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FROM QUANTUM FIELDS TO FRACTAL STRUCTURES: INTERMITTENCY IN PARTICLE PHYSICS

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by

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Abstract

Some features and theoretical interpretations of the intermittency phenomenon observed in high-energy multi-particle production are recalled. One develops on the various connections found with fractal structuration of fluctuations in turbulence, spin-glass physics and aggregation phenomena described by the non-linear Smoluchowski equation. This may lead to a new approach to quantum field properties.

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1. Introduction: The intermittency phenomenon in particle physics

Intermittency has been first invoked in Particle Physics in the study[1] of *dynamical* fluctuations observed in the particle density distribution in small intervals (bins) in relativistic phase-space variables. These fluctuations have been called *dynamical* by contrast with the purely *statistical* ones due to the limited (often small) number of particles registered in small bins. Indeed, current high-energy collisions produce dozens to hundreds of particles per event, which is large considered by particle physics standards but is rather few for applying statistical concepts, such as intermittency, without care. Hence, the first step of ref. [1] was to propose a method for distinguishing *dynamical* fluctuations from the *statistical* "noise". Assuming a simple Poissonian noise, or Bernouilli one if the total multiplicity of events is constrained, one can write a formula for the normalised factorial moments of the multiplicity distribution, namely:

$$\langle F_q\rangle \equiv \frac{\langle k_m\,(k_m-1)\ldots(k_m-q+1)\rangle}{\langle k_m\rangle^q} = \frac{\langle \rho_m^q\rangle}{\langle \rho_m\rangle^q}, \tag{1}$$

where k_m is the observed number of particles in the bin [m] and the "density" ρ_m , with its "ordinary" moments, would correspond to the absence of statistical bias. This is nothing other than a simple application of the difference between frequency and probability weight in Statistics. The assumption about the noise has been surprisingly wel adapted to the study of various reactions[2], such as $e^+ - e^-$annihilations into hadrons and those involving incident hadrons and/or nuclei. In all cases, realistic models except for eventual dynamical fluctuations were found consistent with the noise assumption.

The main yet unexpected outcome of these studies is the power-law behaviour of moments with the binsize δ in the suitable variables, namely the *rapidity* in the standard case displayed on the Figure 1. *Rapidity*[3] is the variable analoguous to the velocity in a relativistic (Lorentzian) frame of reference oriented along the preferred production axis. It is related to the angular distribution of particles with respect to the axis in the total center-of-mass frame. One finds:

$$\langle F_q\rangle \sim \delta^{-f_q} \tag{2}$$

where f_q is constant in some binsize range. In fact the inverse binsize δ^{-1} is a measure of the chosen experimental resolution. Note that other variables, such as the azimuthal angle with respect to the axis, has revealed the same behaviour, and it is even more pronounced, in a two or three-dimensional analysis[4] combining these variables. As we shall develop further on, formula (2) was suggestive of a fractal behaviour of fluctuations similar to fluid turbulence and related to intermittency, i.e. the property of dynamical fluctuations with a hierarchy of scales, showing a fractal dependence on the experimental resolution.

In the next section **2**, we show how the quantum field theory of particle interactions is put into question by the intermittency phenomenon, impredictably. In section **3**, one introduces the random cascading models by analogy with fluid turbulence, but with some clear difference due to relativistic kinematics. Section **4** discusses the connections with phase transitions and also spin-glass systems using the fruitful example of Statistical Mechanics. Section **5** is devoted to the unexpected relations with fractal growth, aggregation processes, and the non-linear Smoluchowski equation. Conclusions and outlook are given in section **6**.

It must be clear to the reader that the following sections mainly reflect our personal views, much influenced of course by the numerous friends and colleagues with whom the author shares the passion for the present subject, and whose names can be found in part in the references list. It is a pleasure to have been invited to talk at the workshop since it allows one to be less rigorous but perhaps more intuitive than for a registered paper. I also took this opportunity to add new pieces of material concerning aggregation phenomena which seem to fit well the subject of the workshop. Many thanks for the organizers of this nice interdisciplinary meeting.

2. Intermittency and the quantum field theory of particles.

The experimentally observed behaviour compatible with (2) has been largely commented in the theoretical literature[5]. So-called "conventional" mechanisms have been proposed to take into account the behaviour (2) while using already known models. Depending on the reaction, short-range and/or long-range rapidity correlations, a hierarchy of resonance decays, and the Bose-Einstein enhancement effect for identical pions, have been invoked[5]. However the higher-dimensional analysis[4] is difficult to explain in this context and the "universal" presence of the phenomenon is difficult to understand, since, "conventionnally", different mechanisms are to be introduced for different reactions. However a "conventional" model is not completely excluded but, as we shall see now, this is another way to formulate our ignorance of quantum fields at strong coupling.

Particle interactions are expected to be well-described by quantum field theories. Indeed the so-called "standard model" of fundamental interactions has met considerable success in the unified description of electro-magnetic interactions. In the domain of strong interactions, the success is also remarkable with the important restriction that the theory remains incomplete: at short time-distances the interactions between quarks and gluons the fundamental building blocks of matter or *partons* is well understood; However at longer distances the transformation of quarks and gluons into the observed hadrons (pions, nucleons, resonances etc..) is rather problematic. Technically, the difficulty is in treating the strong coupling regime of Quantum-Chromo-Dynamics (QCD), the interaction theory of quarks and gluons. Most probably, the difficulty is basic and related to the problem of *confinement*, that is the formulation of a theory where the objects existing at short distances are not the asymptotic particles, the hadrons, which posess a complex composite structure in terms of partons.

The relation of the intermittency phenomenon with QCD has naturally been a constant subject of interest. Even if the only strict theoretical understanding of strong interactions is given by the week coupling limit of QCD, that is the short-distance behaviour, it is interesting to adopt first this framework. Two ways have been investigated: One is to look for an eventual intermittent behaviour of the interaction of quarks and gluons; Another one is to add to the parton-interaction stage a phenomenological description of a second "hadronisation" stage and compare the results of the simulation to the experimental data on factorial models. Both ways have led to interesting results showing better the limitation between the known and the unknown in QCD.

Interestingly enough, it has been known since a long time[6] that QCD admits a hierarchical solution at short-distances, under the form of a parton cascading structure[6], at least for quark and glu jets produced e.g. in e^+e^-reactions. However, while the fractal character of this process was noticed[7] early, its intermittency properties have only recently been numerically proven[8]. On a more phenomenological ground, a reasonably good description of the observed factorial moments has been obtained for Monte-Carlo simulations based on the Lund Model for LEP data[9] on quark jets, where the large production of the famous intermediate boson $Z^0$and its decay into quark jets allows a detailed analysis. It is to be noticed that the underlying mechanism of the appropriate Lund Model is based on the QCD cascading structure. For all other reactions, for which the first quark-gluon cascading stage is not proven to exist, no satisfactory simulation including factorial moments has been found so far.

These facts, together with the unresolved problem of the confinment of quark and gluon jets, points towards the long-lasting problem of quantum field theory at strong coupling. Indeed it is known from renormalization group properties of QCD that this theory is asymptotically free, that is its effective coupling is weak at small distances and becomes strong at long distances, precisely the region where hadrons are formed. Hence, the observed intermittency pattern gives a new angle of attack for the strong coupling problem.

3. Turbulence and random cascading models of particle production

Fluctuation structures leading to the power-law (2), section **2.** is not unknown in Physics. In fully-developed turbulence, as observed in fluids, moments of the eddy velocity distribution are compatible with such a behaviour up to high values of the rank q. One finds general models[10] of cascading which fulfill relation (2) with a phase-space bin δ corresponding to small volumes of the fluid. This is very different from the intermittency for particles, which is present in momentum space and not in coordinate space. Moreover, the problem of statistical noise, as mentionned in introduction, is very different. However the structure of the cascading models of refs. [10] can be adapted to the case of particles and transposed in the appropriate relativistic kinematics.

Let us introduce the specific α-models, introduced by D. Schertzer and S. Lovejoy,*(see the third ref. [10])*, in atmospheric turbulence and considered in ref. [1] in the context of particle physics. Following the scheme of Fig. 2, one considers a series of cascading steps n, ranging from 1 to ν, each of them corresponding to a new λ-partition of phase-space (λ is 2, for simplicity, on the Fig.). In this way, one establishes a correspondence between the value of ν and the desired binning resolution $\frac{\Delta}{\delta}$, where δ is as previously the smaller bin unit and Δ, the larger one. One has the identities:

$$\frac{\Delta}{\delta} \equiv \lambda^{\nu} \equiv M \tag{3}$$

where M is the total number of bins. Let assume that at each step n, density fluctuations may occur and are represented by random factors W for each link of the tree structure (see Fig. 2): One gets after ν steps, for the bin [m] :

$$\frac{\langle \rho_m^q \rangle}{\langle \rho_m \rangle^q} \equiv \left\langle \prod_{n=1}^{\nu} W^q \right\rangle = \{W^q\}^{\nu} = \left(\frac{\Delta}{\delta}\right)^{\frac{ln\{W^q\}}{ln\lambda}} \tag{4}$$

where one has used the mutual independance of the random factors W, and of their normalization conditions $\{W\} = \{1\} = 1$, the brackets$\{\}$ meaning the averaging over the distribution of W's. Using expression (1) for the factorial moments one gets the required relation (2) with:

$$f_q = \frac{ln\{W^q\}}{ln\lambda} \tag{5}$$

where the exponent f_q has been called the "intermittency index of rank q" and is related in this model to the *local* probability distribution of the density fluctuations in rapidity.

In fact, random cascading models can be shown[1] to be consistent with the relativistic kinematic constraints on particle collisions. After the collision between initial particles, the produced "medium", whatever it is, is subjected to the Lorentz expansion of distances and times of interaction. When this expansion reaches the canonical correlation length of 1 fermi ($10^{-13}cm.$), the system breaks into pieces and dynamical fluctuations can be generated and persist, since they cannot be destroyed by re-interaction. However, contrary to the conventional picture in which the finally observed hadrons are all created at this length scale (see Fig. 3a), the intermittency phenomenon implies a self-similar process: the system develops further on in time and undergoes, after some expansion time, a new breaking into pieces, with new fluctuations superimposed onto the old ones, and again expands etc..., see Fig. 3b. In fact, the 1-fermi scale remains the basic length, not as the absolute scale of hadron production as in the conventional picture, but as the average scale of dynamical fluctuations and the intrinsic repetition scale of the hierarchical fluctuation pattern. On average, the intermediate system lasts 1 fermi, but from event-to-event or inside the same event, its time-life may vary considerably, generating self-similarity.

Then, as a particle collision, considering its space-time development, the process is compatible with the random geometrical structure of cascading, as schematized in *Fig.2*. However, the intermittency structure is observed in the short range of momentum space and thus cannot find any justification from fluid turbulence theory, where the intermittent structuration appears in coordinate space; One has to find the appropriate theoretical approach. Statistical Mechanics, as often in field theory, is of great help, as discussed in the next section.

4. Phase transitions, Spin-Glasses and random cascading

Whilst trying to understand the intermittency mechanism in terms of quantum field theory at strong coupling, it is natural to address the same question to spin systems, in particular, when they posess a phase transition. Indeed, it is known that the behaviour of spin systems at a second-order phase transition point, that is when the correlation length diverges, is related by a scale transformation to a quantum field theory[11]. As an application of the factorial moment method, intermittency patterns have been searched for in numerical simulations of the 2-dimensional Ising system near its (pseudo-)phase transition coupling. The idea[12] is to consider the subdivisions of the Ising lattice as the bins of Fig.2. In each "box" in the lattice corresponds a bin $[m]$, and the number K_m, cf. definition (1), is taken to be the number of spins with same orientation (a magnetic cluster).

One important point of interest of these statistical systems is that the assumptions about the "noise" and analytic predictions for the behaviour of moments can be confronted with accurate computer simulations[12]. Intermittency structuration has indeed been clearly seen and the scaling properties of a 2^{nd} order phase transition lead to a specific prediction for the intermittency indices (5), namely

$$f_q = (q-1)D \tag{6}$$

where D is independent of q and related[13] to the critical indices of the relevant spin Hamiltonian. However, ambiguities seem to persist concerning the last point. While one expected[13] t find $D = \frac{1}{8}$ from the Ising Hamiltonian, the remark was made[14] that the result seems to be dependent of the definition of clusters. For connected (percolation) clusters, one would find a Potts Hamiltonian and the value[14] $D = \frac{5}{96}$ which happens to be confirmed by recent simulations[15]. In fact, the problem remain open, not forgetting its extension to other second-order transitions or, more importantly for particle physics, to a first-order phase transition such as the one predicted for QCD at high temperature.

In fact, formula (6) can also be expressed as[16] the existence of a fixed fractal dimension $\tilde{D}$ of the strucure of fluctuations. This property is specific to a phase transition at equilibrium. However, collision processes, except eventually heavy-ion collisions, are probably far from equilibrium. Thus it is thus important to consider other theoretical schemes, which may lead to fractal dimensions $\tilde{D}$ depending of q, and are known as *multi-fractal* systems. This is generally the case of random cascading as presented in the previous section ($\tilde{D} = \frac{f_q}{q-1}$, see formula 4.) It was found that[17] the relevant statistical systems possess a spin-glass structure, that is spin systems with quenched random interactions instead of deterministic ones as in the case of the first lattice problems considered.

One may introduce the concept of a Partition Function Z by summation of density powers over all bins of same width δ. For random cascading models of the type of Fig. 2, one finds:

$$Z(\beta) \equiv \sum_{m=1}^{M} \rho_m^q = \sum_{m=1}^{M} exp\left(-\beta \sum_{n=1}^{\nu} \varepsilon_n\right) \tag{7}$$

where the si bstitution $exp(-\beta\varepsilon_n) \equiv \left(\frac{W}{\lambda}\right)^q$ using the same definition of ρ_m as in formula (4), allows one to identify ϵ_n as a random energy level and $q \sim \beta$ as the inverse temperature of the spin-glass system[17]. In such a way the identification is proven with the Generalized Energy Spin Models[18]. Among the interesting consequences of this identification, one may quote a non-trivial pattern[19] of phase transitions leading to a hierarchical structure very different from the usual order-disorder transition. There exists a breaking of ergodicity at low temperature (high β), and a specific classification of the multi-fractal spectrum[19].

The interpretation of this phase transition in the context of Particle Physics is under study. However, no Lagrangian or Hamiltonian formulation exists for these systems and the explicit formulation in terms of a field theory appears to be difficult. In fact, one recently found that the link with quantum field theory could be made easier using the emergence of an underlying non-linear equation which is discussed in the next section.

5. Fractal growth, aggregates and random cascading equations

Among the unexpected connections between random cascading models of particle production and Statistical Physics, last but not least is the relation with the Physics of aggregation and gelling via the well-known Smoluchowski's formulation[20], leading to non-linear rate equations. It is well known that these equations were originally proposed for the description of the coagulation of colloids submitted to Brownian motion more than 74 years ago. However, quite recently, these equations met a revived interest in the numerous studies on the fractal growth, in particular for cluster-cluster aggregation[21].

In all these studies, the number N of clusters of a given number k of mass units is followed as a function of time. One writes a very general mean-field equation (the effect of spatial fluctuations of N_k being neglected) for the aggregation rate, namely:

$$\frac{dN_k}{dt} = \sum_{i+j=k} N_i K^{ij} N_j - \sum_i N_i K^{ik} N_k \tag{8}$$

where the *fusion weights* K^{ij} are the dynamical input. Eqn. (8) establishes the balance at each time and for each mass k between clusters aggregating to form the mass k and clusters of mass k transferred to higher mass, by aggregation.

The way the connection is made[22] with random cascading is through the derivation of a non-linear equation for the Partition Function of random cascading, see (7) and an appropriate transformation of the Smoluchowski equation. In fact one can show that the following two generating functionals verify the same equation: $\mathcal{G}$ for aggregation and $\mathcal{H}$ for the partition function Z, defined as follows:

$$\begin{aligned} \mathcal{G} &= \langle 1 - u^l \rangle_{N_l} \equiv \sum_l N_l (1 - u^l) \cdot \frac{1}{\sum_l N_l} \\ \mathcal{H} &= \langle e^{uZ} \rangle_{\mathcal{P}(Z)} \equiv \int_0^\infty \mathcal{P}(Z) e^{uZ} dZ \end{aligned} \tag{9}$$

where $\mathcal{P}(Z)$ is the Z probability distribution, computable for random cascading models

One finds the same non-linear equation, namely:

$$\frac{d\mathcal{H}}{d\nu} = \mathcal{H} * \mathcal{H} - \mathcal{H} \tag{10}$$

where $\nu = ln\,|\sum_l N_l|$ can be identified with the generation number of random cascading, see section **3.** and the convolution * is defined in terms of the fusion coefficients K^{ij}[22]. In the case of "multiplicative" aggregation, with $K^{ij} = K^i K^j$, one finds a random-branching random cascading model, while the "monodisperse" aggregation case[21] with discrete time intervals gives back the random cascading models already discussed.

The identity of the equations does not mean identity of the solutions, due to possible different initial conditions. In the first investigations of this problem, it seems that the so-called "scaling" solutions of the Smoluchowski equation, which can be derived in a quite general way[23], lead to interesting solutions for the Partition Function of random cascading with asymptotic freedom properties and intermittency. It appears that the world of these non-linear equations has not revealed all its secrets!

6. Conclusions and future prospects

Intermittency in Particle Physics has revealed a quite fructuous field of research. In fact, the study of fluctuations has always led to an interesting insight on physical phenomena. The study of these fluctuations in particle physics is at a very early stage and one does not yet possess a complete picture of its properties. Much more experimental work and critical comparisons with existing models are needed; However it is already possible to guess that a better understanding of these patterns of fluctuations, combined with the already known facts about the different distributions of particles in high-energy experiments may lead to a deeper vew on subnuclear physics. Fractals and Chaos have probably something to teach us about quantum field theory and elementary particles.

On the other hand, there is a hope that the field theoretical techniques developed in particle physics and briefly described in the present talk could be useful in other domains where the problem and structures of intermittency appear. For instance the combination of the "local" study of fluctuations in phase-space with the "global" thermodynamical apparatus related to the partition functions of spin systems could be useful in many problems. As an unexpected example, the intermittency analysis in terms of factorial moments in relation with percolation models has been shown[24] to be relevant in the study of nuclear multi-fragmentation. Perhaps the long-standing respectable unsolved problem of fluid turbulence itself, could benefit from some ideas, as a return gift from particle physics which seems to have taken interest in methods inspired by turbulence!

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FIGURE CAPTIONS

Figure 1 : **Intermittency: first example**

a) *Particle number distribution*

observed by the JACEE Collaboration for a 5 Tev/nucleon. Si + Ag(Br) cosmic-ray collision on an emulsion plate carried by a balloon[25]. Cosmic-rays provide the only opportunity at present to reach very large energies. and thus high multiplicity-per-event. The next generation of accelerators(LHC at CERN, SSC in Texas), will allow to reach such energies but in a reproducible and controlable way.

b) *Factorial moment of rank 5.*

The factorial moment obtained from Fig. 1a (black dots) is compared with a simulation with Gaussian statistical noise (crosses). The straightline is a typical prediction for intermittent pattern of fluctuations. The Figure is from ref [1].

c)*Intermittency at accelerator.* The factorial moments of rank 2 and 4 are displayed (black squares) and compared with a simulation with only statistical fluctuations (white squares) for an $^{16}O - emulsion$ experiment at CERN.This reaction is very similar to the previous one. but at smaller energy, with much less particle produced (around 120, compared to more than 1000). The straight lines correspond to the intermittency prediction. These results[26] were the first ones published showing that the methods of factorial moments was applicable at present accelerator energies (with moderate multiplicity-per-event). as proposed shortly before[27]. and led to intermittency-like fluctuations very similar to the cosmic-ray event.

Figure 2 : **Random cascading model of intermittency**

The figure represents three stages of realization of a random cascading process.

a) *The tree structure of the model*

at each step n the branches of the tree are subdivided into λlinks. For a given resolution. i.e a given total number of steps ν. there is a one-to-one correspondence between a "box $[m]$" in the phase space and a series of integers $\{\alpha_1, \alpha_2, \cdots, \alpha_\nu\}$. In the example we have chosen $\lambda = 2$. $\nu = 4$.

b) *The "Rapidity-box" representation of the α-model*

of intermittency. For each box of this diagram. one chooses a random factor $W(\alpha)$. In each box. the sign " + " or " - " represents the enhancement (resp. damping) density factor of an $\alpha - model$ (see text). The final density ρ_m of states in the box $[m]$ is the product of the factors $W(\alpha_1) W(\alpha_2) \cdots, W(\alpha_\nu)$.

c) *The "event".*

The fluctuation pattern obtained after ν steps is displayed, following the random values attributed to the boxes of Fig. b).

Figure 3 : **Space-time representation of intermittent fluctuations**

This figure shows the space-time relativistic Lorentz frame in which a particle collision takes place, at least when projected on the (t, z)plane, where t is the time and z the longitudinal distance. The causal conus ($t^2 - z^2 \geq 0$, $t \geq 0$), the region where particles propagate, is dispayed in both figures, together with the causal hyperbolae, $\tau \equiv \sqrt{t^2 - z^2} = cste$, t$\geq$0, which are the curves of same proper-time, that is which correspond to simultaneity in the intrinsic frame of reference. The figures are from A. Bialas' review, see refs. [5].

a)*In-out conventional picture of hadron production*

The figure shows the conventional picture of hadron production after a high-energy collision. At the origin (O) an interacting "piece" of partonic or hadronic matter (hatched region) is created. After a proper time duration τ of order 1 fermi ($10^{-13}cm$) of relativistic expansion, the "piece" breaks into new pieces of size (on average)equal to the conventional correlation length ξ, giving rise to dynamically independent hadrons. The only scale in the problem is $\tau \sim \xi \sim 1$ in fermi units where the speed of light is unity.

b) *Random cascading in space-time*

In the $(1+1)$space-time frame, Fig. 3b shows the (random) generation of intermittent fluctuations following the geometrical structure of the α-models, but satisfying the constraints of Lorentzian relativistics kinematics. In fact, at each step of the cascade, one iterates the individual process shown in Fig. 3a. A piece of partonic or hadronic matter is locally streched by the relativistic expansion to a length larger than ξ, and breaks into λ new pieces, with each a new value of the density, due to an additional random factor W. Appearing at successive proper time-values τ_n, the fluctuations lead to the superposition of structures at different rapidity scales of size $\delta y \sim \xi/\tau_n$. In the figure, one has chosen for simplicity sake, $\lambda = 2$, $\tau_n = 2^n$, and random factors $W_+$and W_- as in Fig. 2.

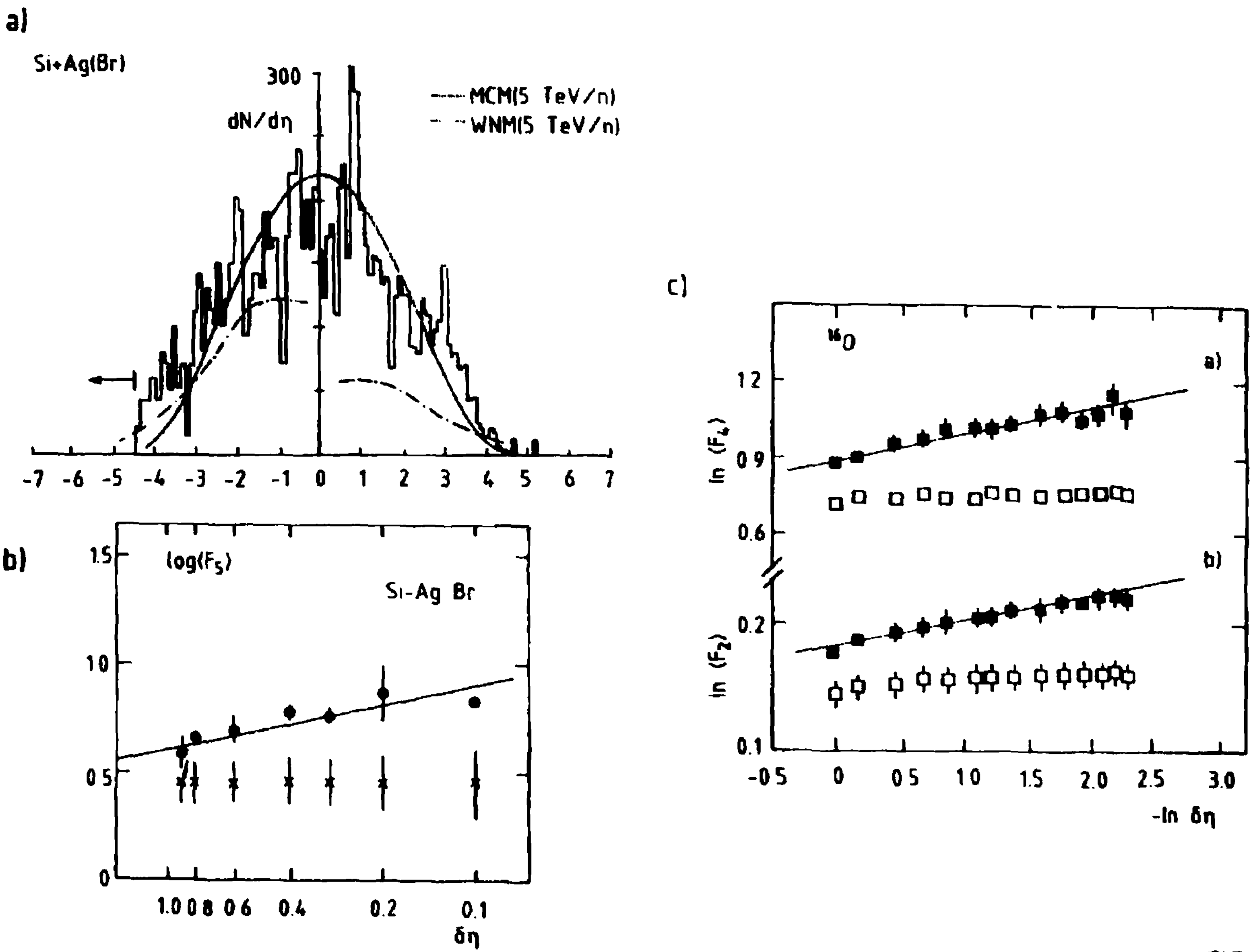

a)
Si+Ag(Br)
300
dN/dη
MCM(5 TeV/n)
WNM(5 TeV/n)
-7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7
b)
log(F5)
Si-Ag Br
1.5
1.0
0.5
0
1.0 0.8 0.6 0.4 0.2 0.1
δη
c)
16O
a)
b)
1.2
0.9
0.6
0.2
0.1
ln (F4)
ln (F2)
-0.5 0 0.5 1.0 1.5 2.0 2.5 3.0
-ln δη

FIG. 1

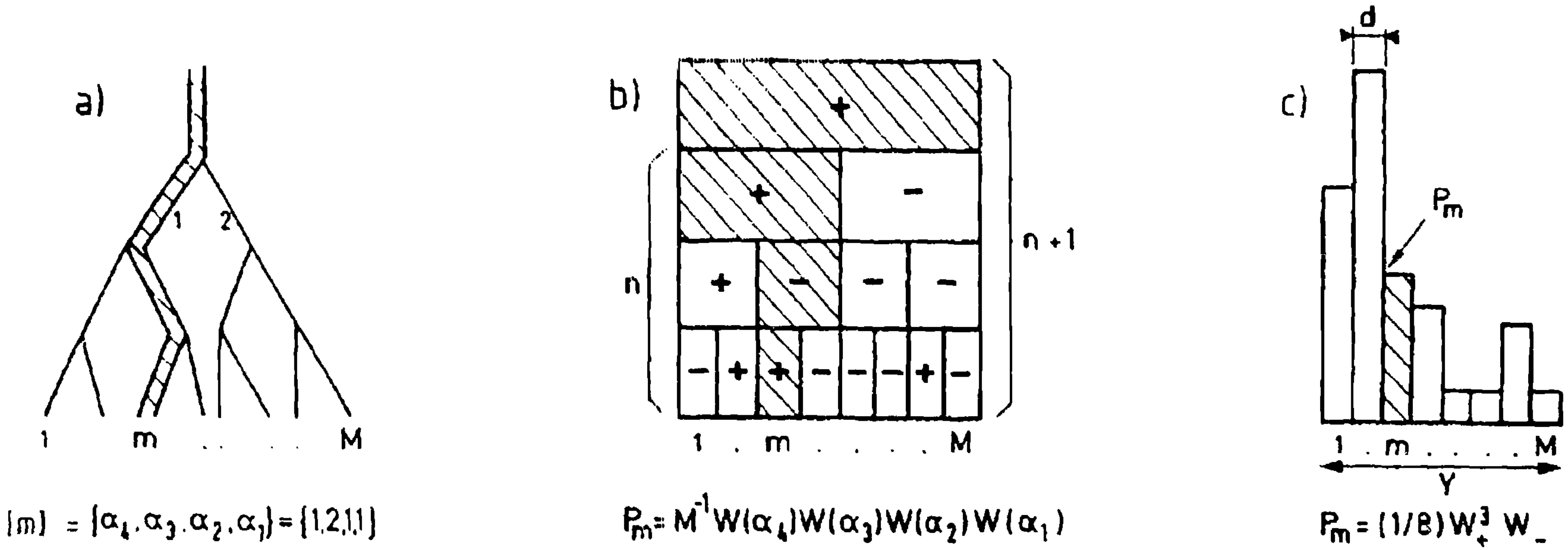
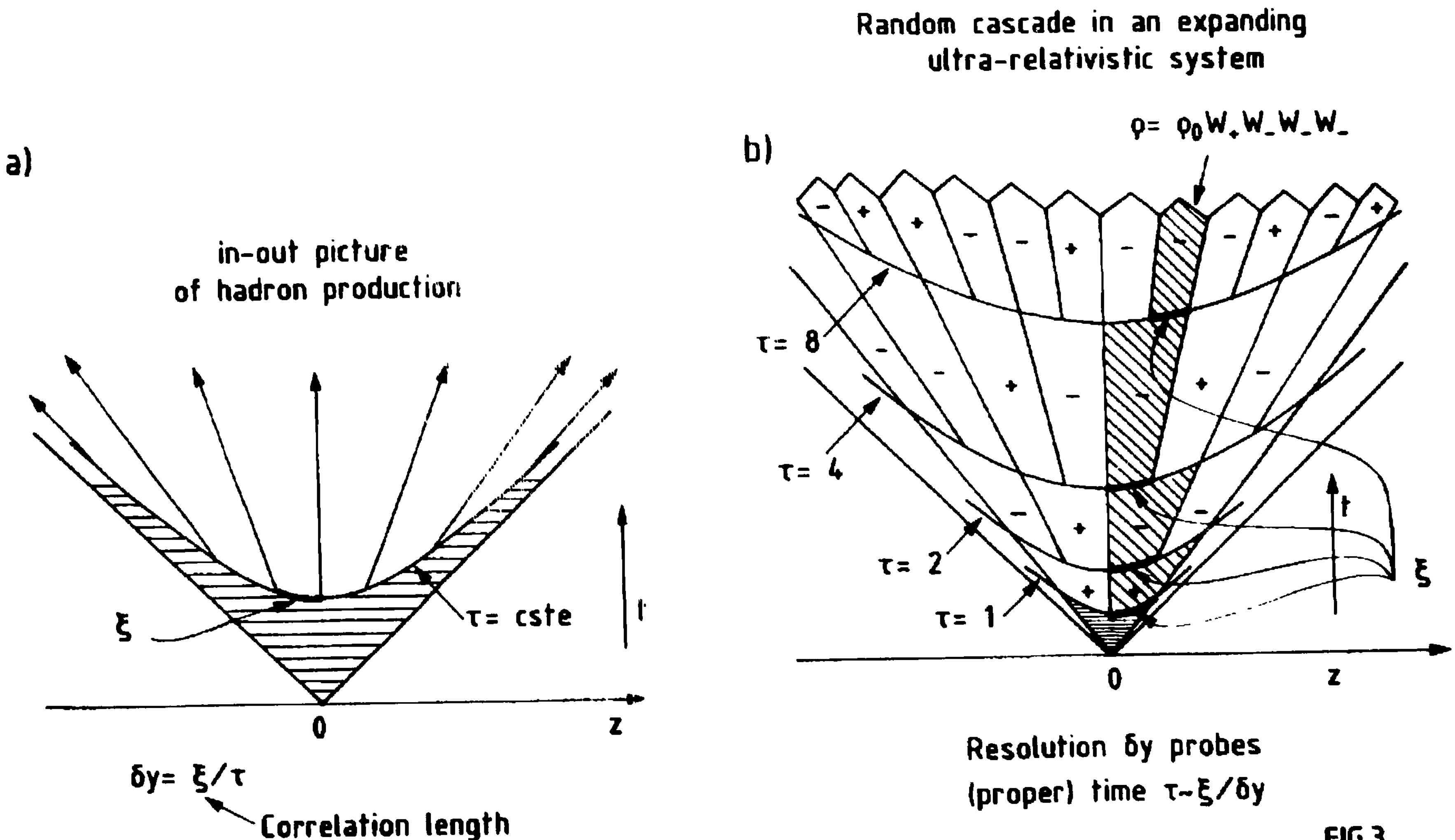

a)
1
2
1
m
M
{m} = {α4, α3, α2, α1} = {1,2,1,1}
b)
+
+
−
+
−
−
−
−
+
+
−
−
−
+
−
n
n+1
1
m
M
Pm = M^-1 W(α4) W(α3) W(α2) W(α1)
c)
d
Pm
1
m
M
Y
Pm = (1/8) W+^3 W−

FIG. 2



a)
in-out picture
of hadron production
ξ
τ= cste
t
0
z
δy= ξ/τ
Correlation length
b)
Random cascade in an expanding
ultra-relativistic system
ρ= ρ0W+W-W-W-
τ= 8
τ= 4
τ= 2
τ= 1
t
ξ
0
z
Resolution δy probes
(proper) time τ~ξ/δy

FIG.3