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# SAWTEETH STABILIZATION BY ENERGETIC TRAPPED IONS

SAMAIN A.; EDERY D., GARBET X., ROUBIN J.P.

Association Euratom-CEA, Centre d'Etudes Nucleaires de Cadarache,
13 - Saint-Paul-lez-Durance (FR). Dept. de Recherches sur la Fusion
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# SAWTEETH STABILIZATION BY ENERGETIC TRAPPED IONS

Samain, A., Edery, D., Garbet, X., Roubin, J.-P.

*Association EURATOM-CEA sur la Fusion Contrôlée*
*Centre d'Etudes Nucléaires de Cadarache B.P. n°1*
*13108 Saint-Paul-lez-Durance, France*

## ABSTRACT AND INTRODUCTION

The analysis of a possible stabilization of sawteeth by a population of energetic ions is performed by using the Lagrangian of the electromagnetic perturbation. It is shown that the trapped component of such a population has a small influence (of order $(1-q)^2$ in the limit $\omega/\omega_d \rightarrow 0$) compared to that of the passing component. The stabilization threshold is calculated assuming a non linear regime in the q=1 resonant layer. The energetic population must create a stable tearing structure ($\Delta'=0$) if the average curvature effect on thermal particles in the layer is small. However, this effect decreases the actual threshold.

## I. ELECTROMAGNETIC LAGRANGIAN FOR ENERGETIC PARTICLES

The mode is described by potentials $(\delta U,\delta A) = (U(x),A(x)) \exp(-i\omega t)$ + c.c., to which each particle species s responds by the current and charge density $j(x),\rho(x)$. The Maxwell equations are equivalent to state that the gauge invariant functionnal:

$$\mathcal{L}(U^*,A^*;U,A) = \mathcal{L}_{vacuum} + \sum_s \mathcal{L}_s ;$$

$$\mathcal{L}_{vacuum} = -\frac{1}{\mu_0}\int d^3x \left|\vec{rot}\, A\right|^2 + \varepsilon_0 \int d^3x \left|i\omega A - \nabla U\right|^2$$

$$\mathcal{L}_s = \int d^3x\, ( jA^* - \rho U^*) \qquad (1)$$

is extremum in $U^*.A^*$. The functional $\mathcal{L}_s$ is simply related to the integral $\int d^3x\, d^3p\, f \cdot h$ where $f(x,p)$ and $h(x,p) = e(U - Av_{//})$ are the perturbations of the distribution function and of the hamiltonian. A standard expression for $\mathcal{L}_s$ is derived by expressing the Vlasov equation relating f to h in angular action space [1]. This itinerary leads to more flexible and more precise forms of $\mathcal{L}_s$ than the extension of the MHD energy principle [2] using the gauge ($U=0$, $A=\xi \times B_0$), leading to $\mathcal{L}_s = -\int d^3x\, (j \times B_0).\xi^*$ where $j \times B_0$ is related to the pressure tensor perturbation.

We apply the variational principle (1) to the study of the mode triggering the sawteeth, exhibiting $N_{toroidal}=1$ and a MHD structure outside a resonant layer on the surface $r=r_1$, where $q=1$. The stabilization of this mode by a species s=H of hot ions requires that the contribution $\mathcal{L}_H$ balances the destabilizing terms in $\mathcal{L}_{thermal}$, namely the tearing term ($\propto \partial j/\partial r$) and the interchange term ($\propto \partial p/\partial r$) in the MHD region. We dispose of a general expression for $\mathcal{L}_H$ applicable for A parallel to $B_0$:

$$\mathcal{L}_H = \int d^3x\, d^3p\, \left\{ -\frac{\partial F}{\partial H} e^2 |U|^2 + \sum_p \frac{\omega \frac{\partial F}{\partial H} + N \frac{\partial F}{\partial \mathcal{M}}}{\omega - p\omega_\theta - N\omega_\varphi} |h_p|^2 \right\} \qquad (2)$$

where $F(\mu,H,\mathcal{M})$ is the unperturbed distribution function ($\mu=mv_\perp^2/2B$, $H=mv^2/2$, $\mathcal{M}=Rp_\varphi$), $\omega_\theta(\mu,H,\mathcal{M})$ and $\omega_\varphi(\mu,H,\mathcal{M})$ are the time average poloidal and toroidal frequencies (for trapped particles $\omega_\theta$ is the bounce frequency and $\omega_\varphi=\omega_d$ is the banana precession frequency). The subscript p indicates the Fourier component $\exp i(p\theta^* + N\varphi^*)$ along the angular variables for the poloidal and toroidal motion (such that $d(\theta^*,\varphi^*)/dt = \omega_{\theta,\varphi}$ ; for trapped particles $\theta^*$ is the phase of the bounce motion and $\varphi^* = \varphi - q\theta$ ). A more specific expression for $\mathcal{L}_H$ is applicable under the MHD constraint: $i\omega A - \nabla_{//}U=0$

$$\mathcal{L}_H = \mathcal{L}_{int} + \mathcal{L}_{res}$$

$$\mathcal{L}_{int} = -\mathrm{Re}\int d^3x\, d^3p\, \frac{e^2}{\omega^2}\frac{\partial F}{\partial \mathcal{M}}\left(\frac{\nabla U \times B_0}{B_0^2}\cdot\nabla\Psi\right)\left(\nabla U \cdot v_{G\perp}\right)^*$$

$$\mathcal{L}_{res} = \sum_p \int d^3x\, d^3p\, \frac{e^2}{\omega^2}\, \frac{\omega\frac{\partial F}{\partial H} + N\frac{\partial F}{\partial \mathcal{M}}}{\omega - p\omega_\theta - N\omega_\varphi} \left|(v_{G\perp}\cdot\nabla U)_p\right|^2$$

where $v_{G\perp}$ is the curvature drift velocity and $\Psi$ the poloidal flux. The functional $\mathcal{L}_{int}$ accounts for the traditional interchange MHD effects, while the functional $\mathcal{L}_{res}$ reflects the toroidal magnetic pumping effect by the mode [3]. For $\omega$ in the thermal diamagnetic range of frequency, the leading term for trapped particles in $\mathcal{L}_{res}$ is the term p=0, for which the resonant denominator $\omega - p\omega_\theta - N\omega_\varphi$ reduces to the small value $\omega - N\omega_d$. For $\omega - N\omega_d \approx \omega_d$, a stabilizing role may be played by this term if it is dominant compared to $\mathcal{L}_{int}$ [4,5,6]. This is the case if the energetic ion population contains the same number of passing and trapped particles, because the compensation of the unfavorable and favorable curvature then lowers the value of $\mathcal{L}_{int}$. Then, it is very important to notice that we have $\mathcal{L}_H \approx \mathcal{L}_{res}$ proportional to $|U|^2/\omega^2$ quasi independently of the value of (1-q) in the domain r<r(q=1). In view of the MHD constraint $i\omega A = \nabla_{//}U = ((1-q)/R)\, U$, the functional $\mathcal{L}_H$ is proportional to $|A|^2/(1-q)^2$ and dominates the tearing and thermal interchange effects (proportional to $|A|^2$ and $|A|^2/(1-q)$ respectively) at low (1-q). However, in the experimentally important limit $\omega \ll \omega_d$, if the energetic population

consists of trapped particles only, the value of $\mathcal{L}_H$ may be derived from (2) in the limit ($\nabla_{//}U = i\omega A$, $\omega \rightarrow 0$ ) and therefore

$$\mathcal{L}_H = \int d^3x\, d^3p \sum_p \frac{N\, \partial F/\partial \mathcal{M}}{-p\omega_b - N\omega_d} \left| e^2 (A v_{//})_p \right|^2$$

clearly independent of (1-q) for given A. It may be shown that in this case $\mathcal{L}_H$ is proportional to $|A|^2$ instead of $|A|^2/(1-q)^2$. For $\omega << \omega_d$, the influence of the topology (trapped or passing) of the energetic particles on the stabilization efficiency provides an important experimental check of the model.

## II. STABILIZATION THRESHOLDS

By combining $\mathcal{L}_H$ with $\mathcal{L}_{vacuum}$ and $\mathcal{L}_{thermal}$ in the MHD domain, and forming the Euler variational equations, one obtains the mode structure and the contribution $\mathcal{L}_{MHD}$. We essentially assume in what follows a tearing structure, i.e., a non cancelling component $A(r)\exp i(\varphi - \theta)$ of the potential A(x). We may then write:

$$\mathcal{L}_{MHD} = \frac{\mathcal{S}}{\mu_0} \Delta' \, A(r_1) A^*(r_1) \; ; \; \mathcal{S} = 2\pi R \; 2\pi r_1 \qquad (3)$$

where $\Delta'$ is the usual jump across the surface $r=r_1$. Neglecting toroidal and thermal pressure effects, we have $\Delta' = +\infty$ in the absence of hot particles, meaning that the mode is strongly unstable as a tearing mode, while it is marginally unstable under the MHD constraint in the layer $A(r_1)=0$. The energetic ions introduce a finite value of $1/\Delta'$, which, in the typical case of an isotropic pressure profile $p_H=p_{H0}$ for $r<r_H<r_1$ and $p_H=0$ for $r>r_H$, is given by [6]:

$$\frac{1}{\Delta'_H} = \frac{r_1}{s_1^2} \frac{\bar{\beta}_H \beta_{crit}}{\beta_{crit} - \bar{\beta}_H}$$

where $\bar{\beta}_H = p_{H0}\,(2\mu_0/B^2)\pi^{1/2} r_H^{3/2} / 2r_1^2$, $s_1=(d\mathrm{Log}\, q/d\mathrm{Log}\, r)$ and $\beta_{crit} \approx (1-q_0)^2/G(r_H)$ with $G(r) = (1 - r^2/r_1^2)^{-1} - \mathrm{Ln}\, |1-r^2/r_1^2| - 1/2$. The toroidal and thermal pressure effects in the MHD bulk also play a role in the value of $\Delta'$. For small $1/\Delta'$ ($1/\Delta' << r_1$), one may write:

$$\frac{1}{\Delta'} = \frac{1}{\Delta'_H} + \frac{1}{\Delta'_{thermal}} \qquad (4)$$

where $\Delta'_{thermal}$ is related to the MHD energy $\delta\hat{W}\,\xi^2$ given in [7]:

$$\frac{1}{\Delta'_{thermal}} \approx \frac{\delta\hat{W}\, \mu_0 R^2}{\mathcal{S} r_1^2 s_1^2 B^2} \approx -\frac{3}{2} \frac{r_1}{s_1^2} (1-q_0) \frac{r_1^2}{R^2} (\beta_{pol}^2 - \beta_{cr}^2)$$

where $\beta_{pol}$ refers to the pressure difference within $r=r_1$ and $\beta_{cr} \approx 0.4$.

The contribution $\mathcal{L}_{layer}$ of the resonant layer to the functional (1), calculated in a non linear island regime, involves the Rutherford currents proportional to the mode growth rate $\gamma$ and the stabilizing interchange effects due to the average curvature within the layer proportional to $\partial\beta/\partial r$ [8].

$$\mathcal{L}_{layer} = \frac{S}{\mu_0}\left( \frac{-0.8\ \delta_{isl}}{\eta}\gamma + \frac{3}{s_1}\frac{r}{\delta_{isl}}\frac{\partial\beta_{tor}}{\partial r}(\beta_{pol} + \frac{l_i}{2})\right) A(r_1)A^*(r_1) \qquad (5)$$

where $\delta_{isl}$ is the half island width. Expressing that $\mathcal{L}_{MHD} + \mathcal{L}_{layer}$ derived from (3) (4) (5) is an extremum in $A^*(r_1)$ provides the value of $\gamma$. If one neglects the average curvature term ($\propto \partial\beta_{tor}/\partial r$) in (5), one obtains $\gamma = 0$ for $\Delta' = \Delta'_H = 0$, i.e. $\bar{\beta}_H = \beta_{crit}$ [6]. On the other hand, retaining that term in the limit $\delta_{isl} \rightarrow 0$, one obtains $\gamma = 0$ for $\Delta'=\infty$. For $1/\Delta'_H \ll r_1$, i.e. $\bar{\beta}_H \ll \beta_{crit}$, this means that stabilization is achieved for $1/\Delta'_H \approx r_1\, \bar{\beta}_H / s_1^2 = -1/\Delta'_{thermal}$ which is essentially the condition given in [4],[5]. In fact for small islands, further important effects linked to the transverse diffusion of particles contribute to the value of $\mathcal{L}_{layer}$. These effects first produces an imaginary term proportional to $(\omega-\omega^*)$, whose cancellation imposes that the mode frequency is of order of the thermal diamagnetic frequencies $\omega^*$. On the other hand, if $\delta_{isl} < \delta_{cr} = (D/\omega)^{1/2}$, where D is the transverse particle diffusion coefficient, a large destabilizing term

$$\frac{S}{\mu_0}\left( 1.5\, \frac{1}{s_1^{\,2}}\frac{1}{\delta_{isl}}\beta_{tor}R^2 \left(\frac{\partial T}{T\partial r}\right)^2\right) A(r_1)A^*(r_1)$$ superimposes in $\mathcal{L}_{layer}$ [9].

Therefore, the average curvature effect in the layer can only stabilize the mode for $\delta_{isl} > \delta_{cr}$. In these conditions, the stabilization threshold is given by the expression:

$$\frac{1}{\Delta'_H} + \frac{1}{\Delta'_{thermal}} = \frac{1}{\Delta'_{layer}} \quad \text{where } \Delta'_{layer} = -\frac{3}{s_1}\frac{r_1}{\delta_{cr}}\frac{\partial\beta_{tor}}{\partial r}(\beta_{pol} + \frac{l_i}{2})$$

Even for small ratios $\delta_{cr} / r_1$, the stabilization threshold may be determined by $\Delta'_{layer}$ rather than by $\Delta'_{thermal}$. The observed stabilization thresholds could in fact give an useful information on the stabilizing or destabilizing effects taking place (with a divergent stucture $\propto 1/\delta_{isl}$) near the resonant layer of the tearing modes.

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