INSTITUTE FOR HIGH ENERGY PHYSICS

IHEP-OTF. 91-87 (IFVE - OTF--91- 87)

И Ф В Э 91-87
ОТФ

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FEIGENBAUM CONSTANTS IN HADRON COLLISIONS

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Protvino 1991

УДК 539.I.0I М-24

Abstract

Batunin A.V. Feigenbaum Constants in Hadron Collisions: IHEP Preprint 91-87. - Protvino, 1991, p. 6, fig. 1. , refs. 12.

The coincidence is found between the law $\langle n_{ch}(s)\rangle$ growth in hadron collisions for symmetric rapidity intervals and the law of growth of the number of elements in limit 2^m-cycles for one-dimensional quadratic maps when a govering parameter is varied. Fractal structure of the corresponding attractor underlies intermittency phenomenon in the multiplicity distribution of particles.

Аннотация

Батунин А.В. Константы Фейгенбаума в столкновениях адронов: Препринт 91-87. - Протвино, I99I. - р. 6, I рис., библиогр. I2.

Обнаружено совпадение закона роста $\langle n_{ch}(s)\rangle$ в столкновениях адронов в полном и ограниченных интервалах по быстроте и закона роста числа элементов предельных 2^m-циклов для одномерных квадратичных отображений при изменении управляющего параметра. Перемежаемость в распределении по множественности заряженных частиц связана с фрактальной структурой соответствующего аттрактора.

In this paper we show that the universality characterizing the transition of nonlinear systems from order to chaos through an infinite sequence of period-doubling bifurcations is also a feature of observables in hadron collisions.

Some words about this universality. It was known that in large classes of nonlinear systems (hydrodynamic, optical, acoustical, etc.), the corresponding phase-space trajectories undergo fine splittings (period-doubling bifurcations) as the governing parameter is varied: circular trajectories transform into "pretzel" ones and so on. In the late seventies, Feigenbaum discovered [1] a universality (i.e., independency of a particular physical system) of these splittings:

1) the parameter convergence is universal;

2) the relative scale of successive branch splittings is universal.

In other words, write λ_m for the values of the governing parameter λ at which the period-doubling bifurcations $2^m \rightarrow 2^{m+1}$ occur (m is the number of the bifurcation, $m=0$ - for a circular trajectory, $m=1$ - for a "pretzel", etc.). Then

$$|\lambda_m - \lambda_\infty| \propto \delta^{-m} \quad \text{as } m \rightarrow \infty , \tag{1}$$

where λ_∞ is the limit value of λ corresponding to $m = \infty$, δ is the first Feigenbaum constant.

Note, that instead of investigating the entire phase-space trajectory, one usually looks at its points x_n of intersection with a given surface (the so-called Poincaré surface [2]). The dependence

$$x_{n+1} = \lambda f(x_n) , \tag{2}$$

is known as a Poincaré map. The number of the points of intersection n_m doubles after every bifurcation, so one has

$$m \propto |\lambda_m - \lambda_\infty|^{-\Delta_F}, \tag{3}$$

where $\Delta = \ln 2/\ln\delta$. In particular, for a logistic map [1,2]

$$x_{n+1} = \lambda x_n(1-x_n), \tag{4}$$

with a single quadratic maximum in [0,1], one has $\delta \cong 4.669201609$, so that $\Delta_F = 0.449806966...$.

We have found a striking coincidence between Δ_F and the exponent Δ in the law of growth with energy of the mean multiplicity of charged particles in the entire rapidity interval $\langle n_{ch}(s)\rangle$.

Indeed, for $\langle n_{ch}(s)\rangle$ we have a fit [3,4]:

$$\langle n_{ch}(s)\rangle \propto (\sqrt{s/s_0})^{\Delta}, \tag{5}$$

with $s_0 = 1$ GeV^2, $\Delta = 0.449 \pm 0.018$ for $p^{\pm}p$ collisions at the energies $\sqrt{s} = 5 - 900$ GeV.

In hadron collisions, the inverse energy $s^{-1/2}$ plays the role of λ, and the value λ_∞ is associated with $s^{-1/2} = 0$ ($\sqrt{s} \to \infty$). Hence at any finite energy, a finite number of hadrons will be produced.

Note, the experimental dependecies $\langle n_{ch}(s)\rangle$ in $p^{\pm}p$ collisions and in e^+e^- annihilation coincide [5] under the substitution

$$\langle n_{ch}(s)\rangle_{pp} = n_0 + \langle n_{ch}(s/k^2)_{e^+e^-}, $$

where $n_0 = 2.57 \pm 0.72$, $k = 3.00 \pm 0.32$, so we can take into consiration e^+e^- annihilation as well.

Thus, as a first approximation we can draw on the following analogy: the number of observed particles is equal to the number of modes (splittings) of a phase-space trajectory corresponding to a solution of some unknown master equation governing the dynamics of hadron generation. Really, we do not know this equation but we have a possibility to study its solutions!

Certainly, the coincidence of only Δ and Δ_F is insufficient to accept the analogy between hadron production and the birth of 2^m-cycles in nonlinear systems, for which Poincaré map is a quadratic function. However, we show that other fine characteristics of these processes also coincide.

In hadron collisions, we can take the rapidity axis as a one-dimensional - cross section of the entire phase space by some Poincaré surface. In fig.1, we have plotted the dependence of the exponent $\Delta(y_c)$ of the growth by form.(5) of the mean multiplicity of charged particles in $p^{\pm}p$ collisions for symmetric rapidity intervals $|y| \leq y_c$ against y_c. Instead of y_c we lay off the quantity $\xi = y_c/y_{max}$ as abcissa, with $y_{max} = \ln[(\sqrt{s} - 2m_N)/m_\pi]$. The experimental points (taken from the remarkable theoretical work [6]) correspond to the quantity S/y_{max}, where

$$S = (\langle n_{ch}\rangle+1)\ln(\langle n_{ch}\rangle+1) - \langle n_{ch}\rangle\ln\langle n_{ch}\rangle \; . \qquad (6)$$

(The dependencies on s and y_c are dropped out.) Evidently, $S \to \ln\langle n_{ch}\rangle$ as $\sqrt{s} \to \infty$, so that $S/y_{max} \to \Delta(y_c)$. Actually, already at $\xi \geq 0.5$ the values of $\Delta(\xi)$ reach the limit value $\Delta(1) = \Delta \simeq 0.45$.

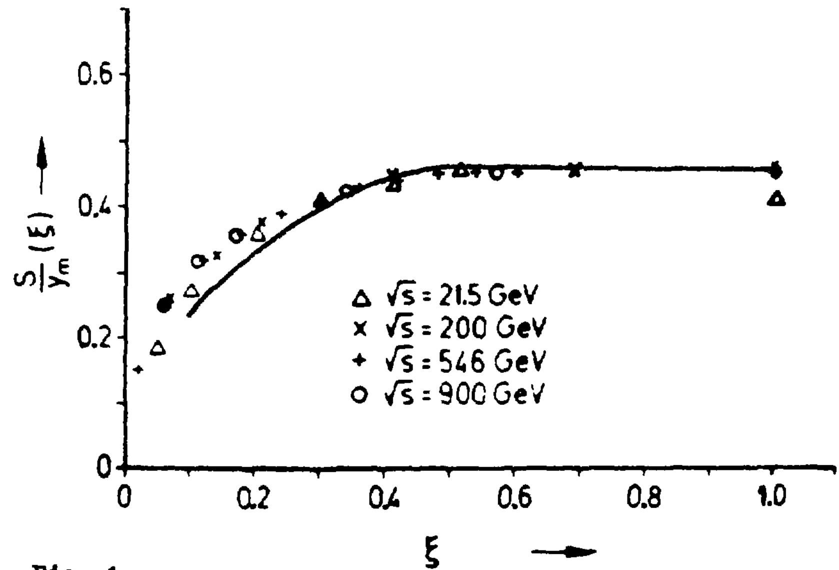

Fig. 1.

The curve in fig.1 has been *calculated* for the exponent of the growth of the number of elements n_m of 2^m-cycles for map (4) from m=1 to m=5 ($2^5 = 32$, which is approximately equal to $\langle n_{ch}\rangle$ at $\sqrt{s}$ = 900 GeV) falling within the interval $\xi|x_{max}-x_{min}|$ with the centre at the point $x = 0.5$. Again, the coincidence of the experimental points with the theoretical curve is striking, the characteristics of both processes look the same.

Note, any infinitesimal interval containing the point $x=0.5$ reproduces the fractal structure of the whole interval $|x_{max} - x_{min}|$ as $m\to\infty$ [1,2,7], since

$$\lim_{m\to\infty} \frac{|x_m - 0.5|}{|x_{m+1} - 0.5|} = \alpha = 2.50290787\ldots \quad . \tag{7}$$

Here α is the second Feigenbaum constant for map (4), controlling the splitting of the phase-space trajectory, and x_m is the nearest to the point $x=0.5$ element of the 2^m-cycle. Therefore, the curve in fig.1 will reach the limit value $\ln 2/\ln\delta$ at still smaller and smaller values of ξ as $m\to\infty$. The experimental points do show this tendency.

The constant α determines [8] Hausdorff dimension D of the attractor for map (4):

$$D = 0.538\ldots \cong \frac{-\ln 2}{\ln[(\alpha^{-1} + \alpha^{-2})/2]} \quad . \tag{8}$$

It would be interesting to compare D with the dimension of a set formed by the coordinates of observed particles on the rapidity (or any other, for example, $p_\perp$) axis as $\langle n_{ch}\rangle \to \infty$. High nonuniformity of the attractor for map (4) could manifest itself in the intermittency phenomenon [7,9], i.e., events with a large number of particles in small rapidity intervals which cannot be accounted for by statistical fluctuations.

If one draws on the analogy "particle coordinates on the rapidity axis - x-coordinates of elements in limit 2^m-cycles", then one has to answer the question: how the continuous experimental dependence (arbitrary real number) $\langle n_{ch}(s)\rangle$ could be put in agreement with the descrete $(2^m, m \in N)$ dependence $n_m(\lambda)$? For $\langle n_{ch}(s)\rangle$ the answer is clear, since averaging over events at fixed $\sqrt{s}$ we, in fact, average over different energy values because the true governing parameter in an isolated event will be the quantity $\hat{s}^{1/2} = (x_1 x_2 s)^{1/2}$, where x_1, x_2 are the momentum fractions of the initial hadrons carried by individual partons. In turn, the product $x_1 x_2$ can vary from 0 to 1.

The answer becomes vague when we pass to the multiplicity in an individual event where $\hat{s}$ is fixed but the number of particles is not necessarily 2^m. This difficulty arises also for $\langle n_{ch}(s)\rangle$ in e^+e^- annihilation.

We may assume the existence of two sources for particle production in the spirit of a two-component model [6,10]. In this model, the emission of particles occurs from a convolution of a coherent one-mode source and a chaotic one. Even if the chaotic source generates only 2^m particles one can, nevertheless, observe an arbitrary number of particles in an isolated event owing to the coherent source.

On the other hand, only the chaotic source contributes to the quantity plotted in fig.1, since S from (6) is none other than a Kolmogorov entropý of the chaotic source [6]. The coherent source has a zero entropy [6], therefore, the entropy comes entirely from the chaotic component and the experimental points in fig.1 do not "feel" the contribution from the coherent source. Thus, the *power law* energy dependence of the mean multiplicity is a reflection of the existence of a *chaotic* source.

It would be very interesting to separate "chaotically" produced particles from others in individual events. If one finds a way to do it, then, after necessary normalization and a shift of the origin of coordinates, the rapidities of "chaotically" produced particles must satisfy some quadratic relation similar to (4).

To conclude, we have found the quantitative coincidence between the dependence of the multiplicity of the elements in 2^m-cycles for map (4) on the governing nonlinearity parameter λ and the dependence of $\langle n_{ch}\rangle$ on the inverse energy $s^{-1/2}$ in $p^{\pm}p$ collisions for all symmetric rapidity intervals. The rapidity axis plays the role of a one-dimensionalcross section of the entire phase space of solutions of a (still unknown) nonlinear master equation for particle generation.

We predict that the exponents of the growth of the mean multiplicity in any symmetric rapidity intervals will reach the limit value $\ln 2/\ln\delta = 0.4498...$ as $s\to\infty$. The dimension of the fractal set formed by the coordinates of chaotically produced particles on the rapidity axis will tend to 0.538... The intermittency in hadron collisions is a direct consequence of the noninteger dimension of this set corresponding to the attractor of map (4) as $\lambda\to\lambda_\infty$.

The appearance of Feigenbaum constants in the characteristics of multiple particle production is a striking evidence for nonlinearity of differential equations governing the dynamics of the process, like the famous Lorenz [11] model with truncated Navier-Stokes equations. Perhaps, our observation will breathe new life into hydrodynamic models of particle generation [12].

I would like to thank Prof. S.S.Gershtein and S.N.Storchak for helpful discussions.

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Received 07 June, 1991

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Константы Фейгенбаума в столкновениях адронов.

Редактор А.А.Антипова. Технический редактор Л.П.Тимкина.

Подписано к печати 31. 07. 91. Формат 60х90/16.
Офсетная печать. Печ.л. 0,38. Уч.-изд.л. 0,53. Тираж 260.
Заказ 476. Индекс 3649. Цена 8 коп.

Институт физики высоких энергий, 142284, Протвино
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ПРЕПРИНТ 91-87, ИФВЭ, 1991