·05
0·01
0
ω
R_a=0·5
R_a=0·1
0 0·1 0·2 0·3 0·4 0·5 0·6 0·7 0·8 0·9 1·0
r

Fig.1



θ
2·0
1·90
1·80
1·70
1·60
R_a= 0·5
R_a=0·1
0 0·2 0·4 0·6 0·8 1·0
r

Fig.2