IC/92/60

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

DETERMINISTIC DYNAMICS OF PLASMA FOCUS DISCHARGES

J. Gratton

M.A. Alabraba

A.G. Warmate

and

G. Giudice

INTERNATIONAL ATOMIC ENERGY AGENCY

UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION

MIRAMARE-TRIESTE

IC/92/60

International Atomic Enegy Agency
and
United Nations Educational Scientific and Cultural Organization
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

DETERMINISTIC DYNAMICS OF PLASMA FOCUS DISCHARGES

J. Gratton *, M.A. Alabraba **, A.G. Warmate **
International Centre for Theoretical Physics, Trieste, Italy

and

G. Giudice ***
INFIP, Facultad de Ciencias Exactas y Naturales, UBA,
Pabellón I, Ciudad Universitaria, 1428 Buenos Aires, Argentina.

MIRAMARE – TRIESTE
April 1992

* Researcher of CONICET, Argentina.
On leave of absence from: INFIP, FCEyN, UBA and CONICET.
Permanent address: INFIP, Facultad de Ciencias Exactas y Naturales, UBA, Pabellón I, Ciudad Universitaria, 1428 Buenos Aires, Argentina.

** Permanent address: Physics Department, Rivers State University of Science and Technology, PMB 5080, Port Harcourt, Nigeria.

*** Fellow of CONICET, Argentina.

Deterministic Dynamics of Plasma Focus discharges

J. Gratton, M. A. Alabraba, A. G. Warmate, and G. Giudice

Keywords:

chaos

dynamical system

modeling

nonlinear physics

plasma focus

PACS numbers:

05.45.+b

52.55.Ez

ABSTRACT

The performance (neutron yield, X-ray production, etc.) of plasma focus discharges fluctuates strongly in series performed with fixed experimental conditions. Previous work suggests that these fluctuations are due to a deterministic "internal" dynamics involving degrees of freedom not controlled by the operator, possibly related to adsorption and desorption of impurities from the electrodes. According to this dynamics the yield of a discharge depends on the outcome of the previous ones. We study 8 series of discharges in three different facilities, with various electrode materials and operating conditions. More evidence of a deterministic internal dynamics is found. The fluctuation pattern depends on the electrode materials and other characteristics of the experiment. A heuristic mathematical model that describes adsorption and desorption of impurities from the electrodes and their consequences on the yield is presented. The model predicts steady yield or periodic and chaotic fluctuations, depending on parameters related to the experimental conditions.

1. INTRODUCTION

If a capacitor bank is discharged onto a pair of coaxial, cylindrical electrodes within a chamber in which a suitable filling gas has been introduced, a small (≈ 1cm) short lived (≈ 50-100ns) dense (≈ 10^{18} cm^{-3}), hot (≈ 1keV) plasma (called the "plasma focus") can be produced [1]. If Deuterium is used as the filling gas, a large number of fusion reactions occur and a neutron pulse is produced by the focus; emission of an X-ray pulse, as well as of energetic ion and electron beams are also produced by the focus [1],[2],[3]. The dynamics and structure of the plasma focus have been the object of studies since many years, however the details of the generation of the emissions and of the fusion reactions are not fully known and certainly very complex [4],[5],[6].

A key reason of interest of the plasma focus is that it allows to produce, with an inexpensive device, a very localized, intense pulsed source of radiations, which could find various applications in technology [7],[8],[9],[10]. Clearly, in considering any application, the reproducibility of the emissions from the focus is a very important issue. It is a well known fact that the performance (as measured by means of some convenient parameter, such as the neutron yield for Deuterium discharges, or the intensity of the X-ray emission) of a plasma focus operated with fixed experimental conditions (materials and geometry of the electrodes, insulator and chamber, voltage of the capacitor bank, filling pressure, etc.) can fluctuate strongly around the average in the normal operating regime. The magnitude and the pattern of these fluctuations as well as their origin are not well known and have been poorly investigated. A possibly related fact that has been also known for a long time (but never thoroughly investigated) is that when experiments are started after the chamber and the electrodes have been exposed to the atmosphere, the performance is very poor at the beginning, and a certain number of discharges has to be performed without opening the device before the "normal" operating regime is attained. In the jargon of the experimentalists this first group of discharges is called "conditioning".

During research carried out years ago in which one of the authors (J. G.) participated it was found evidence indicating that the fluctuations of the neutron yield n could be an effect of the balance between adsorption and desorption of impurities from the surface of the electrode. This means that the fluctuations are a consequence of deterministic (i. e., not random) processes, not controlled by the operator [11]. A consequence of such an "internal" mechanism is that the outcome of a discharge is influenced by the previous ones. However, the full implications of this result were not further developed in that earlier work. Since then great advances have been made in the application of the theory of dynamical systems, in particular, it has been realized that it is possible to analyze the dynamics of an experimental system without any "a priori" knowledge about the governing equations or the relevant degrees of freedom (as is the situation in the present case). Procedures have been developed that allow to reconstruct the system's full motion in phase space from the observation of a single degree of freedom [12],[13]. As we shall discuss below, the postulated impurity adsorption-desorption mechanism could in principle lead to various different behaviours of the yield of plasma focus discharges: steady, periodic (simply or multiply), and chaotic, depending on the values of certain parameters (the properties of the electrodes, insulator and chamber materials, the filling gas, the geometrical and electrical parameters of the discharge, etc.). Thus the different fluctuation patterns that are observed in the experiments, as well as the "conditioning" process may find a unified explanation.

In this paper we investigate the fluctuation pattern of series of plasma focus discharges. First, we summarize our previous results and discuss some inferences that can be drawn from this evidence. Second, we investigate 8 series of discharges performed in plasma focus devices at the INFIP (Buenos Aires) and at the ICTP (Trieste) in which the control parameters were kept as constant as possible in order to ascertain if the observed fluctuation patterns are indeed the result of a deterministic dynamics, as suggested by the results of [11], or are just random effects to be attributed wholly to the lack of accurate reproducibility of the control parameters in each individual discharge and to the measurement errors. We find that it can be confidently concluded that deterministic effects are present, but unfortunately, the experimental errors and uncertainties as well as the small number of discharges of the series do not allow phase space reconstructions sufficiently accurate to determine the dimensions of the attractor and the details of its geometry. It is also shown that the fluctuation patterns depend on various details of the experiment, in addition to the materials of the electrodes. Third, we consider an elementary, global mathematical model, that describes heuristically the essentials of the interplay between adsorption and desorption of contaminants on the electrodes and its influence on the yield of the discharges. We show that the model predicts steady, periodic, and chaotic behaviour, according to the values of three parameters that describe the cleaning efficiency of the discharge, the contaminating effect of a high-yield discharge, and the influence of the contamination on the yield. Thus it is shown that, at least in principle, the putative mechanism can explain the whole range of behaviours observed in the experiments.

We do not attempt more fundamental approaches, such as detailed theoretical models that include the full time-dependent plasma dynamics coupled with the circuit, as well as the geometrical and material properties of the experiment, as this endeavour is clearly beyond practical possibilities given the present state of knowledge.

2. SUMMARY OF PREVIOUS RESULTS

For convenience we summarize the main results of previous work [11]. Experiments comprising more than 1000 discharges in D_2 were performed using electrodes having the same geometry, but made of different materials: OFHC (Oxygen Free High Conductivity) Cu, Brass, and ordinary Cu. Any experiment consists of a series of a large number (> 100) of consecutive discharges. In any series the geometry and materials of the electrodes, insulator and chamber as well as the parameters of the external circuit were fixed, the same kind of filling gas (Deuterium) was employed, and the chamber was not opened, to avoid contamination of the electrodes by exposure to the atmosphere. Only two parameters, the charging voltage V_c of the capacitor bank and the filling pressure p_f, could be controlled by the operator, and were adjusted to given fixed values for all the discharges of the series. The neutron yield n was measured for each discharge. The main results are summarized in Table 1.

We now comment several relevant facts then observed:

(a) The length of the "conditioning" process (first shots of a series before the normal operating regime is achieved, and characterized by an extremely poor neutron yield) depends strongly on the electrode material: it is very short (some 5 discharges) for brass electrodes, and much longer (≈ 100 discharges) in the case of OFHC copper. With ordinary Cu electrodes, more than 350 shots were performed without attaining a regime of significant neutron yield. There are two alternative explanation of this last result: either (i) the conditioning is much longer than 300 discharges, or (ii) the conditioning is extremely short (≈ 1 discharges) but the yield of the normal operating regime is very low.

(b) in the normal regime, n varies strongly from shot to shot in all series, ranging over more than one order of magnitude, with $<n> \approx 1/3$ of the best n. Also, $<n>$ is much larger for OFHC Cu than for brass electrodes. For ordinary Cu electrodes large fluctuations were observed, with the best individual shots attaining only very low n values ($< 0.7\times10^7$). It must be remarked the very large magnitude of the fluctuations (that occour in all the series), that can not be easily explained as a result of the unavoidable changes of the control parameters V_c and p_f from shot to shot (see Section 2.3 below). This is strong evidence indicating that "internal" mechanisms involving degrees of freedom not controlled by the experimenter are operating. The fluctuations appear to be random in the case of OFHC Cu electrodes, but with Brass electrodes a striking alternating pattern is observed, in which a high yield shot tends to be followed by a low yield discharge and vice-versa. A statistical analysis was performed in this case, showing that random effects should be ruled out as possible causes of this alternating behaviour. This alternating pattern is further evidence in favour of internal deterministic mechanisms as the causes of the fluctuations.

(c) it was observed that "bad" shots are delayed by 50 ns with respect to "good" ones; this delay can be explained by a 2.5% high Z contamination of the plasma. Such a quantity of contaminants is consistent with the removal of a monoatomic layer from the electrode.

(d) it was verified that the fluctuations are unaffected by the replacement of the filling gas and/or the flushing of the chamber. This means that the bulk of the contamination is not present in the gas prior to the discharge, but is picked up by the plasma during its evolution. Also, cleaning the electrodes by means of a RF discharge did not have any effect on the fluctuation pattern. This means that the conditions of the electrodes at the beginning of a series are not relevant for the yield fluctuations (except perhaps for the first few shots, i.e. the conditioning).

Clearly the yield of a discharge depends critically on the presence of contaminants in the plasma. In view of the evidence we have discussed, it is reasonable to assume that before a discharge takes place they are adsorbed on the surface of the electrodes. The deterministic character of the fluctuations indicates that their quantity must depend on the outcome of previous discharges. The origin and dynamics of the contamination and fluctuations can then be explained in the following way:

As the discharge sweeps the electrodes the surface contaminants are (partly or totally) desorbed and incorporated into the plasma (which, it must be noted, is formed from the gas initially contained in the interelectrode volume, which is usually a small fraction of the volume of the chamber). If the resulting plasma contamination is large, the discharge will result in a "bad" (i. e., a low temperature, low yield) focus. As the plasma quenches after the discharge its contaminants are redistributed between the chamber walls, the whole volume of gas, and the electrodes, so that only a small fraction will be redeposited onto the latter, which in consequence will be cleaner than before. Then the conditions will be more favourable for the next discharge. In this way after one or more "bad" discharges the electrodes will be sufficiently clean so that a "good" (i. e., high temperature, high yield) focus can occour. On the other hand a high yield focus generates an intense flux of energetic particles and radiations, that on impinging on the chamber walls can liberate a considerable amount of new impurities, part of which are again deposited on the electrode, thus increasing the contamination of the latter. If this effect is large,

after a "good" shot the conditions of the electrode will be such that one, or several, "bad" shots are likely to occour. As a result of such a mechanism the yield of a series of discharges will in general fluctuate in a very complicate manner. The pattern of these fluctuations will depend on the materials, geometry, and other characteristics of the device that determine (also in a complicate manner) the balance of impurities on the electrodes. In particular, these factors may combine in such a way as to produce an alternating pattern of fluctuations as was observed in the series with brass electrodes. They might also combine to produce more complex patterns such as those observed with OFHC Cu electrodes, or they might even compensate so as to produce a constant yield. The proposed adsorption-desorption mechanism, it was concluded, could in principle explain the different patterns of fluctuations observed in the normal regime, and the lengths of the conditioning processes [11].

3. EXPERIMENTS

To achieve a better understanding of the fluctuation pattern we shall presently analyze neutron production data from series of discharges performed in the INFIP, Buenos Aires, in 1977, 1986 and 1988, as well as X-ray data from three series performed in Trieste with the ICTP-UM facility in 1991. Each series consists of a sequence of N_T discharges performed without opening the chamber, and *after* conditioning. In each series the electrode, insulator and chamber materials, their geometry, and the filling gas were the same, and the control parameters V_c and p_f were kept as constant as possible.

3.1. Neutron production experiments

The series performed in the INFIP will be called "Buenos Aires experiments" in the following. The data were extracted from the logbooks at INFIP, that cover 15 year's work and record about $3x10^4$ discharges. Only 5 series of data were found that conformed to the requirements stated above. They consist of two series with brass electrodes in a Pyrex chamber performed in 1977 with the PF-I device [14], designated Ex77a (N_T = 133) and Ex77b (N_T = 100), and three series with OFHC Copper in a s.s. (stainless steel) chamber with the PF-II device [15], designated Ex86 (N_T = 331), Ex88a (N_T = 291), Ex88b (N_T = 91). In Ex88b a set of electrodes different from those of Ex86 and Ex88a was employed. The series Ex77a and Ex77b are part of the data that were studied in [11]. The filling gas was Deuterium in all cases. In each series, the chamber was flushed and new gas was introduced after any 5-10 discharges. In each discharge, neutron yield measurements, as well as various other measurements not relevant for the present study, were made.

3.2 X-ray production experiments

These experiments were performed in June 1991 at the ICTP, on the 3.3 kJ ICTP-UM facility which is provided with OFHC electrodes and a s.s. chamber. A description of this device can be found elsewhere [16],[17]. Additional details of the set-up and operation relevant for the present experiment have also been previously reported [18],[19]. In what follows we shall designate these experiments as the "Trieste experiments". Three series designated Ex91a (N_T = 101), Ex91b (N_T = 100) and Ex91c (N_T = 97) were performed with V_c =12 kV. The filling gas was Argon (p_f = 0.6, 0.7, and 0.8mb in Ex91a, Ex91b, and Ex91c, respectively) and it was changed after each discharge. The X-ray emission was registered by means of two X-ray diodes, one with a 24 μm Mylar foil in front, and the other with a 24 μm Mylar foil plus an additional 10 μm Cu foil. We call X_3 and X_4 the corresponding signals. The ratio $y = X_3/X_4$ between the two signals depends on the plasma temperature [19], and was taken as a measure of the "yield".

It should be noticed that by virtue of Takens's theorem, a series of data corresponding to any single degree of freedom of the system is sufficient for the reconstruction of the full phase space [13]. The choice of this degree of freedom is in principle arbitrary, being in practice dictated by practical reasons and convenience. Then, for the present purposes, it is irrelevant whether the "yield" is defined by the neutron production (Buenos Aires experiments) or by a certain function of the temperature (Trieste experiments).

The Buenos Aires and Trieste series are summarized in Table 2. In total, the data base consists of 1244 discharges, of which 233 with the PF-I facility (brass electrodes in Pyrex chamber), 713 with the PF-II (OFHC Cu electrodes in s.s. chamber), and 298 with the ICTP-UM device (OFHC Cu electrodes, s.s. chamber).

3.3 Experimental errors and other limitations of the present studies and their consequence on the analysis of the data

Various sources of error can be identified in the measurements:

(a) Uncertainties in the control parameters:

(i) V_c: the voltage actually measured is that of the power source that is charging the capacitor bank; between the moment the source is disconnected and the moment when the discharge is triggered there is an certain time interval during which V_c varies, since the capacitors are slowly discharging; furthermore the interelectrode voltage V_e (that is the truly relevant parameter) depends not only on V_c, but also on the voltage drop in the rest of the circuit, which in turn depends on the current (that may be slightly different in each discharge). In consequence, even if

V_c were constant it could not be assured that V_e is strictly the same in each discharge. The uncertainty in V_e owing to these causes can be estimated to be as large as 5-10%. It is reasonable to assume that the true V_e fluctuates randomly within this margin; since the neutron yield of the Plasma Focus depends very strongly on V_e ($n \sim V_e^2$) this is then an important source of error.

(ii) p_f: the pressure reading is not very accurate ($\Delta p_f/p_f \approx$ 5-10%) but this uncertainty is not expected to have a large influence on the neutron yield, since the Buenos Aires experiments were performed at, or near, the maximum of the $n(p_f)$ curve, where n is not very sensitive to small changes of p_f.

(b) Errors of the measurement of the yield:

(i) In the Buenos Aires experiments y ($\equiv n$) was measured by means of a silver activation counter; the errors were small (< 5%) since the number of counts was fairly large giving good statistics.

(ii) As said before, in the Trieste experiments y was defined as the ratio of two X-ray signals. These were recorded by means of a dual trace oscilloscope that sampled the signals from both diodes. Since the X-ray signals consist of very sharp spikes, they can be severely distorted in an uncontrolled way owing to the strobe sampling. To further complicate matters, the signals were contaminated by a clearly visible, large amplitude noise consisting of a 60 ns oscillation. We notice that in deriving y the errors of each signal are compounded. It is difficult to estimate accurately the magnitude of the resulting error in y, but it may be easily as large as 20%.

It should be remarked that the characteristics of the Trieste experiments are not the best choice to bring in good evidence the effects we are trying to study. In the first place, the earlier results [11] indicate that the deterministic behaviour should be much more clearly displayed with brass, rather than Cu electrodes; second, should the contamination of the electrodes (that subsequently passes into the plasma) by Oxygen or other high Z impurities be the cause of the fluctuations, its effect should be more marked if the device is operated with Hydrogen, Deuterium, or any other low Z gas, rather than with Argon. We have been aware of these facts since the beginning, but we could not improve the situation since we had to operate the device in its current condition, accepting the scarce diagnostics, instrumentation and materials available, and within a very strict time limit, as the ICTP-UM facility was bound to be disactivated by July 1991.

A drawback of the present investigation is the small number (100-300) of discharges of each series. In fact, to reconstruct attractors and/or determine their dimension, it is recommended to use data bases larger than ours. However we notice that it has been shown that it is possible to arrive at significant results with quite small series (less than 500 data points) [20],[21]. We would also like to point out that it is quite difficult and time demanding to carry out a program of several series each of, say, 10^3 discharges. It must be kept in mind that during each series it is not allowed to open the vacuum chamber (and/or to change the electrodes, insulator, etc.); in actual practice it is difficult to satisfy this requirement since much too frequently some components of the device break down or fail thus interrupting the series. In the case of the Trieste experiments, in addition, time limits did not allow allow more than 300 discharges. In the light of the preceding comments, it makes sense to attempt to extract as much information as possible from the available data. If the results appear promising, then it may be worthwhile for experimenters to undertake the much more costly and lengthy systematic studies that are needed to fully investigate this interesting problem. In any case, since the present experiments comprise all together more than 1200 discharges and, as will be shown below, the results of the analysis of the various series are consistent with our main assumptions concerning the fluctuation dynamics, the conclusions that might be drawn from the separate study of each one of them are mutually reinforced.

4. ANALYSIS OF EXPERIMENTAL RESULTS

In Figure 1 the raw yield data are shown for typical series. It can be observed quite clearly the alternating pattern of fluctuations in the EX77 series (brass electrodes), and the irregular fluctuations (apparently random) typical of the discharges with OFHC Cu electrodes. We list in Table 3 the fluctuation levels of the series (= standard deviation/mean) and the parameters σ (= internal surface of chamber/electrode surface) and ε (= volume of chamber/interelectrode volume) of the experiments. It can be observed from Figure 1 and Table 3 that the fluctuation level of the series performed with the PF-I facility is higher than that of the PF-II series. This is probably related to the much larger values of σ, ε of the PF-II machine. In fact if our assumptions are correct, as σ, ε increase, the electrode contamination must decrease (everything else being the same), and accordingly the amplitude of the fluctuations should decrease.

The analysis of the Buenos Aires and Trieste data is complicated by the uncertainties and measurement errors discussed in Section 2.3. Summarizing: (i) the fact that the control parameters are not strictly the same for all the discharges of the series, but vary randomly (within certain limits), induces variations of y not related to the deterministic effects of the "internal" dynamics that we are trying to isolate, and (ii), the yield measurements themselves are subject to random errors.

Any deterministic effect on the fluctuations will be obscured by the admixture of this experimental "noise". The first problem is then to ascertain if the observed fluctuations can be explained entirely as arising from random effects, or if there is in addition a deterministic effect. If a deterministic effect does in fact exist, the next problem is to isolate it from the noise. Various techniques are available to extract the deterministic part from a noisy signal, see for example [22]; among these we used the Grassberger-Procaccia method for measuring dimension [23] and the singular system analysis [24].

The analysis starts from the raw data, that is, the experimental values $y_1, y_2, y_3, \ldots, y_{N_T}$ of the "yield" of each of the N_T discharges of a series. An embedding of dimension d of the data is constructed defining

$$x_i \equiv (y_{i+1}, y_{i+2}, \ldots, y_{i+d-1}),\ x_i \in \Re^d,\ i = 1, \ldots, N,\ N = N_T - (d-1) \qquad (1)$$

Such embeddings are the basis for the reconstruction of the phase space of the system and the determination of the properties of the attractor. The sequence $X \equiv \{x_i\}$ is called the trajectory of the system. In the following we shall omit for brevity the detailed description of the methods of analysis, since it is readily available in the literature [22],[23],[24], we shall only present the relevant formulae in order to introduce and clarify the notation.

4.1 Measure of the local dimension of the attractor

A measure of the dimension of the trajectory can be obtained by considering

$$C(r) = \lim_{N \to \infty} \frac{1}{N^2} \left\{ \begin{array}{c} \text{number of pairs } (i, j) \text{ such} \\ \text{that } |x_i - x_j| < r \end{array} \right\} \qquad (2)$$

In general, for r small enough

$$C(r) \cong A r^{\nu} \qquad (3)$$

where A is a constant and $\nu \le d$. According to (3), for a given d the slope of $C(r)$ vs r in a log-log plot should tend, for small r, to ν, thus allowing to determine the latter.

For a deterministic series of data, as d becomes sufficiently large, ν saturates (for small r) to a maximum ν_m ($\nu_m < d$), which is then the (local) dimension of the attractor. On the other hand, for a purely random series one has always $\nu = d$. For a noisy deterministic series, as d becomes larger the slope will tend to $\nu = d$ for very small r (where the noise dominates), but will show a plateau $\nu = \nu_m$ for sufficiently large values of r (but still small, to ensure the validity of the scaling (3)) [21],[25]. A limitation of this method is that if the noise amplitude is large and/or the data base is too small, it may not be possible to detect a plateau, since it should appear in an interval in which r is not sufficiently small for (3) to be valid, or the curves $d(\ln C)/d(\ln r)$ vs r for large ν are not smooth enough for the plateau to be visible.

We applied this type of analysis to the Trieste data. In Figure 2 we represent $\nu = d(\ln C)/d(\ln r)$ vs $\ln r$ for $d = 2, 3, \dots, 6$ for Ex91a, b, c. It can be observed that the results are disappointing owing to the insufficient number of data points: no plateau is observed except for Ex91a, where it is barely visible. Even in this case it is not possible to estimate ν_m, since this requires to calculate $C(r)$ for embeddings of higher dimensionality (> 6), which cannot be done with sufficient accuracy for very small data bases. However the results of this analysis are not in contradiction with the assumption that the fluctuations are partly deterministic.

4.2 Measure of the embedding dimension of the trajectory (Singular system analysis)

For very noisy data, a more effective method is the singular system analysis [24] which is based on the covariant matrix $\Xi \equiv X^T X$ or, in terms of the data,

$$\Xi = \frac{1}{N}\begin{bmatrix} \sum_{i=1}^{N} y_i y_i & \sum_{i=1}^{N} y_i y_{i+1} & \cdots & \sum_{i=1}^{N} y_i y_{i+d-1} \\ \cdots & \cdots & & \cdots \\ \sum_{i=1}^{N} y_{i+d-1} y_i & \sum_{i=1}^{N} y_{i+d-1} y_{i+1} & \cdots & \sum_{i=1}^{N} y_{i+d-1} y_{i+d-1} \end{bmatrix} \tag{4}$$

in which it has been assumed that $<y_j> = 0$. If the data are free of noise, there will be in general n' ($n' \le d$) nonzero eigenvalues σ_i^2 of Ξ; n' is then the dimensionality of the minimum subspace containing the trajectory. The σ_i are in this case the lengths of the half axes of the n'-dimensional ellipsoid that encompasses the data points, and n' is a measure of the global dimensionality of the attractor (that need not in general coincide with the local dimension derived by means of the method described in Section 4.1). On the other hand, for a purely random series of data $y_j = \xi_j$, one will have $n' = d$ for any d and all the eigenvalues of the covariant matrix will be equal: $\sigma_i^2 = <\xi^2>$. For a noisy deterministic series we can write

$$y_j = y_{dj} + \xi_j, \tag{5}$$

where y_{dj} denotes the deterministic part of the signal, and ξ_j the overimposed noise. Then the eigenvalues of Ξ are given by:

$$\sigma_i^2 = \sigma_{di}^2 + <\xi^2> \tag{6}$$

in which σ_{di}^2 are the eigenvalues associated with the deterministic part of Ξ. In this case, then, the eigenvalues σ_i^2 will be all nonzero, but only n' ($n' \le d$) among them will be different, and larger than the remaining $d - n'$, which are related to the noisy part of the signal. The latter will be equal (approximately equal in a practical case in which N is not very large) to $<\xi^2>$. The first n' eigenvectors corresponding to the n' larger eigenvalues then span the n'-dimensional minimum subspace containing the deterministic part of the trajectory. In this way the effect of the deterministic part of the signal can be recognized and extracted from noisy data.

In Figure 3 are represented the eigenvalues of Ξ for $d < 10$ for all series. The application of this technique to the experimental data shows that in all cases the spread (defined by $s_d = [\sigma_{i,max}^2 - \sigma_{i,min}^2]/\sigma_{i,min}^2$) of the eigenvalues σ_i^2 increases almost linearly with d, being usually quite large for $d = 9$. In comparison, for an artificial random series the spread increases slowly and is always considerably less ($s = 0.7$ for $d = 9$, see Figure 3,(i)) than that observed in the experiments (see Table 4). In most cases, as d increases a small number of eigenvalues (n' = 1-4) grows larger and separates clearly from the remaining σ_i^2 which remain small. Exceptions are Ex77b and Ex86 where this separation is not as clearly observed, although the spread ($s_9 = 2.6$ and $s_9 = 1.5$, respectively) is significantly larger than that of the artificial random series.

4.3 Reconstruction of trajectories

Figure 4 shows some examples of 3-D embeddings of trajectories. Figure 5 shows projections of X on the 3-D subspace spanned by the eigenvectors corresponding to the 3 largest eigenvalues of Ξ, $d = 9$, for the same trajectories depicted in Figure 4. We notice some similarities between the Trieste experiments and the Buenos Aires OFHC experiment Ex88b, in spite of the fact that the experimentally measured quantities are physically different. Inspection of the trajectories and their projections do not always reveal with sufficient clarity the structure of the attractors, although they show clearly deterministic signatures in most cases. Some features of the structure can be fairly well recognized in Ex77a, Ex88b and Ex91b. As d increases, the 3-D projection of a trajectory folds over itself (see also Section 6 and Figure 7); the effect of the noise is less in the projections, but the folding of the trajectory obscures the details of the geometry.

5. DISCUSSION OF EXPERIMENTAL RESULTS

The results of the measure of local dimensions, the singular system analysis, and the inspection of trajectories are summarized in Table 4. Taken together they clearly show the (partly) deterministic origin of the fluctuations for all experiments (some specific comments will be made below for the cases of Ex86 and Ex88a), and corroborate the conclusions of the previous work [11]. As expected, the materials are most important in determining the fluctuation pattern: there is a marked difference between Brass and OFHC Cu electrodes (in the latter case, the pattern can be similar even if very different devices are compared) both in the level of fluctuations and in the geometry of the attractor. The effects of the geometry and of the volume and surface ratios ε, σ, are also noticeable and in qualitative agreement with what should be expected from the assumed mechanism of fluctuations. The behaviour of the eigenvalues of the covariant matrix and the inspection of the trajectories indicate that low dimensional attractors do exist, although their properties cannot be determined with sufficient accuracy, since the experimental noise is large and the small number of data do not allow a sufficiently accurate isolation of the deterministic effect.

The cases of Ex86 and Ex88a deserve a special discussion since the interpretation of the evidence is less obvious (see Table 4). In both cases the spread of the eigenvalues of Ξ is small, and no structure can be recognized in the trajectories and their projections. The smaller spread ($s_0 = 0.9$) is that of Ex88a, for which however a small number of eigenvalues ($n' = 3$) grows noticeably larger and separates clearly from the other σ_i^2. First of all we notice that the paucity of positive evidence in these series does not by itself negate the possibility that the fluctuations are (partly) due to a deterministic mechanism: it is still perfectly possible, for example, that one is in presence of noisy deterministic fluctuations for which it is not easy to separate the deterministic effects from the noise. It might however be argued that the simplest explanation of Ex86 and Ex88a is that the fluctuations are purely random noise. Even if this were true, it would not be contrary to our assumed deterministic mechanism. We remark that all the remaining series indicate clearly that the yield dynamics is deterministic, although obscured by random noise. It is difficult to explain why the deterministic dynamics should be absent in these two doubtful cases. A possible, and in our opinion, better explanation might be that in Ex86 and Ex88a the deterministic dynamics is not chaotic, but is such as to produce a constant yield (see also the next Section); in this case the observed fluctuations could be the result of the purely random experimental noise (see Section 3.3). This interpretation is in fact consistent with the relatively small fluctuations of Ex86 and Ex88a, and with the large σ, ε of the PF-II experiments; in addition the fact that $<y>$ is largest for OFHC Cu electrodes (everything else being equal), together with the model predictions (see below) that $<y>$ decreases in going from the steady to the periodic and to the chaotic fluctuation regimes, tend to sustain this explanation.

6. SIMPLE GLOBAL MATHEMATICAL MODEL

In this Section we present a heuristic mathematical model to show how adsorption-desorption processes can produce yield fluctuation patterns of various kinds, as observed in the experiments. It is to be expected that independently of the details, an interplay between absorption and desorption such as we have described above should in general lead to a highly nonlinear relationship between the yields y_j and y_{j+1} of successive shots, that can be represented as a nonlinear mapping of the form

$$y_{j+1} = f(y_j) \,, \tag{7}$$

where f is a function that depends on the details of the adsorption-desorption mechanism, on the characteristics of the experiment (geometry, materials, filling gas, etc.) and on the values of the control parameters. According to the values of the controlling parameters and the form of f, it is to be expected that the iterates of (7) may in principle exhibit various basically different behaviours as is well known from the theory of nonlinear mappings [26]. The possibilities are the following:

(a) after an initial series of fluctuations the yield finally settles to a constant value.

(b) after an initial transient series of fluctuations, the yield may attain a periodic regime; for example, "good" shots may alternate with "bad" ones, or y could display a more complicate multiple periodicity.

(c) the yield may vary nonperiodically (i. e., chaotically).

The first transient iterations of the mapping (7) should correspond to the conditioning process, and the subsequent ones to the normal regime of the experiments.

To be specific, let us consider an example that shows how a simple idealized adsorption-desorption mechanism can be represented by a mapping of the form (7). To this purpose, let us introduce a parameter z to describe the "contamination state" of the electrodes; accordingly we shall assume that $z = 0$ corresponds to perfectly "clean" electrodes, and that z increases as the quantity of adsorbed impurities is increased. Then y_{j+1} must be a decreasing function of z_j, the contamination remaining after the previous discharge: $y_{j+1} = g(z_j)$. For mathematical simplicity we shall normalize the yield, assuming $y(z = 0) = 1$. Of course, for want of a detailed physical model, we do not know g, but it seems reasonable to assume that there should be some critical value of z, say z_0, such that for $z \ll z_0$ $g(z) \approx 1$, and for $z \gg z_0$ $g(z) \approx 0$. A simple choice of g with these properties is:

$$y_{j+1} = 1/[1 + (z_j/z_0)^{\lambda}], \tag{8}$$

in which λ is an exponent to be chosen appropriately. This formula describes the effect of the contamination on the yield.

To close our model we need a formula to compute the contamination z_j in terms of the outcome of the previous discharges. This formula must describe the desorption-adsorption mechanism. We shall take into account the cleaning effect of the discharge by the assumption that the contamination remaining after the passage of the current sheath over the electrodes is a constant fraction $\gamma\,(\gamma < 1)$ of that previously present (i. e., z_{j-1}, the contamination left over after the preceding discharge), independently of the yield of the focus, that occours *after* the passage of the current sheath over the electrode. In a similar way, the contamination added by the effect of a high yield focus must be an increasing function of y_j (since a high yield implies a large flux of particles and radiation impinging on the chamber walls, and so a larger amount of new impurities). For simplicity it can be assumed that this effect is directly proportional to y through a coefficient βz_0. Then, the contamination after the discharge j with its corresponding focus have taken place will be the sum of what is left of the previous contamination after the passage of the current sheath, plus that added by the effect of the outcome of discharge j:

$$z_j = \gamma z_{j-1} + \beta z_0 y_j, \tag{9}$$

In this formula, γ and β are constant (for fixed experimental conditions) parameters and the z_0 factor has been introduced for convenience. One can combine (8) and (9) to eliminate z and obtain a formula giving y_{j+1} in terms of y_j:

$$y_{j+1} = 1/\{1 + [\gamma(y_j^{-1}-1)^{1/\lambda} + \beta y_j]^{\lambda}\}. \tag{10}$$

This formula describes a nonlinear mapping as anticipated. The mapping (10) depends only on the parameters γ, β, λ that represent, respectively, the "cleaning efficiency" of a discharge, the "contaminating efficiency" of a high yield focus, and the dependence of the yield on the contamination. We observe that γ, β, λ should in general depend on the characteristics of the experiment and on the control parameters (for example, we should guess that γ should depend mainly on the electrodes materials, and that β should also depend on the materials of the chamber walls, and be a decreasing function of (σ, ε). A detailed physical model should (in principle, at least) allow to determine their values. At the present stage, however, we shall consider them as free parameters to be adjusted empirically. In any case, it can be safely assumed that for each series γ, β, λ are constant.

Figure 6 shows a bifurcation diagram of the mapping (10), varying β for fixed γ, λ. It can be observed that the diagram is qualitatively similar to that of the familiar logistic map [27] presenting as β is increased the typical Feigenbaum sequence of bifurcations leading to chaos. Windows of periodicity appear for larger values of β. It can be observed that according to the model $\langle y \rangle$ decreases in going from the steady to the periodic and to the chaotic regime. In Figure 7a we show an artificial series of "shots" generated by 150 iterations of (10) (the first 5000 iterates were discarded to eliminate transients). No noise was added. The series was computed for $\beta = 13.33$, $\gamma = 0.3$, $\lambda = 4$, that are inside a chaotic interval. Figure 7b shows the application of the singular system analysis to this artificial series. In Figure 7c, d we show, respectively a 3-D embedding of the trajectory, and the projection of the trajectory on the 3-D subspace spanned by the eigenvectors corresponding to the three largest eigenvalues of Ξ (d = 9). Comparing Figures 7 (c') and 7 (d') one observes that for large d the projection of the trajectory folds over itself in such a way that it is more difficult to recognize its geometry if N_T is small. Comparing with Figures 3, 4, and 5, that show the same for the experiments, a certain similarity can be noticed between the artificial series of Figure 7 and Ex77a. However, one must be cautious in drawing definite conclusions from any such similarity. The fact that the experiments do support our contention of a deterministic adsorption-desorption mechanism (plus a qualitative similarity between the model and some experimental pattern) does not warrant that (10) is its correct description. The model (10) may be too simple to explain the details of the experimental fluctuation patterns. In addition, since it is not possible to estimate starting from first principles the parameters γ, β, λ the model is, to a considerable extent, arbitrary. For these reasons, and because the lack of a sufficiently large data base does not allow to determine accurately the attractors, and so make more meaningful and critical comparisons, it is not worthwhile at the present stage to attempt to formulate more sophisticated models.

7. CONCLUSIONS

We have investigated the fluctuations of 8 series of discharges performed in plasma focus devices at the INFIP (Buenos Aires) and at the ICTP (Trieste) to ascertain if the observed patterns are indeed the result of a deterministic dynamics, as suggested by the results of ref. [11]. We find that it can be confidently concluded that deterministic effects are present, but unfortunately, the experimental errors and uncertainties as well as the small number of discharges of the series do not allow phase space reconstructions sufficiently accurate to determine the geometry and the dimensions of the attractor. We have formulated an elementary mathematical model, that describes heuristically the essentials of the interplay between adsorption and desorption of contaminants on the electrodes and its influence on the yield of the discharges. The model predicts steady, periodic, and chaotic behaviour, according to the values of three parameters that describe the cleaning efficiency of the discharge, the contaminating effect of a high-yield discharge, and the influence on contamination on the yield. The predictions of the model are qualitatively compatible with the experiments, thus showing that, at least in principle, the putative mechanism can explain the whole range of behaviours observed in the experiments.

Owing to various limitations, the present study must be considered as a first attempt to find a unifying explanation of the fluctuations of Plasma Focus discharges, and more extensive work should be desirable to get a clear picture of this interesting problem well as to obtain more details of the structure of the attractors under various experimental conditions.

More fundamental approaches, such as detailed theoretical models that include the full time-dependent plasma dynamics coupled with the circuit, as well as the geometrical and material properties of the experiment, are still premature and must wait for more complete experimental evidence, beside being clearly beyond practical possibilities given the present state of knowledge.

ACKNOWLEDGEMENTS

Thanks are due to S. Lee for permitting us to use the ICTP-UM plasma focus facility, to G. Denardo for lending important instruments and to A. Richetti for giving access to a microcomputer of the Department of Physics, Trieste University. It is a pleasure to acknowledge helpful discussions with H. Cerdeira. Thanks are also due to F. Aversa for helping with the computations of Section 5. Three of the authors (J.G.),(M.A.A.) and (A.G.W.) would like to thank Professor Abdus Salam, the International Atomic Energy Agency and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste.

REFERENCES

[1] J. Mather, Dense Plasma Focus, in Methods of Experimental Physics, Vol. 9, Part B, Academic Press (1971) p. 202.

[2] H. Schmidt, Plasma Focus and z-pinch, Proc. 2nd Latin American Workshop on Plasma Physics and Controlled Thermonuclear Fusion, CIF Series, Vol. 12, World Scientific Pub. Co., Singapore (1987) p. 1.

[3] M. Sadowski, Progress in studies of X-rays and ion beam emissions from Plasma Focus facilities, Proc. 2nd Latin American Workshop on Plasma Physics and Controlled Thermonuclear Fusion, CIF Series, Vol 12., World Scientific Pub. Co., Singapore (1987) p. 52.

[4] A. M. J. Bernard, Plasma Focus, in Pulsed High Beta Plasmas, D. E. Evans ed., Pergamon (1976) p. 69.

[5] V. V. Vikhrov, S. I. Braginskii, Dynamics of the z-pinch, Rev. Plasma Phys. 10 (1985) 425.

[6] H. Herold, H. J. Kaeppeler ed., Proc. 3d Int. Workshop on Plasma Focus Research Institut für Plasmaforschung, Stuttgart, FRG (1983).

[7] G. Decker, R. Wienecke, Plasma Focus Devices, Physica 82C (1976) 155.

[8] G. Decker, H. Herold, Der Plasmafokus, Phys. Bl. 36 (1980) 328.

[9] H. Conrads, Dense Plasma Focus as a Neutron Source for Fusion Research, Nucl. Sci. Eng. 106 (1990) 299.

[10] V. Nardi, Advanced Plasma Focus, in Proc. Japan-US Works. P-119 on 14 MeV Neutron Source for Materials Research and Development Based on Plasma Devices, A. Miyahara, F. H. Coensgen ed., Inst. of Plasma Phys., Nagoya University, Nagoya, Japan (1988) p. 213.

[11] A. Bañuelos, H. Bruzzone, R. Delellis, J. Gratton, R. Gratton, H. Kelly, M. Milanese, J. Pouzo, F. Rodríguez Trelles, Recent Plasma Focus Research, in Plasma Physics and Controlled Nuclear Fusion Research 1978, Vol II, IAEA, Vienna (1979) p. 173.

[12] N. H. Packard, J. P. Crutchfield, J. D. Farmer, R. S. Shaw, Geometry from a time series, Phys. Rev. Lett. 45 (1980) 712.

[13] F. Takens, Detecting strange attractors in turbulence, in Dynamical systems and Turbulence, Warwick 1980, Vol. 898 of Lecture Notes in Mathematics, D. A. Rand, L. S. Young eds., Springer, Berlin (1981) p. 366.

[14] H. Bruzzone, R. Gratton, H. Kelly, M. Milanese, J. Pouzo, Experimental Results of a Low Energy Plasma Focus, in Energy Storage, Compression and Switching, Vol. I, W. H. Bostick, V. Nardi, O. S. F. Zucker eds., Plenum, New York (1976) p. 255.

[15] J. Pouzo, J. Gratton, Design and Construction of the PF-II Device, in Energy Storage, Compression and Switching, Vol. II, W. H. Bostick, V. Nardi, O. S. F. Zucker eds., Plenum, New York (1983) p. 643.

[16] S. Lee, T. Y. Tou, S. P. Moo, M. A. Eissa, A. V. Gholap, K. H. Kwek, S. Mulyodrono, A. J. Smith, Suryadi, W. Usada, M. Zakaullah, A simple facility for the teaching of plasma dynamics and plasma nuclear fusion, Am. J. Phys. 56 (1988) 62.

[17] S. Lee, Experiments with the ICTP-UM 3.3 kJ Plasma Fusion facility, ICTP H4-SMR 554/4 (1991).

[18] M. A. Alabraba, K. D. Alagoa, T. Chera, O. H. Chin, W. P. De Sa, J. Singh, S. Lee, Z. L. Li, M. M. Masoud, C. H. Moreno, E. Negro, A. M. Punithavelu, H. Ramos, A. Rodrigo, A. Serban, S. Sobhanian, A. G. Warmate, C. S. Wong, Report of Experimental and Diagnostic Group, Spring College on Plasma Physics, ICTP (1991).

[19] C. S. Wong, S. Lee, Five channel diode X-ray spectrometer, Instruction Manual, ICAC-UM, enquiries can be directed to Plasma Res. Lab., Phys. Dept., University of Malaya, 59100 Kuala Lumpur, Malaysia (1991).

[20] N. B. Abraham, A. M. Albano, B. Das, G. de Guzman, S. Yong, R. S. Gioggia, G. P. Puccioni, J. R. Tredicce, Calculating the dimension of attractors from small data sets, Phys. Lett. 114A (1986) 217.

[21] H. Atmanspacher, H. Scheingraber, W. Voges, Global scaling properties of a chaotic attractor reconstructed from experimental data, Phys. Rev. A37 (1988) 1314.

[22] N. Gershenfeld, An experimentalist's introduction to the observation of dynamical systems, in Directions in Chaos, Hao Bai-lin ed., World Scientific, Singapore (1988) p. 310.

[23] P. Grassberger, I. Procaccia, Characterization of strange attractors, Phys. Rev. Lett. 50 (1983) 346.

[24] D. S. Broomhead, G. P. King, Extracting qualitative dynamics from experimental data, Physica 20D (1986) 217.

[25] H. L. Swinney, Chemical Chaos, in Nonequilibrium Dynamics in Chemical Systems, C. Vidal ed., Springer, New York (1984) p. 124.

[26] see for example R. M. May, Simple mathematical models with very complicated dynamics, Nature 261 (1976) 459.

[27] see for example J. M. T. Thompson, H. B. Stewart, Nonlinear Dynamics and Chaos, John Wiley (1986) p. 174.

Table 1. Summary of previous experiments [11]:

Electrode material	OFHC Cu	Brass	Ordinary Cu
n° of conditioning discharges	~100	~5	1? - >350?
$<n>$ $(\times 10^7)$	2.4	1	< 0.1?
Fluctuation pattern	irregular	alternating	irregular

Table 2. Summary of experiments:

Series	Facility	Electrodes	Chamber	Data	N_T
Buenos Aires series:					
Ex77a	PF-I	Brass	Pyrex	n	133
Ex77b	PF-I	Brass	Pyrex	n	100
Ex86	PF-II	OFHC Cu	s.s.	n	331
Ex88a	PF-II	OFHC Cu	s.s.	n	291
Ex88b	PF-II	OFHC Cu	s.s.	n	91
Trieste series:					
Ex91a	ICTP-UM	OFHC Cu	s.s.	X rays	101
Ex91b	ICTP-UM	OFHC Cu	s.s.	X rays	100
Ex91c	ICTP-UM	OFHC Cu	s.s.	X rays	97

Table 3. Fluctuation level (= standard deviation/mean), σ (= internal surface of chamber/electrode surface), ε (= volume of chamber/interelectrode volume) of the experiments (the values of σ and ε are approximate):

Facility and experiments:	Fluctuation level:	σ	ε
PF-I			
Ex77a	1.24	10	20
Ex77b	1.00	10	20
PF-II			
Ex86	0.68	25	200
Ex88a	0.67	25	200
Ex88b	0.59	25	200
ICTP-UM			
Ex91a	0.71	2.5	6
Ex91b	0.54	2.5	6
Ex91c	0.59	2.5	6

Table 4. Summary of the analysis of the experiments:

Series:	Method:				
	Local dimension measurement		Trajectory dimension		Trajectory inspection
	plateau:	ν_m:	n^*:	s_0:	structure recognizable?
Buenos Aires series:					
Ex77a, Brass	-	-	3	6.7	yes, fairly well
Ex77b, Brass	-	-	2-5	2.6	yes, poorly
Ex86, OFHC Cu	-	-	3-5	1.5	no
Ex88a, OFHC Cu	-	-	3	0.9	no
Ex88b, OFHC Cu	-	-	1	6.8	yes, fairly well
Trieste series:					
Ex91a, OFHC Cu	yes	< 1?	3	13.7	yes, poorly
Ex91b, OFHC Cu	?	-	3-4	1.8	yes, fairly well
Ex91c, OFHC Cu	?	-	2	5.8	yes, poorly

FIGURE CAPTIONS

Fig. 1. Yield data for: (a) Ex77a, (b) Ex86, (c) Ex91b. The yield has been normalized so that $\langle y \rangle = 0.5$.

Fig. 2. Local dimension measurements; $\nu = d(\ln C)/d(\ln r)$ vs $\ln r$ for: (a) Ex91a, (b) Ex91b, (c) Ex91c. The label on the curves indicates the value of d.

Fig. 3. Eigenvalues of the covariant matrix for $d = 2, 3, \ldots, 9$: (a) Ex77a, (b) Ex77b, (c) Ex86, (d) Ex88a, (e) Ex88b, (f) Ex91a, (g) Ex91b, (h) Ex91c, (i) Random artificial series ($N_T = 150$).

Fig. 4. 3-D embeddings of trajectories: (a) Ex77a, (b) Ex77b, (c) Ex88a, (d) Ex88b, (e) Ex91a, (f) Ex91b.

Fig. 5. Projection of trajectories on the 3-D subspace spanned by the eigenvectors corresponding to the 3 largest eigenvalues of Ξ, $d = 9$: (a) Ex77a, (b) Ex77b, (c) Ex88a, (d) Ex88b, (e) Ex91a, (f) Ex91b.

Fig. 6. Bifurcation diagram for the mapping (10) ($\gamma = 0.3$, $\beta = 13.33$, $\lambda = 4$).

Fig. 7. Artificial series generated by 151 iterations of (10). The first 5000 iterates were discarded to eliminate transients. (a) Raw data. (b) Eigenvalues of the covariant matrix for $d = 2, 3, \ldots, 9$. (c), (c') 3-D embeddings of the trajectory. (d), (d') Projection of the trajectory on the 3-D subspace spanned by the eigenvectors corresponding to the 3 largest eigenvalues of Ξ, $d = 9$.

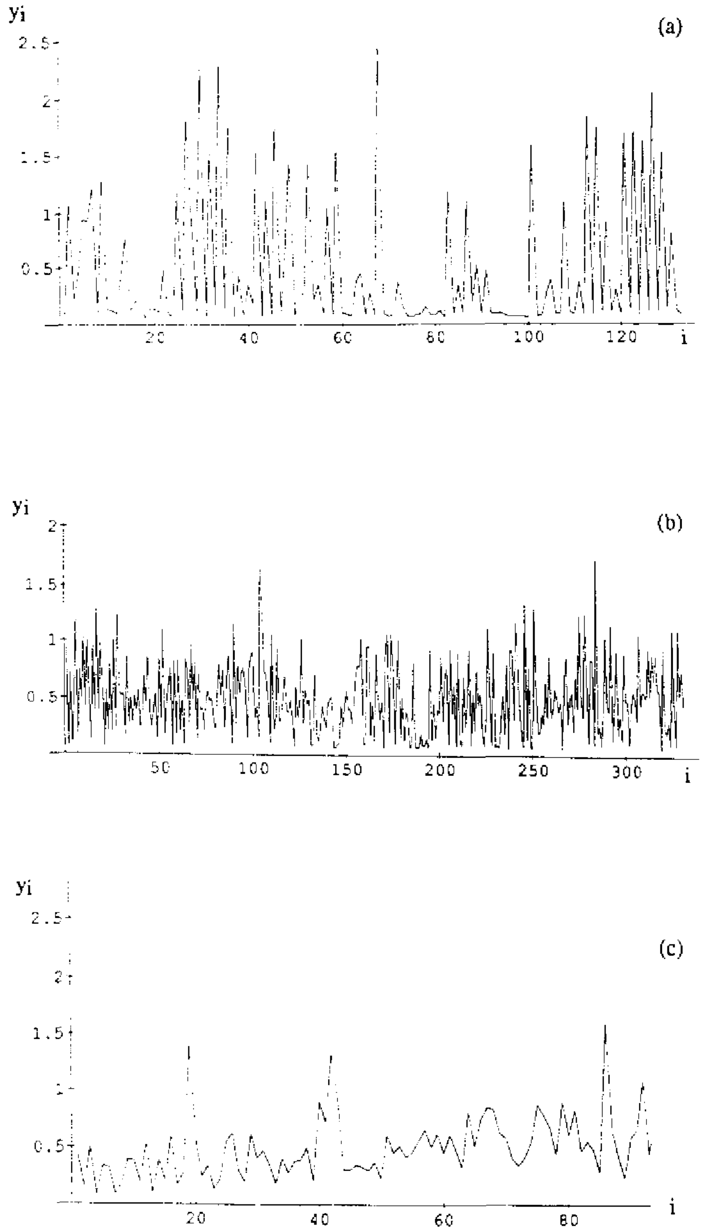

Fig.1

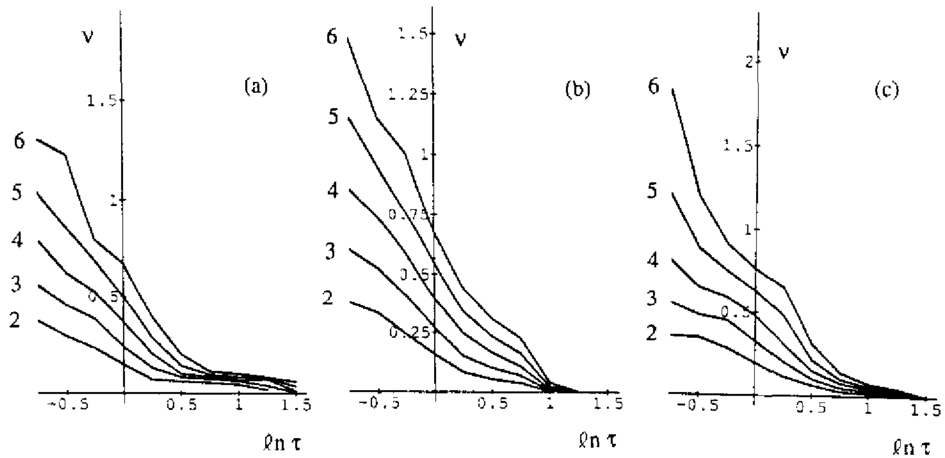

Fig.2

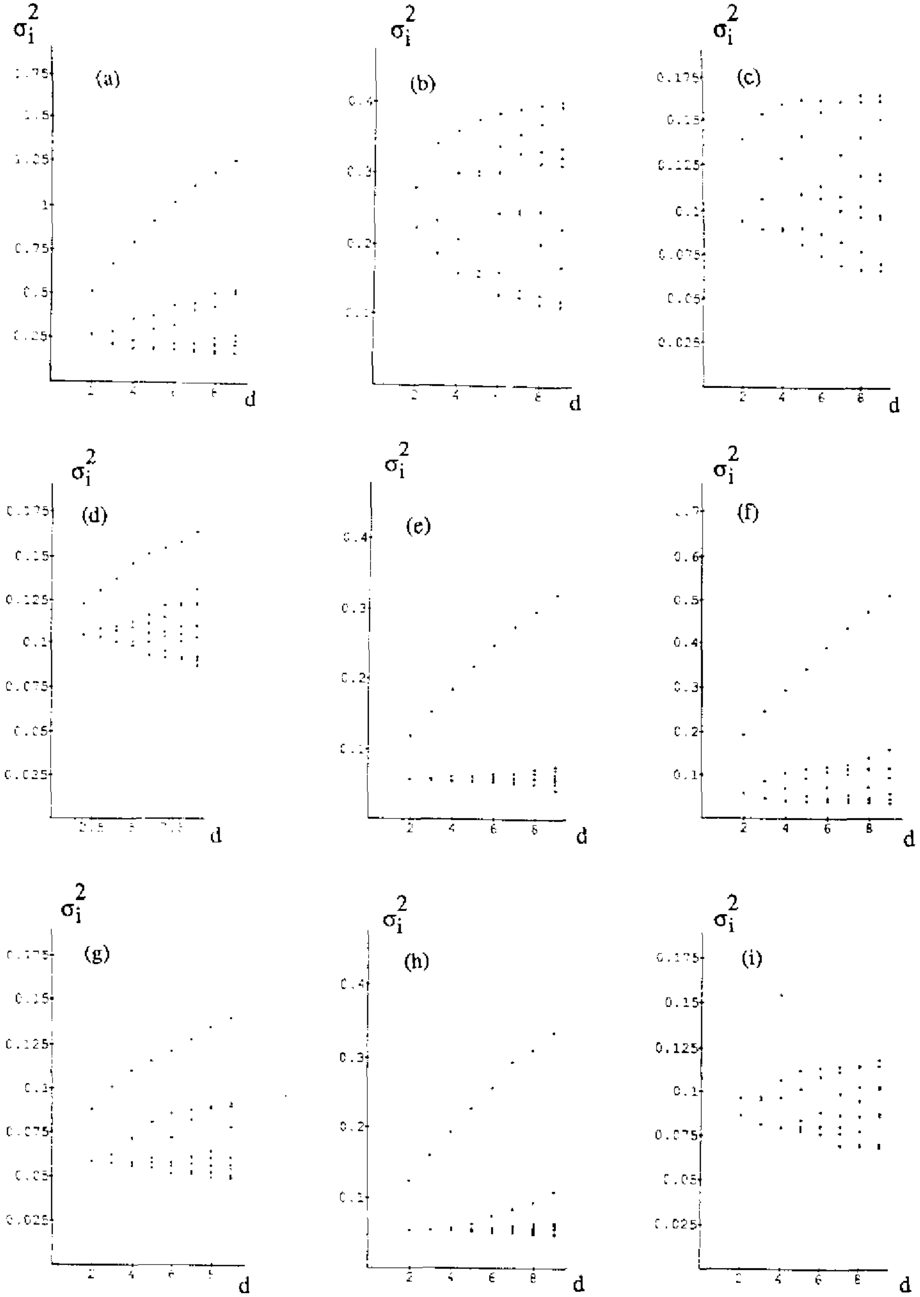

Fig.3

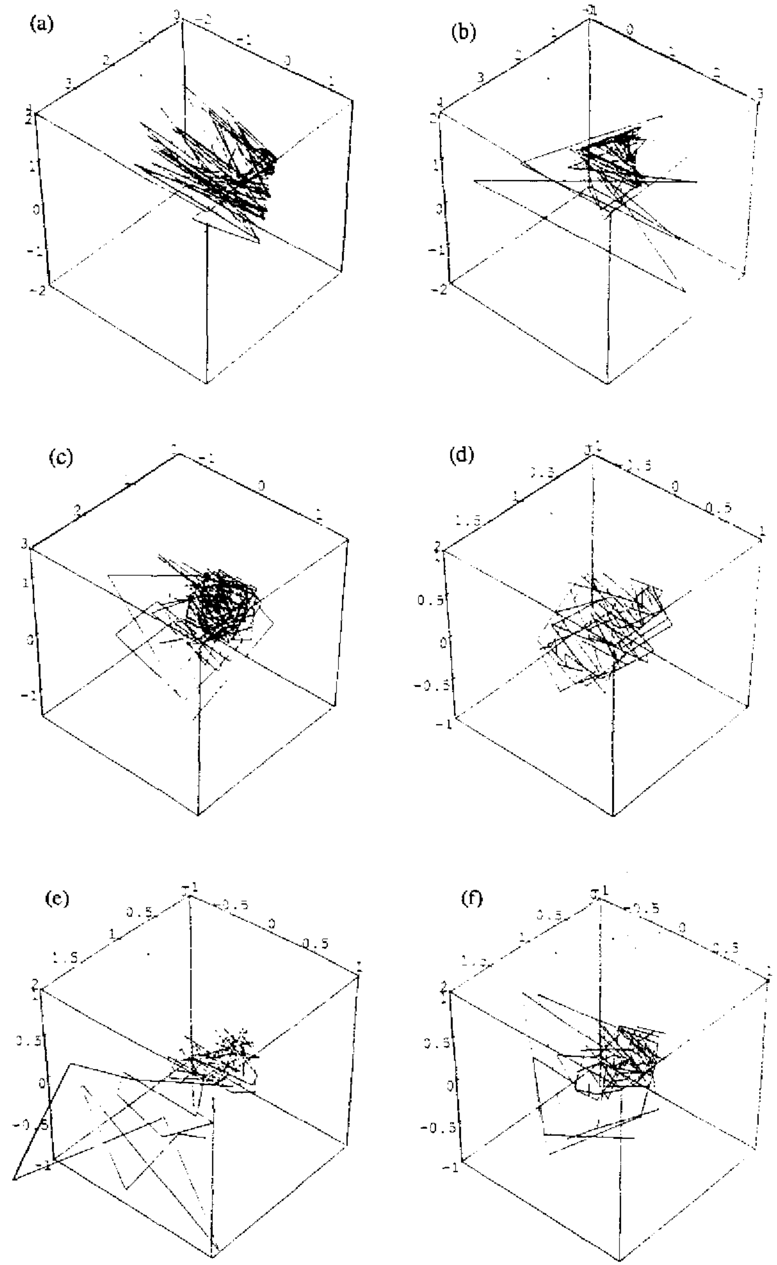

Fig.4

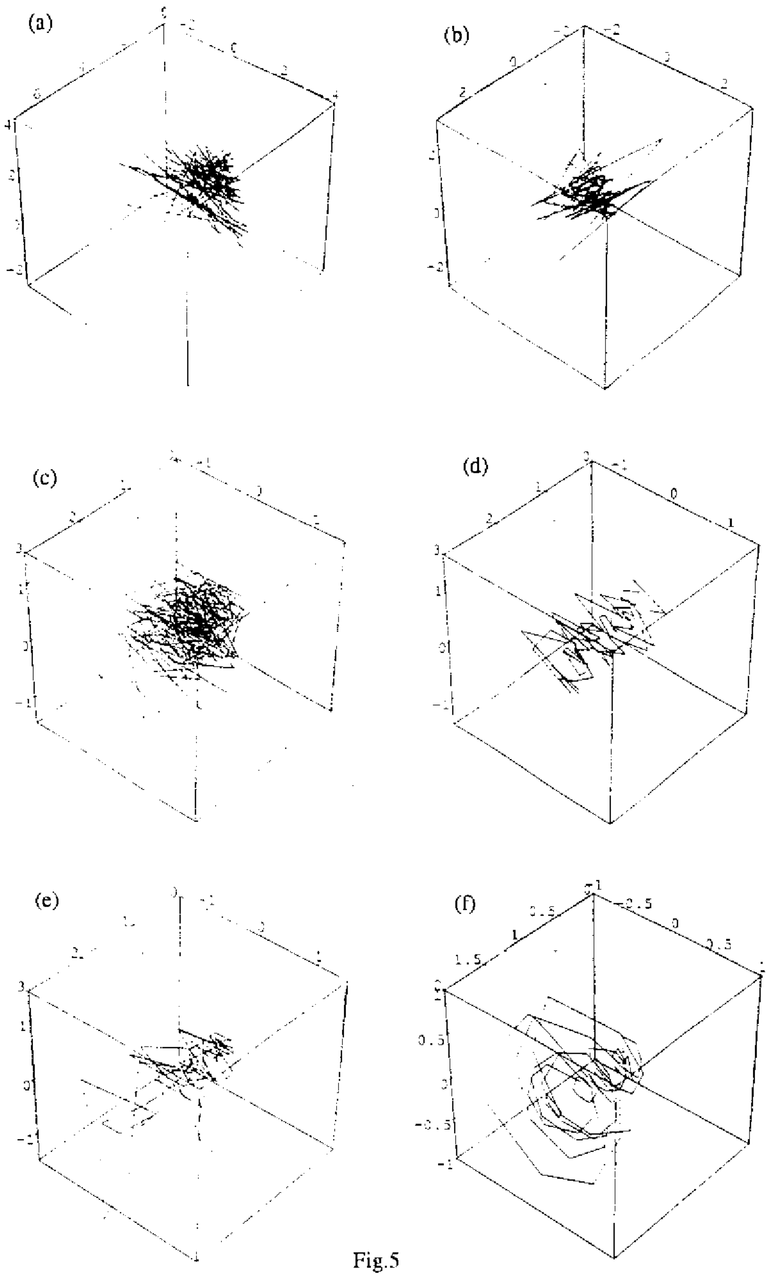

Fig.5

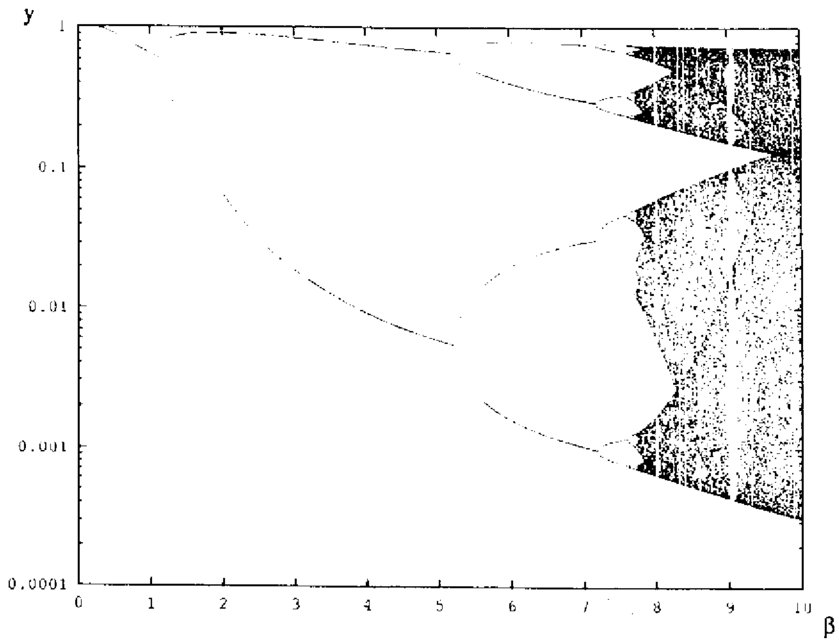

Fig.6

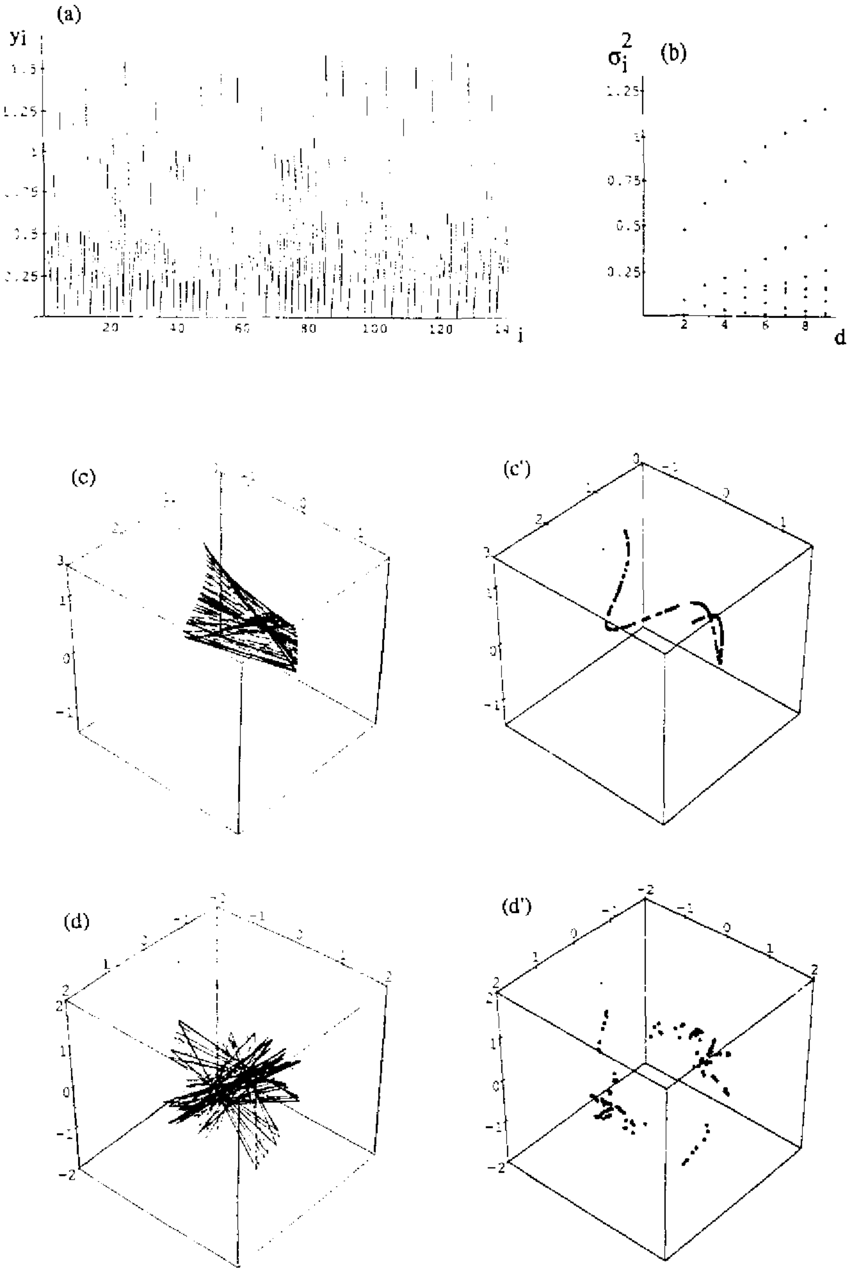

Fig.7