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Vortices in Nonuniform Upper–Hybrid Field

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Abstract: The equations describing the interaction of an upper-hybrid pump wave with small low-frequency density perturbations are discussed under assumption that the pump is spatially nonuniform. The conditions for the modulational instability are investigated. Instead of a dispersion relation, describing the growth of perturbations in the case of an uniform pump, in our case of nonuniform pump a differential equation is obtained and from its eigenvalues are found the instability criteria. Taking into account the slow-frequency self-interaction terms some localized solutions similar to dipole vortices are found, but described by analytic functions in all space. It is shown that their characteristic size and speed are determined by the pump intensity and its spatial structure.

1 Introduction

The interaction of a high-power electromagnetic wave with a plasma at the upper-hybrid resonance has attracted a great deal of attention mainly in connection with electron-cyclotron heating in tokamaks [1, 2]. Some experiments have also been carried out on influence of high intensity radio waves on the ionosphere [3, 4].

Near upper-hybrid layer an extraordinary electromagnetic wave significantly increases and can excite parametric instabilities such as parametric decay [2, 5, 6] or modulational (oscillating two-stream) instability [2], [7]-[9]. The nonlinear stage of modulational instability can lead to the formation of solitons [3], [7]-[9] or upper-hybrid vortices [10]. Upper-hybrid vortices are similar to drift vortices [11]-[14], Alfven vortices [15, 16] or vortices in self-gravitating plasmas [17].

Formation of vortices near upper-hybrid resonance is beleived to be a final stage of the instability when the upper-hybrid and drift perturbations saturate because of their self-interactions [10]. It is also regarded as a process in plasmas responsible for anomalous transport [18]-[21]. Some computer simulations [22] have shown that such vortices can be rather stable and preserve their shape after direct collisions. Vortices have been assumed to exist as the initial condition for a set of nonlinear equations. Nevertheless it is difficult to imagine the formation of these localized structures as a result of linear or nonlinear instability. Density and potential profiles of many vortices are described by different cylindrical functions for their inner and outer parts. Outside a circle with a definite radius the vortex potential decays exponentially. These functions and several first derivatives are matched on a cylindrical surface with a definite radius a, but some finite order derivatives remain discontinuous. These artificial surfaces were needed under assumption that all plasma parameters are constant or depend linearly on coordinates all over the space. The boundedness of real plasma systems and pump wave sources leading to pump inhomogeneity can play an essential role to create smooth localized structures. So we should consider a nonuniform pump field.

As it is known [23] a nonuniform localized pump field near the upper–hybrid resonance is unstable under modulational (or oscillating two–stream) instability when the peak intensity exceeds some threshold. The essential properties of unstable perturbations excited by a localized pump were first examined for unmagnetized plasmas [24]– [26]. It was shown that the spatial structure of these perturbations is determined by the intensity and spatial structure of the pump field. The size of the most unstable perturbations arizes much smaller than the size of the pump localization region if the instability threshold is significantly exceeded. The unstable perturbations grow up simultaneously and form a contracting caviton during the linear stage of the instability yet after the switch on of nonuniform pump [27], [28]. Then the nonlinear stage of modulational instability can lead to either a stationary soliton and formation of vortices or to a wave collapse [29].

The purpose of this article is to consider a possibility of formation of double–vortices by a nonuniform pump at the upper–hybrid frequency.

In section 2 we show that the modulational instability can develop in a nonuniform pump field under some instability conditions.

In section 3 we find localized solutions of a nonlinear set of equations, described by analytic functions in all space, which correspond to dipole vortices. The number of vortices, their characteristic size and speed are completely determined by the pump intensity and spatial structure.

2 Modulational instability of a nonuniform upper–hybrid pump field

Let us consider the interaction of an upper–hybrid wave, propagating through a plasma perpendicularly to the external uniform magnetic field $B_0\vec{e}_z$, with low frequency electrostatic perturbations ($\omega \ll \Omega_i$, Ω_i being the ion cyclotron frequency). The plasma is assumed to be weakly inhomogeneous so that we can use a linear approximation

$$n_0(x) = n_0(0) + \frac{dn_0}{dx}x = n_0\left(1 + \frac{x}{L}\right) \tag{1}$$

Suppose that the electric field of an extraordinary wave, propagating perpendicularly to the external magnetic field, can be approximately written in the form

$$\vec{E}_0 = -\vec{e}_y E_0 \exp(-i\omega_0 t) + c.c., \qquad \omega_0 = \omega_{UH} = \left(\omega_{pe}^2 + \Omega_e^2\right)^{1/2} \tag{2}$$

where ω_{pe}, Ω_e are Langmuir and cyclotron frequencies. Here, in contrast to [10] we take into account pump wave inhomogeneity. The nonuniformity of the pump can be connected for example with boundedness of the electromagnetic beam or with the standing nature of the wave or with linear conversion of an obliquely incident extraordinary electromagnetic wave [30, 31]. For perturbations localized near the peak of the pump intensity we can use the parabolic approximation

$$E_0(x,y) = E_0 \cdot \left(1 - \frac{x^2}{r_x^2} - \frac{y^2}{r_y^2}\right) \tag{3}$$

We have chosen the pump in the form (2),(3) for simpler comparison with the work [10] where upper–hybrid vortices were considered.

To investigate the possibility for modulational (oscillating two–stream) instability we shall use the basic equations derived in [10]. For the case of weakly magnetized plasmas $\omega_{pe}^2 \gg \Omega_e^2$ and for high–frequency perturbations ($\omega \geq \omega_{pe} \gg \Omega_e$) and

$\partial/\partial z \ll \nabla_\perp$, the wave potential satisfies the equation

$$\nabla_\perp \cdot \left\{ \left[2i\omega_{pe}\frac{\partial}{\partial t} - \Omega_e^2 - \omega_{pe}^2\left(\frac{x}{L} + \frac{\tilde{n}_e}{n_0}\right) + 3v_{Te}^2\nabla^2 \right] \nabla_\perp \Phi \right\} = 0 \qquad (4)$$

Here v_{Te} is the electron thermal velocity, $\tilde{n}_e$ the low-frequency electron density perturbation. For low-frequency potential perturbations $\partial/\partial t \ll \Omega_i$, $\partial/\partial z \ll \nabla_\perp$, we can use the equation (10) from [10] for the low-frequency potential $\tilde{\Phi}$

$$\left[\frac{\partial}{\partial t} + \frac{1}{B_0}(\vec{e}_z \times \nabla_\perp \tilde{\Phi}) \cdot \nabla_\perp\right] \cdot \left[\frac{dn_0}{dx}x + \tilde{n}_e + \frac{\varepsilon}{e}\left(1 + \frac{\omega_{pi}^2}{\Omega_i^2}\right)\nabla_\perp^2 \tilde{\Phi}\right] = 0 \qquad (5)$$

Here ω_{pi} is the ion Langmuir frequency. For Boltzmann electron distribution one can obtain low-frequency connection between $\tilde{n}_e$ and potentials $\tilde{\Phi}$, Φ_p

$$\tilde{\Phi} + \Phi_p + \frac{T_e}{n_0 e}\tilde{n}_e = 0 \qquad (6)$$

where T_e is the electron temperature, Φ_p is the ponderomotive potential produced by the high-frequency electric field

$$\Phi_p = \frac{e}{2m_e\omega_{pe}^2}|\nabla\Phi|^2 \qquad (7)$$

Equations (4)–(7) are a closed nonlinear system of equations describing high- and low-frequency interaction in weakly magnetized nonuniform plasmas (compare to (8)–(13) in [10]).

In order to investigate the linear stage of modulational instability we linearize this system of equations putting

$$\nabla\Phi \rightarrow \nabla\Phi_0 + \nabla\Phi, \qquad |\nabla\Phi| \ll |\nabla\Phi_0|$$

The low-frequency ponderomotive potential Φ_p, eq.(7), takes the form

$$\Phi_p = \frac{e}{2m_e\omega_{pe}^2}2\nabla\Phi_0\nabla\Phi = -\frac{e}{m_e\omega_{pe}^2}E_0\frac{\partial\Phi}{\partial y} \qquad (8)$$

and equation (5) can be written in the form

$$\frac{\partial}{\partial t}\left(\tilde{\Phi} + \Phi_p - \lambda_T^2\nabla_\perp^2\tilde{\Phi}\right) + \frac{\partial}{\partial y}u\tilde{\Phi} = 0 \qquad (9)$$

or

$$\frac{\partial}{\partial t}\left[\tilde{n}_e - \lambda_T^2 \nabla_\perp^2 \left(\tilde{n}_e - \frac{en_0}{T_e}\Phi_p\right)\right] = -\frac{\epsilon n_0}{T_e}\frac{\partial}{\partial y}u\tilde{\Phi} \tag{10}$$

where

$$\lambda_T^2 = \frac{\varepsilon_0 T_e}{n_0 e^2}\left(1 + \frac{\omega_{pi}^2}{\Omega_i^2}\right), \quad u = \frac{T_e}{eB_0 n_0}\frac{dn_0}{dx}.$$

For plane wave perturbations $\sim \exp(-i\omega t + ik_x x + ik_y y)$,

$$\tilde{n}_e = -\frac{n_0 e}{T_e}\frac{\omega k_\perp^2 \lambda_T^2 - uk_y}{\omega(1 + k_\perp^2\lambda_T^2) - uk_y}\Phi_p$$

where $k_\perp^2 = k_x^2 + k_y^2$. For $k_\perp^2\lambda_T^2 \ll 1$, what we suppose further, it follows

$$\tilde{n}_e = -\frac{n_0 e}{T_e}\frac{\omega k_\perp^2\lambda_T^2 - uk_y}{\omega - uk_y}\Phi_p \tag{11}$$

For simplicity we can neglect the term Ω_e^2 in eq.(4) for high-frequency perturbations with $\omega_{pe}^2 \gg k_\perp^2 v_{Te}^2 \gg \Omega_e^2$, and the term $\omega_{pe}^2 x/L$ under assumption $k_x r_x \gg 1$, $k_y r_y \gg 1$. Introducing notations

$$\delta\Phi_p = -\frac{\Phi_{p0}}{E_0}\frac{\partial}{\partial y}(\Phi + \Phi^*), \quad \Phi_p = \Phi_{p0} + \delta\Phi_p, \quad \Phi_{p0} = \frac{eE_0^2}{2m_*\omega_{pe}^2}$$

we obtain from eq.(4)

$$\left(9\frac{v_{Te}^2}{\omega_{pe}^2}\nabla_\perp^4 + \frac{4}{v_{Te}^2}\frac{\partial^2}{\partial t^2}\right)\delta\Phi_p = \frac{6}{n_0}\frac{\partial^2}{\partial y^2}\tilde{n}_e\Phi_{p0} \tag{12}$$

Substituting eq.(11) into eq.(12) we obtain a dispersion relation for modulational instability describing the growth of perturbations $\tilde{n}_e$, $\delta\Phi_p$ for the case when $E_0 =$ *const.*

$$-4\frac{\omega^2}{v_{Te}^2}(\omega - uk_y) = W_0 k_y^2(k_\perp^2\lambda_T^2\omega - uk_y) - 9k_\perp^2 r_D^2(\omega - uk_y)$$

where $W_0 = E_0^2/4\pi n_0 T_e$, $r_D = v_{Te}/\omega_{pe}$.

This equation has only one real root if

$$W_0 > 9\frac{k_\perp^2}{k_y^2\left(1 + \frac{\omega_{pi}^2}{\Omega_i^2}\right)} + \frac{4}{3}\frac{u^2}{v_{Te}^2}\frac{1}{k_\perp^2\lambda_T^2} \tag{13}$$

The condition (13) is a sufficient condition for complex conjugate roots to exist. The instability threshold is the smallest in the case of a uniform plasma and equal to

$$W_{0th} = \frac{9}{1 + \frac{\omega_{pi}^2}{\Omega_i^2}}$$

For the aperiodic instability we obtain easily the growth rate $\gamma = Im\,\omega$ (if $\gamma \gg uk_y$)

$$\gamma^2 = \frac{1}{4} k_\perp^2 r_D^2 v_{Te}^2 \left[W_0 k_y^2 \left(1 + \frac{\omega_{pi}^2}{\Omega_i^2}\right) - 9k_\perp^2 \right] \tag{14}$$

Let us consider now the simplest form of the pump wave inhomogeneity $W_0 = W_0(1 - x^2/r_x^2)$ and assume that $\delta\Phi_p \sim \exp(-i\omega t + ik_y y)$, $u = 0$. Then from eqs (11),(12) instead of an algebraic dispersion relation we obtain an ordinary differential equation

$$\left\{ 9r_D^2 \left(k_y^2 - \frac{d^2}{dx^2}\right)^2 - 4\frac{\omega^2}{v_{Te}^2} - 6W_0 k_y^2 \lambda_T^2 \left(1 - \frac{x^2}{r_x^2}\right) \left(k_y^2 - \frac{d^2}{dx^2}\right) \right\} \delta\Phi_p = 0 \tag{15}$$

Here we take into account the assumption that the scale length of perturbations is much smaller than the pump scale length r_x. We seek for localized solutions of this equation in $\vec{e}_x$ direction, that is particular solutions $|\delta\Phi_p| \to 0$ when $|x| \to \infty$. These solutions correspond to eigenfunctions of eq.(15). If there are eigenvalues ω with $Im\,\omega > 0$ the corresponding states are absolutely unstable, with the growth rate $\gamma = Im\,\omega$.

For simplicity let us investigate two opposite cases;

1. case: $k_y \gg \frac{\partial}{\partial x} \gg r_x^{-1}$

Then we obtain from eq.(15)

$$\left\{ \frac{d^2}{dx^2} + \frac{1}{3} W_0 k_y^2 \left(1 + \frac{\omega_{pi}^2}{\Omega_i^2}\right) \left(1 - \frac{x^2}{r_x^2}\right) + 4\frac{\omega^2}{v_{Te}^2} \right\} \delta\Phi_p = 0$$

The eigenfunctions of this equation are Parabolic Cylinder functions [32] and the eigenvalues are equal to

$$-\omega_n^2 = \gamma_n^2 = v_{Te}^2 \left\{ \frac{1}{3} W_0 k_y^2 \left(1 + \frac{\omega_{pi}^2}{\Omega_i^2}\right) - \frac{2n+1}{r_x} k_y \sqrt{\frac{W_0}{3}\left(1 + \frac{\omega_{pi}^2}{\Omega_i^2}\right)} \right\}.$$

The instability condition for n-th mode

$$W_0 \left(1 + \frac{\omega_{pi}^2}{\Omega_i^2}\right) > \frac{3(2n+1)^2}{k_y^2 r_x^2} \tag{16}$$

generalizes condition (13) to the case of one–dimensional nonuniform pump field. The scale of perturbations localized in $\vec{e}_x$–direction depends on the number n and on the ratio W_0/r_x^2

$$<\delta x> = \frac{\sqrt{n}\, r_x}{k_y \sqrt{W_0 \left(1 + \frac{\omega_{pi}^2}{\Omega_i^2}\right)}}$$

We see that it is smaller than r_x above threshold.

2. case: $r_x^{-1} \ll k_y \ll \frac{d}{dx}$.

Then the equation (15) becomes

$$\left\{ \frac{d^4}{dx^4} - \frac{4}{9} \frac{\omega^2}{r_D^2 v_{Te}^2} + \frac{2}{3} \frac{W_0 k_y^2 \lambda_T^2}{r_D^2} \left(1 - \frac{x^2}{r_x^2}\right) \frac{d^2}{dx^2} \right\} \delta\Phi_p = 0$$

As in [26] a Fourier transformation of this equation enables one to reduce the eigenvalue problem of the second order in k–space if we assume parabolic approximation of the pump field

$$\left\{ \frac{d^2}{dx^2} - \left(\xi^2 + \frac{B}{\xi^2} - A\right) \right\} \delta\Phi_p(\xi) = 0$$

where we introduced variables

$$\delta\Phi_p(k) = \int_{-\infty}^{\infty} e^{ikx} \delta\Phi_p(x) dx, \quad \xi = k \left(\frac{k_y}{r_x}\right)^{-1/2} \cdot \left[\frac{2}{3} W_0 \left(1 + \frac{\omega_{pi}^2}{\Omega_i^2}\right)\right]^{-1/4}$$

and A, B are constants

$$A = k_y r_x \left(\frac{2}{3}W_0\right)^{1/2} \cdot \left(1 + \frac{\omega_{pi}^2}{\Omega_i^2}\right)^{1/2}, \qquad B = \frac{2\gamma^2 r_x^2}{3W_0\omega_{pe}^2 k_y^2 \lambda_T^2}.$$

This equation was considered in [26]. It has eigenfunctions of the form

$$\delta\Phi_p^{n,s}(\xi) = \xi^s u_{2n}(\xi) e^{-\frac{\xi^2}{2}}$$

where $u_{2n}(\xi)$ is a polynomial of power $2n$ and $s = 1/2 + \sqrt{1/4 + B}$. The last condition gives for the instability growth rate

$$\gamma_n^2 = \frac{3}{8}\omega_{pe}^2 k_y^2 \lambda_T^2 W_0 \left\{ \left[-\frac{4n}{r_x} + k_y \left(\frac{2}{3}W_0\right)^{1/2} \cdot \left(1 + \frac{\omega_{pi}^2}{\Omega_i^2}\right)^{1/2} \right]^2 - \frac{1}{r_x^2} \right\}$$

So now the instability of the n–th mode takes place for the pump intensities:

$$W_0 \left(1 + \frac{\omega_{pi}^2}{\Omega_i^2}\right) > \frac{3}{2} \frac{(4n+1)^2}{k_y^2 r_x^2} \qquad (17)$$

The last criterion is similar to that expressed by eq.(16).

3 Vortices in nonuniform pump field

As it is shown in [10] the nonlinear system of equations (4)–(7) in the presence of a constant pump

$$\vec{E}_0 = -\nabla\Phi_0$$

has solutions in the form of double vortices moving with a constant velocity v_g, with potential and density perturbations given by

$$\Phi = \Phi_1(r)\exp(i\varphi), \quad n = n_1(r)\exp(i\varphi)$$

where $r^2 = x^2 + (y - v_g t)^2$, $\varphi = \arccos x/r$, with small corrections of fifth and higher cylindrical harmonics.

Let now the pulse move with the velocity v_g so that

$$\vec{E}_0(r) = \vec{E}_0 \cdot \left(1 - \frac{r^2}{r_0^2}\right)$$

near the pump peak.

For the stationary case wich is more often considered we can put $v_g = 0$. Then we can seek a solution of our nonlinear problem in the form of vortices moving with the same velocity v_g, or immobile one for the stationary case. Now we do not need to do any matching of different kind of solutions inside and outside some artificial surface. We assume that vortices are localized at the pump peak so that potential perturbations Φ, n smoothly tends to zero when r tends to infinity and the vortex scale δr is much smaller than the pump scale inhomogeneity. On the other hand pump peak size is assumed to be much larger than the characteristic scale of the upper-hybrid wave perturbations in uniform plasmas.

To simplify the nonlinear system (4)-(7) we suppose that the same linear condition is fulfiled as in [10]

$$\left(\lambda_T^2 \nabla_\perp^2 - 1\right)\left(\delta\Phi_p + \frac{T_e}{n_0 e}\tilde{n}_e\right) + \delta\Phi_p - B_0 u x = \mathcal{F}' \cdot \left(\delta\Phi_p + \frac{T_e \tilde{n}_e}{n_0 e} + B_0 v_g x\right)$$

where $\mathcal{F}'$ is some constant (in linear approximation, using (10) and $\partial/\partial t = -v_g \partial/\partial y$ we can show that $\mathcal{F}' = 1$).

Then equation for the density perturbation can be written in the form (compare to eq.(28) in [10])

$$\left(\nabla_1^2 - A_1^2\right)\left(\nabla_1^2 - A_1^2\right)\mathcal{N} = 0 \tag{18}$$

where

$$\nabla_1^2 = \frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} - \frac{1}{\rho^2}, \qquad \rho = \frac{r}{\lambda_T},$$

$$\mathcal{N} = n_1 + a\rho, \qquad A_1^2 + A_2^2 = 1 + f + V - W(\rho),$$

$$f = \mathcal{F}', \qquad V = \kappa\frac{v_g^2}{v_{Te}^2},$$

$$A_1^2 A_2^2 = -\left[V(1+f) + W(\rho)f\right], \qquad \kappa = \frac{1}{9}\left(1 + \frac{\omega_{pi}^2}{\Omega_i^2}\right),$$

$$W(\rho) = \frac{1}{6}\kappa\frac{\Phi_{p0}(\rho)}{T_e} = W_m\left(1 - \frac{\rho^2}{\rho_0^2}\right), \quad a = \lambda_T\frac{eN_0B_0}{2T_e}\frac{u + fv_g}{f + 1 + \frac{3}{2}\left(\frac{e\Phi_{p0}}{mv_g^2}\right)}.$$

We take into account here that under assumption $r_0 \gg \lambda_T, \delta r$ we can change the order of differentiations so that $\nabla(W\Psi) = W\nabla\Psi$.

Strictly speaking we further assume not the function n_1 but the function $\mathcal{N}$ to be a localized function of ρ so that $\mathcal{N} \to 0$ if $\rho \to \infty$. Evidently such an assumption can be true in the region where linear approximation for the inperturbed plasma density is true.

Then for localized solutions $\mathcal{N}$ of equation (18) we can obtain the equivalent integral equation

$$\int_0^\infty \left(\nabla_1^2\mathcal{N}\right)^2 \rho d\rho + \int_0^\infty (1 + f + W - V)\left[\left(\frac{d\mathcal{N}}{d\rho}\right)^2 + \frac{\mathcal{N}^2}{\rho^2}\right]\rho d\rho =$$
$$\int_0^\infty [V(f+1) + Wf]\mathcal{N}^2 \rho d\rho \tag{19}$$

We can conclude from the equation (19) that for $f > 0$, what we further suppose, localized solutions can only exist in the presence of the pump ($W \neq 0$) at least for the immobile case ($V = 0$).

It is more convinient further to use Fourier transform $\mathcal{N}_k e^{i\theta}$ of the function $\mathcal{N} e^{i\varphi}$

$$\mathcal{N}_k e^{i\theta} = \int_0^\infty \rho \mathcal{N}(\rho) d\rho \int_0^{2\pi} e^{ik\rho\cos(\theta-\varphi)} e^{i\varphi} d\varphi =$$

$$ie^{i\theta} \int_0^\infty \rho \mathcal{N}(\rho) d\rho J_1(\rho k) d\rho$$

For the function $\mathcal{N}_k$ from the equation (18) one can obtain the equation

$$\left\{ \frac{W_m}{\rho_0^2} \left(k^2 + f\right) \cdot \left(\frac{d^2}{dk^2} + \frac{1}{k}\frac{d}{dk} - \frac{1}{k^2} \right) - k^4 - k^2 (1 + f - W_0 - V) + \right.$$

$$\left. + \ [V(f+1) + W_m f] \right\} \mathcal{N}_k = 0 \qquad (20)$$

where $\rho_0 = r_0/\lambda_T$ and we use parabolic approximation for the pump. Under assumption $f > 0$ the last equation is a regular differential equation and can have regular eigenfunctions localized in k–space.

Let us rewrite the equation (20) in a more convinient form

$$\left(\frac{d^2}{dk^2} + \frac{1}{k}\frac{d}{dk} - \frac{1}{k^2} \right) \mathcal{N}_k + \frac{\rho_0^2}{W_m} C(k) \left(k_t^2 - k^2\right) \mathcal{N}_k = 0 \qquad (21)$$

where

$$C(k) = \frac{k^2 + k_1^2}{k^2 + f}, \qquad k_1^2 = \sqrt{\frac{B^2}{4} + A} - \frac{B}{2},$$

$$k_t^2 = \sqrt{\frac{B^2}{4} + A} + \frac{B}{2}, \qquad A = V(f+1) + W_m f,$$

$$B = W_m + V - 1 - f,$$

and $A > 0$, $k_1^2 > 0$, $k_t^2 > 0$. Here the function $C(k)$ changes monotonously between two positive constants k_1^2/f and 1, and can be regarded approximately as a constant C_0. We are going to estimate its value later.

Equation (21) can be then reduced to the Whitteker equation [32]. Using variables

$$z = k^2 \rho \sqrt{\frac{C_0}{W_m}}, \quad \chi = \frac{\mathcal{N}_k}{k}$$

we have

$$\left\{ \frac{d^2}{dz^2} - \frac{1}{4} + \frac{\rho}{4} \sqrt{\frac{C_0}{W_m} \frac{k_t^2}{z}} \right\} \chi = 0 \tag{22}$$

Equation (22) has regular solutions $\chi = W_{n,1/2}(z)$ bounded on infinity if and only if

$$\frac{1}{4} \rho_0 \sqrt{\frac{C_0}{W_m}} k_t^2 = n, \quad n = 1,\ 2,\ 3 \ldots$$

or

$$(W_m + V - 1 - f) + \sqrt{(W_m + V - 1 - f)^2 + 4\left[V(f+1) + W_m f\right]} =$$
$$= \frac{8n}{\rho_0} \sqrt{\frac{W_m}{C_0}} \tag{23}$$

Using asymptotic representations of the Whitteker function ($z \gg n$)

$$W_{n,\frac{1}{2}}(z) \sim e^{-\frac{z}{2}} z^n \cdot \left[1 + \frac{\frac{1}{4} - \left(n - \frac{1}{2}\right)^2}{1!\, z} + \cdots \right]$$

we obtain for the function $\mathcal{N}_k$

$$\mathcal{N}_k \sim \left(k^2 \alpha\right)^n k e^{-\alpha k^2}, \quad \alpha = \frac{\rho_0}{2} \sqrt{\frac{C_0}{W_m}} \tag{24}$$

Performing Fourier transform of the function $\mathcal{N}_k e^{i\theta}$ we obtain

$$\mathcal{N}(\rho, \varphi) \sim e^{i\varphi} \rho^n e^{-\frac{\rho^2}{4\alpha}} \tag{25}$$

So the characteristic perurbation scale in the real space is of the order

$$\delta r \sim (r_0 \lambda_T n)^{1/2} (C_0/W_m)^{1/4}$$

and because of our assumptions

$$\lambda_T^2 \ll (\delta r)^2 \ll r_0^2$$

or

$$1 \ll (\delta\rho)^2 \ll \rho_0^2 \tag{26}$$

we find a restriction on the pump intensity to excite n–th localized nonlinear structure (with n zeros along the radius)

$$n\rho_0 > \sqrt{\frac{W_m}{C_0}} > \frac{n}{\rho_0}.$$

For a given normalized pump intensity W_m and given V we can obtain from (23) an expression for f at which n–th perturbation is localized

$$f_n = \left\{ \frac{4n}{\rho_0}\sqrt{\frac{W_m}{C_0}} \left[\frac{4n}{\rho_0}\sqrt{\frac{W_m}{C_0}} - (W_m + V - 1) \right] - V \right\} \cdot \left(W_m + V - \frac{4n}{\rho_0}\sqrt{\frac{W_m}{C_0}} \right)^{-1}.$$

As we assumed before $f_n > 0$ which demands

$$\frac{4n}{\rho_0}\sqrt{\frac{W_m}{C_0}} < W_m + V < 1 + \frac{4n}{\rho_0}\sqrt{\frac{W_m}{C_0}} - \frac{\rho_0 V}{4n}\sqrt{\frac{C_0}{W_m}} \tag{27}$$

The left–hand side condition in (27) is similar to (16), (17) for the modulational instability of the n–th mode of one–dimensional pump field.

Let us consider the stationary case $V = 0$ in more details. It follows from eq.(27)

$$\frac{4n}{\sqrt{C_0}\rho_0} < \sqrt{W_m} < \frac{2n}{\sqrt{C_0}\,\rho_0} + \sqrt{1 + \frac{4n^2}{C_0\rho_0^2}}. \tag{28}$$

In eqs (22)–(28) C_0 is an intermidiate value of $C(k)$ when k changes from zero to infinity. We can write $C(k)$ in the form

$$C(k) = \left(k^2 + \frac{A}{k_t^2}\right) \cdot \left(k^2 + f_n\right)^{-1}$$

and for the case $V = 0$

$$C(k) = \left(k^2 + \frac{\sqrt{W_m C_0}\,\rho_0}{4n} f_n\right) \left(k^2 + f_n\right)^{-1}$$

It follows from (28) that

$$\frac{\sqrt{W_m C_0}\,\rho_0}{4n} \geq 1$$

and then

$$C(k) \geq 1, \quad C_0 \approx 1$$

Thus for $V = 0$, n–th mode vortex is to be excited by a nonuniform pump if conditions (28) are fulfilled and

$$f_n = \frac{4n\sqrt{W_m}}{\rho_0}\left\{\left(W_m - \frac{4n}{\rho_0}\sqrt{W_m}\right)^{-1} - 1\right\}$$

For the case $V \neq 0$ there is also a possibility of having vortices moving with a pulse shaped high frequency wave if conditions (27) are fulfilled for V and W_m. They are to a certain extent similar to some velocity restrictions on the possibility of a soliton existance moving with high frequency wave pulse in magnetized plasma [3]. But the restrictions on soliton velocity found in [3] did not depend on the soliton amplitude and width because of the approximate method used there to solve a set of nonlinear equations.

4 Conclusion

In this article we have discussed the nonlinear interaction of a spatially nonuniform upper-hybrid pump wave with small, low–frequency density perturbations in a plasma. This interaction is expressed by a ponderomotive force term coming from the electron parallel momentum equation. The plasma is assumed to be weakly inhomogeneous and placed in an external homogeneous magnetic field.

In the first part of the article are derived some criteria for modulational instability. It is shown that in the case when the pump wave amplitude is constant the instability criterion follows from a dispersion relation. The threshold for the instability is the smallest for a homogeneous plasma. In the case when the pump is spatially nonuniform instead of the dispersion relation a differential equation is obtained. The eigenfunctions of this differential equation are Parabolic Cylinder functions, and the instability conditions are found from the eigenvalues. It is shown that the scale of localized perturbations depends on the eigennumber n, the pump amplitude and its scale length.

Further, starting with the same equations as in [10] but with the assumption of a nonuniform pump we have found some localized solutions, corresponding to dipole vortices. Instead of using an ansatz, and matching conditions on an artificial surface, as in previous works devoted to the problems of plasma vortices [10]–[22], we have found vortices described by analytic functions in any finite region of space. For the density perturbation $\mathcal{N}$ this is Fourier transform of

$$\mathcal{N}_k e^{i\theta} = W_{n,\frac{1}{2}}\left(\frac{k^2\rho}{\sqrt{W_m}}\right)\cdot e^{i\theta}$$

W_n being the Whitteker function. The size and the shape of the vortices are complitely determined by the pump intensity and the inhomogeneity scale.

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